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CALCULATION OF PION FORM FACTOR *

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ABSTRACT

The pion form factor is calculated using the structure function W_2, which incorporates kinematical constraints, threshold behaviour and scaling. The Bloom-Gilman sum rule is used and only the two leading Regge trajectories are taken into account.

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Introduction

Hite and Müller [1] performed a detailed investigation of the kinematics and reggeization of πV scattering and its crossed reactions. In particular they established the behaviour of the dominant Regge pole residues under the condition that: a) the kinematical constraints be satisfied, b) the total photo-absorption cross-section has a finite non-zero value, c) the amplitude becomes gauge invariant in the limit $m_V \to 0$.

It is well known that unitarity relates the structure functions for electroproduction to the forward Compton amplitude for massive photons. Since the structure functions may be related to the form factor, it is interesting to calculate the latter in a model which incorporates explicitly the constraints mentioned above.

Such a calculation of the pion form factor is the object of this note. In addition to the above constraints, this model also incorporates scaling and is therefore similar to that of Moffat and Snell [2].

Calculation

The forward virtual Compton scattering of pions is given by [2]

$$M_{fi} = \varepsilon_q^{\mu} t^{*}_{\mu\nu} \varepsilon_p^{\nu} ,$$

where the momenta are chosen as in Fig.1, with

$$t^{*}_{\mu\nu} \equiv \left(p_\mu - \frac{p.q}{q^2} q_\mu\right)\left(p_\nu - \frac{p.q}{q^2} q_\nu\right) \frac{T_2 \gamma}{m} - \\ - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) m T_1 \gamma \tag{1}$$

and

$$\gamma := \frac{e^2 m}{E_\pi} ,$$

m is the mass of the pion, ε_1^{μ} and ε_2^{ν} are the polarization vectors of the incoming and outgoing V mesons.

The structure functions W_i are defined by the relation

$$\mathrm{Im}\, T_i = \pi W_i \; . \tag{2}$$

Now in Ref.1 it is shown that the forward amplitude $T_2(t = 0)$ is related to the crossed channel kinematical singularity-free helicity double flip amplitude $\tilde{f}^{+}_{1-1}$ (of spin parity +) by

$$T_2 = \frac{m}{\gamma} \tilde{f}^{+}_{1-1}, \quad (t=0) \quad . \tag{3}$$

The amplitude $\tilde{f}^{+}_{1-1}$ is known [1] to possess the following Regge pole expansion:

$$\tilde{f}^{+}_{1-1} \simeq \sum_{\tau\alpha} \frac{8}{\sqrt{\pi}\, s_0} \gamma^{+}_{1-1} \left(\frac{1+\tau e^{-i\pi\alpha}}{\sin \pi\alpha} \right) \frac{1}{(\alpha-2)!} \left(\frac{s}{s_0}\right)^{\alpha-2} . \tag{4}$$

Here τ is the signature and γ is the reduced residue. We have

$$-q^2 = M^2 \leqslant 0$$

and ν, the energy of the photon with momentum q, in the reference frame in which the pion is at rest, is given by

$$s = m^2 + M^2 + 2m\nu \quad . \tag{5}$$

Bjorken's scaling variable is defined by

$$\omega \equiv - \frac{2m\nu}{M^2} \quad . \tag{6}$$

We now construct our model amplitude for νW_2. We require this function to exhibit the following properties:

a) Satisfaction of the kinematical constraints derived in Ref.1, as well as a non-vanishing total photo-absorption cross-section in the limit $M \to 0$;

b) Correct threshold behaviour;

c) Scaling in the scaling limit.

From Ref.1 we know that in order to guaranetee (c) we must replace $(\gamma^{+}_{1-1})_{t} = 0$ by $\left(\frac{M^2 \gamma^{+}_{1-1}}{\alpha-1}\right)_{t=0}$ where γ^{+}_{1-1} is free of any singularities. Here the factor M^2 is necessary to ensure gauge invariance in the limit $M^2 \to 0$ and $\frac{1}{\alpha - 1}$ is required so that the total photo-absorption cross-section has a finite non-zero value.

The threshold for elastic scattering in s-channel is $\sqrt{s} = m$ or (see (5)) $M^2 = -2m\nu$ and $\omega = +1$. In the vicinity of the threshold we therefore expect our amplitude to have some behaviour like

$$\left(\frac{\omega^2 - 1}{\omega^2 + \eta^2}\right)^p ,$$

where p is some positive power and η^2 a constant $\geqslant 0$. The choice of $p = 2$ corresponds to a pion form factor which behaves as $F_\pi(q^2) \sim (-q^2)^{-3/2}$, as $-q^2 \to \infty$ which is consistent with the data available at present; and also W_2 does not change sign in going through $\omega = 1$.

We assume scaling in the well-known manner:

$$\begin{matrix} \nu \to \infty \\ M^2 \to -\infty \end{matrix}, \quad \omega \text{ fixed} \left\{ \begin{aligned} \nu W_2(\nu, M^2) &= F_2(\omega) \\ 2m W_1(\nu, M^2) &= F_1(\omega) \end{aligned} \right. \tag{7}$$

Remembering that $\frac{E_\pi}{m} = 1$ in the pion laboratory, we may write

$$\nu W_2 = \left(\frac{\omega^2 - 1}{\omega^2 + \eta^2}\right)^2 \frac{4}{\pi^{3/2} e^2} \sum_\alpha \frac{\tilde{\gamma}^{+}_{1-1}}{(\alpha-1)!} \left(\frac{-M^2}{-M^2 + 2m\nu_0}\right) \left(\frac{2m\nu}{-M^2 + 2m\nu_0}\right)^{\alpha-1} . \tag{8}$$

We observe that for $M^2 \to -\infty$, $2m\nu_0 \simeq \frac{1}{4}\,(\mathrm{GeV})^2$, this expression becomes a function of ω as required by (7). It should be observed that (8) is not a normal Regge pole model, because Regge behaviour implies that both ν and ω are large.

It is convenient to write

$$\frac{4}{\pi^{3/2} e^2} \cdot \frac{\tilde{\gamma}^{+}_{1-1}}{(\alpha-1)!} \equiv \beta_\alpha \quad (\text{for } t=0) \, . \tag{9}$$

then

$$\nu W_2 \simeq \left(\frac{\omega^2 - 1}{\omega^2 + \eta^2}\right)^2 \sum_\alpha \beta_\alpha \left(\frac{\omega}{2m}\right)^{\alpha-1} \tag{10}$$

in the scaling limit, where the factor $2m$ has been inserted for convenience.

Next we use the Bloom-Gilman [3] finite energy sum rule extended to $\gamma\pi$ scattering

$$\left(\frac{2m}{-M^2}\right)\int_{\frac{-M^2}{2m}}^{\nu} d\nu\, \nu W_2(\nu, M^2) = \int_1^{\frac{2m\nu}{-M^2}} d\omega\, \nu W_2(\omega)\,, \tag{11}$$

where $\nu = \frac{s - m^2 - M^2}{2m}$. The sum rule implies that for $\nu < \nu_\pi$, $\nu W_2(\omega)$ acts as a smooth function which averages $\nu W_2(\nu, M^2)$ in the sense of finite energy sum rules. By making the very strong assumption of locality, i.e. in the vicinity of a resonance (including the pion pole in (11)) Bloom and Gilman [3] showed that $\nu W_2(\omega)$ is still a good average of the resonance bump which appears in $\nu W_2(\nu, M^2)$.

The contribution of the pion form factor to νW_2 is [4]:

$$\nu W_2 = 2m\nu \left[F_\pi(M^2)\right]^2 \delta(s - m^2)\ . \tag{12}$$

Substituting (12) into (11) we obtain

$$\left[F_\pi(M^2)\right]^2 = \int_1^{1+\frac{s-m^2}{-M^2}} d\omega\, \nu W_2(\omega) \cong \int_1^{1+\frac{s-m^2}{-M^2}} d\omega \left(\frac{\omega^2-1}{\omega^2+\eta^2}\right)^2 \times \sum \beta_\alpha \left(\frac{\omega}{2m}\right)^{\alpha-1}, \tag{13}$$

through (10).

Taking into account only the two most dominant trajectories ρ, ρ' with $\alpha_\rho(0) = 1$ and $\alpha_{\rho'}(0) = \frac{1}{2}$, the integrals in (13) may be evaluated exactly. Writing $\Delta = \frac{s - m^2}{-M^2}$ we find that

$$\left[F_\pi(M^2)\right]^2 = \beta_\rho(0)\left[\omega + \frac{\omega(1+\eta^2)^2}{2\eta^2(\omega^2+\eta^2)} + \frac{(1-3\eta^2)(1+\eta^2)}{2\eta^3}\,\mathrm{tg}^{-1}\frac{\omega}{\eta}\right]_1^{1+\Delta} +$$

$$+ \sqrt{2m}\,\beta_{\rho'}(0)\Bigg[2\sqrt{\omega} + \frac{\sqrt{\omega}(1+\eta^2)}{2\eta^2} + \left(\frac{3-5\eta^2}{4\eta^2}\right)\left(\frac{1+\eta^2}{4\eta^2}\right)\sqrt{\eta}\ \times$$

$$\times \left\{\log\left(\frac{4\omega+4\sqrt{\omega}+2\eta}{4\omega-4\sqrt{\omega}+2\eta}\right) + 2\,\mathrm{tg}^{-1}\left(\frac{4\sqrt{\omega\eta}}{2\eta-4\omega}\right)\right\}\Bigg]_1^{1+\Delta}\ . \tag{14}$$

Here the residues $\beta_{\rho,\rho'}(0)$ are known. They have been estimated by Moffat and Snell [2] to be

$$\beta_\rho(0) = 0.18$$

$$\beta_{\rho'}(0) = 0.15\ (\mathrm{GeV})^{-\frac{1}{2}}\ . \tag{15}$$

Using $m = 0.14$ GeV and $S = 1.29\ (\mathrm{GeV})^2$ for the π meson, we find a good fit of the known data points [5),6),7)] shown in Fig.2 for $\eta = 0.05$. It should be remembered that our model cannot really be expected to be good in the region of small M^2. Neglecting $\alpha = 0$ fixed pole contributions, however, seems a reasonable approximation.

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FIGURE CAPTIONS

Fig.1 Forward virtual Compton scattering of pion.

Fig.2 The pion form factor → $-M^2$ $(GeV/c)^2$.

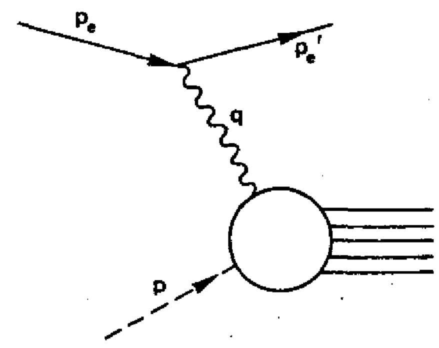

Fig. 1

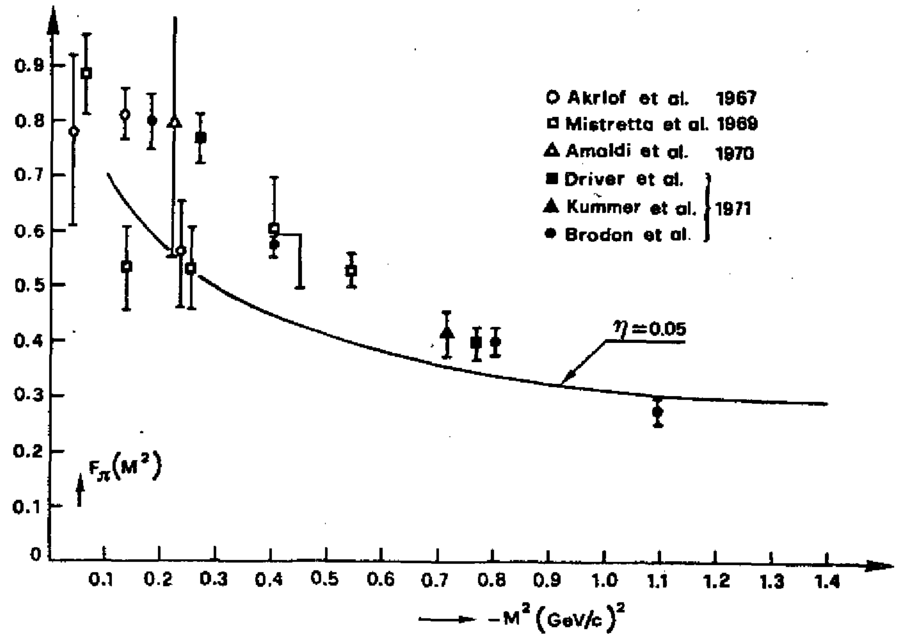

Fig. 2