CREEP FATIGUE DAMAGE UNDER MULTIAXIAL CONDITIONS

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When structural components are subjected to severe cyclic loading conditions with intermittent periods of sustained loading at elevated temperature, the designer must guard against a failure mode caused by the interaction of time-dependent and time-independent deformation. This phenomena is referred to as creep-fatigue interaction. A recent compendium by Conway, Stenz, and Berling [1] offers a historical perspective for a decade of work aimed at providing a physical/mathematical description of this process.

Although low-cycle fatigue data can generally be correlated as a function of the nominal strain range and the number of cycles, and stress rupture data correlated as a function of constant stress magnitude and time, the interaction problem has not been treated with uniform success. The most elementary form of interaction theory (called linear damage summation) is now embodied in the ASME Boiler and Pressure Vessel Code [2] to provide design rules against creep-fatigue failure in Class 1 nuclear construction. This simple interaction theory can be shown to give reasonable results (and, therefore, provide sufficient conservatism) for a class of uniaxial experiments that are strain-controlled for the cyclic deformation and stress-controlled during hold periods. More complex uniaxial strain histories have not been as amenable to simple treatment, however, and, except for the quasi-multiaxial experiments of Zamrik, et al. [3], no multiaxial examination of creep-fatigue damage has been attempted.

In recent years, a competitor for the linear damage summation theory has emerged—called strainrange partitioning [4]. This procedure is based upon the visualization of the cyclic strain in a uniaxial creep-fatigue test as a hysteresis loop, with the inelastic strains in the loop counter-balanced in one of two ways:

(a) tensile creep strain balanced by compressive creep strain, tensile plastic strain balanced by compressive plastic strain, and tensile plastic strain balanced by compressive creep strain; or (b) tensile creep strain balanced by compressive plastic strain. The former of these would constitute a repeated cycle partitioned into \( \Delta e_{cc} \), \( \Delta e_{pp} \), and \( \Delta e_{pc} \); the latter would be partitioned into \( \Delta e_{cc} \), \( \Delta e_{pp} \), and \( \Delta e_{cp} \). A more complete discussion of the method can be found in [5].

Since most design analysis of elevated temperature components is multiaxial in nature, both the linear damage summation and strainrange partitioning methods have been extended from the uniaxial visualizations, primarily through the use of inelastic strain invariants. In this way, "effective" plastic strain and "effective" creep strain play the same roles as uniaxial plastic and creep strains. However, the terminology "tensile" and "compressive" require interpretation and some procedure for treating the
strain that is accumulated (unbalanced) during the cycle is needed. Linear damage summation avoids these two difficulties but introduces others. For instance, since the creep damage term is written

\[ \sum \frac{t}{T_d} \]  

where \( T_d \) is the creep rupture time at stress and temperature, and since the stresses in a component during cyclic operation are both time-varying and multiaxial, some critical assumptions must be made. First, \( T_d \) is written as a function of effective stress, \( \bar{\sigma} \); second, inaccuracy in the integral evaluation due to the large variations in \( T_d \) with respect to small changes in temperature, is ignored. Both linear damage summation and strainrange partitioning are subject to difficulties regarding the definition of a "cycle."

In order to test these two damage theories, measurements from a multiaxial experiment are needed. None are yet available, although some crucial questions may be answered by the program begun by the Japanese [6]. Rather than wait for these results, however, an analytical comparison has been developed here, with its primary focus on the application of both theories to a set of verified multiaxial finite element calculations. In the next section, the analysis of a thin-walled, high-temperature piping segment is described and the calculations are compared to experimental results in order to verify their accuracy. Sections after that describe the application of linear damage summation and of a modified version of strainrange partitioning to these calculations. The two theories are then compared and contrasted in terms of ease of use, possible inconsistencies, and component life prediction. Future work to further test the damage theories is recommended.

2. Description of the Physical Model

The physical configuration for which the damage calculations are done is the thermal pipe ratchetting experiment conducted at ORNL [7]. In this experiment a 304 stainless steel (Heat 9T2796) pipe segment (radius of 3.85 mm, wall thickness of 9.53 mm) is subjected to cyclic loadings resulting from internal liquid sodium pressure and thermal histories depicted in Fig. 1.

Quantities required for the damage calculations are obtained through a stress analysis of a mathematical model of the experiment. To this end, the pipe is modeled as a generalized plane strain problem, which corresponds to treating the pipe as though it were infinitely long and capped at both ends. Temperature histories in the pipe wall resulting from temperature variations in the sodium are determined by an independent transient thermal analysis. Material properties are input as functions of temperature.
Plasticity with kinematic hardening and creep with strain hardening are used to represent the inelastic material response. Instead of a functional form for the creep law, empirical creep data given in [7] is supplied in tabular form (conventional creep laws for 304 stainless steel do not adequately represent the creep data for this particular heat).

The analysis is carried through three loading cycles using a general purpose, nonlinear, structural finite element code. Results for the outer fiber hoop stress-strain path are given in Fig. 2. The lower case letters on the curve correspond to equivalent letters on thermal and pressure histories given in Fig. 1. Focusing on the first cycle (the first a-f series) the "ab" segment of the curve indicates significant plastic deformations. The major creep deformation occurs in the "ef" segment where the pipe is held under essentially constant load for 160 hours at the highest temperature of the cycle, 593°C (1100°F). The three ranges marked on the abscissa in Fig. 2 represent the hoop strain at the ends of the first, second, and third cycles, as measured in the ORNL experiment. Comparing similar data from the analysis, denoted by the three D's on the curve in Fig. 2, to these ranges, good correlation is evident between the experimental and analytical results.

The pipe wall model is appropriate for damage calculations for several reasons. It represents a real world problem in that it simulates structural behavior that occurs in the primary piping system of an LMFBR during a series of reactor transients. The level of complexity of the model is sufficient to pose many of the problems that might arise in attempting to perform damage computations on a component. For example, in addition to multiaxial stress states, the temperature varies with position and time throughout the cycle, plastic and creep material behaviors are present, both of which contribute to strain accumulation, and the loading is non-proportional. The two methods for computing damage are, therefore, meaningfully tested when applied to this model.

One drawback of the model is that the experiment at ORNL was not carried out to failure. Consequently, quantitative comparisons between experimental life cycle data and that predicted by damage calculations are not possible at this time.

3. Damage by the Linear Damage Summation Rule

The linear damage summation rule which is set down in the ASME Boiler and Pressure Vessel Code [2] is given by the relation

\[
\sum_{n=1}^{P} \left( \frac{n}{t_{n}} \right) + \sum_{i=1}^{Q} \left( \frac{t}{t_{n}} \right) \leq D.
\]
In eq. (1), \( n \) is the number of applied cycles of loading condition \( j \). The symbol \( t \) represents the time duration of the \( k \)th load condition. The quantity \( N_d \) is the number of allowable cycles corresponding to the equivalent strain-range, \( \Delta \varepsilon_{\text{equiv}} \), defined by

\[
\Delta \varepsilon_{\text{equiv}} = \max \left\{ \frac{1}{3} \left[ (\Delta \varepsilon_x - \Delta \varepsilon_y)^2 + (\Delta \varepsilon_y - \Delta \varepsilon_z)^2 + (\Delta \varepsilon_z - \Delta \varepsilon_x)^2 \right]^{1/2} \right. \\
+ \left. 6(\Delta \varepsilon_{xy}^2 + \Delta \varepsilon_{yz}^2 + \Delta \varepsilon_{zx}^2) \right\}^{1/2},
\]  

where \( \Delta \varepsilon_x = \varepsilon_x - \varepsilon_{x_i} \), \( \Delta \varepsilon_y = \varepsilon_y - \varepsilon_{y_i} \), etc. (the subscript "i" denotes an extreme condition within the cycle). The relationship between \( N_d \) and \( \Delta \varepsilon_{\text{equiv}} \) for both 304 and 316 stainless steels is shown graphically in Fig. T-1420.1 [2], for various temperatures. The quantity \( T_d \) in eq. (2) is the allowable time at the stress value of \( \sigma_{\text{eff}}/9 \), where \( \sigma_{\text{eff}} \) is the effective stress induced by the \( k \)th load. For various temperatures, the dependence of \( T_d \) on \( \sigma_{\text{eff}}/9 \) is displayed graphically in Fig. I-14.6 [2]. The symbol, \( D \), denotes the allowable creep-fatigue damage. Its functional relationship to the two sums in eq. (2) is depicted in the graph of Fig. T-1420.2 [2].

The first sum in eq (2) accounts for fatigue damage and the second represents creep-rupture damage. Damage due to accumulated plastic strain is not incorporated in this damage rule. This omission may result from the fact that strain limits exist elsewhere in the ASME Code to restrict the accumulation of inelastic strain.

Noting the manner in which the strain-range is determined above, fatigue damage is predicted even when the strain is not cyclic. Although inaccurate, this procedure is conservative. Additional conservatism occurs in the creep-rupture term as creep reversals within the cycle are treated as strictly damage generating without accounting for damage healing processes.

Linear damage summation has come under attack in recent years due to its over-conservatism. Consequently, the method of strain-range partitioning has arisen as a possible alternative for damage evaluation. The next section proposes a technique for extending strain-range partitioning to multiaxial stress states.

4. Damage by Strainrange Partitioning

At the outset, two forms of strainrange partitioning for multiaxial stress states were considered for these damage calculations, one proposed by S. S. Manson and G. K. Halford [8] and the other by J. M. Duke [9]. As it stands, the fatigue portion of the Manson procedure is not directly applicable in this case because of its restriction to proportional loading. Although not sharing this restriction, the method proposed by Duke is unsettling in that it predicts fatigue damage even when strains are not
Consequently, a new method for damage assessment was constructed to circumvent these difficulties.

Basically, the fatigue portion of the new method is modeled after that proposed by Manson and Halford in that the notion of a dominant direction based on stress is retained. However, in contrast to using principal stresses to define this direction, principal stress deviators are used, consistent with plasticity and creep material models for metals. This insures that the dominant direction corresponds to the direction of the largest inelastic principal strain component.

Also in contrast to the Hanson procedure the new method is not restricted to proportional loading. This restriction occurs in the Manson technique because a dominant direction is defined for the complete cycle based on principal stress directions. Thus, it is important that the principal stress directions remain fixed throughout the cycle; hence the necessity of proportional loading. In the current method, this restriction is relaxed by defining the dominant direction at a number of temporal points throughout the cycle (using principal stress deviators). The sign determined from the dominant direction at each point is applied to the effective strain increment occurring between the previous and current points. In this manner the dominant inelastic strain is driven by the corresponding dominant stress deviator throughout the cycle, regardless of the direction of this deviator. Consequently, the method is not restricted to loadings which are proportional. This procedure is described in greater detail in steps "a" through "e" below.

The damage caused by accumulated strain in the new method is assessed by the technique proposed by Duke. Although the procedure set forth by Manson and Halford for determining this damage is conceptually similar, Duke's technique is more precisely defined and thus easier to implement. This technique is reproduced in steps "f" and "g" below.

The following list itemizes the details of the present method.

(a) Each effective plastic incremental strain in the cycle is given a sign based on the sign of the principal stress deviator with the largest magnitude at the end of the increment. Because of the flow rule normality assumption, this sign corresponds to that of the principal incremental plastic strain component with the largest magnitude.

(b) Accumulating both the positive and negative effective strain increments throughout the cycle, the plastic strain range, \( \Delta \varepsilon^{pp} \), is defined as the smaller (in magnitude) of these two sums. The plastic strain residual is defined as the algebraic sum of the positive and negative effective strain accumulations.

(c) The creep strain range, \( \Delta \varepsilon^{cc} \), and creep strain residual are determined in an entirely similar manner by following steps (a) and (b) above.

(d) The mixed creep-plasticity strain range, \( \Delta \varepsilon^{cp} \), or \( \Delta \varepsilon^{pc} \), is determined
through the plastic and creep strain residuals. If the residuals are of the same sign, no mixed strainrange exists for the cycle. If they are of opposite signs, the mixed strainrange is the smaller in magnitude of these two residuals. The mixed strainrange is designated either "cp" for a positive creep residual or "pc" for a positive plastic residual.

(e) Having computed the various strainranges, the fatigue damage for the nth cycle, $D^n_f$ is given by

$$D^n_f = \frac{1}{N^{pp}} + \frac{1}{N^{cc}} + \frac{1}{N^{cp/pc}} , \quad (4)$$

where the $N^{pp}$, $N^{cc}$, and $N^{cp/pc}$ are the life cycles for the respective $\Delta \epsilon^{pp}$, $\Delta \epsilon^{cc}$, and $\Delta \epsilon^{cp/pc}$ (obtained from partitioned strainrange-life relations for the material at the maximum temperature occurring in the cycle).

(f) Damage caused by ductility exhaustion, $D_\epsilon$, is computed by the method proposed by Duke [9]. The total equivalent plastic and creep strains $\epsilon^{p\text{equiv}}$ and $\epsilon^{c\text{equiv}}$ are computed (using the plastic and creep strain components) at the time of interest. The damage is then given by

$$D_\epsilon = CTF \left[ \left( \frac{\epsilon^{c\text{equiv}}}{\epsilon_f^c} \right)^2 + \left( \frac{\epsilon^{p\text{equiv}}}{\epsilon_f^p} \right)^2 \right]^{1/2} \quad (5)$$

where

$$\epsilon_f^p = \ln \left( \frac{100}{100-RA} \right) \quad (RA \text{ is the reduction in area};$$

$\epsilon_f^c$ is the uniaxial creep strain at rupture determined by multiplying the creep strain corresponding to stress, elapsed time, and temperature at the time of interest (obtained from the appropriate isochronous stress-strain curve), by the time to rupture (also corresponding to conditions at the time of interest), and dividing by the elapsed time;

CTF is the triaxiality factor, $3H/\sqrt{J_2}$ where $H$ is the mean normal stress and $J_2$ is the second invariant of the deviatoric stress tensor.

The triaxiality factor in the above relation is conservatively restricted such that $CTF \geq 1$.

(g) Using the damages $D^n_f$ and $D_\epsilon$, the total damage, $D$, for $N$ cycles is given by

$$D = D_\epsilon + \sum_{n=1}^{N} D^n_f .$$

Here $D_\epsilon$ is evaluated at the end of the $N^{th}$ cycle.
The method of extending strainrange partitioning to multiaxial stress states has several attractive features. It predicts the correct strainrange in the uniaxial case (a necessary property). The partitioned strainranges are independent of the mean normal stress in accordance with plasticity and creep theories for metals. The method is not restricted to proportional loading. Consistent with experimental observation, the triaxiality factor is applied such that as it increases the ductility decreases. The method is easily automated (for example, in a post-processor), since the required quantities are directly calculable from quantities that normally result from the stress analysis.

The damage caused by accumulation of creep strain in this method can be erroneous as the damage is computed on the basis of conditions existing at the time of interest, which may not be representative of the total history. As a possible alternative, which accounts for damage healing processes, compute the creep damage as in linear damage summation and apply a sign to each damage increment, designating either damage generation (+) or damage healing (-). The sign is taken as that of the principal stress deviator with the largest magnitude after the current loading condition has been applied. Care must be taken to interpret the net damage computed as positive, if it turns out to be otherwise.

As with the other proposed techniques for damage evaluation which use a triaxial strainrange partitioning approach, this method has not been experimentally verified. However, its simplicity, versatility, and ease of implementation make it worthy of consideration.

5. Results and Conclusions

The damage associated with the pipe ratcheting experiment described in Section 2 is evaluated by each of the techniques presented in Sections 3 and 4. Special data required in the linear damage summation rule for 304 stainless steel (e.g., stress-to-rupture and fatigue curves) can be found in the ASME Boiler and Pressure Vessel Code [2]. Data necessary for the strainrange partitioning approach is available from several different sources. Partitioned strainrange-life relations for 316 stainless steel reported in [10] are used in the damage calculations as life relations for 304 stainless steel are not yet available. The reduction in area at fracture (42 percent) is taken from [1], Table 7.4. The uniaxial creep strain at rupture, $\epsilon_f^c$, is obtained through the use of isochronons stress-strain curves and creep rupture curves given in Code Case 1592 [2], Fig. T-1800-A-7 and I-14-6A, respectively.

Results of the damage computations are displayed in Figs. 3-5 as a function of the radial position in the pipe wall. These curves show that the fatigue damage is small, compared to the total damage, for both methods of damage evaluation. The qualitative behavior of the
curves for fatigue damage shown in Fig. 3 is similar, although significantly greater fatigue damage is predicted by the linear damage summation rule. This difference can be explained by noting that the strain-range in the linear damage summation rule includes strain accumulation as well as cyclic strain whereas only cyclic strains are included in the strain-range partitioning method.

Significant differences also are evident in the strain accumulation damage shown in Fig. 4. The main reason for this is that the damage mechanisms in this particular application are different for the two damage evaluation methods. The damage predicted by the strain-range partitioning approach results from accumulated plastic strain as well as creep strain, whereas the linear damage summation rule accounts only for damage caused by creep strain accumulation. Thus, near the inner radius of the pipe where the accumulated strains are predominantly plastic, the damage predicted by the strain-range partitioning method is significantly greater than that predicted by the linear damage summation rule.

Focusing on the damage caused by the accumulation of creep strain, as determined by the two methods, significant quantitative differences are noted (damage predicted by the linear damage summation rule is a factor of two times that by the strain-range partitioning approach). This is not surprising when one considers the differences between the two methods. In the strain-range partitioning approach, the accumulated creep strain is averaged over the elapsed time based on stresses and temperatures at the time of interest. In addition to being inaccurate, this technique can produce results which are non-conservative. On the other hand, in linear damage summation, all creep strain increments throughout the cycle cause damage, regardless of creep strain reversals. A final difference is that ductility reductions resulting from triaxial stress states are taken into account by the strain-range partitioning approach but not by linear damage summation.

The combined fatigue and strain accumulation damage assessed by the two methods is illustrated in Fig. 5. The various curves show the damage occurring in the first, second, and third cycles for each method. As both methods predict relatively small fatigue damage, these curves primarily represent the damage caused by accumulated strain. Note that near the inner wall the damage per cycle assessed by the strain-range partitioning approach decreases markedly with increasing cycle number. This reflects the fact that the plastic strain accumulation per cycle, which accounts for most of the damage in this region, is also decreasing. On the other hand, over the outer half of the pipe wall linear damage summation predicts increasing damage per cycle versus cycle number. In this region, because of plastic strain-hardening, stress levels are rising, resulting in increased creep strain accumulation per cycle and, therefore, greater damage. This effect is not so apparent in the damage determined by the strain-range partitioning approach as the creep strain accumulation damage is assessed using the stress level at the end of
the cycle which is comparatively low (at low stress levels, the damage per cycle is a much weaker function of the stress).

As the plastic strain accumulation per cycle converges to a constant value at all points in the pipe wall, it is evident that the strainrange partitioning damage per cycle will also approach a constant value. On the other hand, due to increasing stress levels, the damage per cycle predicted by linear damage summation will continue to increase, eventually surpassing the strainrange partitioning damage in some regions of the pipe wall. Thus in this instance, the linear damage summation rule would provide a more conservative estimate of the damage. In passing, it is noted that in situations such as these where damage per cycle increases with each cycle, care must be taken to conservatively extrapolate the damage to failure.

Comparing the ease of application of the two methods, one aspect of linear damage summation makes it particularly cumbersome. To compute the cyclic strainrange, the linear damage summation rule vaguely directs the analyst to "select a point when conditions are at an extreme for the cycle" to use as a strain reference point. Since it is difficult a priori to determine this point, all points of the cycle must be considered as possible reference points and the one yielding the largest strainrange selected. This selection process makes linear damage summation more difficult to apply than strainrange partitioning. Other features of the two methods are roughly equivalent from an "ease of application" viewpoint.

In summary, differences in approach between the methods of damage evaluation lead to differences in damage prediction. Moreover, the lack of experimental failure data for triaxial stress states makes it impossible to judge which of the methods is the most accurate at this time. Nevertheless, this study is valuable in that the differences in the two approaches are discussed, in detail, and for this particular application, the effects of these differences are quantified. Before any decision can be reached as to which method is the more correct in predicting damage, it is essential that multiaxial experimental failure data be available for verification purposes.


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Fig. 2  Outer fiber hoop stress-strain path (3 cycles).

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Fig. 5  Total damage/cycle versus radius.
Hoop Stress $\times 10^7$ Pa

- Data predicted by ZAPATA analysis.

- Ranges of experimental data at the end of the 1st, 2nd, & 3rd cycles.

Hoop Strain (%)
Fatigue Damage

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2x10^4

Strainrange Partitioning

Linear Damage Summation

Radius (mm.)
Accumulation Damage

• Strainrange Partitioning
• Linear Damage Summation

Strain Accumulation Damage

Radius (mm)