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THE PION-NUCLEUS INTERACTION

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ABSTRACT

The latest developments in the construction of pion-nucleus optical potential are presented and a comparison with the latest data on $\pi + {}^{12}C$ is made. The suggested mechanisms for the (p,π) reaction are discussed with a comparison of the theoretical results with experiment.

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I. INTRODUCTION

Since the pion is the mediator of the nucleon-nucleon interaction, the study of nuclei with pion beams is in many ways similar to the study of atomic structure with photons. However, in the case of pions there are several added features not present in the atomic case: (i) The pion, with a mass of ~140 MeV, can be absorbed or produced on nuclei with a resultant high momentum transferred to the nucleus. For example for (p,π) reactions with 150 MeV proton, we get a momentum transfer of ~500 MeV/c[1]. Such high momentum transfer allows us to study the short range behaviour of the nuclear wave function. (ii) The pion, with isospin I = 1, can excite certain degrees of freedom commonly not excited by other projectiles. Thus, in double charge exchange (π^+, π^-), we can get to final nuclear states not accessible by other projectiles. For example, we might be able to identify $\Delta I_z = 2$ isobar analog states, and nuclei off the line of stability[2]. (iii) Although nuclear physics usually treats the nucleons as point particles, in actual fact, the nucleon can be excited both in free space and in the nuclear media. These excited states, which are pion-nucleon resonances (or isobar), have attracted considerable attention in recent years[3] as a means of reducing the discrepancy between theory and experiment in such observables as the binding energy, charge, and magnetic form factor of the deuteron and ^{3}He. If such components of the nuclear wave function (having nucleons replaced by their isobar) are important in the ground state then they should be vital in most reactions involving pions.

For the present talk I would like to concentrate my attention on two reactions involving pions which have their analogue in low energy nuclear physics. In this way I will be able to show how one can extract spectroscopic information from the reaction that is comparable to what we can get from other reactions such as those discussed by Ames in this summer school. In Sec. II, we consider pion-nucleus elastic scattering, with particular attention to the construction of the optical potential for pions. As has been the case with protons, deuterons, ..., most reactions which involve a pion + nucleus in either initial or final state, will require the knowledge of the pion distorted wave. Thus, before we can proceed to the more interesting reactions such as (π,p) or charge exchange scattering, we need to understand elastic scattering. We will show how the standard approximations to the optical potential, that give reasonable results at high energies, fail for pion energies below ~100 MeV, and possible reasons for this failure will be discussed. In particular, we will find that we need to include the Fermi motion of the nucleons, and that pion can get absorbed and re-emitted on nucleons in the nucleus.

In Sec. III, we discuss possibly the most interesting reactions involving pions i.e. (p,π) and (π,p). Here, we find the experimental evidence to favour a stripping mechanism similar to (d,p). However, the fact that both (p,π^+) and (p,π^-) reactions take place is an indication that the stripping mechanism is not the whole story, and more complicated reaction mechanism may be present. Furthermore, we find that in treating the (p,π^+) reaction as a stripping process we face the ambiguity of treating the $N\leftrightarrow N+\pi$ vertex in a non-relativistic scheme. Finally in Sec. IV, we present some concluding remarks.

II. PION-NUCLEUS ELASTIC SCATTERING

There are several approaches to the description of pion-nucleus scattering. In this lecture we will concentrate on the optical potential approach, since it is the one most familiar to the low energy nuclear physicist. Ideally we would like to describe π-A scattering as a two-body problem with the pion-nucleus potential derived from the basic π-N interaction. A similar program for nucleon-nucleus scattering (particularly at low energies) is doomed to failure, because of the richness of structure in the total cross section as a function of energy. This is partly due to the long range nature of the N-N interaction. In fact it is not until we get to relatively high energies >100 MeV that we can derive an optical potential for protons from a multiple scattering theory.[4] We hope that the short range nature of the π-N interaction and the fact that it is weak with respect to N-N interaction at low energies will enable us to achieve our aim of deriving the optical potential from multiple scattering theory.

Experimentally it is rather difficult to deal with pion beams of energy less than ~30 MeV for scattering experiments. This means we should treat the pion as relativistic, which creates some problems, as we are used to treating nucleons in nuclei using non-relativistic kinematics and dynamics. This historic precedence has led to a number of attempts at deriving the optical potential, or the multiple scattering series, by treating the pions as relativistic, while the nuclear structure is treated in the usual non-relativistic manner. This has introduced a number of problems that will be discussed in this talk.

Multiple Scattering Theory. The multiple scattering series was first developed by Watson[5], and later applied to nucleon-nucleus scattering by Kerman, McManus and Thaler[6] after some modifications. In the present talk I will derive the Watson multiple scattering series following the procedure of Tandy *et al*[7]. The Hamiltonian for pion-nucleus scattering is given by

$$H = K_0 + H_A + V, \tag{1}$$

with

$$V = \sum_{i=1}^{A} V_{0i}. \tag{2}$$

In Eq. (1) H_A is the target Hamiltonian, K_0 is the pion kinetic energy, and V_{0i} is the interaction of the pion with the i^{th} nucleon in the target. The amplitude for π-A scattering satisfies the many-body Lippmann-Schwinger equation

$$T(E) = V + V\, G_A\, T(E) \tag{3}$$

with

$$G_A(E) = (E^+ - K_0 - H_A)^{-1}. \tag{4}$$

Introducing the target ground state projection operator $P_A = |\phi_0{}^A\rangle\langle\phi_0{}^A|$ with $H_A\phi_0{}^A = \varepsilon_0\phi_0{}^A$ and $Q_A = 1-P_A$, we can reduce Eq. (3) to a two-body equation of the form

$$T(E) = U(E) + U(E)G_A(E)P_A T(E), \tag{5}$$

where the optical potential U(E) satisfies the equation

$$U(E) = V + VG_A(E)Q_A U(E). \tag{6}$$

Making use of the fact that V is a sum of A terms we can write

$$U(E) = \sum_{i=1}^{A} U_i(E) \tag{7}$$

where $U_i(E)$ satisfies the equation

$$U_i(E) = \tau_i(E) + \tau_i(E)G_A(E)Q_A \sum_{\substack{j=1 \\ j\neq i}}^{A} U_j(E) \tag{8}$$

with $\tau_i(E)$ the π-N amplitude in the nuclear medium satisfying the integral equation

$$\tau_i(E) = V_{0i} + V_{0i}G_A(E)Q_A\tau_i(E). \tag{9}$$

By substituting the interated version of Eq. (8) into Eq. (7) we get the standard Watson multiple scattering series for the optical potential[5]. We note that $\tau_i(E)$, as defined in equation (9), is an operator in the Hilbert space of the pi... and A nucleon because of the presence of $G_A(E)$ in Eq. (9).

So far we have not achieved our aim of writing the optical potential U(E) in terms of the free π-N amplitude t(ω) which satisfies the two-body equation

$$t_i(\omega) = V_{0i} + V_{0i}G_0(\omega)t_i(\omega) \tag{10}$$

with

$$G_0(\omega) = (\omega-K_0-K_i+i\epsilon)^{-1} \tag{11}$$

where K_i is the kinetic energy of the i^{th} nucleon. There are two ways of achieving this final link between U(E) and t(ω):

a) We can make use of the fact that we can write

$$\tau_i(E) = t_i(\omega) + t_i(\omega)[G_A(E)Q_A-G_0(\omega)]\tau_i(E) \tag{12}$$

and then adjust the parameter ω such that:

$$\tau_i(E) \approx t_i(\omega). \tag{13}$$

This is very similar to the reference spectrum method used in nuclear matter[8]. At high energies, we expect $G_A(E)Q_A \approx G_A(E)$, and the effect of the binding of the i^{th} nucleon to the rest of the core, to be small. In that case, Eq. (13)

might be a good approximation. However, at low energies, where the binding energy of the nucleon to the core may be important, we can make the approximation of describing the target nucleus as a particle plus a core. Which brings us to the second method of relating $\tau(E)$ with $t(\omega)$.

b) In the model where the target is a particle plus a core, $H_A \rightarrow (K_1+K_c+H_c+u)$, where u is the nucleon-core potential. In this case $\tau_i \approx \hat{\tau}_c$, where $\hat{\tau}_e$ satisfies the equation†

$$\hat{\tau}_e = v + v\hat{G}_e\hat{Q}_e\hat{\tau}_e \tag{14}$$

where $\hat{G}_c(E) = (E^+-K_0-K_1-K_c-H_c-u)^{-1}$, $v = V_{0i}$, and $\hat{Q}_e$ is the corresponding projection operator to Q_A in the three-particle Hilbert space. Following Tandy *et al*[7] we can set Eq. (14) into a set of Faddeev equations with no interaction between the core and the pion, and of the form

$$\hat{\tau}_e = \hat{t}_p\hat{G}_0\hat{\tau}_p \tag{15a}$$

$$\hat{\tau}_p = (E-H_0) + [\hat{t}_e\hat{G}_0-\hat{G}_0^{-1}P_e\hat{G}_0]\tau_e \tag{16b}$$

where

$$\hat{G}_0(E) = (E-H_0)^{-1} = (E-K_0-K_1-K_c-H_c)^{-1}, \tag{16}$$

and $\hat{t}_c(E)$, $\hat{t}_p(E)$ the π-N and N-C amplitudes in the three-body Hilbert space defined by the equations

$$\hat{t}_c(E) = u + u\hat{G}_0(E)\hat{t}_e(E) \tag{17a}$$

$$\hat{t}_p(E) = v + v\hat{G}_0(E)\hat{t}_p(E) \tag{17b}$$

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† Operators defined in the model three-body space (pion, nucleon, and core) are distinguished by a hat, e.g. $\hat{G}_e$.

In this three-body model, the lowest order approximation to $\hat{\tau}_e$ is

$$\hat{\tau}_e(E) = \hat{t}_p(E) = t(\omega) \qquad (18)$$

In this case the energy variable ω in the π-N amplitude is that dictated by the three-body kinematics. Thus, in lowest order, both approximations are equivalent, except for the choice of energy variable in the off-shell π-N amplitude.

Before we proceed to a comparison of experiment with theory, we should examine what we have neglected, by taking for the optical potential

$$U(E) \simeq \sum_{i=1}^{A} t_i(\omega). \qquad (19)$$

From Eq. (8) we see that by taking $U_i \simeq \tau_i$, we are neglecting intermediate states in which the target nucleus is in an excited state. On the other hand, taking $\tau_i = t_i$ involves replacing the π-N amplitude in the medium by the free π-N amplitude. This not only ignores the fact that the nucleon in intermediate states can interact with the rest of the nucleons in the core, which is partly taken care of in the three-body approximation, but there is the effect of the Pauli principle. This latter part has two components; the first, is when we have an intermediate π-N state the nucleon should be antisymmetrized with the core particles. In other words, we need to calculate the Brueckner G-matrix rather than the usual T-matrix. This in principle is included in Eq. (9) if we write Q_A in terms of antisymmetric states. The second Pauli effect which is not included in the above formulation and is unique to pion scattering is due to the fact that $\pi+N \leftrightarrow N$, and thus the π-N amplitude should in effect have intermediate states of bare nucleons with no pions, and that nucleon should be antisymmetrized with the core nucleons.

Optical Potential for Pions in the Impulse Approximation. To lowest order the pion-nucleus optical potential is given by

$$U(E) = \sum_{i=1}^{A} t_i(\omega) \tag{20}$$

where ω can be chosen either to satisfy Eq. (13), or as the total energy of the system minus the kinetic energy of the core plus the c.m. kinetic energy of the π-N pair. This latter choice corresponds to that dictated by the three-body approximation. Diagramatically, Eq. (20) is given in Fig. 1 and is sometimes referred to as the Rayleigh-Lax potential[9]. This potential is to be used in a two-body Lippmann-Schwinger equation which we get from Eq. (5) by taking matrix elements of the operator with respect to the asymptotic π-A states $|k,\phi_0{}^A\rangle$. The resultant two-body equation is of the form

$$\langle\vec{k}'|T(E)|\vec{k}\rangle = \langle\vec{k}'|U(E)|\vec{k}\rangle + \int d^3k'' \frac{\langle\vec{k}'|U(E)|\vec{k}''\rangle\langle\vec{k}''|T(E)|\vec{k}\rangle}{E-\varepsilon_\pi(k'')-\varepsilon_A(k'')+i\varepsilon}, \tag{21}$$

where $\varepsilon_\pi(k'')$ and $\varepsilon_A(k'')$ are the pion and target nucleus energies in intermediate states. In most calculations $\varepsilon_{\pi,A}(k) = \sqrt{k^2+m^2_{\pi,A}}$, i.e. we take the relativistic expressions for the pion energy as well as the nucleus energy. The potential $\langle\vec{k}'|U(E)|\vec{k}\rangle$ in the approximation of Eq. (20) is given as

$$\langle\vec{k}'|U(E)|\vec{k}\rangle = \langle\vec{k}',\phi_0{}^A|\sum_{i=1}^{A} t_i(\omega)|\vec{k},\phi_0{}^A\rangle$$

$$= \sum_n \int d^3p_3\, \rho_n(\vec{q}_1{}',\vec{q}_1)\langle\vec{q}_3{}'|t(\omega)|\vec{q}_3\rangle \tag{22}$$

where $\rho_n(\vec{q}_1{}',\vec{q}_1)$ is the density matrix and is given by

$$\rho_n(\vec{q}_1{}',\vec{q}_1) = \psi_n^*(q_1') \; \psi_n(q_1) \tag{23}$$

where ψ_n is the ... core wave function or in many-body terms is given by $\langle\phi_n^{(A-1)}|a_{q_1}|\phi_0^{(A)}\rangle$ where ... is the nucleon destruction operator. Thus the ... potential ... related to ... single particle spec...oscopic factors. The momenta $\vec{q}_1$ and q_3 are related to the core momentum p_3 and the final pion momentum k by the relations (see Fig. 1)

$$\vec{q}_1 = \vec{p}_3 - \frac{(A-1)m_\pi}{m_\pi+m_N}\vec{k} \quad \text{and} \quad \vec{q}_3 = \vec{k} + \frac{m_\pi}{Am_N}\vec{p}_3 \tag{24}$$

with similar relations for $\vec{q}_1'$ and $\vec{q}_3'$ in terms of $\vec{k}'$ and $\vec{p}_3$. We now have to decide on our choice of ω. If we take the three-body model, then ω is given by

$$\omega = \frac{k^2}{2\mu_1} - |\epsilon_n| - p_3^2/2\mu_3 \tag{25}$$

with

$$\mu_1 = Am_\pi m_N/(m_\pi+Am_N)$$

$$\mu_3 = (A-1)m_N(m_N+m_\pi)/(m_\pi+Am_N), \tag{26}$$

where the target and core masses are taken as Am_N and $(A-1)m_N$ respectively. From Eq. (25) we see that the energy variable in the π-N amplitude depends not only on ϵ_n, the binding of the nucleon to the core, but also on p_3 the momentum of the core. This means we need to know the π-N t-matrix off-shell and for all energies $\omega < \frac{k^2}{2\mu_1} - |\epsilon_n|$.

... ... of the π-N amplitude with respect to the density matrix Eq. (22) is relatively complicated and we would like, if possible, to simplify Eq. (..) still further. This is achieved by realising that ψ_n is to a good approximation ... and independent of n in which case[10]

$$\langle\vec{k}'|U(E)|\vec{k}\rangle \cong A\ F(K) \int d^3p|\psi(p)|^2\langle\vec{k},\vec{p}+\vec{p}_0|t(\omega)|\vec{k},\vec{p}+\vec{p}_0-\vec{K}\rangle \qquad (27)$$

where $\vec{K} = \vec{k}' - \vec{k}$ and $\vec{p}_0 = -\frac{\vec{k}}{A} + \frac{A-1}{2A}\vec{K}$. In Eq. (27), $F(K)$ is the average nuclear form factor. Taking into consideration the fact that $|\psi(p)|^2$ is peaked at p=0, we can further simplify this result to get

$$\langle\vec{k}'|U(E)|\vec{k}\rangle \cong A\ F(K)\ \langle\vec{k},\vec{p}_0|t(\omega_0)|\vec{k},\vec{p}_0-\vec{K}\rangle. \qquad (28)$$

In going from Eq. (27) to Eq. (28) we have taken the nucleon to have zero momentum. In other words, Eq. (28) is the static approximation to Eq. (27). This approximation is commonly referred to as the static or factored approximation

In Fig. (2) we present the results of Kujawski and Aitken[11] who used both eqs. (22) (solid line) and (28) (dashed line) to calculate π^--^{12}C elastic cross section at 120 MeV. It is clear from the results, that except in the forward direction, the two theoretical approximations are quite different and both are in disagreement with experiment. However, at such an energy we should be treating the pions as relativistic. This can be achieved in the factored approximation by taking the relativistic definition of ω_0 as the sum of the pion and nucleon energy in the π-A c.m. We then have to relate the amplitude in the π-A c.m. to the "experimentally" known π-N amplitude in the π-N c.m. This is achieved through the transformation[10]

$$\langle\vec{k},\vec{p}_0|t(\omega_0)|\vec{k}',\vec{p}_0-\vec{K}\rangle = \gamma\ \langle\vec{\kappa}|t(\tilde{\omega}_0)|\vec{\kappa}'\rangle \qquad (29)$$

where κ and κ' are the π-N relative momentum in the π-N c.m. system and

$$\gamma = \left\{\frac{E_\pi(\vec{\kappa})E_\pi(\vec{\kappa}')E_N(\vec{\kappa})E_N(\vec{\kappa}')}{E_\pi(\vec{k})E_\pi(\vec{k}')E_N(\vec{p}_0)E_N(\vec{p}_0-\vec{K})}\right\}^{1/2} \qquad (30)$$

where $\omega_\pi(q) = (q^2+m_\pi^2)^{1/2}$ and $E_N(q) = (q^2+m_N^2)^{1/2}$. In Eq. (29) we have a generalization of an on-shell relation for constructing a Lorentz invariant amplitude from the non-relativistic T-matrix. To completely define the relation between the amplitudes in the π-N and π-A c.m., we need to establish relationship between the π-N c.m. variables $\{|\vec{\kappa}|,|\vec{\kappa}'|,\hat{\kappa}.\hat{\kappa}',\tilde{\omega}_0\}$ and the π-A c.m. variables $(\vec{k},\vec{k}',\omega_0)$. This is achieved by making use of the Lorentz invariance of the Mandelstam variables $s = (p_\pi+p_N)^2$ and $t = (p_\pi-p'_\pi)^2$. Unfortunately this procedure leads to violation of energy conservation for ... π-N T-matrix, and $\cos(\hat{\kappa}.\hat{\kappa}')$ can take unphysical values >1.0. Both of these physically unacceptable conditions are the result of using a combination of relativistic and non-relativistic kinematics[12].

With all the above approximations to the optical potential the resultant fit to the data is quite good at high energies, as illustrated in Fig. 3 for 120 MeV π-^{12}C scattering[10,13], where the further approximation of taking $\vec{p}_0 = -\vec{k}/A$ in Eq. (27) was employed[13]. This result is surprising, considering the difference found by Kujawski and Aitken[11] on comparing the results using the potential in Eq. (22) and the corresponding factored approximation Eq. (28). Unfortunately, this success of the optical potential fails again on going to pion energies of 50 MeV as illustrated in Fig. 4[11]. If we attempt to fit the π-^{12}C data by adjusting the π-N amplitude, we find that such a fit (see Fig. 5) gives rise to a π-N amplitude which has far more absorption than the π-N data dictates. This means that in the factored approximation we are neglecting some contributions to the imaginary part of the potential. These missing contributions are partly due to processes such as (πNN→NN) as we will see, and partly due to the factored approximation. An alternative method of improving the agreement between experiment and theory at low energy is by proper choice of the energy variable ω_0 in Eq. (29). This was first illustrated by Landau and Thomas[16] who used a three-body definition of the energy i.e.

$$\tilde{\omega}_0 = E_\pi(k) + \frac{k^2}{2Am_N} + m_N - |\varepsilon_n| - p_3{}^2/2m_3. \qquad (31)$$

This is basically the same as Eq. (25), the only change being the pion energy is treated relativistic. They found (see Fig. 6) that they could fit both the ^{4}He and ^{12}C data with $|\varepsilon_n| = 5$ MeV which is too small if interpreted as the average separation energy of a nucleon from the core.

More recently a covariant calculation of the diagram in Fig. 1 has been performed by Liu and Shakin[17]. In their calculation they have taken the target nucleus and the core to be on-mass-shell. This reduces the dimensionality of the integrals both in the covariant two-body integral equation for π-A scattering, and the determination of the potential from Fig. to three-dimensions. By performing a totally covariant calculation they avoid all problems associated with treating the pion relativistic and the nucleon non-relativistic. However, in this case their single nucleon wave function or the nucleon-core wave function $\psi_n(\vec{q}_1)$ has to come from a relativistic nuclear structure calculation. Their results for $\pi^+ + {}^{12}$C are presented in Fig. 7 (dashed line), the dotted line is the result of a covariant factored approximation. We note that by performing the integral over the Fermi motion they lowered the magnitude of the cross section and moved the minimum to smaller angles which is in closer agreement with experiment. However, we find that the discrepancy between theory and experiment is still appreciable.

We know from the analysis of pionic atoms[18], that we need to include "true" absorption (i.e. the pion gets absorbed on two nucleons) to fit the experimental data. In the simplest possible approximation this absorption is proportional to the nuclear density square. Such a term was added by Liu and Shakin[17] to their covariant potential, with the strength of "true" absorption adjusted to fit the data. Their best fit is given in Fig. 7 by the solid line, and it is clear that with this additional absorption we can achieve a reasonable agreement with experiment.

A more extensive study of the importance of this absorption with energy has been done at TRIUMF[19], where they have taken π^+-^{12}C data at 30, 40, and 50 MeV and π^+-^4He at 24, 50, 60, 75 MeV. They find that the importance of "true" absorption decreases as one increases the incident pion energy. This is illustrated in Fig. 8 where some preliminary results for π^+-^{12}C are presented. The theoretical results are based on the approximation of Landau and Thomas[16] with an additional absorptive term which depends on the density square. The energy dependence of the "true" absorption is taken from the analysis of Spuller and Measday[20] for $pp \to \pi^+ d$, while the strength at low energy is adjusted to the results of Krell and Ericson[18] for pionic atoms. From the results in Fig. 8 we see that the fit to the cross section at the three energies is reasonable provided we include "true" absorption.

Thus we see by taking the lowest order in the multiple scattering series, and the inclusion of "true" absorption we may soon be able to construct an optical potential that will describe pion-nucleus scattering at all energies. Of course, to describe both low and high energy pion elastic scattering, we need a relativistic theory for the optical potential. There are at present two possible formulations; one is the covariant approach[17] which has been applied to pion scattering, and the second involves the use of Hamiltonian relativistic quantum mechanics[21]. Another source of ... have not discussed is the choice of π-N amplitude and its ... The most common choice is a separable potential which is used in most of the results we have discussed. The other approach is based on the Low-Chew theory and has the advantage that may incorporate crossing ... the π-N amplitude as well as the effect of Pauli principle.

III. PION PRODUCTION IN NUCLEI.

Because of the pion mass (~140 MeV), we expect pion production (absorption), due to the large momentum transfer >2.5 fm^{-1}, to give us considerable information on short range behaviour of the nuclear wave function. The kind of information we will eventually get depends to a large extent on the reaction mechanism for this process. Also, since the reaction mechanism may depend on the energy of the incident projectile, we might get different nuclear structure information at different energies. In the present section, I would like to discuss the possible reaction mechanism, the kind of information they give and some of the problems involved.

Before we can establish the mechanism that dominates a given reaction, we should examine the general features of the experimental data. At present there is some very good data on (p,π) reaction from Uppsala[22-24], and more data is coming from the new meson facility using higher energy protons. In Fig. 9 we present the Uppsala angular distribution for $^{13}C(p,\pi^+)$ ... and $^{13}C(p,\pi^-)^{14}O$ with 185 MeV protons. These results have two distinct features: (i) the cross section for π^+ production is peaked in the forward d... while the π^- cross section is almost isotropic. ...nce th... in... ...

basically the same structure, the mechanism for π^+ production is different from that of π^- production. (ii) The cross section for π^+ production is much larger than that of the π^-. The same situation is present in $^9Be(p,\pi^{\pm})$ as seen in Fig. 10. The fact that the π^+ is forward peaked, means that the reaction mechanism is the old stripping mechanism, as depicted in Fig. 11, with the corresponding amplitude given by

$$M \sim \langle k,(A-1), N|G_0(E)|k',A\rangle$$

$$= \langle N|\vec{q}\rangle \left\{E - k^2\left(\frac{m_\pi + Am_N}{2m_\pi Am_N}\right) - \frac{A}{A-1}\frac{q'^2}{2m_N}\right\}^{-1} \langle \vec{q}' \ldots \rangle$$

$$= \langle N|\vec{q}\rangle\, \psi(\vec{q}') \qquad (32)$$

where $\langle N|\vec{q}\rangle$ is the vertex function for $N \to N + \pi$ and $\psi(\vec{q}')$ is the wave function of the exchange nucleon with respect to the target. Thus if we know $\langle N|\vec{q}\rangle$, and the single nucleon mechanism is dominant, then we can measure $\psi(\vec{q}') \propto \langle \phi_0^{(A-1)}| \ldots |\phi_0^{(A)}\rangle$ and then measure the single particle spectroscopic factor. At threshold, i.e. $k' = 0$, $q = k \simeq 2.5\ fm^{-1}$. In practice, the amplitude in Eq. (32) gives the wrong results by orders of magnitude, and we need to introduce distortion in the initial and final state to get anything close to the experimental result.

The fact that (p,π^-) cross section is not zero indicates that proton stripping as described above, is not the full story. However, the fact that we need distortion to fit the experimental data, also implies that π^- production is possible through charge exchange scattering in the final state, as illustrated in Fig. 12. Experimentally, we know that the charge exchange cross section is down by a factor of ~200 from the elastic cross section. This implies that (p,π^-) cross section, using single nucleon exchange plus dist... ...nal state (Fig. 12), should ... by ... is a little too large compared to the experiment... processes that may be considered single nucleon mechanism, ... π^- production, are given in Fig. 13, where Fig. 13a is configuration mixing[2], and Fig. 13b is Δ^{++} exchange[25]. In the latter process the contribution is significant if the final nucleus has a reasonable comp...t of its wave function with one nucleon in the Δ^{++} state.

How does theory based on single nucleon exchange compare with experiment? If we consider the Uppsala experiments with 185 MeV proton, then the final pion is low energy, and our knowledge of the pion optical potential at these energies is not very good. In other words, we do not expect very ... With this note of warning, we present in Fig. 14 a comparison of theory[26] and experiment for $^{12}C(p,\pi^+)^{13}C$ for several pion optical potentials. Although the overall agreement looks good there are some major sources of discrepancy. If we consider these as an indication for the failure of the single nucleon mechanism, we should recall that there are sources of ambiguity that could explain some of these discrepancies. The most obvious of these ambiguities are: (i) the choice of $N\to N\pi$ vertex. If we use a covariant formulation then we have a choice of two possible interactions, the pseudoscalar (γ^5) or pseudovector ($\gamma^\mu\gamma^5$). However, if we treat the reaction in a non-relativistic scheme, then ... must reduce these vertices through a Foldy-Wouthuysen transformation, which unfortunately is not unique[27-30]. In most calculations, the $N\to N\pi$ vertex is represented by either the static approximation (i.e. ... or the Galilean-invariant model, (i.e. $H'_{NN\pi} \sim \vec{\sigma}.\left[\vec{\nabla}_\pi + \frac{\omega_\pi}{2E_N}(\vec{\nabla}_N - ...)\right]$). ...ly, it has been suggested that if (p,π^+) reactions at more than one energy are examined, the static approximation is favoured[31]. (ii) While ... reacti...

[illegible] range of momentum transfer, the forward angles are dominated by [illegible] momentum transfer i.e. $\psi(q')$ for small q'. On the other hand, (p,π) [illegible] essentially starts where (d,p) stripping leaves. We therefore need [illegible] for large $q' \gtrsim 2\ \mathrm{fm}^{-1}$. Although this might be the source of [illegible] between theory and experiment, it is the kind of nuclear [illegible] information we hope to extract from (p,π) reactions.

[illegible] turn to the two nucleon mechanism which is represented in Fig. 15. [illegible] for this diagram is given for $d(p,\pi)T$ by

$$\langle k, d|G_0\, t_{NN\to\pi NN}\, G_0|k'T\rangle$$

$$\sim \int d^3Q\, d^3 \ldots\, \psi(\vec{Q})\,\Psi(\vec{Q}', \vec{P})\, \langle \vec{p}|t_{NN\to\pi NN}|\vec{p}'\vec{P}'\rangle, \tag{33}$$

[illegible] relative momentum of the two nucleons in d^*. To calculate [illegible] we need to know the amplitude for $NN\to\pi NN$ off-shell. This can be derived from one of several models available for describing pion production [illegible] scattering[32-34] or by parametrizing the amplitude to fit the data for [illegible][28]. The wave function $\psi(\vec{Q})$ is that used in the definition of the [illegible] Eq. (22), while $\Psi(\vec{Q}',\vec{P})$ is the wave function of two nucleons relative to a core of (A-2) nucleons. If this mechanism is dominant, then we [illegible] about short range two-body correlation through the $\vec{P}$ dependence of Ψ [illegible]. In Fig. 16 we present a typical result[35] for the two-[illegible] where the $NN\to NN\pi$ amplitude is taken to be that in Fig. 17. The [illegible] this case is not bad considering the fact that single nucleon [illegible] give a much better result.

[illegible] the two reaction mechanisms seem quite different in certain [illegible] relate the two mechanisms. This is particularly true near the (3,3) π-N resonance[36].

From the above results we see that experimentally we are at a stage where that one can measure both (p,π⁺) and (p,π⁻) differential cross sections not only to the ground state but excited states of the final nucleus. The next step is to study the energy dependence of the cross section as we go through the (3,3) resonance. This may not only give us information on the optimum choice of N→Nπ vertex, but may give more insight to the reaction mechanism. In the meantime, the theory has to be improved by removing the sources of uncertainty such as the pion optical potential, the N→Nπ vertex, and finally determine the short range behaviour of the single particle wave function for large q.

IV. CONCLUSION

In conclusion, I would like to state some of the main obstacles to a better understanding of both π-A elastic scattering and pion production. (1) It is clear that we can describe pion-nucleus scattering at all energies using optical potentials. We have gone a long way towards achieving our aim of deriving an optical potential in terms of the basic π-N amplitude and the amplitude for NN→NN. Most of the kinematic difficulties can be removed by performing completely relativistic calculations i.e. treating both pions and nucleons relativistically. There is already some nuclear structure results based on Hartree-Fock calculations using the Dirac equations[37]. These can be combined with the available relativistic formulation of scattering theory[17,20], to give us totally relativistic calculation of the pion optical potential. More recently there have been some covariant (p,π) calculations without distortion with the hope of resolving some of the ambiguities associated with the N→πN vertex[38].

(ii) To get any nuclear structure information we need to understand both the elastic π-N amplitude and the production amplitude for NN→πNN, at least at a phenomenological level, as both of these amplitudes are the input to both π-A elastic scattering and pion production in nuclei.

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FIGURE CAPTIONS

Fig. 1. Diagramatic representation of the impulse approximation to the pion-nucleus optical potential.

Fig. 2. Differential cross section for π^{-}-^{12}C at 120 MeV. • experimental points (Ref. 39); —— results of three-body model (Eq. (22)); X results of factored approximation Eq. (28); ---- results for $\omega = q_3^2/2\mu_{12}$ with $\mu_{12} = m_\pi m_N/(m_\pi+m_N)$. This results in the use of the half-off-shell π-N T-matrix in Eq. (22). [Results from Ref. 11.]

Fig. 3. Differential cross section for π^{-}-^{12}C at 120 MeV using the factored approximation, relativistic kinematics for the pion, and different π-N amplitudes; —— Chew Model; ····· Kisslinger Model; ---- Landau-Tabakin. [Results from Ref. 13.]

Fig. 4. Differential cross section for π^{+}-^{12}C at 50 MeV. Experimental points of Ref. 14. Theory for different π-N amplitudes; —— separable potential of Londergan, McVoy and Moniz (Ref. 40), ---- Local Laplacian potential (Ref. 41); Kisslinger potential (Ref. 41). [Results from Ref. 14.]

Fig. 5. Differential cross section for π^{+}-^{12}C at 50 MeV. Best fit by adjusting parameters of π-N potential; Kisslinger potential (----); Local Laplacian (——). [Results from Ref. 14,15.]

Fig. 6. Differential cross section for π^{+}-^{12}C and π^{+}-^{4}He at 50 and 51 MeV respectively. The optimal energy $\omega_0^2 = (p_\pi+p_N)^2$ is used with two set of π-N phases and the three-body choice of energy in Eq. (31). [Results from Ref. 16.]

Fig. 7. Comparison of data for π^{+}-^{12}C differential cross section at 50 MeV with covariant optical potential results: ..... covariant factored approximation; ---- covariant calculation of diagram in Fig. 1;

—— covariant calculation of Fig. 1 plus an absorptive term depending on the square of nuclear density. [Results from Ref. 17.]

Fig. 8. Comparison of π^+-^{12}C differential cross section at 30, 40 and 50 MeV with optical potential of Landau and Thomas Ref. 16. ---- No "true" absorption; —— with absorption which depends on nuclear density square. The experimental points (|) are TRIUMF (preliminary) (Ref. 19) (ϕ) Marchal *et al* (Ref. 43); (¦) Amann *et al* Ref. 14; (ϕ) Kane *et al* (Ref. 44).

Fig. 9. Differential cross section for $^{13}C(p,\pi^+)^{14}C$ and $^{13}C(p,\pi^-)^{14}O$ with 185 MeV protons. [Results from Ref. 23.]

Fig. 10. Differential cross section for $^{9}Be(p,\pi^+)^{10}Be$ and $^{9}Be(p,\pi^-)^{10}C$ with 185 MeV protons. [Results from Ref. 23.]

Fig. 11. Diagramatic representation of the one nucleon mechanism.

Fig. 12. One nucleon mechanism plus charge exchange scattering in final state.

Fig. 13 (a) One nucleon mechanism in which the excited nucleus emits a π^-.
(b) One nucleon mechanism - π^- production through the exchange of Δ^{++}.

Fig. 14. Comparison of the differential cross section for pion production to three final states, with theoretical results with pion optical potential based on different π-N interaction. —— the π-N interaction of Ref. 40; – – – – the π-N interaction of Ref. 40 modified to improve the fit to the elastic π^+-^{12}C data; ---- the π-N interaction of Ref. 42. [Results from Ref. 26.]

Fig. 15. Diagramatic representation of the two nucleon mechanism.

Fig. 16. Differential cross section for $^{12}C(p,\pi^+)^{13}C$ using the two nucleon mechanism. The input $NN \to NN\pi$ amplitude is based on the diagram in Fig. 17. [Results from Ref. 35.]

Fig. 17. Diagramatic representation of the model amplitude for $NN \to NN\pi$ used in calculating the cross section for pion production in the two nucleon mechanism.

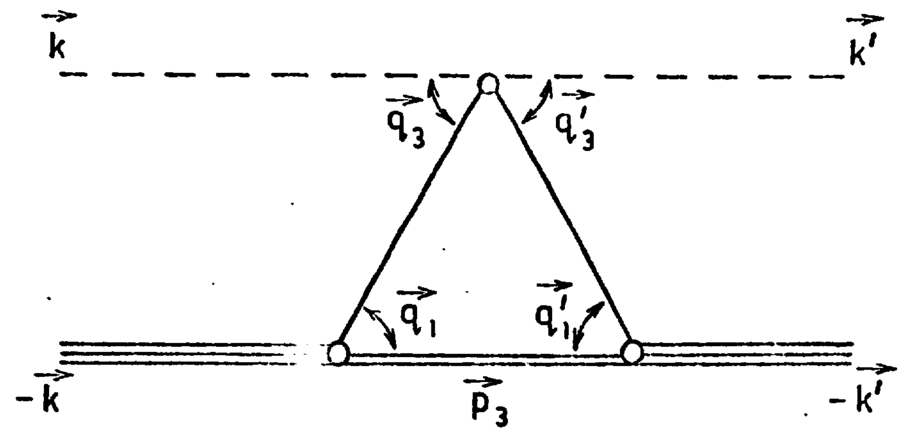
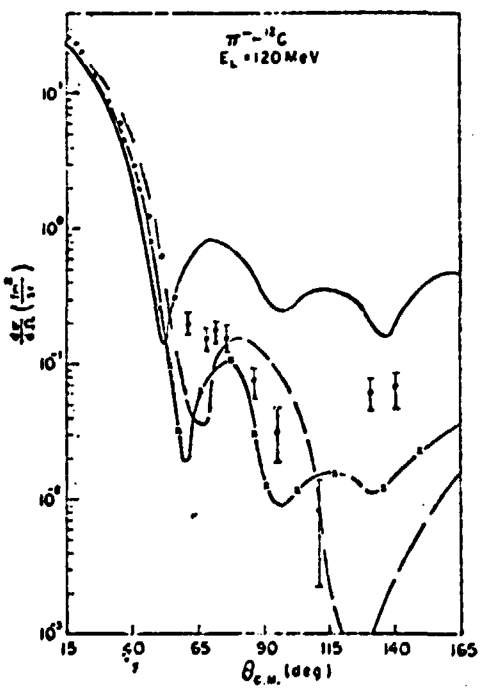


Fig. 1



$\pi^- - {}^{12}C$
$E_L = 120$ MeV
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
$\frac{d\sigma}{d\Omega}\left(\frac{fm^2}{sr}\right)$
15
40
65
90
115
140
165
$\theta_{c.m.}$ (deg)

Fig. 2

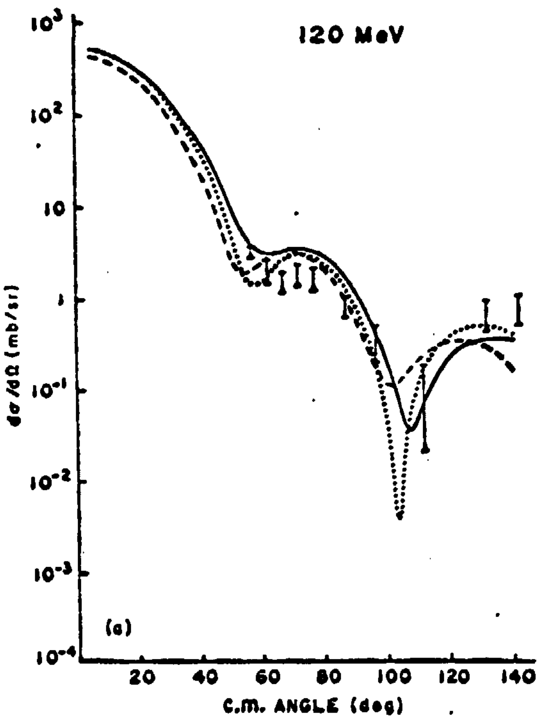

120 MeV
dσ/dΩ (mb/sr)
C.M. ANGLE (deg)
(a)
10³
10²
10
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
20
40
60
80
100
120
140

Fig 3

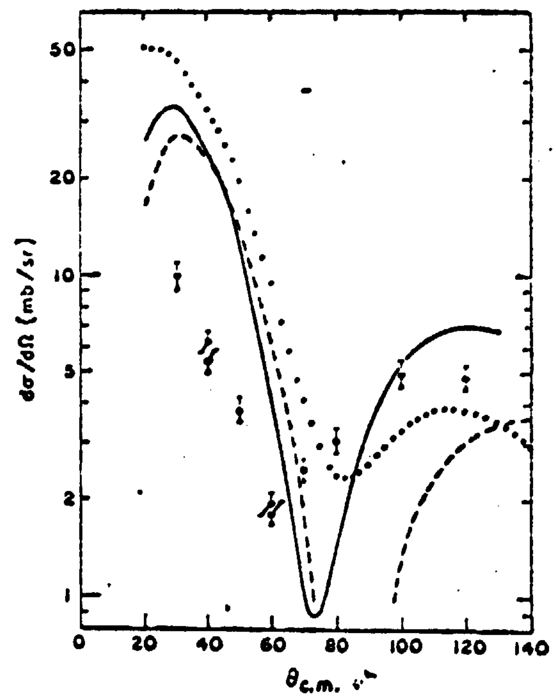


Fig. 4

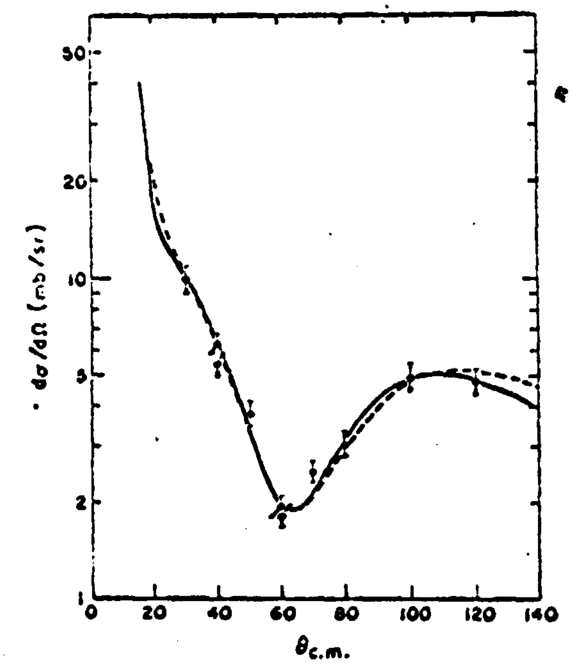
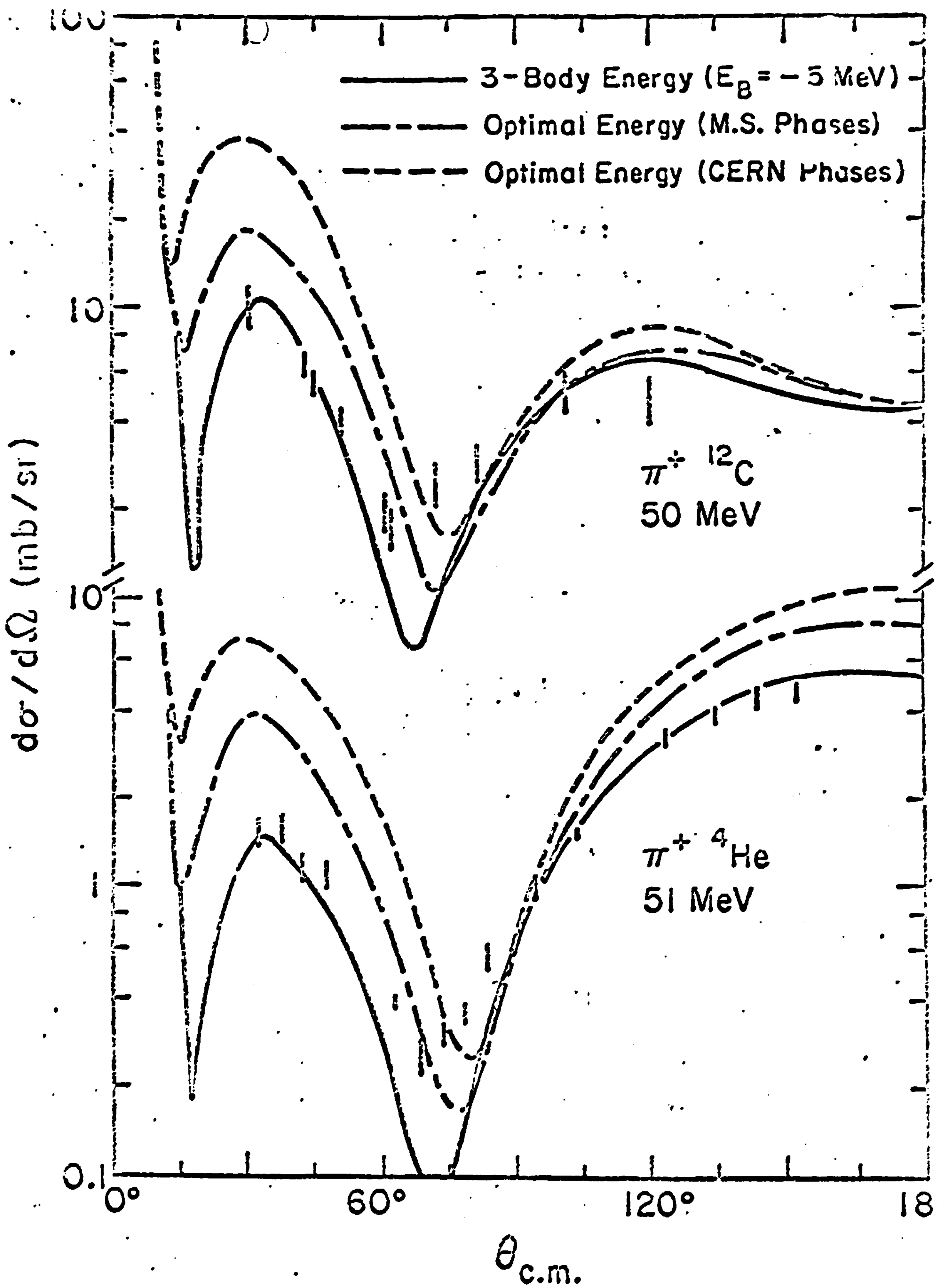


Fig 5



3-Body Energy (E_B = −5 MeV)
Optimal Energy (M.S. Phases)
Optimal Energy (CERN Phases)
π^+ ^{12}C
50 MeV
π^+ ^4He
51 MeV
$d\sigma/d\Omega$ (mb/sr)
100
10
10
1
0.1
0°
60°
120°
180
$\theta_{c.m.}$

Fig. 6

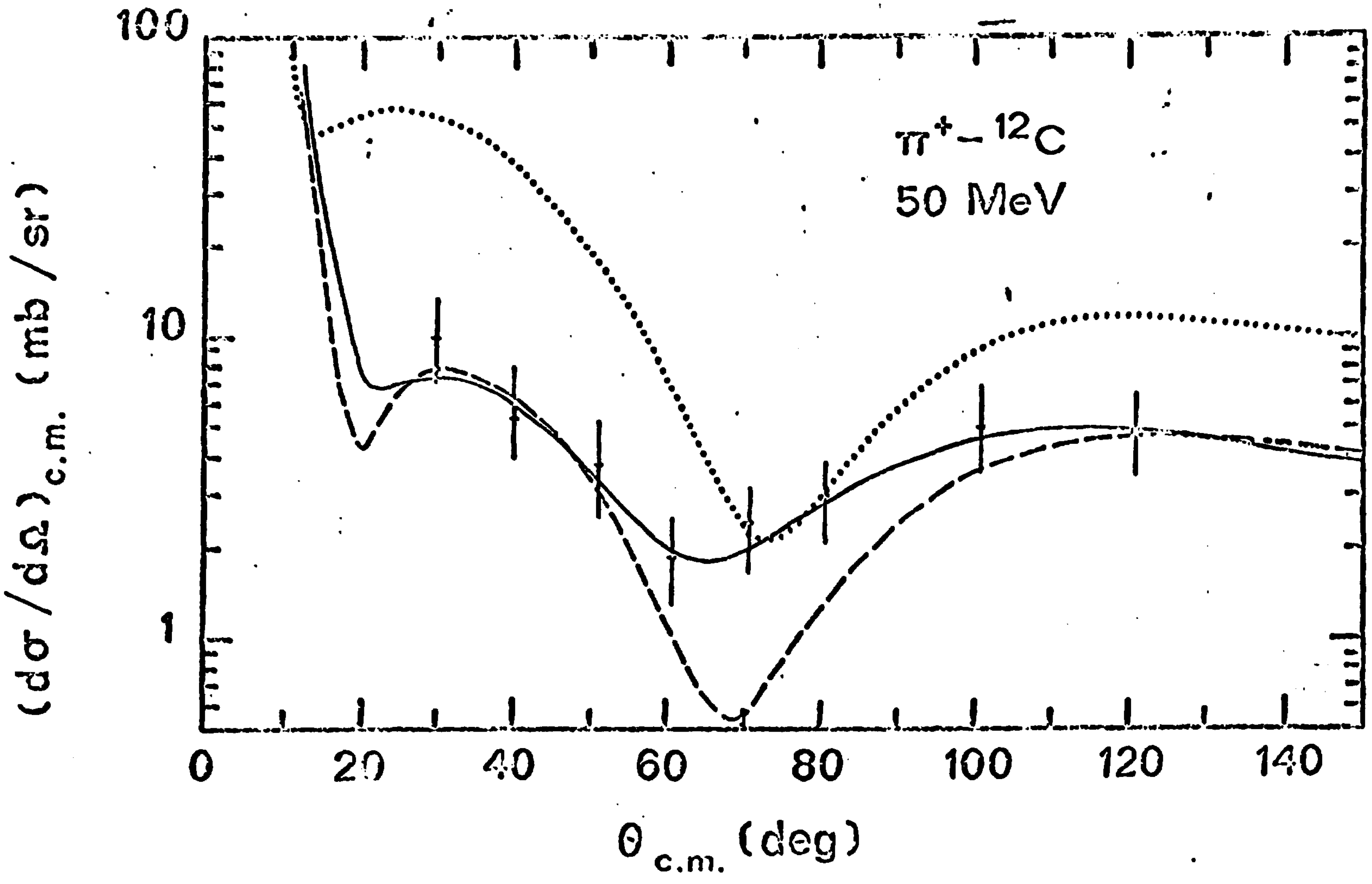
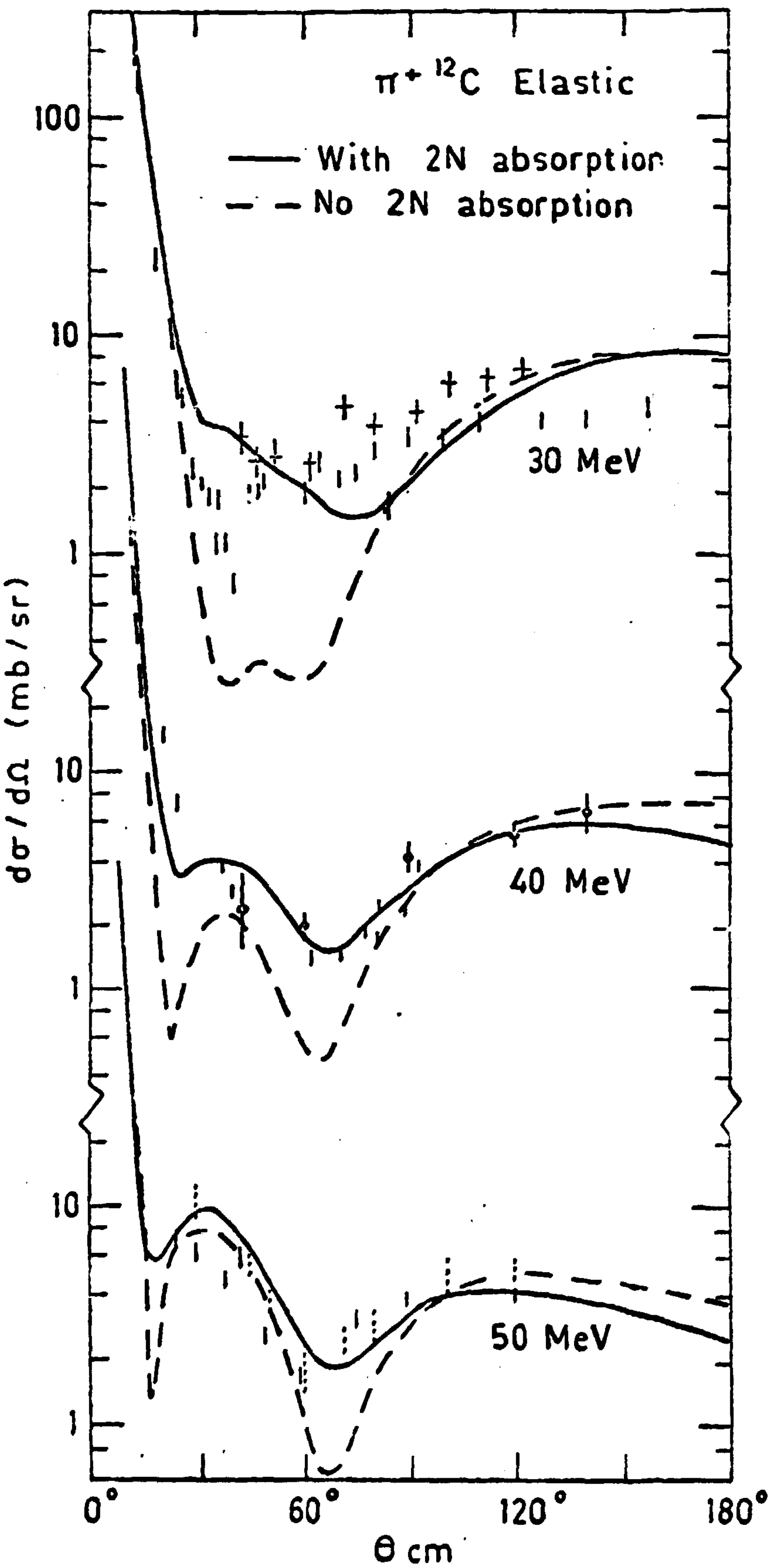
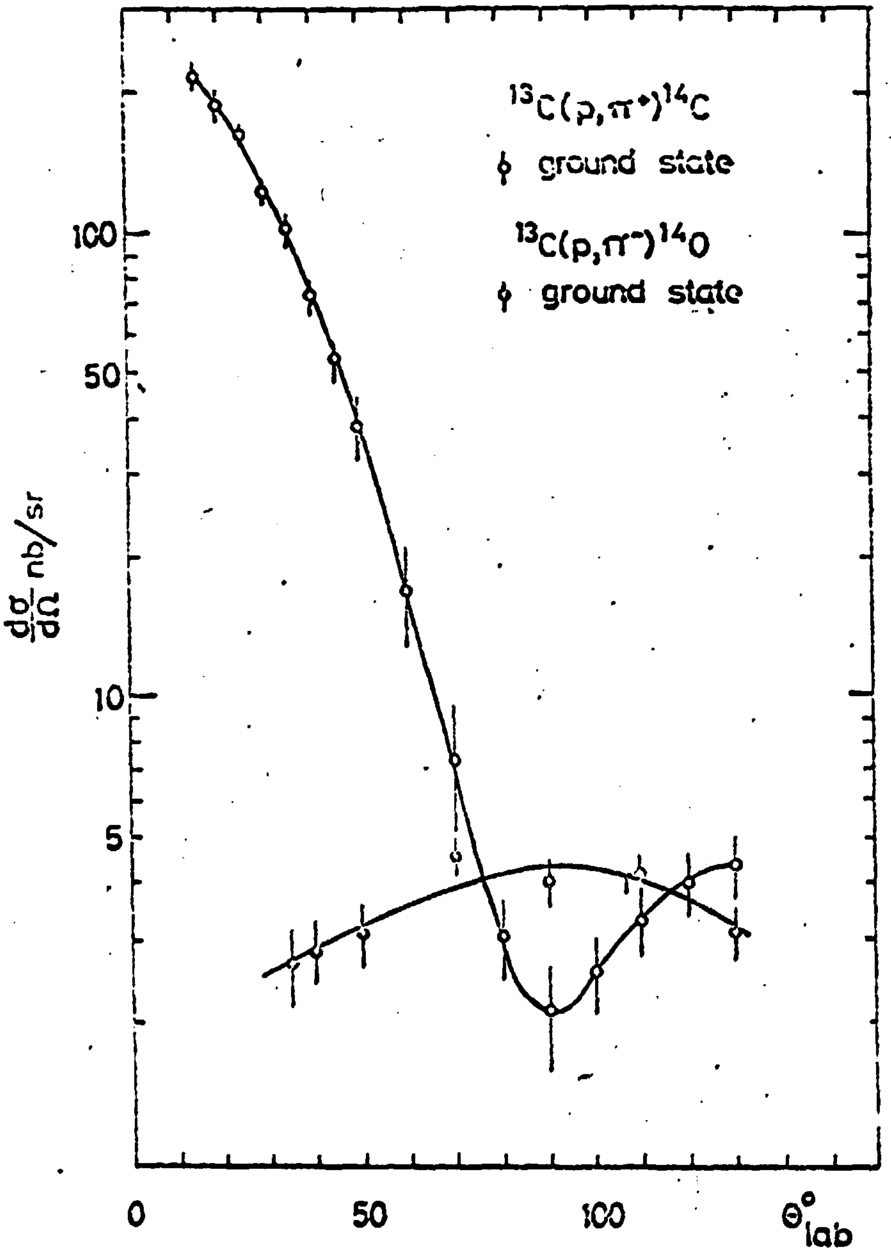

$\pi^+ - {}^{12}C$
50 MeV
$(d\sigma/d\Omega)_{c.m.}$ (mb / sr)
$\theta_{c.m.}$ (deg)
100
10
1
0
20
40
60
80
100
120
140

Fig. 7



π+ 12C Elastic
With 2N absorption
No 2N absorption
100
10
1
10
1
10
1
30 MeV
40 MeV
50 MeV
dσ / dΩ (mb / sr)
0°
60°
120°
180°
θ cm

Fig.8



$^{13}C(p,\pi^+)^{14}C$
ground state
$^{13}C(p,\pi^-)^{14}O$
ground state
$\frac{d\sigma}{d\Omega}$ nb/sr
100
50
10
5
0
50
100
Θ^o_{lab}

Fig 9

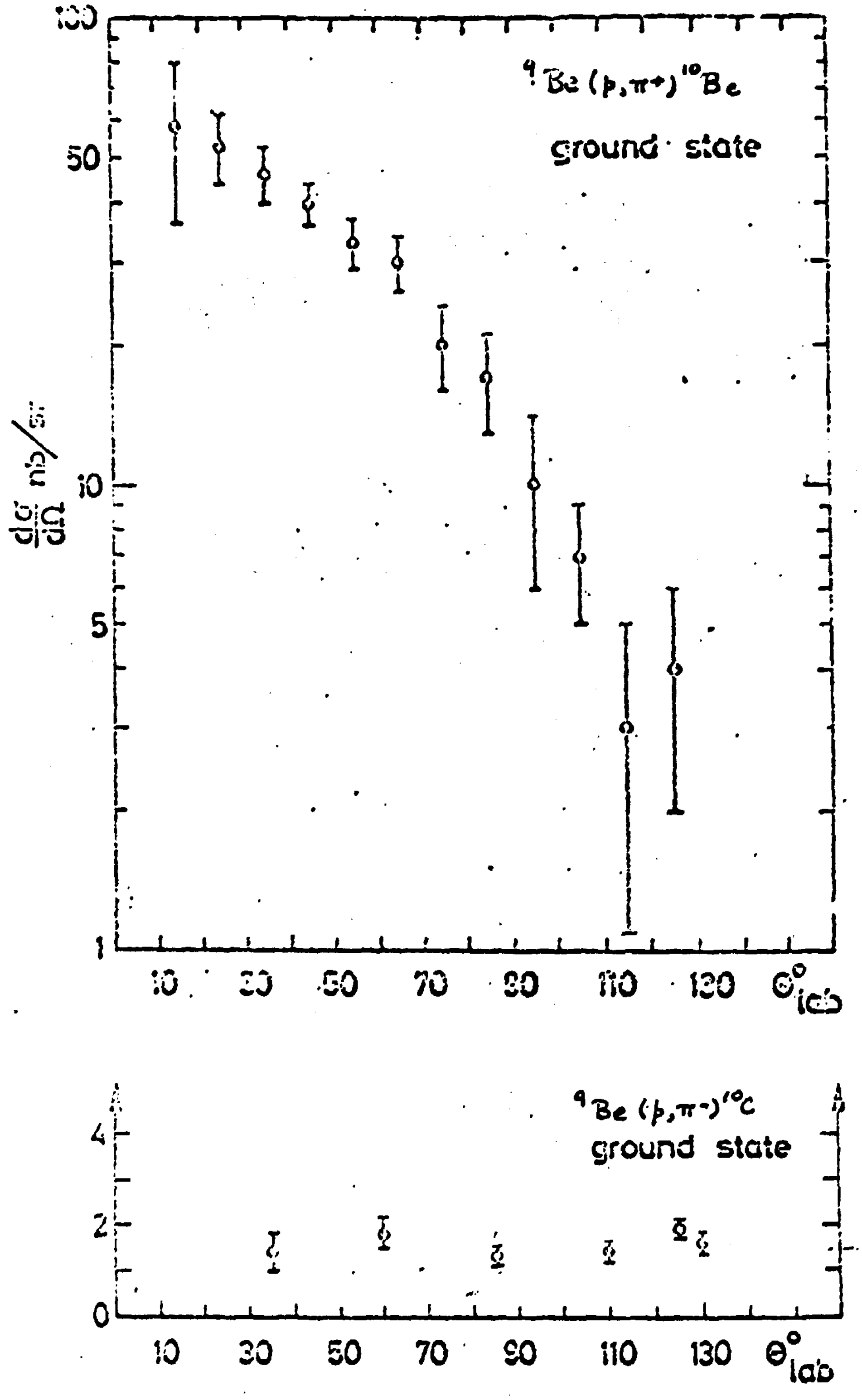


Fig 10

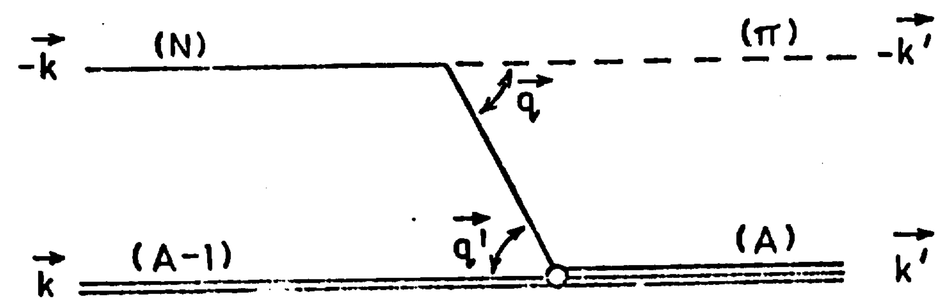


Fig 11

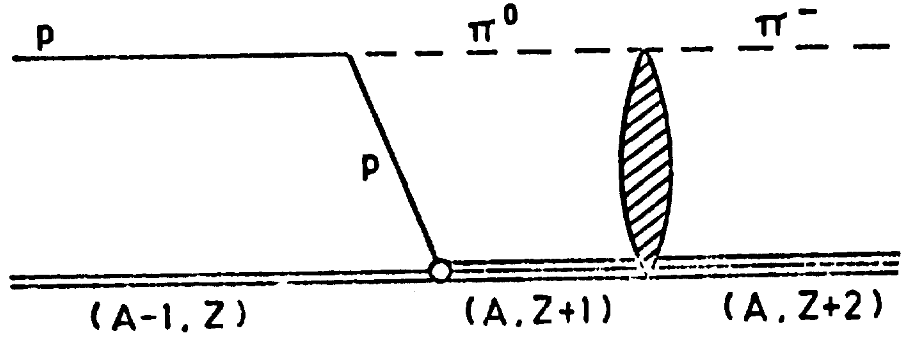


Fig 12

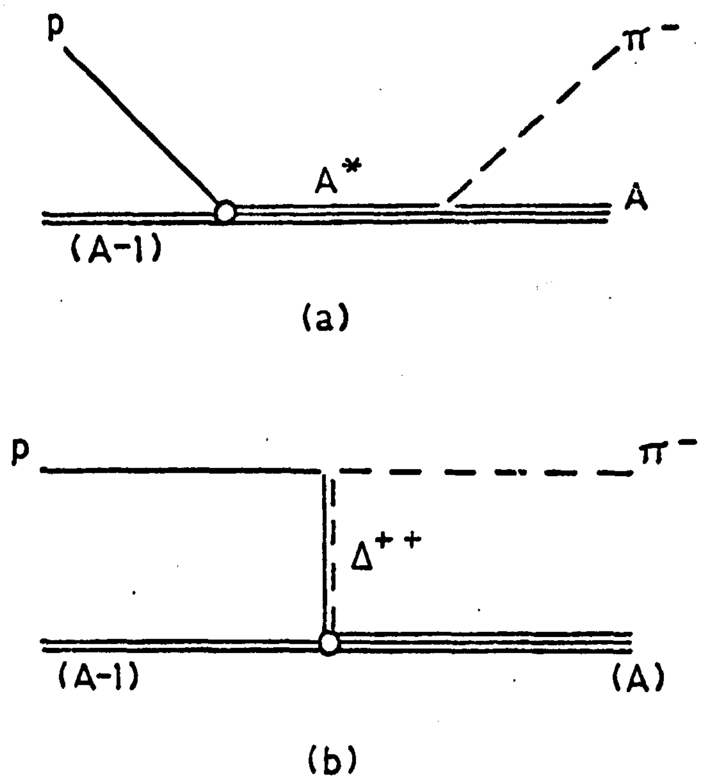


Fig 13

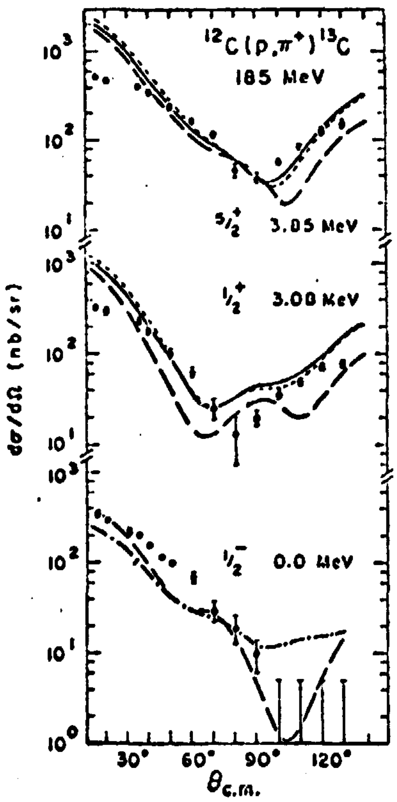


Fig. 14

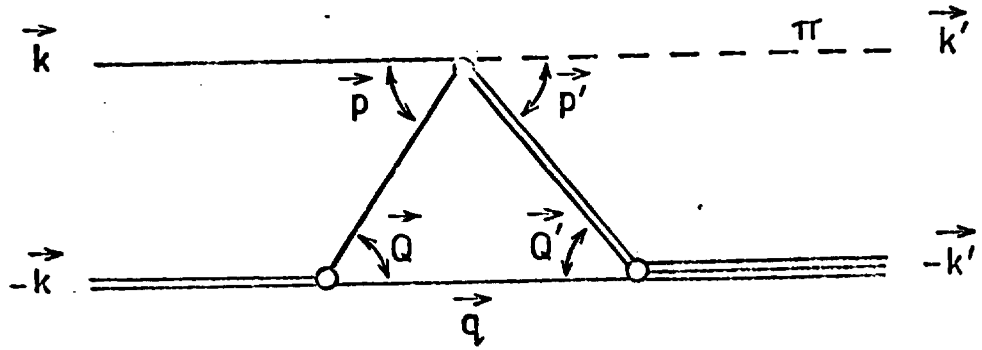


Fig. 15

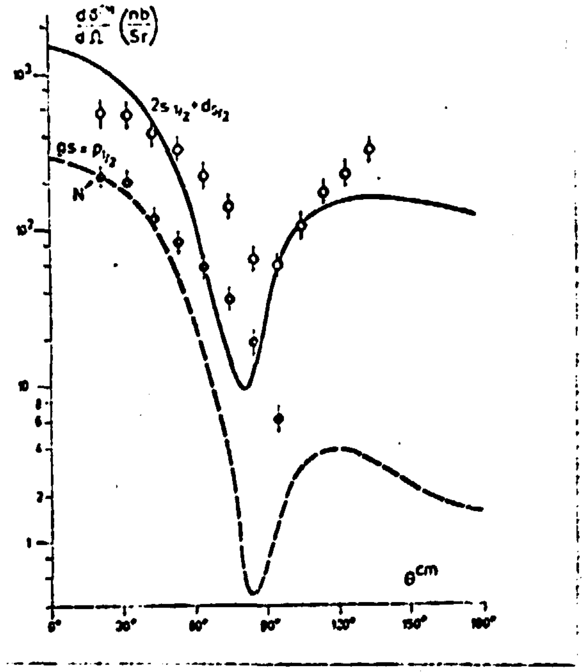


Fig 16

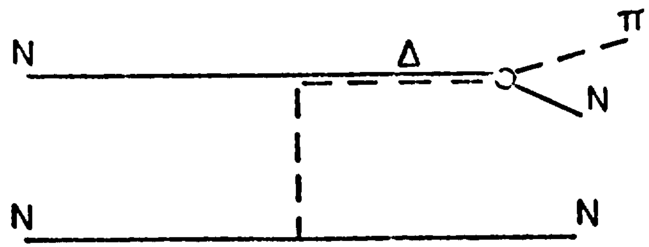


Fig 17