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E1 TRANSITIONS IN THE HARARI QUARK MODEL *

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ABSTRACT

The radiative decays $\psi'(3.684) \rightarrow \gamma\chi(^3P_J)$ and $\chi(^3P_J) \rightarrow \gamma\psi(3.1)$ have been analysed within the framework of the Harari quark model. The spatial matrix elements describing these $L = 1$ to $L = 0$ transitions have been estimated from the $A_2(1310) \rightarrow \gamma\rho(770)$ mode by applying U(6) symmetry at the quark level. The resulting decay widths, which compare very well with experimental data, have subsequently been used to determine the $SU(3)_H$ assignments for the χ states.

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There have been fairly extensive analyses of the intermediate χ states observed [1] in the sequence $\psi'(3.684) \rightarrow \gamma\chi(^3P_J) \rightarrow \gamma\gamma\psi(3.1)$. It is generally conceded that these even parity $(L = 1)$ mesons conform to the following pattern [2),3)]:

$$\chi(^3P_0) : 3.41 \text{ GeV} \quad ; \quad \chi(^3P_1) : 3.50 \text{ GeV} \quad ; \quad \chi(^3P_2) : 3.55 \text{ GeV} \ . \qquad (1)$$

This picture has received broad support experimentally [4] for the $\chi(3.41)$ and $\chi(3.55)$ states through their $\pi\pi$ and $K\bar{K}$ decays, and the angular distribution of the photon in $\psi'(3.684) \rightarrow \gamma\chi(^3P_J)$, the latter being expected [5] to be of the form $(1 + \cos^2\theta)$ for $J = 0$ and $(1 + \frac{1}{13}\cos^2\theta)$ for $J = 2$.

It is the purpose of this letter to continue the earlier analysis [6] of the radiative decays of the "new" mesons successfully carried out for M1 transitions, to include the E1 transitions (we are neglecting the M2 contribution) of $\psi'(3.684) \rightarrow \gamma\chi(^3P_J)$ and $\chi(^3P_J) \rightarrow \gamma\psi(3.1)$. This programme has been carried out, as will be seen later in this paper, through the relatively simple assumption of U(6) symmetry for the "reduced" matrix elements of heavy quark transition operators between the 1^{--} and J^{++} $(J = 0,1$ and $2)$ states. While retaining the Harari [7] assignments for the $\psi(3.1)$ and $\psi'(3.7)$, we shall also attempt to understand the $SU(3)_H$ assignments ($\underline{1}$ or $\underline{8}$) for the χ states through their radiative decays.

Despite the conventional thinking [2),3)] of the even parity states as p-wave excitations of a charmed quark-anti-quark pair, we believe the results reported herein deserve attention for the following reason, viz. the growing interest in the literature [8)-11)] of quark models with flavours beyond the usual four (udsc). This has been motivated by some experimental support, e.g. from phenomenological analyses [10),11)] of dilepton production and y-anomalies in νN and $\bar{\nu}N$ reactions [12),13)]. There have also been clues [10),14)] from the "structure" of $\sigma(e^+e^- \rightarrow \text{hadrons})$ in the 4 GeV region that could motivate new flavours of quarks. We also note here that some of the earlier calculations [5),15)] on radiative decays of $\psi'(3.684) \rightarrow \gamma\chi(^3P_J)$ generally tended to overestimate [16] the experimental values by factors of 4-5. In the more recent calculation by Eichten et al. [17] these partial widths have compared quite favourably with experiment, but only after taking into account the coupling of the ψ' states with the continuum $D\bar{D}$, states whose threshold is at 3.72 GeV.

The two main aspects of the Harari [7] quark model which will concern us here are:

a) The (uds) quarks and the (tbr) quarks form a six-dimensional representation of the U(6) group. While the full symmetry is badly broken, an approximate version could be, as conjectured by Harari, the $SU(3)_L \times SU(3)_H \times U(1) \times U(1)$, where L and H refer to light and heavy quarks and the two U(1) groups to baryon number and heaviness respectively.

b) The $\psi(3.1)$ and $\psi'(3.684)$ are regarded in the model as the $SU(3)_H$ 1^{--} singlet and 1^{--} octet ($I_H = 0$) respectively. By taking this pattern of singlet vs. octet mass breaking among states composed of (tbr) quarks to be just as consistent as among states made of (uds) quarks, we were able to explain successfully the $\psi(3.1) \to \gamma\eta_c(2.8)$ partial width in an earlier investigation [6].

For the present calculation, we note that unlike the charmonium interpretation for the χ states, the orbital matrix element in the Harari model for the $\psi' \to \gamma\chi(^3P_J)$ and $\chi(^3P_J) \to \gamma\psi$ transitions is now the same. What is not clear, however, are the $SU(3)_H$ representations (1 or 8) to which the χ states belong. Recall that this question was resolved by Harari for the ψ and ψ' through a consideration of their respective leptonic widths. Assuming that all the χ states are composed of identical (heavy) quark-antiquark mixtures, we shall show that the radiative transitions $\psi' \to \gamma\chi(^3P_J)$ clearly prefer the singlet over the octet assignments.

With the quark charges given by $Q_t = Q_r = +\frac{2}{3}$ and $Q_b = -\frac{1}{3}$, we note that the E1 quark operator for $SU(3)_H$ $\underline{1} \leftrightarrow \underline{1}$, $\underline{1} \leftrightarrow \underline{8}$ and $\underline{8} \leftrightarrow \underline{8}$ transitions is given by [3]:

$$\begin{aligned} \underline{1} \leftrightarrow \underline{1} &: \frac{1}{3}(\vec{x}_1 - \vec{x}_2)\cdot\frac{\vec{\epsilon}}{\sqrt{2k_0}} , \\ \underline{1} \leftrightarrow \underline{8} &: -\frac{1}{\sqrt{18}}(\vec{x}_1 - \vec{x}_2)\cdot\frac{\vec{\epsilon}}{\sqrt{2k_0}} \\ \underline{8} \leftrightarrow \underline{8} &: \frac{1}{2}(\vec{x}_1 - \vec{x}_2)\cdot\frac{\vec{\epsilon}}{\sqrt{2k_0}} , \end{aligned} \qquad (2)$$

where $\vec{x}_1$ and $\vec{x}_2$ refer to the position vectors of the quarks in the hadron, and $\vec{\epsilon}$ is the photon vector. The $L = 1$ to $L = 0$ transitions are then determined essentially by the following "reduced" matrix element for heavy quark transitions:

$$|M_{fi}|^2 \equiv |\langle L = 1|(\vec{x}_1 - \vec{x}_2)\cdot\vec{\epsilon}|L = 0\rangle|^2 . \qquad (3)$$

We estimate it here by noting that the same Eq.(3) enters the calculation of the radiative width $A_2 \to \rho\gamma$ for which the E1 quark operator (for (uds)

quarks) takes the form $\frac{1}{3}(\vec{x}_1 - \vec{x}_2)\cdot\frac{\vec{\epsilon}}{\sqrt{2k_0}}$. In the limit of U(6) symmetry for the (uds) and (tbr) quarks, the matrix element for heavy quark transitions can then easily be obtained from $\Gamma(A_2 \to \rho\gamma)$ leading to

$$|M_{fi}|^2 = 0.3943 \ \Gamma(A_2 \to \rho\gamma) , \qquad (4)$$

with Γ expressed in units of keV. It has been calculated recently by Levy et al. [18] in an analysis of the e.m. decays of the tensor mesons from $PV \to P'V'$ processes to be 80 keV. An earlier investigation by Berger and Feld,[19] using a non-relativistic harmonic oscillator model for mesons, yielded a value of 110 keV (or 290 keV), the latter uncertainty arising mainly from the magnitude of the level spacing parameter in their h.o. model. For our purposes, it is enough therefore to take an approximate value of 100 keV. The resulting widths for the decays $\psi' \to \gamma\chi(^3P_J)$ and $\chi(^3P_J) \to \psi\gamma$ are given in Table I.

The experimental numbers have been taken from the data reported recently by Feldman, quoted in Ref.3, with the masses of the χ states given by Eq.(1). The numbers in brackets in column two of Table I show the expected widths if the $\chi(^3P_J)$ states were taken to belong to the $SU(3)_H$ octet instead of the singlet. It is clear therefore that the results clearly favour the singlet assignment for the χ states in the framework of the Harari quark model.

The branching ratios for the $\chi(^3P_J) \to \psi\gamma$ have also been tentatively quoted by Feldman, in Ref.3, namely $5 \pm 3\%$, $30 \pm 8\%$ and $12.5 \pm 7.5\%$ for J = 0,1 and 2 respectively. Using the central values, this would lead to approximate total widths of the order of 2.6 MeV, 0.86 MeV and 3.2 MeV for the J = 0,1 and 2 χ states respectively. Jackson [20] has recently estimated the total widths from dipole sum rules in the charmonium picture with a charmed quark mass of 1.65 GeV. Apart from the fact that the upper and lower bounds on the total widths of the χ states obtained in Ref.20 agree quite well with our values, we also note that the smaller total width for the J = 1 χ states harmonizes with Yang's theorem for the emission of three gluons (as against two gluons for J = 0 and 2) in the colour gauge gluon picture.

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TABLE I

Decay mode	Γ_{theor} in keV	Γ_{expt} in keV
$\psi'(3.684) \to \gamma\chi(^3P_0)$	16.0 (72.0)	17.5 ± 6.0
$\psi'(3.684) \to \gamma\chi(^3P_1)$	13.4 (60.0)	20.0 ± 7.0
$\psi'(3.684) \to \gamma\chi(^3P_2)$	10.0 (45.0)	18.0 ± 7.0
$\chi(^3P_0) \to \gamma\psi(3.1)$	132.0	-
$\chi(^3P_1) \to \gamma\psi(3.1)$	285.0	-
$\chi(^3P_2) \to \gamma\psi(3.1)$	381.0	-

Radiative widths obtained in the limit of E1 dominance. The numbers in brackets denote the widths when $\chi(^3P_J)$ are treated as $SU(3)_H$ octet ($I_H = 0$) states. The experimental data have been taken from Jackson (Ref.20).