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RESTRICTED MASS ENERGY ABSORPTION COEFFICIENTS FOR USE IN DOSIMETRY

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1. Introduction

The spatial distribution of absorbed dose in matter irradiated by a photon beam is almost entirely due to the diffusion and absorption of the secondary electrons that are set in motion by the photons in collisions with the atoms of the medium. The energy dissipated by a photon is therefore not absorbed close to its interaction centres but generally distributed over a large volume by the diffusion of the secondary electrons.

Only a small fraction of the photon energy is thus absorbed close to the photon "path". This fraction may be calculated when the differential cross sections in electron energy are known for the production of photo electrons, compton electrons and electron-positron pairs. These cross sections may be integrated with appropriate weight factors over the whole spectrum of secondary electrons to yield the restricted mass energy absorption coefficient which expresses the fraction of the photon energy which is absorbed locally in some region R_Δ. For computational reasons the size of the region R_Δ is most simply related to the range of secondary electrons of some restriction energy Δ. The energy restriction, Δ, of the mass energy absorption coefficient to be used together with a region of interest of given size and shape is therefore related to the energy restriction of the mass stopping power to be used with the same region when being irradiated by electrons.

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The restricted mass energy absorption coefficient can be used in situations where there is a lack of charged particle equilibrium and the usual mass energy transfer coefficient cannot be used to calculate the locally absorbed energy from a photon fluence.

An example, of how restricted mass energy absorption coefficients can be calculated from existing differential photon interaction cross sections, is given in the following section. In the last sections some applications are given of the use of restricted mass energy absorption coefficients in dosimetry.

2. Theory

2.1 Fundamental considerations

The restricted energy absorption cross section for monoenergetic photons may be specified as a certain fraction of the total interaction cross section. This fraction is here defined as the ratio of the energy on the average absorbed locally within some region R_Δ from the photon fluence through that region to the total photon energy that has passed through the same region. A more stringent name for this entity would perhaps be, "restricted mass energy transfer coefficient" (ICRU 1971) as the energy which is lost from the region of interest as Bremsstrahlung etc is disregarded. However, this fraction of the electron energy is fairly small particularly when the energy restriction is small and the probability for two interactions in the volume of interest almost negligable. For computational reasons the region of interest is below assumed to be spherical with a radius R_Δ equal to the range of electrons of energy Δ.

The major part of the locally absorbed energy from the photon fluence is due to secondary and higher order electrons set in motion by the photons. The energy distribution of these secondary electrons (and positrons) may be divided in two parts depending on how they contribute to the locally absorbed energy.

The low energy electrons with energies from the restriction energy and below dissipates almost all their energy in the region R_Δ around the interaction centres of the photons as these electrons are almost completely stopped inside R_Δ. The high energy electrons with energies above Δ only dissipates a small fraction of their energy inside R_Δ as most of these electrons escape the region of interest.

The contribution from electrons with energies above the restriction energy Δ can be estimated by using the restricted stopping power at the restriction energy (L_Δ, Berger and Seltzer (1964)) for these high energy electrons. The fraction of the electron energy which is lost inside the region R_Δ by an electron of energy, T, far above the restriction energy is with fair accuracy given by

$$\frac{\Delta T}{T} = \frac{L_\Delta(T)\ R_\Delta}{T} \tag{1}$$

This simple relation overestimates the energy loss for electron energies T far above the restriction energy, Δ, and underestimates the energy loss when T is below Δ (cf. Fig. 1 and below).

The restricted cross section for a certain photon interaction may thus be divided in two parts which become

$$\Sigma_\Delta(E) = \int_0^\delta \frac{T}{E} \frac{d\Sigma(E,T)}{dT} dT + \int_\delta^E \frac{R_\Delta L_\Delta(T)}{E} \frac{d\Sigma(E,T)}{dT} dT \tag{2}$$

where $\delta = L_\Delta(\delta) R_\Delta \approx \Delta$

and $\frac{d\Sigma(E,T)}{dT}$ is the differential cross section for a photon of energy E to generate a secondary electron or positron of energy T. In the calculations below δ is for simplicity approximated by Δ. For a wide range of secondary electron energies (T) the restricted mass stopping power $L_\Delta(T)$ can be regarded as a constant independent on the energy. In particular this is so when the restriction energy is low and the electron energy is high (cf. ICRU (1970)). Under this assumption the integrals of above can be evaluated by straight forward calculations.

When more accurate values are desired the above approximations have to be refined and for example the escape of electrons with energy below Δ when generated close to the border of R_Δ has to be considered. All these problems, with escape of low energy electrons and neglection of some of the energy lost by higher energy electrons when using L_Δ, can also be brought back to the selection of an appropriate energy restriction Δ for the region of interest as discussed by Spencer (1971) in connection with energy deposition in cavities. However, it is possible to improve the simple relation (1) by quite elementary considerations if the mean cord length of the volume of interest is fairly independent of the cord direction

$$\bar{d} = \int_0^{\pi} \frac{\bar{d}_\theta}{\pi}\, d\theta = \frac{4V}{A} \simeq \bar{d}_\theta \qquad (3)$$

where V is the volume and A the surface area of the volume of interest (ICRU 1971). The mean track length of secondary electrons generated in the volume of interest is then given simply by

$$\bar{t} = \begin{cases} = R\left(1 - \dfrac{R}{2\bar{d}}\right) & R \leq \bar{d} \\ = \dfrac{\bar{d}}{2} & R \geq \bar{d} \end{cases} \qquad (4)$$

where R is the range of the secondary electrons. The fraction of the electron energy which is absorbed inside the volume of interest when a linear energy-range relation is used becomes

$$\frac{\Delta T}{T} = \begin{cases} = 1 - \dfrac{R}{2\bar{d}} = 1 - \dfrac{3T}{8\Lambda} & T \leq \dfrac{4}{3}\Lambda \\ = \dfrac{\bar{d}}{2R} = \dfrac{2\Lambda}{3T} & T \geq \dfrac{4}{3}\Lambda \end{cases} \quad (5)$$

assuming $\bar{d} = 4/3 \; R_{\Lambda}$.

If instead a more accurate exponential energy range relation is used the above expression takes the form,

$$\frac{\Delta T}{T} = 1 - \frac{15R}{32\bar{d}} \qquad R \leq \bar{d} \quad (6)$$

which differs less than 3 % from (5). The energy range relation used,

$$E = k_1 \; R^{k_2} \quad (7)$$

with $k_1 = 1.36$ MeV $(g/cm^2)^{-k_2}$

$k_2 = 0.6$

is accurate to better than 10 % from 0.5 MeV down to 500 eV when compared to values tabulated by ICRU (1970) for water.

Equation (5) is plotted in Fig. 1 as dashed curves. Also included are results from Monte Carlo calculations for a 1 mm thick silicon semiconductor detector of 2.5 cm^2 (Waino and Knoll 1966). It is believed that eq. (5) gives a fairly accurate discription of the transition region from low to high energies but eq. (1) is more accurate at energies far above Λ.

To get an estimation of the order of magnitude of the restricted mass energy absorption coefficient the approximations of above (eq. (1) and eq. (2)) will be used to calculate the contributions from photo electrons, compton electrons and electron-positron pairs.

2.2 Photo electric interactions

For the low energy region below some 50 keV in water and about 500 keV in lead the photo electric effect dominates the photon interactions with matter. In the photo electric interactions most of the photon energy is transferred to kinetic energy of the electrons. This is a good approximation except for photon energies near the binding energies of the electrons in heavy elements where a large fraction of the photon energy can escape as fluorescence radiation. Assuming that this is not the case and that the whole photon energy is transferred to one electron, disregarding Auger processes etc. the restricted photo electric cross section can be approximated by

$$\tau_\Delta = \begin{cases} = \tau_a \simeq \tau & E \leq \Delta \\ = \dfrac{R_\Delta L_\Delta(E)}{E} \tau_a \simeq \dfrac{R_\Delta L_\Delta(E)}{E} \tau & E > \Delta \end{cases} \qquad (8)$$

where τ_a is the cross section for photo electric absorption e.g. according to Storm and Israel (1970).

2.3 Compton interactions

In the medium energy range from about one MeV to about ten MeV the Compton or Klein-Nishina scattering dominates the photon interactions. In these interactions a photon of initial energy E is inelastically scattered on an electron so its energy is decreased to E' and the electron gets an energy of E-E', when the binding energy of the electron is neglected. The differential cross section with respect to the energy of the secondary photon may be written (Roy and Reed 1968a),

$$\frac{d\sigma}{dE'} = \pi r_o^2 Z \frac{E_o}{E^2} \left\{ \frac{E}{E'} + \frac{E'}{E} + E_o^2 \left(\frac{1}{E'} - \frac{1}{E}\right)^2 - 2E_o \left(\frac{1}{E'} - \frac{1}{E}\right) \right\} \qquad (9)$$

where r_o is the Bohr radius and E_o is the rest energy of the electron. The restricted Klein-Nishina cross section thus takes the form,

$$\sigma_\Delta(E) = \begin{cases} = \displaystyle\int_{\frac{EE_o}{2E+E_o}}^{E} \frac{E-E'}{E} \frac{d\sigma}{dE'} dE' \equiv \sigma_a & E \le \frac{\Delta}{2}\left\{ 1 + \left(1 + \frac{2E_o}{\Delta}\right)^{1/2} \right\} \\[2ex] = \displaystyle\int_{E-\Delta}^{E} \frac{E-E'}{E} \frac{d\sigma}{dE'} dE' + \int_{\frac{EE_o}{2E+E_o}}^{E-\Delta} \frac{R_\Delta L_\Delta(E-E')}{E} \frac{d\sigma}{dE'} dE' & E > \frac{\Delta}{2}\left\{ 1 + \left(1 + \frac{2E_o}{\Delta}\right)^{1/2} \right\} \end{cases} \qquad (10)$$

as $EE_o/2E+E_o$ is the minimum energy of the scattered photon.

After insertion of eq. (9) and assuming $L_\Delta(E-E') = L_\Delta(E)$ the integrals in eq. (10) become,

$$\sigma_\Delta = \begin{cases} = \sigma_a = 2\pi r_o^2 Z\varepsilon \left\{ 3\varepsilon - 1 + \dfrac{\varepsilon^2+5\varepsilon+22/3}{(2+\varepsilon)^3} + \dfrac{1-2\varepsilon-3\varepsilon^2}{2} \ln\left(1+\dfrac{2}{\varepsilon}\right) \right\} & E \le \frac{\Delta}{2}\left\{ 1 + \left(\frac{2\varepsilon}{\delta}+1\right)^{1/2} \right\} \\[2ex] = \pi r_o^2 Z\varepsilon\delta \left\{ 2\varepsilon^2 + 2\varepsilon - 1 + \dfrac{\varepsilon^2}{1-\delta} + \dfrac{\delta}{2}\left(1+2\varepsilon+\varepsilon^2 - \dfrac{2}{3}\delta\right) - \dfrac{1-2\varepsilon-\varepsilon^2}{\delta} \ln(1-\delta) \right\} + & \\ \pi r_o^2 Z \dfrac{R_\Delta L_\Delta(E)}{E} \left\{ 4\varepsilon - \delta\left(1 - \dfrac{\delta}{2} + 2\varepsilon - \varepsilon^2 \dfrac{2-\delta}{1-\delta}\right) + \dfrac{2(1+\varepsilon)}{(2+\varepsilon)^2} + (1-2\varepsilon-\varepsilon^2)\ln(1-\delta)\left(1+\dfrac{2}{\varepsilon}\right) \right\} & E > \frac{\Delta}{2}\left\{ 1 + \left(\frac{2\varepsilon}{\delta}+1\right)^{1/2} \right\} \end{cases} \qquad (11)$$

wher $\varepsilon = \dfrac{E_o}{E}$ and $\delta = \dfrac{\Delta}{E}$

2.4 Pair production

For high energy photons of some 10 MeV and above the dominating interactions are due to pair production in the field of the nucleus. In these reactions both the electron and the positron may dissipate kinetic energy in the volume element of interest and both these possibilities must therefore be taken into account. If a simplified energy relation is assumed disregarding the recoil energy and the coulumb force of the nucleus on the electron positron pair the relation between the photon energy E and the energy of the positron (E_+) and the electron (E_-) becomes

$$E = E_+ + E_- + 2E_o \tag{12}$$

where E_o is the rest energy of the electron and the positron. The differential pair production cross section with respect to the positron energy may now be written (to simplify calculations complete screening is assumed, Roy and Reed, 1968b):

$$\frac{d\kappa}{dE_+} = \frac{4\alpha(Zr_o)^2}{E^3}\{(E_+{}^2+E_-{}^2+\tfrac{2}{3}E_+E_-)(\ln\frac{183}{Z^{1/3}}-f(Z)) - \frac{E_+E_-}{9}\} \tag{13}$$

Under the above assumptions the cross section is symmetric with respect to the energies of the electron and the positron. This implies that the total restricted cross section is twice the contribution from the positron if it is further assumed that the restricted stopping power of the electron and the positron are the same, thus:

$$\kappa_\Delta = \begin{cases} = 0 & E \le 2E_o \\ = \displaystyle\int_0^{E-2E_o} \frac{E_+ + E_-}{E} \frac{d\kappa}{dE_+} dE_+ & 2E_o < E \le 2E_o + \Delta \\ = 2\displaystyle\int_0^{\Delta} \frac{E_+}{E} \frac{d\kappa}{dE_+} dE_+ + 2\int_\Delta^{E-2E_o} \frac{R_\Delta L_\Delta(E_+)}{E} \frac{d\kappa}{dE_+} & E > 2E_o + \Delta \end{cases} \quad (14)$$

If now $L_\Delta(E_+)$ is regarded as a constant above $2E_o + \Delta$ and the cross section from eq (13) is used eq (14) becomes:

$$\kappa_\Delta = \begin{cases} =0 & 1-2\varepsilon \le 0 \\ =(1-2\varepsilon)\kappa = \frac{28}{9} Z^2 r_o^2 (1-2\varepsilon)^4 \{\ln\frac{183}{Z^{1/3}} - f(Z) - \frac{1}{42}\} & 0 < 1-2\varepsilon \le \delta \\ =8\alpha Z^2 r_o^2\ \delta^2 \{ (\frac{\delta^2}{3} + (\frac{1}{2} - \varepsilon - \frac{\delta}{9})(1-2\varepsilon)(\ln\frac{183}{Z^{1/3}} - f(Z)\ ) + (\frac{\delta}{4} - \frac{1-2\varepsilon}{3})\frac{\delta}{9}\} + & \\ 8\alpha Z^2 r_o^2 \frac{R_\Delta L_\Delta(E)}{E} \{ (\frac{7}{9}(1-2\varepsilon)^3 - (1-2\varepsilon)^2 + \frac{2\delta^2}{3}(1-2\varepsilon - \frac{2\delta}{3}))(\ln\frac{183}{Z^{1/3}} - f(Z)\ ) - & \\ \frac{(1-2\varepsilon)^3 + (3-6\varepsilon-2\delta)\delta}{54} \} & \delta < 1-2\varepsilon \end{cases} \quad (15)$$

2.5 The restricted energy absorption coefficient

From the restricted cross sections defined above restricted energy absorption coefficients may be defined by multiplication with the number of scattering centres per unit volume. Thus the restricted energy absorption coefficient can be written:

$$\mu_\Delta = N(\tau_\Delta + \sigma_\Delta + \kappa_\Delta) = \frac{N_a \rho}{A}(\tau_\Delta + \sigma_\Delta + \kappa_\Delta) \tag{16}$$

and the restricted mass energy absorption coefficient becomes

$$\frac{\mu_\Delta}{\rho} = \frac{N_a}{A}(\tau_\Delta + \sigma_\Delta + \kappa_\Delta) \tag{17}$$

In Figure 2 has this latter quantity been plotted as a function of the photon energy for three different values of the restriction energy Δ. The contributions from the elementary processes are calculated from the expressions given above and indicated by dashed lines in the Figure.

3. Application on cavity chambers

The absorbed dose to a medium irradiated by high energy photons can be expressed in two different ways when charged particle equilibrium is assumed

$$D_m = \int_\Delta^E \Phi_m(E) \left(\frac{S_\Delta(E)}{\rho}\right)_m dE \tag{18}$$

or

$$D_m = \int_0^E \Psi_m(E) \left(\frac{\mu_{en}}{\rho}\right)_m dE \tag{19}$$

where the first integral is taken over the spherical fluence of secondary and higher order electrons $\Phi(E)$ generated by the photons and their respective restricted mass stopping powers $\frac{S_\Delta(E)}{\rho}$ and the second integral is taken over the energy fluence of photons $\Psi(E)$ and their respective mass energy absorption coefficients $\frac{\mu_{en}(E)}{\rho}$. When charged particle equilibrium is not prevalent the second relation no longer holds and the first relation has to be used. However, the restricted energy absorption coefficient defined above can be used in a similar way as in the second equation above to give the locally absorbed dose from the photon fluence.

If for example a detector is placed in the medium and the detector size, R_Δ, is small compared to the range of the secondary electrons and the mean free path of the photons the absorbed dose to the detector may be divided in two parts. The first part is caused by the electron fluence from the medium passing through the detector. This contribution becomes:

$$D_{d,e} = \int_\Delta^E \Phi_m(E) \left(\frac{S_\Delta(E)}{\rho}\right)_d dE \qquad (20)$$

The second part is due to secondary electrons generated and absorbed in the detector. This contribution can now be written:

$$D_{d,X} = \int_0^E \Psi_m(E) \left(\frac{\mu_\Delta(E)}{\rho}\right)_d dE \qquad (21)$$

The ratio of the absorbed dose in the detector to the absorbed dose in the medium thus becomes:

$$\frac{D_d}{D_m} = \frac{\int_\Delta^E \Phi_m(E) \left(\frac{S_\Delta(E)}{\rho}\right)_d + \int_0^E \Psi_m(E) \left(\frac{\mu_\Delta(E)}{\rho}\right)_d dE}{D_m} \qquad (22)$$

This expression may be simplified further by introducing the appropriate ratios of mass stopping powers and mass energy absorption coefficients and assuming charged particle equilibrium:

$$\frac{D_d}{D_m} = \frac{\int_{\Delta}^{E} \Phi_m(E) \left(\frac{S_\Delta(E)}{\rho}\right)_m^d \left(\frac{S_\Delta(E)}{\rho}\right)_m dE}{\int_{\Delta}^{E} \Phi_m(E) \left(\frac{S_\Delta(E)}{\rho}\right)_m dE} + \frac{\int_{o}^{E} \Psi_m(E) \frac{\mu_\Delta(E)}{\mu_{en}(E)} \left(\frac{\mu_{en}(E)}{\rho}\right)_m^d \left(\frac{\mu_{en}(E)}{\rho}\right)_m dE}{\int_{o}^{E} \Psi_m(E) \left(\frac{\mu_{en}(E)}{\rho}\right)_m dE} \qquad (23)$$

which after introduction of suitable average values takes the form

$$\frac{D_d}{D_m} = \overline{\left(\frac{S_\Delta}{\rho}\right)_m^d} + \overline{\left(\frac{\mu_\Delta}{\mu_{en}}\right)_d \left(\frac{\mu_{en}}{\rho}\right)_m^d} \qquad (24)$$

As might be expected this expression has some resemblance with the results of Burlin (1968) for chambers of intermediate sizes. However, in the expression given by Burlin the weight factor in front of the average mass energy absorption coefficient ratio is taken indirectly from the absorption of the electron slowing down spectra and not from the direct contribution due to photon interactions in the detector. Equation (24) is therefore a fairly accurate expression when the correction term is only a small fraction of D_d/D_m and transition effects on the electron spectrum can be disregarded. When this no longer is the case the first term of eq (24) generally has to be corrected, but the second term may still be used. The correction may be done by a similar way of reasoning as used in eq (3) through (7) above.

The most straight forward configuration for application or measurement of restricted mass energy absorption coefficients is when a small detector of uniform composition is placed in vacuum and irradiated by a clean photon beam. In this configuration the absorbed dose in the detector is given simply by eq(21). If instead the absorbed dose and the energy fluence is known the restricted mass energy absorption coefficient can be determined. The restricted absorption coefficient obtained in this way is of course that connected to the particular detector size that has been used to measure the absorbed dose.

4. The distribution of absorbed dose around a small compton electron source

In many radiation geometries with high energy photons no charged particle equilibrium is obtained, particularly near boarders between greatly differring densities or near boarders of the radiation beam. An estimation of the absorbed dose distribution in these situations can often be made when the differential distribution in energy and direction of secondary electrons is taken into account.

In materials of low atomic number like water and tissue etc the compton interactions will predominate the photon interactions over a wide range of energies from about 0.1 MeV to about 10 MeV. Under these conditions the major part of the absorbed dose in the medium is deposited in the slowing down and diffusion of the compton electrons.

This process can be studied in two complementary ways either by determining the distribution of absorbed dose around a small source of compton electrons or by determining the fraction of the absorbed dose to a small volume element that originates from different volumes around the element of interest. Due to the reciprocity between the radiation source and the irradiated volume are these two distributions equal except for that their orientations are turned 180° relative to each other, that is parallel and antiparallel to the photon fluence (Loevinger, 1959). Of course this reciprocity only holds as long as the medium is homogeneous and uniformly irradiated with photons over a volume with a diameter not less than the maximum range of secondary electrons.

The absorbed dose in the source volume can be calculated directly by using the restricted mass energy absorption coefficient appropriate for the size of the source volume according to eq (21). The distribution of absorbed dose around the source can be estimated from the diffusion of the electrons out of the source volume.

The necessary information for the diffusion calculation is the distribution of the electrons in energy and direction which in the Klein-Nishina approximation becomes (Nelms, 1953):

$$\frac{T}{E_o} = \frac{2\alpha^2}{1+2\alpha+(1+\alpha)^2\tan^2\Psi} \quad (25)$$

and

$$\frac{d\sigma}{d\Omega} = 2r_o^{\,2} Z\left(\frac{1+\alpha}{1+2\alpha+(1+\alpha)^2\tan^2\Psi}\right)^2 \cos^{-3}\Psi \quad (26)$$

$$\left\{1+\left(\frac{(1+\alpha)^2\tan^2\Psi-1}{(1+\alpha)^2\tan^2\Psi+1}\right)^2+\frac{4\alpha^2}{(1+(1+\alpha)^2\tan^2\Psi)(1+2\alpha+(1+\alpha)^2\tan^2\Psi)}\right\}$$

where $\alpha=\frac{E}{E_o}$ and Ψ is the angle between the primary photon and the secondary electron. The motion of the electrons may be described by the simple diffusion model (Rossi, 1952):

$$f(t,x) = \frac{e^{-\frac{x^2}{\bar{x}^2(t)}}}{(\pi\bar{x}^2(t))^{1/2}} \quad (27)$$

where f(t,x) is the spatial distribution function, t is the penetration depth, x is the projected lateral displacement from the initial path, and finally

$$\overline{x^2}(t) = R^2 + \theta(T)\frac{\rho t^3}{3} \tag{28}$$

is the mean square radius of the lateral particle distribution which at the centre of the source has been set equal to the source radius. The mass scattering power of the medium for the particular electron energy has been denoted $\theta(T)$. By the use of polar coordinates (r,Θ) with origo at the source centre (cf Fig 3) the electron fluence may now be written:

$$F(r,\Theta) = N\Phi\int_{-\pi/2}^{\pi/2} \frac{d\sigma}{d\Omega}\;\frac{f(r,r(\Psi-\Theta))}{r}\;d\Psi \tag{29}$$

which after putting $\tan\Psi = s$ becomes:

$$F(r,\Theta)=\frac{2r_o^2 ZN\Phi}{\pi^{1/2}}\int_{-\infty}^{\infty}\frac{1}{r}\left(\frac{1+\alpha}{(1+\alpha)^2 s^2+2\alpha+1}\right)^2\left\{1+\left(\frac{(1+\alpha)^2 s^2-1}{(1+\alpha)^2 s^2+1}\right)^2 + \right.$$

$$\left. + \frac{4\alpha^2}{((1+\alpha)^2 s^2+1)((1+\alpha)^2 s^2+2\alpha+1)}\right\} \tag{30}$$

$$e^{-\frac{r^2(\tan^{-1}(s)-\Theta)^2}{\overline{x^2}(r,s)}}\quad\left(\frac{1+s^2}{\overline{x^2}(r,s)}\right)^{1/2}\;ds$$

where the mean square radius is

$$\overline{x^2}(r,s) = R^2 + \theta(T)\frac{\rho r^3}{3} \tag{31}$$

and the variation of the initial mass scattering power with angle can be derived from eq (25):

$$\theta(T) \simeq \theta(E)\left(\frac{E}{T}\right)^2 = \theta(E) \cdot \left(\frac{1+2\alpha+(1+\alpha)^2 s^2}{2\alpha}\right)^2 \qquad (32)$$

The integral in eq (30) has been evaluated by numerical integration for 3 different photon energies representative for ^{60}Co, 6 MV and 42 MV X-ray beams. The photon energies used in these calculations are 1.25, 2.0 and 14 MeV respectively, corresponding to the mean photon energies in the three beams. The results are given in the Table for a source diameter of 1 mm (R = 0.5 mm) in water.

In Fig 4 and 5 are the isofluence curves drawn for ^{60}Co and 42 MV photons respectively. The curves are normalized to 100 % at 0.5 mm from the source centre. It is evident that the shape of the curves more reflects the multiple scattering and square law attentuation of the electrons than their maximum ranges.

It should be pointed out that the results are obtained through several approximations: the problem has been two-dimensionalized the mean photon energy has been used instead of the complete photon spectrum, the limited range and energy loss of the electrons has been disregarded, and the source is described by a mean square radius and not a sharp cut of radius.

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The electron fluence from a 1 mm diameter compton electron source as given by the integral of eq (30).

Energy	θ	r= 0.5	1	2	5	10	20	50	100 (mm)
^{60}Co	0^{o}	4.65E 6	1.88E 6	6.62E 5	1.31E 5	3.36E 4	8.10E 3	1.16E 3	2.46E 2
α=2.45	10^{o}	4.54E 6	1.75E 6	5.50E 5	9.30E 4	2.34E 4	5.98E 3	9.25E 2	2.12E 2
$\theta=\frac{2.4 rad^2}{g/cm^2}$	30^{o}	3.75E 6	1.02E 6	1.95E 5	2.54E 4	6.29E 3	1.58E 3	2.62E 2	7.69E 1
	60^{o}	2.00E 6	2.50E 5	3.96E 4	4.90E 3	1.16E 3	2.92E 2	5.05E 1	1.33E 1
	90^{o}	7.30E 5	4.33E 4	5.84E 3	9.55E 2	2.73E 2	8.02E 1	1.62E 1	5.00E 1
	120^{o}	1.92E 5	7.37E 3	1.55E 3	3.19E 2	1.01E 2	3.22E 1	7.20E 0	2.69E 0
	180^{o}	8.85E 3	1.16E 3	3.68E 2	8.75E 1	2.97E 1	1.01E 1	2.44E 0	8.34E-1
6MV	0^{o}	4.79E 6	2.05E 6	7.74E 5	1.68E 5	4.46E 4	1.09E 4	1.57E 3	3.47E 2
α=3.91	10^{o}	4.66E 6	1.88E 6	6.12E 5	9.95E 4	2.43E 4	6.20E 3	1.03E 3	2.56E 2
$\theta=\frac{1.16 rad^2}{g/cm^2}$	30^{o}	3.80E 6	9.95E 5	1.60E 5	1.82E 4	4.92E 3	1.18E 3	1.84E 2	5.04E 1
	60^{o}	1.91E 6	1.80E 5	2.38E 4	2.55E 3	6.65E 2	1.67E 2	2.87E 1	8.07E 0
	90^{o}	6.30E 5	2.46E 4	2.90E 3	4.62E 2	1.34E 2	3.94E 1	9.80E 0	2.49E 0
	120^{o}	1.45E 5	3.56E 3	7.10E 2	1.46E 2	4.67E 1	1.50E 1	3.37E 0	1.20E 0
	180^{o}	4.65E 3	5.15E 2	1.63E 2	3.87E 1	1.32E 1	4.52E 0	1.09E 0	3.76E-1
42MV	0^{o}	5.55E 6	2.72E 6	1.29E 6	4.29E 5	1.62E 5	4.30E 4	6.90E 3	1.70E 3
α=27.4	10^{o}	5.40E 6	2.43E 6	8.42E 5	6.80E 4	8.88E 3	2.00E 3	3.23E 1	8.21E 1
$\theta=\frac{0.04 rad^2}{g/cm^2}$	30^{o}	4.26E 6	9.69E 5	4.14E 4	1.92E 3	4.44E 2	1.07E 2	1.56E 1	3.88E 0
	60^{o}	1.90E 6	4.92E 4	1.10E 3	1.14E 2	2.62E 1	6.62E 0	1.15E 0	3.77E-1
	90^{o}	4.98E 5	1.27E 3	9.10E 1	1.41E 1	4.09E 0	1.21E 0	2.52E-1	7.88E-2
	120^{o}	7.60E 4	1.09E 2	1.99E 1	4.10E 0	1.32E 0	4.26E-1	9.70E-2	3.20E-2
	180^{o}	4.76E 2	1.31E 1	4.36E 0	1.04E 0	3.58E-1	1.23E-1	2.98E-2	1.02E-2

Legends to the figures

Fig 1. The fraction $\Delta T/T$ of the electron energy generated in a sphere of water of radius R_Δ which is absorbed inside the sphere. At low electron energies almost all the electron energy is lost inside the sphere, but when the range of the electrons increases considerably above R_Δ an increased loss of electrons is observed. The Monte Carlo results are taken from calculations by Wainc and Knoll (1966) for a Silicon detector.

Fig 2. Restricted mass energy absorption coefficients for water as calculated in this paper for energy restrictions of 1, 10 and 200 keV. At 200 keV the elementary contributions from photo electric (τ_Δ), compton (σ_Δ) and pair production (κ_Δ) processes are indicated.

Fig 3. Description of the variables used for estimation of the distribution of absorbed dose around a small compton electron source.

Fig 4. Isofluence curves in water for compton electrons generated in a small spherical volume irradiated by ^{60}Co photons (1.25 MeV). The isofluence curves are normalized in the forward direction to 100 % at 0.5mm from the source centre. The diameter of the source is 1 mm and the scale is 10:1.

Fig 5. As Fig 4 but for 42 Mv Bremsstrahlung. The calculation was made for 14 MeV monoenergetic photons which is approximately the mean photon energy of the 42 MV spectrum.

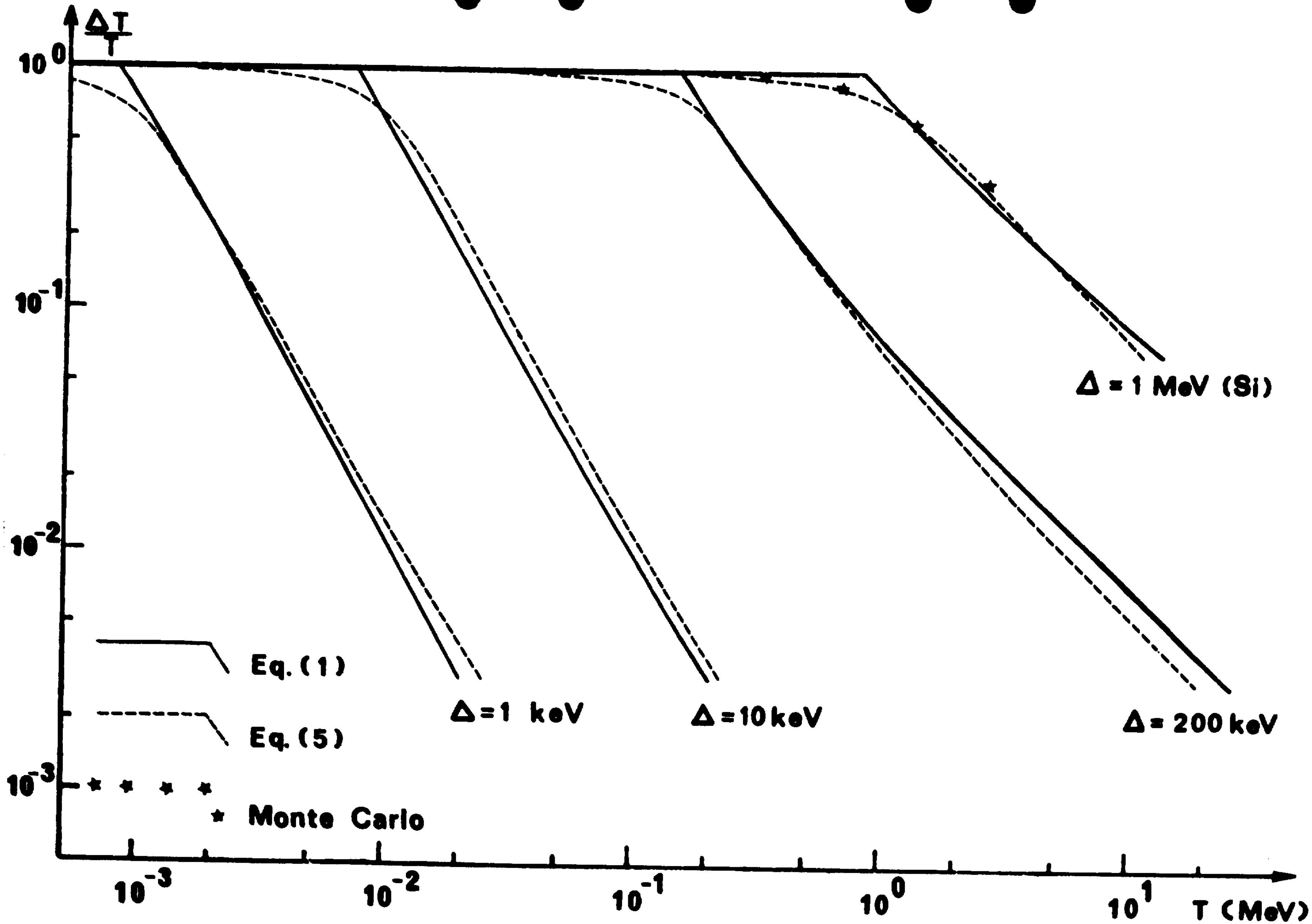
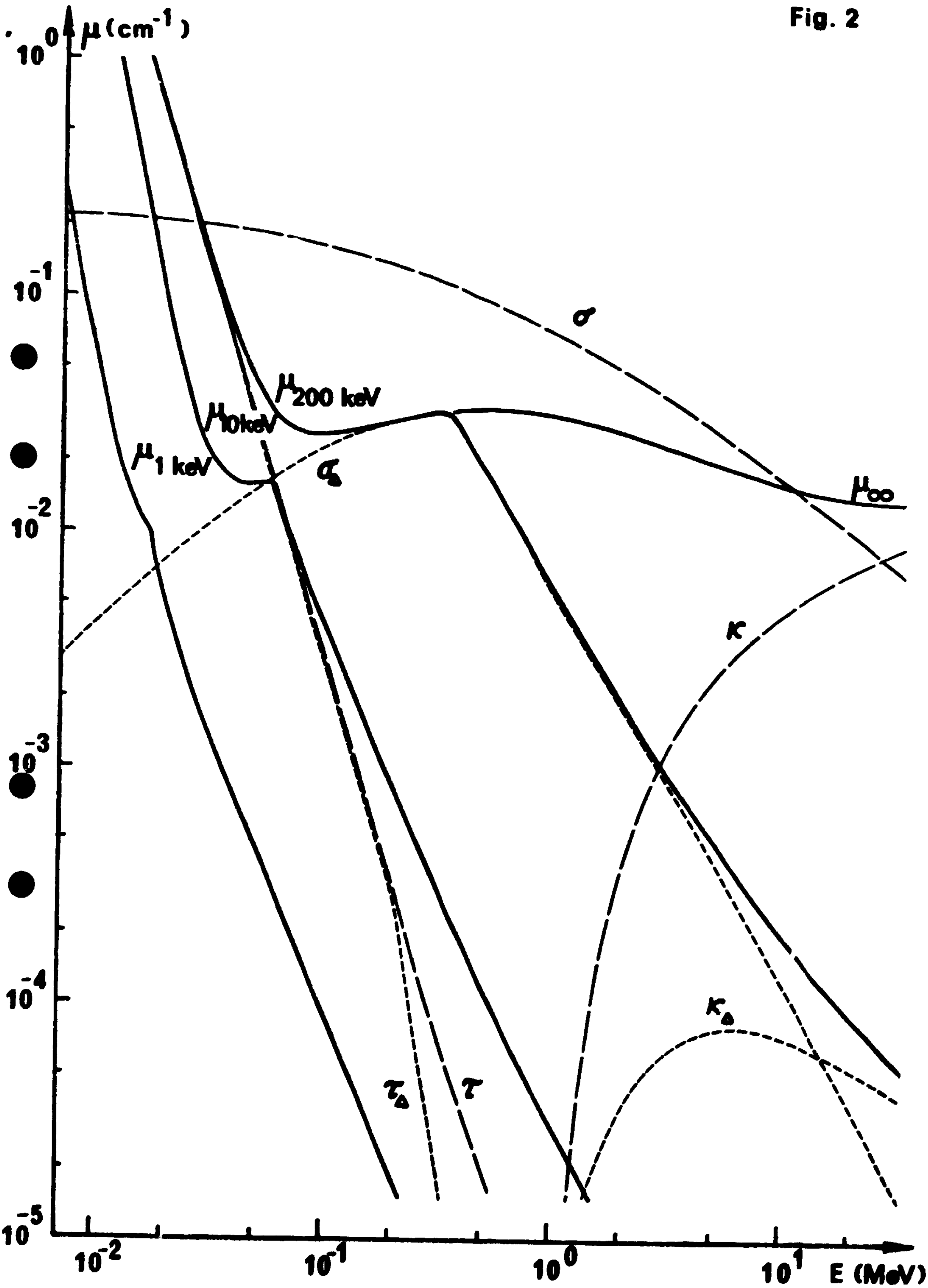

ΔT/T
10^0
10^-1
10^-2
10^-3
Δ = 1 MeV (Si)
Δ = 1 keV
Δ = 10 keV
Δ = 200 keV
Eq. (1)
Eq. (5)
Monte Carlo
10^-3
10^-2
10^-1
10^0
10^1
T (MeV)

Fig. 1



μ (cm-1)
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-2
10^-1
10^0
10^1
E (MeV)
σ
μ200 keV
μ10keV
μ1 keV
σΔ
μ∞
κ
κΔ
τΔ
τ

Fig. 2

Fig. 3

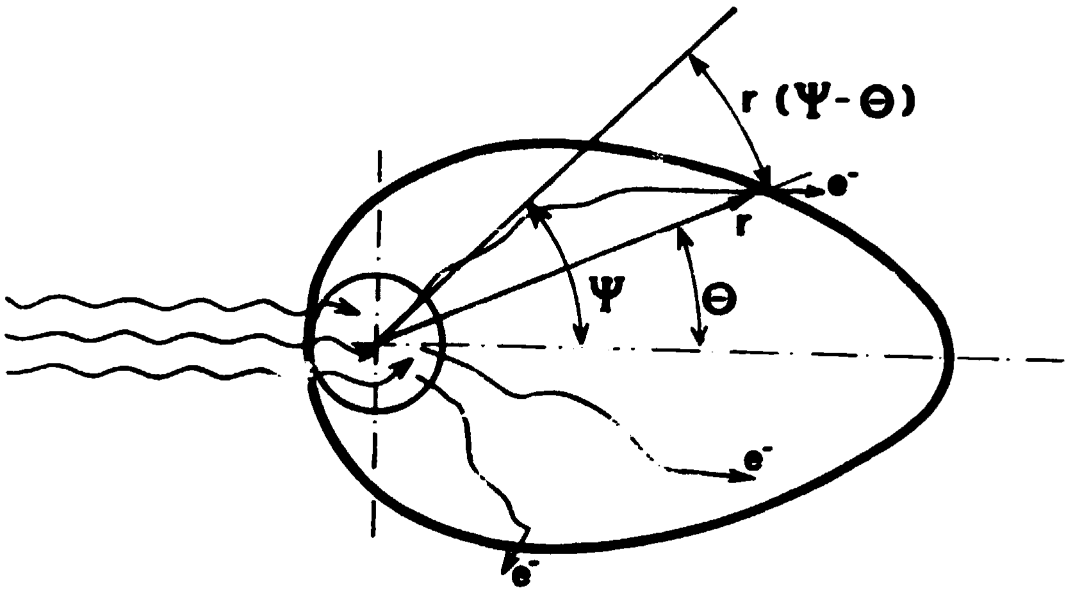
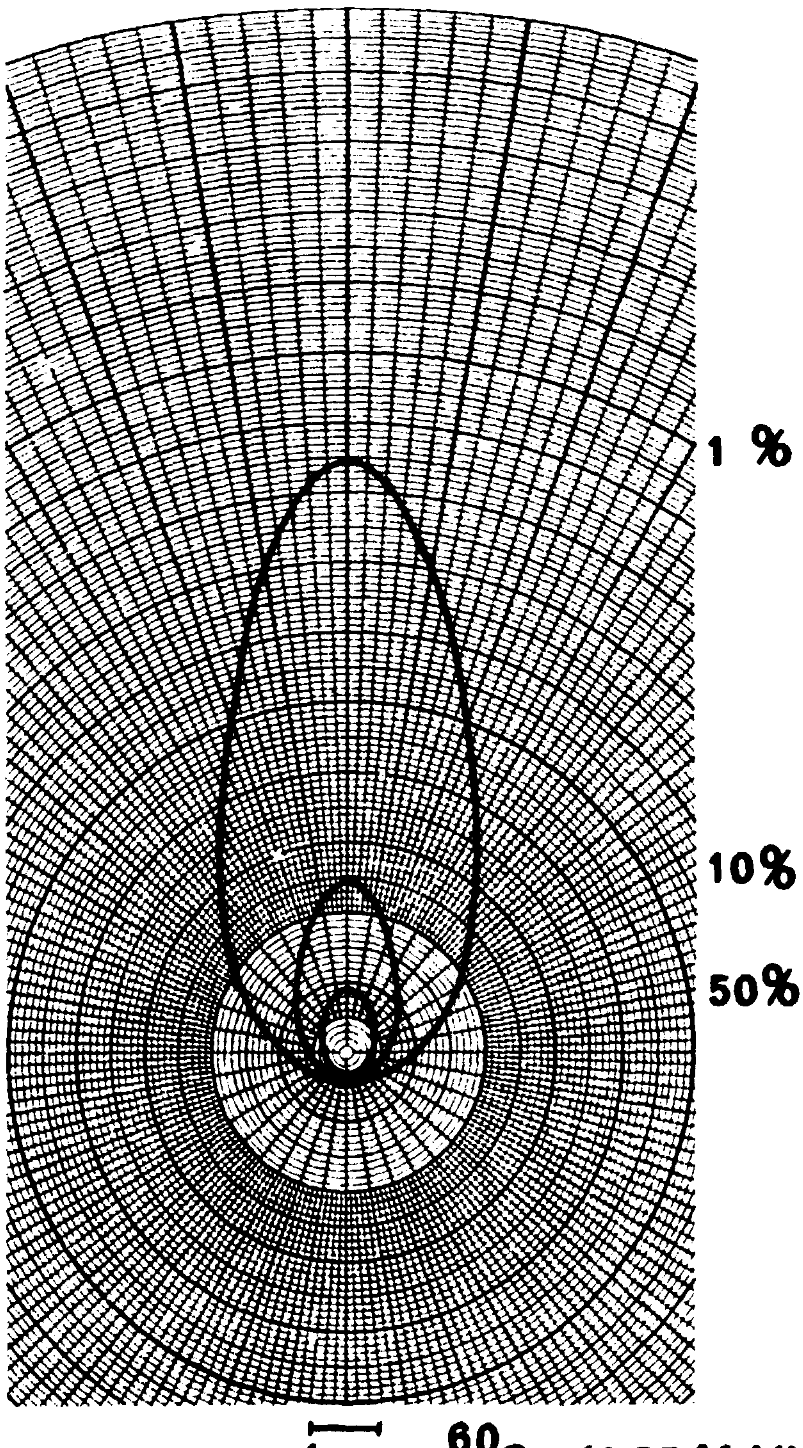
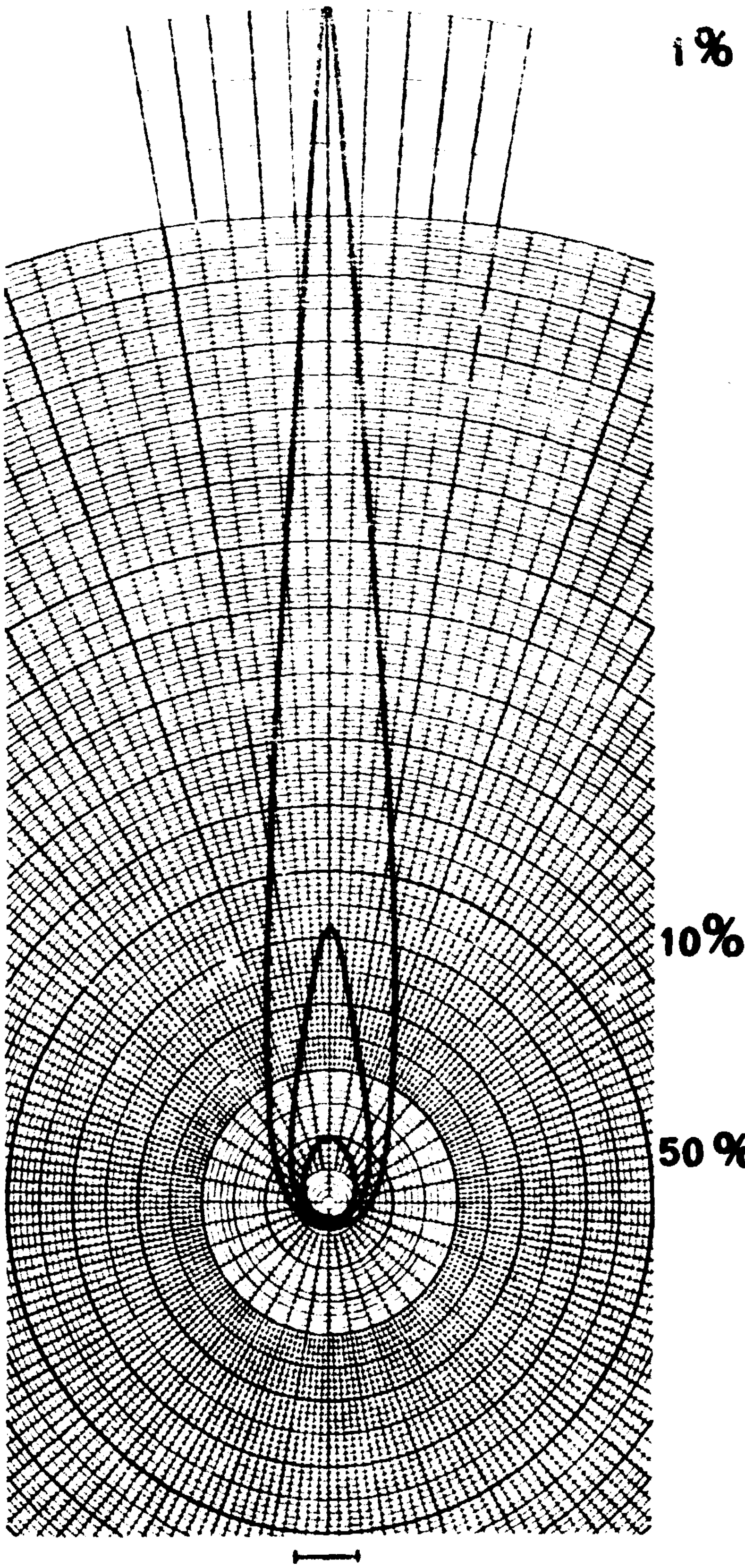


Fig. 4



^{60}Co (1.25 MeV)

Fig. 5



42 MV (14 MeV)