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L'ÉNERGIE ATOMIQUE DU CANADA LIMITÉE

A MATHEMATICAL MODEL OF STEAM-DRUM DYNAMICS

by

E.O. MOECK and H.W. HINDS

Presented at the 1975 Summer Computer Simulation Conference,

San Francisco, July 21-23, 1975

Chalk River Nuclear Laboratories

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Modèle mathématique de la dynamique d'un réservoir de vapeur*

Par

E.O. Moeck et H.W. Hinds

*Rapport présenté au Congrès de la simulation sur ordinateur tenu à San Francisco du 21 au 23 juillet 1976. Tiré à part avec l'autorisation de Simulation Councils, Inc., LaJolla, California.

Résumé

Pour étudier la contrôlabilité des centrales nucléaires il est nécessaire de faire des simulations pour tous les composants majeurs de la centrale. L'un de ces composants est le réservoir de vapeur.

Des équations mathématiques décrivant le comportement dynamique de la pression, de la masse de l'eau, etc. dans un réservoir de vapeur sont établies à partir de principes de base. Le modèle qui en résulte renseigne sur des effets comme le surchauffage de la vapeur et le sous-refroidissement de l'eau ainsi que le rejaillissement spontané du liquide et la condensation de la vapeur.

Les données expérimentales provenant d'un pressuriseur sont prédites adéquatement par le modèle. L'augmentation de pression faisant suite à un arrêt de turbine peut être prédite par le modèle à compression isentropique mais non par le modèle à équilibre thermodynamique.

Les équations sont linéarisées individuellement et appliquées sur un ordinateur analogique de telle façon que leur comportement non linéaire est retenu pour l'étude des petites perturbations.

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ABSTRACT

To study the controllability of nuclear generating stations, simulations comprising all the major components of the plant are required. One such component is a steam drum.

Mathematical equations describing the dynamic behaviour of pressure, water mass, etc. in a steam drum are derived from basic principles. The resultant model includes such effects as steam superheating and water subcooling as well as spontaneous flashing of liquid and condensation of vapour.

Experimental data from a pressurizer are adequately predicted by the model. The pressure rise following a turbine trip can be predicted by the isentropic-compression model but not by the thermodynamic-equilibrium model.

The equations are individually linearized and implemented on an analog computer in such a way that their non-linear behaviour is retained for small-perturbation studies.

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ABSTRACT

To study the controllability of nuclear generating stations, simulations comprising all the major components of the plant are required. One such component is a steam drum.

Mathematical equations describing the dynamic behaviour of pressure, water mass, etc. in a steam drum are derived from basic principles. The resultant model includes such effects as steam superheating and water subcooling as well as spontaneous flashing of liquid and condensation of vapour.

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INTRODUCTION

The analysis described in this paper was carried out during the development of dynamic simulations of advanced CANDU-BLW* nuclear generating stations. The simulations are used to study the controllability of the advanced BLW concept, as described in the companion paper by Lepp and Hinds [1]. The steam-drum model is one block in the overall plant simulation.

To clarify our terms of reference, we define a steam drum as a stationary, adiabatic vessel of fixed volume, containing single-component liquid and its vapour and having inflows and outflows of both liquid and vapour. In the example under consideration, shown in Fig. 1, steam (vapour) and water (liquid) enter the inner drum via separators, while a steam main feeds steam to the turbine and a downcomer supplies water to the reactor coolant pumps. On passing through the core, some water boils, and a two-phase mixture discharges from the reactor and enters a mixing region surrounding the inner drum. Here, condensate, returning from the turbine, mixes with the reactor coolant and is thus heated to saturation temperature. Hence, in the remainder of this paper, the steam-drum model refers to the inner drum region, receiving saturated steam and saturated water from the separators.

The purpose of the model is to generate the dynamic behaviour of
- drum pressure
- drum water-mass
- enthalpy of steam entering the steam main
- enthalpy of water entering the downcomer

in response to changes in the various flows.

Numerous simulations of nuclear and fossil-fired power plants have been carried out during the past several decades, and since these plants are based on the steam cycle, each simulation must have had a model

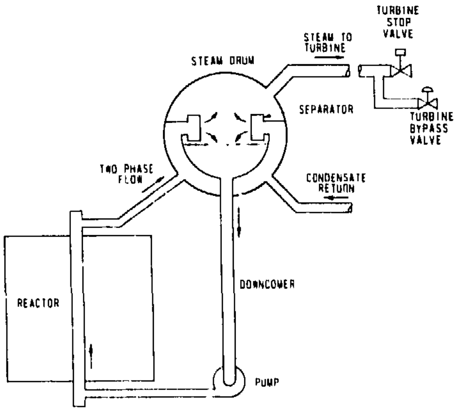


FIG. 1 SCHEMATIC OF CANDU-BLW COOLANT CIRCUIT

of a steam drum and possibly also of a pressurizer. Common practice has been to assume that the steam and water in these large vessels remain in a permanently saturated state [2] to [7]. This assumption results in simple equations that can be readily solved for given physical dimensions, fluid properties and operating conditions. However, observed pressure changes are at times more severe than those predicted by such models, hence there is an incentive to develop more accurate simulations.

M'Pherson [8] was probably the first to derive very detailed equations for both phases in a steam drum. He included the possibility of
- steam superheat
- spontaneous flashing
- spontaneous condensation
- heat transfer at the liquid surface
- heat transfer between each phase and the vessel walls.

His general model is highly complex, requiring some inputs based on engineering judgement, hence he suggested that the steam be considered permanently saturated, to reduce the complexity of the model.

Collins [9] attempted to apply M'Pherson's model to experimental data but was unable to obtain acceptable agreement, partly because of uncertainties in the data and partly because of two mutually compensating parameters in the model. Subsequently Hopkinson [10] derived a simple set of steam-drum equations based on the assumption of permanently wet steam.

Goemans [11] not only derived and programmed a detailed set of equations describing the dynamics of a pressurizer, but also conducted a series of precise measurements on a large-scale test apparatus. He was able to accurately predict multiple insurges and outsurges by assuming
- isentropic compression of the steam during an insurge
- negligible heat transfer between the steam and vessel walls
- negligible heat transfer across the liquid surface
- heat conduction from the liquid to the vessel.

*CANada Deuterium Uranium, Boiling Light Water Cooled

His model was only slightly in error when the last effect was neglected.

However, Gorman [12] found that an outsurge could be accurately predicted by a model based on saturation conditions, neglecting heat transfer between fluids and walls, but a single insurge could not be adequately represented by an isentropic-compression model [13].

A possible explanation for the opposing conclusions reached by Goemans [11] and Gorman and Gupta [13] is as follows:

> during multiple surges the vessel walls reach a temperature somewhere between saturation and maximum superheat, thus having only limited heat exchange with the steam, whereas during a single insurge the walls are initially at saturation temperature and hence extract an appreciable amount of heat from the steam throughout the duration of the insurge.

Kulkarni's [14] model is similar to that described in [12] and [13] except that it does not require an experimentally-determined heat-transfer coefficient.

Poletaev and Sulkhanishvili [15] also derived a pressurizer model but appear to rely heavily on experimental heat-transfer correlations.

MATHEMATICAL MODEL OF THE STEAM DRUM

The dynamic behaviour of the steam-drum variables is obtained from the equations of conservation of mass and energy, written separately for the liquid and vapour phases. Equations of state, boundary conditions, and assumptions regarding the thermodynamic processes complete the mathematical set.

The steam space is considered to be homogeneous, i.e., steam arriving from the separators or from flashing water mixes instantaneously with the resident steam and leaves the drum at the mixed-mean enthalpy. Similarly, the liquid phase is considered to be well-stirred at all times so that the water leaving the drum is always at the mixed-mean enthalpy.

Based on these assumptions, the equations for the conservation of mass are

$$\frac{dM_w}{dt} = W_f - W_w + W_{con} - W_{fl} \qquad (1)$$

and

$$\frac{dM_s}{dt} = W_g - W_s - W_{con} + W_{fl} \qquad (2)$$

where t is time and the other symbols are identified in Fig. 2.

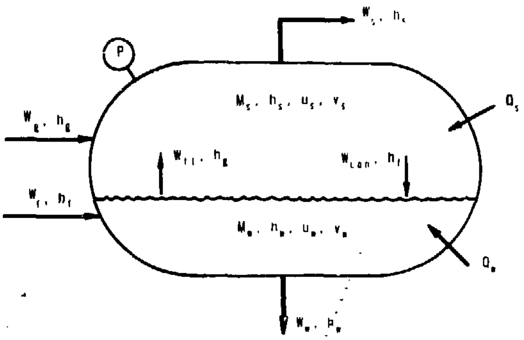
The conservation of energy is given by the following expression, neglecting changes in potential and kinetic energy,

$$\underbrace{\begin{bmatrix}\text{Rate-of-rise}\\ \text{of internal}\\ \text{energy}\end{bmatrix}}_{①} = \underbrace{\begin{bmatrix}\text{net rate of}\\ \text{internal}\\ \text{energy}\\ \text{inflow}\end{bmatrix}}_{②} + \underbrace{\begin{bmatrix}\text{net work}\\ \text{done on}\\ \text{system}\end{bmatrix}}_{③} + \underbrace{\begin{bmatrix}\text{net heat}\\ \text{inflow}\\ \text{into}\\ \text{system}\end{bmatrix}}_{④} \qquad (3)$$

For the liquid phase, these terms are

① $= \frac{d}{dt}(M_w u_w)$

② $= W_f u_f + W_{con} u_f - W_w u_w - W_{fl} u_g$



		SUBSCRIPTS
h = SPECIFIC ENTHALPY	, $J.kg^{-1}$	con CONDENSATION
M = MASS	, kg	f : SATURATED WATER
P = PRESSURE (ABSOLUTE)	, $N.m^{-2}$	fl : FLASHING
Q = HEAT FLOW	, $J.s^{-1}$	g : SATURATED STEAM
u = SPECIFIC INTERNAL ENERGY	, $J.kg^{-1}$	s : STEAM
v = SPECIFIC VOLUME	, $m^3.kg^{-1}$	w : WATER
W = FLOW	, $kg.s^{-1}$	fg = VAPOURIZATION

FIG. 2 IDENTIFICATION OF VARIABLES

③ $= W_f P v_f + W_{con} P v_f - W_w P v_w - W_{fl} P v_g - P\frac{d}{dt}(M_w v_w)$

④ $= Q_w$

The first four terms of ③ are the flow work while the last term is the compression work done by the rising water level.

Before proceeding with the substitution, it is worthwhile to combine terms by making use of the identity

$$h = u + Pv \qquad (4)$$

Hence, after rearrangement,

$$\frac{d}{dt}(M_w h_w) = (W_f + W_{con})h_f - W_w h_w - W_{fl} h_g + M_w v_w \frac{dP}{dt} + Q_w \qquad (5)$$

Similarly, he energy equation for the vapour phase is

$$\frac{d}{dt}(M_s h_s) = (W_g + W_{fl})h_g - W_s h_s - W_{con} h_f + M_s v_s \frac{dP}{dt} + Q_s \qquad (6)$$

The equations of state for the liquid and vapour phases can be written as follows

$$v_w = v_w(P, h_w) \qquad (7)$$

$$v_s = v_s(P, h_s) \qquad (8)$$

The boundary condition of interest is a constant drum volume,

$$\frac{d}{dt}(M_w v_w + M_s v_s) = 0 \qquad (9)$$

The seven basic equations, (1), (2) and (5) to (9), govern the behaviour of eleven unknowns:

P, M_w, M_s, W_{con}, W_{fl}, h_w, h_s, v_w, v_s, Q_w and Q_s.

Hence no general solution is possible unless additional equations are specified or simplifying assumptions are made.

The first assumption is that heat transfer between the fluids and the vessel materials and between the two phases (by conduction) is negligible, i.e.

$$Q_w = Q_s = 0 \tag{10}$$

The second assumption is that neither phase can exist in a metastable form. This means that the vapour can be either saturated or superheated (but not super-saturated) while the liquid can be either saturated or subcooled (but not superheated). This assumption implies that flashing and condensation occur spontaneously within the bulk of the liquid and vapour phases, respectively.

Figure 3 gives an illustration of spontaneous flashing and condensation. If a unit mass of liquid, at pressure P and saturation conditions 2 is depressurized by an amount ΔP, some of the liquid flashes into saturated steam. The horizontal intercept on the temperature-entropy diagram is a measure of the amount of flashing. Similarly, a unit mass of vapour, at pressure P and saturation conditions 3, will partially condense when depressurized by an amount ΔP. Again, the amount of condensation is given by the horizontal intercept. Figure 3 also shows that, initially subcooled liquid at 1 or superheated vapour at 4, when depressurized, must first reach the saturation line before flashing or condensing into the other phase. Finally, a pressure increase from P to $P+\Delta P$ suppresses flashing and condensation, regardless of the initial state of the liquid (1 or 2) or vapour (3 or 4).

The concepts described above are a straightforward application of reversible thermodynamics; they have no relation to the condensation or flashing that may occur at the vessel walls because of heat conduction. Equation (10) disallows these.

The results of the preceding paragraphs can be summarized, as in Table 1, to show under which combinations of pressure changes and thermodynamic conditions of the phases, flashing and condensation must occur. Using Table 1, we can write the missing equations to solve for the nine unknowns.

TABLE 1
FLASHING AND CONDENSATION

Pressure	Condition of Steam	Condition of Water	Condensation	Flashing
rising	saturated or superheated	saturated or subcooled	no	no
falling	saturated	saturated	yes	yes
falling	saturated	subcooled	yes	no
falling	superheated	saturated	no	yes
falling	superheated	subcooled	no	no

To illustrate the application of Table 1 and Fig. 3, we now derive the equations in detail for a rising pressure and present the results for the other conditions as a summary.

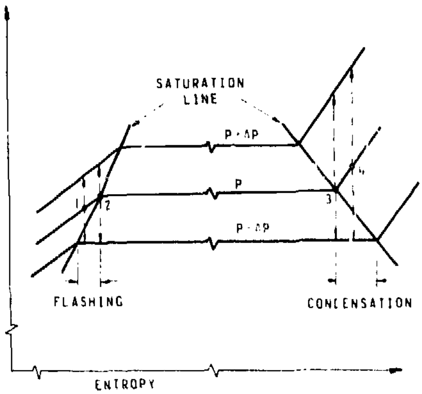


FIG. 3 TEMPERATURE-ENTROPY DIAGRAM SHOWING SPONTANEOUS FLASHING AND CONDENSATION

A rising pressure results in increased subcooling of the liquid and superheating of the vapour, so that a saturated-saturated condition can occur only as a starting point of a transient. Hence the specific volume of each phase must be defined by two coordinates, as was done in equations (7) and (8). Differentiation of these yields

$$\dot v_w = \left(\frac{dv_w}{dh_w}\right)\dot h_w \tag{11**}$$

where the dot superscript signifies d/dt,

and
$$\dot v_s = \left(\frac{\partial v_s}{\partial P}\right)_h \dot P + \left(\frac{\partial v_s}{\partial h_s}\right)_P \dot h_s \tag{12}$$

Substitution of these into equation (9) gives

$$v_w\dot M_w + M_w\left(\frac{dv_w}{dh_w}\right)\dot h_w + v_s\dot M_s + M_s\left[\left(\frac{\partial v_s}{\partial P}\right)_h \dot P + \left(\frac{\partial v_s}{\partial h_s}\right)_P \dot h_s\right] = 0 \tag{13}$$

From Table 1 we also know that

$$W_{fl} = W_{con} = 0 \tag{14}$$

This relation can be substituted into the mass and energy equations, (1), (2), (5) and (6), which together with the volume constraint (equation (13)), constitute a set of five equations in the unknowns $\dot P$, $\dot M_w$, $\dot M_s$, $\dot h_w$ and $\dot h_s$. This set is shown as the matrix M1.0 in Table 2.

The solution for the principal variable, $\dot P$, is shown as equation M1.1 in Table 3 and for the other variables as equations M1.2 to M1.5. The remaining equations M1.6 to M1.9 are transcribed from the preceding paragraphs. This, then, is the steam-drum model for a rising pressure.

**liquid compressibility, i.e. $\left(\frac{\partial v_w}{\partial P}\right)_h$, can be neglected with respect to thermal expansion, $\left(\frac{\partial v_w}{\partial h_w}\right)_P$.

TABLE 2

STEAM-DRUM MODEL IN MATRIX FORM

PRESSURE	CONDITION OF STEAM	CONDITION OF WATER	GOVERNING MATRIX	MATRIX NO.
RISING	SUPER-HEATED	SUB-COOLED	$\begin{bmatrix} v_w & v_s & M_s\left(\frac{\partial v_s}{\partial P}\right)_h & M_w\left(\frac{dv_w}{dh_w}\right) & M_s\left(\frac{\partial v_s}{\partial h_s}\right)_p \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ h_w & 0 & -M_w v_w & M_w & 0 \\ 0 & h_s & -M_s v_s & 0 & M_s \end{bmatrix} \begin{bmatrix} \dot{M}_w \\ \dot{M}_s \\ \dot{P} \\ \dot{h}_w \\ \dot{h}_s \end{bmatrix} = \begin{bmatrix} 0 \\ W_f - W_w \\ W_g - W_s \\ W_f h_f - W_w h_w \\ W_g h_g - W_s h_s \end{bmatrix}$	M1.0
FALLING	SATU-RATED	SATU-RATED	$\begin{bmatrix} v_f & v_g & \left[\left(\frac{dv_f}{dP}\right) M_w + \left(\frac{dv_g}{dP}\right) M_s\right] & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ h_f & 0 & \left(\frac{dh_f}{dP} - v_f\right) M_w & -h_f & h_g \\ 0 & h_g & \left(\frac{dh_g}{dP} - v_g\right) M_s & h_f & -h_g \end{bmatrix} \begin{bmatrix} \dot{M}_w \\ \dot{M}_s \\ \dot{P} \\ W_{con} \\ W_{fl} \end{bmatrix} = \begin{bmatrix} 0 \\ W_f - W_w \\ W_g - W_s \\ h_f(W_f - W_w) \\ h_g(W_g - W_s) \end{bmatrix}$	M2.0
FALLING	SATU-RATED	SUB-COOLED	$\begin{bmatrix} v_w & v_g & \left(\frac{dv_g}{dP}\right) M_s & \left(\frac{dv_w}{dh_w}\right) M_w & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ h_w & 0 & -v_w M_w & M_w & -h_f \\ 0 & h_g & \left(\frac{dh_g}{dP} - v_g\right) M_s & 0 & h_f \end{bmatrix} \begin{bmatrix} \dot{M}_w \\ \dot{M}_s \\ \dot{P} \\ \dot{h}_w \\ W_{con} \end{bmatrix} = \begin{bmatrix} 0 \\ W_f - W_w \\ W_g - W_s \\ h_f W_f - h_w W_w \\ h_g(W_g - W_s) \end{bmatrix}$	M3.0
FALLING	SUPER-HEATED	SATU-RATED	$\begin{bmatrix} v_f & v_s & \left[\left(\frac{dv_f}{dP}\right) M_w + \left(\frac{\partial v_s}{\partial P}\right)_h M_s\right] & \left(\frac{\partial v_s}{\partial h_s}\right)_p M_s & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ h_f & 0 & \left(\frac{dh_f}{dP} - v_f\right) M_w & 0 & h_g \\ 0 & h_s & -v_s M_s & M_s & -h_g \end{bmatrix} \begin{bmatrix} \dot{M}_w \\ \dot{M}_s \\ \dot{P} \\ \dot{h}_s \\ W_{fl} \end{bmatrix} = \begin{bmatrix} 0 \\ W_f - W_w \\ W_g - W_s \\ h_f(W_f - W_w) \\ h_g W_g - h_s W_s \end{bmatrix}$	M4.0
FALLING	SUPER-HEATED	SUB-COOLED	SAME AS MATRIX M1.0	

TABLE 3

MAIN EQUATIONS OF STEAM-DRUM MODEL

PRESSURE	CONDITION OF		MAIN EQUATIONS	EQ'N NO.
	STEAM	WATER		
RISING	SUPER-HEATED	SUB-COOLED	$\dot{P} = \dfrac{\left[v_s(W_g - W_s) + v_w(W_f - W_w) - W_g(h_s - h_g)\left(\frac{\partial v_s}{\partial h_s}\right)_P + W_f(h_f - h_w)\left(\frac{dv_w}{dh_w}\right)\right]}{\left(\frac{1}{P\gamma}\right)v_s M_s - \left(\frac{dv_w}{dh_w}\right)v_w M_w}$	M1.1
			$\dot{M}_w = W_f - W_w$; $\dot{M}_s = W_g - W_s$	M1.2,3
			$\dot{h}_w = v_w \dot{P} + (h_f - h_w)\frac{W_f}{M_w}$; $\dot{h}_s = v_s \dot{P} - (h_s - h_g)\frac{W_g}{M_s}$	M1.4,5
			$\dot{v}_w = \left(\frac{dv_w}{dh_w}\right)\dot{h}_w$; $\dot{v}_s = \left(\frac{\partial v_s}{\partial P}\right)_h \dot{P} + \left(\frac{\partial v_s}{\partial h_s}\right)_P \dot{h}_s$	M1.6,7
			$W_{fl} = 0$; $W_{con} = 0$	M1.8,9
FALLING	SATU-RATED	SATU-RATED	$\dot{P} = \dfrac{\left[v_g(W_g - W_s) + v_f(W_f - W_w)\right]}{\left[\left(\frac{dh_f}{dP} - v_f\right)\frac{v_{fg}}{h_{fg}} - \left(\frac{dv_f}{dP}\right)\right]M_w + \left[\left(\frac{dh_g}{dP} - v_g\right)\frac{v_{fg}}{h_{fg}} - \left(\frac{dv_g}{dP}\right)\right]M_s}$	M2.1
			$\dot{M}_w = W_f - W_w + W_{con} - W_{fl}$; $\dot{M}_s = W_g - W_s - W_{con} + W_{fl}$	M2.2,3
			$h_w = h_f(P)$; $h_s = h_g(P)$	M2.4,5
			$v_w = v_f(P)$; $v_s = v_g(P)$	M2.6,7
			$W_{fl} = -\left[\left(\frac{dh_f}{dP} - v_f\right)\frac{M_w}{h_{fg}}\right]\dot{P}$; $W_{con} = -\left[\left(v_g - \frac{dh_g}{dP}\right)\frac{M_s}{h_{fg}}\right]\dot{P}$	M2.8,9
FALLING	SATU-RATED	SUB-COOLED	$\dot{P} = \dfrac{\left[v_g(W_g - W_s) + v_w(W_f - W_w) + W_f(h_f - h_w)\left(\frac{dv_w}{dh_w}\right)\right]}{\left\{\frac{1}{h_{fg}}\left(\frac{dh_g}{dP} - v_g\right)\left[v_{fg} + (v_f - v_w) - (h_f - h_w)\left(\frac{dv_w}{dh_w}\right)\right] - \left(\frac{dv_g}{dP}\right)\right\}M_s - v_w\left(\frac{dv_w}{dh_w}\right)M_w}$	M3.1
			$\dot{M}_w = W_f - W_w + W_{con}$; $\dot{M}_s = W_g - W_s - W_{con}$	M3.2,3
			$\dot{h}_w = v_w \dot{P} + (h_f - h_w)(W_f + W_{con})\frac{1}{M_w}$; $h_s = h_g(P)$	M3.4,5
			$\dot{v}_w = \left(\frac{dv_w}{dh_w}\right)\dot{h}_w$; $v_s = v_g(P)$	M3.6,7
			$W_{fl} = 0$; $W_{con} = -\left[\left(v_g - \frac{dh_g}{dP}\right)\frac{M_s}{h_{fg}}\right]\dot{P}$	M3.8,9
FALLING	SUPER-HEATED	SATU-RATED	$\dot{P} = \dfrac{\left[v_s(W_g - W_s) + v_f(W_f - W_w) - W_g(h_s - h_g)\left(\frac{\partial v_s}{\partial h_s}\right)_P\right]}{\left\{\frac{1}{h_{fg}}\left(\frac{dh_f}{dP} - v_f\right)\left[v_{fg} + (v_s - v_g) - (h_s - h_g)\left(\frac{\partial v_s}{\partial h_s}\right)_P\right] - \left(\frac{dv_f}{dP}\right)\right\}M_w + \left(\frac{1}{P\gamma}\right)v_s M_s}$	M4.1
			$\dot{M}_w = W_f - W_w - W_{fl}$; $\dot{M}_s = W_g - W_s + W_{fl}$	M4.2,3
			$h_w = h_f(P)$; $\dot{h}_s = v_s \dot{P} - (h_s - h_g)\left(\frac{W_g + W_{fl}}{M_s}\right)$	M4.4,5
			$v_w = v_f(P)$; $\dot{v}_s = \left(\frac{\partial v_s}{\partial P}\right)_h \dot{P} + \left(\frac{\partial v_s}{\partial h_s}\right)_P \dot{h}_s$	M4.6,7
			$W_{fl} = -\left[\left(\frac{dh_f}{dP} - v_f\right)\frac{M_w}{h_{fg}}\right]\dot{P}$; $W_{con} = 0$	M4.8,9
FALLING	SUPER-HEATED	SUB-COOLED	SAME AS EQUATIONS M1.1 to M1.9	

The same procedure was followed to arrive at similar sets of equations for a falling pressure in combination with steam and water in thermodynamic equilibrium or non-equilibrium. The results are given as matrices M2.0 to M4.0 in Table 2 and equations M2.1 to M4.9 in Table 3.

Table 2 shows that of the five variables, the first three ($\dot{P}$, $\dot{M}_w$, $\dot{M}_s$) are always present while the last two are some combination of $\dot{h}_w$, $\dot{h}_s$, W_{con} or W_{fl}. From Table 3 it is evident that during a falling pressure, the thermodynamic properties of the steam (or water) are locked onto the saturation line as functions of a single variable, pressure, provided the steam (or water) is saturated.

The parameter $(1/P\gamma)$, appearing in equations M1.1 and M4.1, is not obvious from an inspection of the corresponding matrices M1.0 and M4.0. However, from basic thermodynamics and Maxwell's relations, it can be obtained as the identity

$$\left[\frac{1}{v_s}\left(\frac{\partial v_s}{\partial P}\right)_h + \left(\frac{\partial v_s}{\partial h_s}\right)_P\right] \equiv -\frac{1}{P\gamma} \qquad (15)$$

where γ is the isentropic expansion exponent,

$$\gamma = -\frac{v_s}{P}\left(\frac{\partial P}{\partial v_s}\right)\Bigg|_{\text{constant entropy}} \qquad (16)$$

This completes the derivation of the equations governing the behaviour of steam-drum variables during various combinations of thermodynamic states.

For most applications, the equations of Tables 2 and 3 can be simplified considerably by neglecting thermal expansion of the liquid, i.e. by assuming $(dv_w/dh_w) = 0$. As an example, for the Gentilly-1† steam-drum, the effect of neglecting this term introduces an error in $\dot{P}$ of about 0.1%.

COMPARISON WITH EXPERIMENTAL DATA FROM THE NPD†† PRESSURIZER

To validate the mathematical model, it is desirable to have relevant experimental data from an actual steam drum but, except for the turbine-trip transient discussed in the following section, we have none. However, we do have a set of measurements obtained on the pressurizer of the NPD generating station [16], [17]. From a thermodynamic viewpoint, a pressurizer resembles a steam drum, except that there are no continuous throughflows of the two phases, i.e.

$$W_f = W_g = W_s = 0 \qquad (17)$$

Only water, W_w, flows into the pressurizer during an insurge and out during an outsurge.

The NPD pressurizer is a vertical, cylindrical tank of total volume V and water volume given by

$$V_w = a + bL \qquad (18)$$

where a and b are constants and L is the water level.

Hence $\quad M_w v_w = V_w = a + bL \qquad (19)$

and $\quad M_s v_s = V - V_w = V - a - bL \qquad (20)$

Separate outsurge [16] and insurge [17] experiments have been conducted during which the pressure and water level were monitored as functions of time. An experiment was usually begun after a period of steady-state operation, so that initially, saturated-saturated conditions can be assumed.

During an outsurge the pressure falls, so that equations M2.1 to M2.9 of Table 3 apply. Before transcribing them here, it is convenient to rearrange them in terms of liquid level, since this variable was easier to monitor than water flow.

Summation of equations M2.2 and M2.3 gives

$$\dot{M}_w + \dot{M}_s = -W_w \qquad (21)$$

Differentiation of equations (19) and (20) and substitution in (21) results in

$$W_w = \left[\left(\frac{dv_f}{dP}\right)\frac{M_w}{v_f} + \left(\frac{dv_g}{dP}\right)\frac{M_s}{v_g}\right]\dot{P} + b\left(\frac{1}{v_g} - \frac{1}{v_f}\right)\dot{L} \qquad (22)$$

Equations (17) to (20) and (22) can now be substituted in equation M2.1 to obtain an expression for the rate-of-change of level with pressure,

$$\frac{dL}{dP} = \frac{1}{b}\left[aF_1 + VF_2\right] + F_1 L \qquad (23)$$

where $$F_1(P) = \left[\left(\frac{v_g}{v_f}\frac{dh_f}{dP} - \frac{dh_g}{dP}\right)\frac{1}{h_{fg}} + \frac{1}{v_g}\frac{dv_g}{dP}\right] \qquad (24)$$

and $$F_2(P) = \left[\left(\frac{dh_g}{dP} - v_g\right)\frac{1}{h_{fg}} - \frac{1}{v_g}\frac{dv_g}{dP}\right] \qquad (25)$$

F_1 and F_2 need be evaluated only once, and a computer program was written to do this. Heavy-water properties were taken from Elliott's tabulations [18].

With the functions F_1 and F_2 specified, equation (23) was integrated numerically, from a given initial condition, to obtain the locus of the outsurge path in the (L, P) plane. Figure 4 illustrates one such calculation. The agreement between the theoretical curve and experimental points is excellent, and is typical of the close correlation obtained for all the experimental outsurge runs.

Gorman's [12] prediction of the same data was equally good, although he solved his model by an iterative technique.

During an insurge the pressure rises, so that equations M1.1 to M1.9 of Table 3 apply. Manipulation of these, together with equations (17) to (20), results in

$$\frac{dL}{dP} = \left(\frac{1}{P\gamma}\right)\left(\frac{V-a-bL}{b}\right) \qquad (26)$$

For ordinary steam [19], the exponent γ is nearly constant over a wide range of pressures and superheats. Thus, throughout the region

$$3.5 \le P \le 7.5 \text{ (MPa)}$$

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† CANDU-BLW-250 MW(e) plant at Gentilly, Quebec

†† Nuclear Power Demonstration 25 MW(e) plant at Rolphton, Ontario

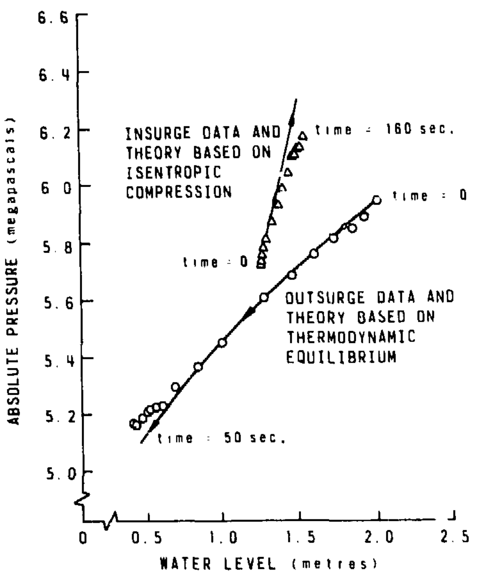

FIG. 4 NPD PRESSURIZER DURING INSURGE AND OUTSURGE

and for temperatures between saturation and 25 kelvins superheat,

$$\gamma = 1.26 \pm 1\%$$

The same value of γ was assumed for heavy water. Since, for a given experiment, γ can be considered constant, equation (26) can be integrated analytically,

$$P(V-a-bL)^{\gamma} = \text{constant} \qquad (27)$$

Equation (27) is simply the isentropic compression relation that could have been used right from the beginning for an enclosed steam volume such as is found in the pressurizer. However, for an open system, such as a steam drum, the detailed analysis, including mass and energy conservation, is necessary because of the continuous inflow of saturated steam.

Figure 4 shows the theoretical insurge locus, calculated from equation (27), and experimental data from a typical run [17]. The insurge and outsurge data, as well as the theoretical curves, clearly illustrate the non-linear pressure behaviour, even for small disturbances. For a unit change in water level, the pressure rises two or three times higher than it falls. This is because, during an outsurge, water flashes into steam, tending to stabilize the pressure.

For the insurge, agreement between data and theory is acceptable near the beginning of the transient, but as the insurge proceeds, the calculated pressure becomes higher than that observed. The reason for the deteriorating agreement is simple. The model makes no allowance for heat transfer from the superheated steam to the vessel walls, whereas some heat transfer, and hence steam condensation, occurs during the 2 to 3 minute test interval.

Gorman and Gupta [13] obtained excellent agreement for the entire duration of the insurge by specifying a steam-to-wall heat transfer coefficient. Their calculations show that during the first 80 seconds of the transient, about 3.9 MJ of heat is transferred from the trapped steam to the vessel walls (conduction to the liquid is about two orders of magnitude less). This heat removal is equivalent to the condensation of about 2.2 percent of the original steam mass.

More recently, Kulkarni [14] also obtained excellent agreement between theoretical and experimental insurge transients by allowing heat transfer to the metal walls. Instead of specifying a heat transfer coefficient, he assumed that the metal surface temperature followed the saturation temperature at the prevailing pressure, and the interior of the metal stored heat according to Fourier's diffusion law.

While these approaches, [13] and [14], are more accurate, they also increase the complexity of the model. The resultant increase in computing costs is justifiable if the model is used to obtain an optimum pressurizer design. However, in a steam drum one would expect that heat transfer to the walls has a negligible influence on the pressure, because the residence time of the throughflowing steam is too short (typically 10 seconds) for any significant fraction to be removed by condensation at the walls.

TURBINE-TRIP TRANSIENT

Predictions from the steam-drum model can also be compared with the observed pressure rise following a turbine trip. In the Gentilly-1 plant, a turbine trip is accompanied by a rapid closure of the turbine stop valve (TSV) and 'simultaneous' opening of a bypass valve to divert the full steam flow into the main condenser. If the manoeuver is successful, the steam-drum pressure stays within acceptable limits and the reactor power remains near the setpoint which can be subsequently ramped down in an orderly fashion. Since the TSV leads the bypass valve, a temporary flow restriction occurs which, at worst, is a complete flow stoppage for a short time interval.

The steam-drum pressure rise resulting from a total blockage of the steam outflow can be readily predicted by the steam-drum model. For the Gentilly-1 reactor at full power, the pressure rate calculated from the isentropic-compression model (equation M1.1) is

1060 kilopascals/second

and from the thermodynamic-equilibrium model (equation M2.1)

400 kilopascals/second.

These two rates are superimposed on the observed pressure transient [22] in Fig. 5, which shows that the observed pressure rise approaches the maximum rate predicted by the isentropic-compression model and exceeds that predicted by the thermodynamic-equilibrium model. Consequently, the latter is incapable of predicting the severity of a turbine-trip transient.

SMALL-DISTURBANCE ANALYSIS

Each equation of Table 3 can be linearized in the neighbourhood of a reference condition, taken for convenience to be the steady-state saturated-saturated condition, and designated by the subscript 'o'. The resultant set of equations is summarized in Table 4.

The term D_3, equation L10, is usually much smaller than either D_2 or D_4 and hence was neglected in the analog computer solution described below.

TABLE 4

LINEARIZED EQUATIONS OF STEAM-DRUM MODEL

PRESSURE		RISING	FALLING	FALLING	FALLING	FALLING
CONDITION OF	STEAM	SUPERHEATED	SATURATED	SATURATED	SUPERHEATED	SUPERHEATED
	WATER	SUBCOOLED	SATURATED	SUBCOOLED	SATURATED	SUBCOOLED
FLASHING		$\Delta W_{fl} = 0$	ΔW_{fl}	$\Delta W_{fl} = 0$	ΔW_{fl}	SAME AS FOR RISING PRESSURE
CONDENSATION		$\Delta W_{con} = 0$	ΔW_{con}	ΔW_{con}	$\Delta W_{con} = 0$	
WATER ENTHALPY		$\Delta \dot{h}_w$	Δh_f	$\Delta \dot{h}_w$	Δh_f	
STEAM ENTHALPY		$\Delta \dot{h}_s$	Δh_g	Δh_g	$\Delta \dot{h}_s$	
WATER SPECIFIC VOLUME		$\Delta \dot{v}_w$	Δv_f	$\Delta \dot{v}_w$	Δv_f	
STEAM SPECIFIC VOLUME		$\Delta \dot{v}_s$	Δv_g	Δv_g	$\Delta \dot{v}_s$	
RATE OF CHANGE OF PRESSURE		$\Delta \dot{P} = \frac{\Delta N_1 + \Delta N_2 + \Delta N_3}{D_4 - D_3}$	$\Delta \dot{P} = \frac{\Delta N_1}{D_1 + D_2}$	$\Delta \dot{P} = \frac{\Delta N_1 + \Delta N_2}{D_2 - D_3}$	$\Delta \dot{P} = \frac{\Delta N_1 + \Delta N_3}{D_1 + D_4}$	
EQUATION NUMBER		L1	L2	L3	L4	

DEFINITION OF TERMS USED ABOVE

TERM	EQ'N NO.	TERM	EQ'N NO.
$\Delta N_1 = v_{go}(\Delta W_g - \Delta W_s) + v_{fo}(\Delta W_f - \Delta W_w)$	L5	$\Delta \dot{h}_s = \frac{1}{\tau_s}(\Delta h_g - \Delta h_s) + v_{go}\,\Delta \dot{P}$	L16
$\Delta N_2 = \left[W_f \left(\frac{dv_f}{dh_f}\right)\right]_o (\Delta h_f - \Delta h_w)$	L6	$\Delta \dot{h}_w = \frac{1}{\tau_w}(\Delta h_f - \Delta h_w) + v_{fo}\,\Delta \dot{P}$	L17
$\Delta N_3 = \left[W_g \left(\frac{\partial v_s}{\partial h_s}\right)_P\right]_o (\Delta h_g - \Delta h_s)$	L7	$\tau_s = \left(\frac{M_s}{W_g}\right)_o$	L18
$D_1 = \left[\left(\frac{dh_f}{dP} - v_f\right)\frac{v_{fg}}{h_{fg}} - \left(\frac{dv_f}{dP}\right)\right]_o M_{wo}$	L8	$\tau_w = \left(\frac{M_w}{W_f}\right)_o$	L19
$D_2 = \left[\left(\frac{dh_g}{dP} - v_g\right)\frac{v_{fg}}{h_{fg}} - \left(\frac{dv_g}{dP}\right)\right]_o M_{so}$	L9	$\Delta h_g = \left(\frac{dh_g}{dP}\right)_o \Delta P$	L20
$D_3 = \left[v_f \left(\frac{dv_f}{dh_f}\right) M_w\right]_o$	L10	$\Delta h_f = \left(\frac{dh_f}{dP}\right)_o \Delta P$	L21
$D_4 = \left[\frac{v_g M_s}{P \gamma}\right]_o$	L11	$\Delta \dot{v}_s = \left(\frac{\partial v_s}{\partial P}\right)_{ho} \Delta \dot{P} + \left(\frac{\partial v_s}{\partial h_s}\right)_{Po} \Delta \dot{h}_s$	L22
$\Delta W_{con} = -\left[\frac{1}{h_{fg}}\left(v_g - \frac{dh_g}{dP}\right) M_s\right]_o \Delta \dot{P}$	L12	$\Delta \dot{v}_w = \left(\frac{dv_w}{dh_w}\right)_o \Delta \dot{h}_w$	L23
$\Delta W_{fl} = -\left[\frac{1}{h_{fg}}\left(\frac{dh_f}{dP} - v_f\right) M_w\right]_o \Delta \dot{P}$	L13	$\Delta v_g = \left(\frac{dv_g}{dP}\right)_o \Delta P$	L24
$\Delta \dot{M}_s = \Delta W_g - \Delta W_s - \Delta W_{con} + \Delta W_{fl}$	L14*	$\Delta v_f = \left(\frac{dv_f}{dP}\right)_o \Delta P$	L25
$\Delta \dot{M}_w = \Delta W_f - \Delta W_w + \Delta W_{con} - \Delta W_{fl}$	L15*		

*These equations apply under all conditions

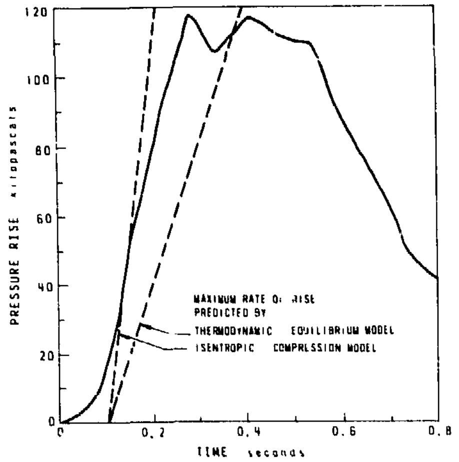

FIG. 5 GENTILLY-1 STEAM-DRUM PRESSURE RISE FOLLOWING A TURBINE TRIP

ANALOG COMPUTER SOLUTION OF SMALL-DISTURBANCE MODEL

The equations of interest from Table 4 have been implemented on an analog computer [20], [21] to obtain the behaviour of the drum variables in response to small perturbations in the various flows. The wiring diagram is shown in Fig. 6.

One comparator is used to determine whether or not the steam is saturated while the other determines the condition of the water. Saturation is equivalent to setting the corresponding time constant, τ_s or τ_w, (equation L18 or L19) to zero. In practice they are set to 1 millisecond as smaller values cause overloads. The problem may also be viewed in another way: a small time constant corresponds to high gain. Due to this high loop gain, the enthalpy difference $(\Delta h_g - \Delta h_s)$ or $(\Delta h_f - \Delta h_w)$ is essentially zero whenever the comparator indicates saturation.

In general, it is easily seen from Table 4 that

$$\Delta \dot{P} = \frac{\Delta N}{D} \qquad (23)$$

where $\Delta N = \Delta N_1$ or a combination of ΔN_1, ΔN_2 and ΔN_3

and $D = D_1$, D_2, or D_4 or a combination thereof, depending on steam and water conditions.

The comparators also switch potentiometers in the feedback loop of the amplifier generating $\Delta \dot{P}$, to yield the correct gain.

The comparators are controlled primarily by the quantities $(\Delta h_g - \Delta h_s)$ and $(\Delta h_w - \Delta h_f)$; if either is equal to (or greater than) zero, the corresponding quantity is saturated. (Neither of these quantities can be positive due to the high loop gain when saturated.) To prevent the system from becoming 'stuck' in the saturated/saturated mode, a diode network gates rising pressure ($-\Delta \dot{P} < 0$) to the comparators, thus forcing the superheated/subcooled condition.

Although each equation is linearized, the small-disturbance model itself is non-linear because it switches from one set of equations to another, as required. Figure 7 shows the response of various [illegible] parameters to a periodic disturbance in [illegible]. During the first third of the cycle ([illegible] at time = ½ s), there is net steam outflow, the pressure falls, and both water and steam are saturated. As soon as the net steam flow crosses zero and becomes an inflow, both comparators switch and the water becomes subcooled and the steam superheated. These conditions continue even though the pressure begins to fall as the net steam flow crosses the zero axis. Eventually the steam reaches saturation, $\Delta h_s = \Delta h_g$ and the corresponding comparator switches to the saturated condition. The step in $\Delta \dot{P}$ is due to spontaneous steam condensation. A short time later, $\Delta h_w = \Delta h_f$ and the other comparator switches to the saturated condition, at which point the $\Delta \dot{P}$ trace shows a second step due to spontaneous water flashing. The cycle repeats with both steam and water at saturation.

The water mass, also shown in Fig. 7, decreases because flashing exceeds condensation every time the water is saturated. Thus in the simulation, as in the actual plant, controllers are required to keep the various parameters from drifting beyond acceptable bounds.

CONCLUSIONS

(1) A non-linear model describing the dynamics of a steam drum has been derived from basic principles, without empirical inputs, except for assumptions regarding the thermodynamic processes occuring in the liquid and vapour phases.

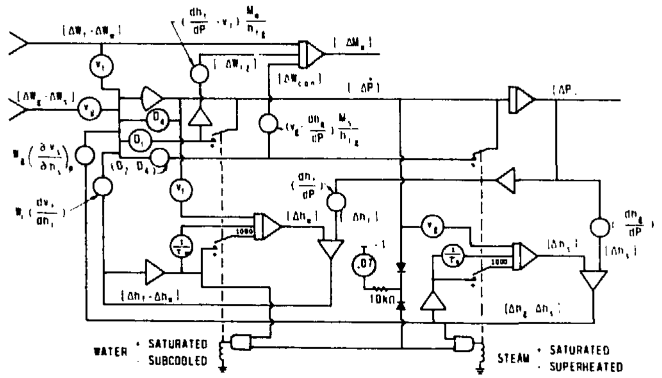

FIG. 6 ANALOG-COMPUTER WIRING DIAGRAM OF LINEARIZED, UNSCALED STEAM-DRUM EQUATIONS

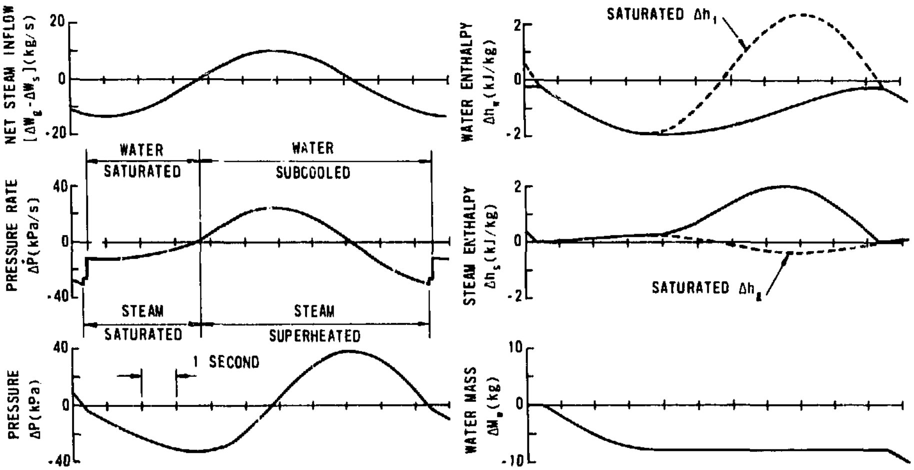

FIG. 7 TRANSIENT RESULTS FROM GENTILLY-1 STEAM-DRUM SIMULATION

(2) The model shows excellent agreement with experimental data obtained from a pressurizer during outsurge, but for an insurge the predicted pressures are higher than those observed experimentally. The difference is attributed to heat transfer from the superheated steam to the metal walls. Better agreement is expected for a steam drum because this heat transfer is a much smaller fraction of the energy content of the throughflowing steam.

(3) The severity of the steam-drum pressure rise following a turbine trip is predicted by the isentropic-compression model but not by the thermodynamic-equilibrium model.

(4) For small disturbances, the mathematical equations can be linearized individually and implemented on an analog computer so that a non-linear model is retained. The computer switches in the correct set of equations, depending on the thermodynamic state of the liquid and vapour phases.

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