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L'ÉNERGIE ATOMIQUE DU CANADA LIMITÉE

A HYBRID COMPUTER SIMULATION OF REACTOR SPATIAL DYNAMICS

by

H.W. HINDS

Paper to be presented at the Third Power Plant Dynamics, Control and Testing Symposium, Knoxville, Tennessee, 7-9 September 1977

Chalk River Nuclear Laboratories

Chalk River, Ontario

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Simulation par ordinateur hybride de la dynamique spatiale des réacteurs

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Résumé

Les équations différentielles partielles qui décrivent la dynamique spatiale à vitesse unique des réacteurs à neutrons thermiques ont été converties en un ensemble d'équations différentielles ordinaires en employant des approximations de différences finies pour les dérivées spatiales. Les variables ont ensuite été normalisées en condition de référence à état stable, d'une nouvelle manière, afin d'avoir un ensemble d'équations particulièrement approprié pour application sur ordinateur hybride.

Une console d'ordinateur analogique AD/FIVE de dynamique appliquée peut résoudre complètement en parallèle jusqu'à 30 équations différentielles simultanées. Cela correspond à peu près à huit noeuds de réacteur ayant chacun deux groupes actifs de neutrons retardés. Pour améliorer la précision, il faut généralement augmenter le nombre des noeuds. Grâce à la technique Hsu-Howe de multiplexage, un module unidimensionnel à 8 modes a été assujeti à un mouvement de va et vient entre la moitié gauche et la moitié droite du réacteur afin de simuler un modèle à 16 noeuds également unidimensionnel.

Les deux versions (8 ou 16 noeuds) du modèle ont été mises à l'essai sur des problèmes de repère du type "perte de caloporteur" lesquels ont également été résolus au moyen du code numérique FORSIM, avec deux groupes d'énergie et 26 noeuds. Un bon accord a été obtenu entre les deux techniques.

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ABSTRACT

The partial differential equations describing the one-speed spatial dynamics of thermal neutron reactors were converted to a set of ordinary differential equations, using finite-difference approximations for the spatial derivatives. The variables were then normalized to a steady-state reference condition in a novel manner, to yield an equation set particularly suitable for implementation on a hybrid computer.

One Applied Dynamics AD/FIVE analog-computer console is capable of solving, all in parallel, up to 30 simultaneous differential equations. This corresponds roughly to eight reactor nodes, each with two active delayed-neutron groups. To improve accuracy, an increase in the number of nodes is usually required. Using the Hsu-Howe multiplexing technique, an 8-node, one-dimensional module was switched back and forth between the left and right halves of the reactor, to simulate a 16-node model, also in one dimension.

These two versions (8 or 16 nodes) of the model were tested on benchmark problems of the loss-of-coolant type, which were also solved using the digital code FORSIM, with two energy groups and 26 nodes. Good agreement was obtained between the two solution techniques.

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NOMENCLATURE

Symbol	Definition	Units
a	coefficient of finite-difference approximation	m^{-2}
C_i	concentration of the i^{th} delayed-neutron precursor	m^{-3}
D	diffusion coefficient	m
P	net precursor contribution	$m^{-3} \cdot s^{-1}$
t	time	s
v	neutron velocity	$m \cdot s^{-1}$
β_i	i^{th} delayed-neutron group fraction	
β	total delayed-neutron fraction $= \sum_i \beta_i$	
Σ_a	macroscopic absorption cross section	m^{-1}
Σ_f	macroscopic fission cross section	m^{-1}
Φ	flux	$m^{-2} \cdot s^{-1}$
ν	number of neutrons/fission	
λ_i	decay constant of i^{th} group of delayed neutrons	s^{-1}
ρ	reactivity	

Subscript

i	delayed-neutron group index
j,k	spatial node index
o	reference condition

1. INTRODUCTION

As nuclear reactors increase in size, spatial dynamics effects become more important. In particular, for a large reactor of the CANDU* type, the coolant circuit may be divided into several independent circuits, each feeding a sector of the core. If one of these circuits should rupture, i.e. a loss-of-coolant accident (LOCA), the positive void reactivity effect would create a large azimuthal flux tilt, as well as a rapidly increasing overall flux. This flux tilt could be accentuated further (in the worst case) by the action of the shutoff rods, if they were to enter a non-voiding core region first.

To investigate such problems, a spatial reactor model, based on one-speed diffusion theory, has been developed. At present, the model is restricted to

- a single dimension, divided into either 8 or 16 nodes,
- 2 or 3 delayed-neutron groups.

In the 8-node version, the model can be implemented all in parallel on a single AD/FIVE analog computer of the Dynamic Analysis Facility at CRNL [1]. For the 16-node version, the Hsu-Howe multiplexing technique [2] is used to switch the 8-node module back and forth between the two halves of the reactor.

The results for two LOCA-type cases are presented in this paper.

2. THEORY

The one-speed diffusion equation describing the neutron flux in a reactor can be written

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = \nabla . D \nabla \Phi - \Sigma_a \Phi + (1-\beta)\nu\Sigma_f \Phi + \sum_i \lambda_i C_i \qquad (1a)$$

with

$$\frac{\partial C_i}{\partial t} = \beta_i \nu \Sigma_f \Phi - \lambda_i C_i \qquad (1b)$$

---

*CANada Deuterium Uranium

where ν, β, λ, and v are constants and all other variables are functions of both time and space.

We now make three assumptions:

(1) The reactor properties are regionwise uniform.

(2) The flux is adequately represented by its values at the region centres, or nodes.

(3) The spatial derivative at each node can be replaced by a finite-difference form,

$$\left(\nabla.D\nabla\phi\right)_j \simeq D_j \sum_k a_{jk}\phi_k \tag{2}$$

Applying equations (1a,b) to node j and substituting equation (2) gives the usual finite-difference form of the diffusion equation. This can be rearranged, by collecting those terms associated with the delayed neutrons, to give

$$\frac{1}{v}\frac{d\phi_j}{dt} = D_j \sum_k a_{jk}\phi_k + (\nu\Sigma_{fj}-\Sigma_{aj})\phi_j + P_j \tag{3a}$$

where the net precursor contribution, P_j, is given by

$$P_j = -\sum_i \left(\beta_i \nu\Sigma_{fj}\phi_j - \lambda_i C_{ij}\right) \tag{3b}$$

and

$$\frac{dC_{ij}}{dt} = \beta_i \nu\Sigma_{fj}\phi_j - \lambda_i C_{ij} \tag{3c}$$

We assume that a static, stable reference condition exists for the reactor and this is found by setting the time derivatives to zero, i.e.

$$0 = D_j \sum_k a_{jk}\phi_{ko} + (\nu\Sigma_{fjo}-\Sigma_{ajo})\phi_{jo} \tag{4a}$$

$$C_{ijo} = \frac{\beta_i}{\lambda_i}\nu\Sigma_{fjo}\phi_{jo} \tag{4b}$$

where we have assumed that the diffusion coefficient is not time-varying. From equation (4a) we obtain

$$0 = D_j \sum_k a_{jk} \frac{\Phi_{ko}}{\Phi_{jo}} \Phi_j + (\nu\Sigma_{fjo} - \Sigma_{ajo})\Phi_j \tag{5}$$

Also, all cross-sections can be expressed as changes from this reference condition,

$$\Sigma_{aj} = \Sigma_{ajo} + \Delta\Sigma_{aj} \tag{6a}$$

$$\Sigma_{fj} = \Sigma_{fjo} + \Delta\Sigma_{fj} \tag{6b}$$

Combining these equations and normalizing to the reference condition gives

$$\frac{1}{v}\frac{d[\Phi_j]}{dt} = D_j \sum_k a_{jk} \frac{\Phi_{ko}}{\Phi_{jo}} \left([\Phi_k] - [\Phi_j]\right) + \left(\nu\Delta\Sigma_{fj} - \Delta\Sigma_{aj}\right)[\Phi_j] + \beta\nu\Sigma_{fjo}[P_j] \tag{7a}$$

$$[P_j] = \sum_i \frac{\beta_i}{\beta} \left\{\left(1 + \frac{\Delta\Sigma_{fj}}{\Sigma_{fjo}}\right)[\Phi_j] - [C_{ij}]\right\} \tag{7b}$$

$$\frac{d[C_{ij}]}{dt} = \lambda_i \left\{\left(1 + \frac{\Delta\Sigma_{fj}}{\Sigma_{fjo}}\right)[\Phi_j] - [C_{ij}]\right\} \tag{7c}$$

where we have used the notation that square brackets contain normalized variables, i.e.

$$[\Phi_j] = \frac{\Phi_j}{\Phi_{jo}} \tag{8}$$

and where, by definition,

$$P_{jo} = \beta\nu\Sigma_{fjo}\Phi_{jo} \tag{9}$$

Note that the same term appears on the right-hand sides of both equations (7b) and (7c).

If the change in fission cross-section is small compared to its reference value, then it can be neglected in equations (7b) and (7c), although it will still be significant in equation (7a). Thus, we obtain the equation set used in our model,

$$\frac{1}{v}\frac{d[\Phi_j]}{dt} = D_j \sum_k a_{jk} \frac{\Phi_{ko}}{\Phi_{jo}} \left\{[\Phi_k] - [\Phi_j]\right\} - \Delta\Sigma'_{aj}[\Phi_j] + \beta_i \nu\Sigma_{fjo}[P_j] \tag{10a}$$

$$[P_j] = -\sum_i \frac{\beta_i}{\beta} \left\{[\Phi_j] - [C_{ij}]\right\} \tag{10b}$$

$$\frac{d[C_{ij}]}{dt} = \lambda_i \left\{[\Phi_j] - [C_{ij}]\right\} \tag{10c}$$

where $$\Delta\Sigma'_{aj} = \Delta\Sigma_{aj} - \nu\Delta\Sigma_{fj} \tag{11}$$

The initial condition is usually chosen to be a multiple of the reference condition. Or conversely, the reference condition is chosen to be a multiple of the initial condition. Thus

$$[\Phi_j] = [C_{ij}] = I \tag{12}$$

independent of j, at time zero.

3. IMPLEMENTATION

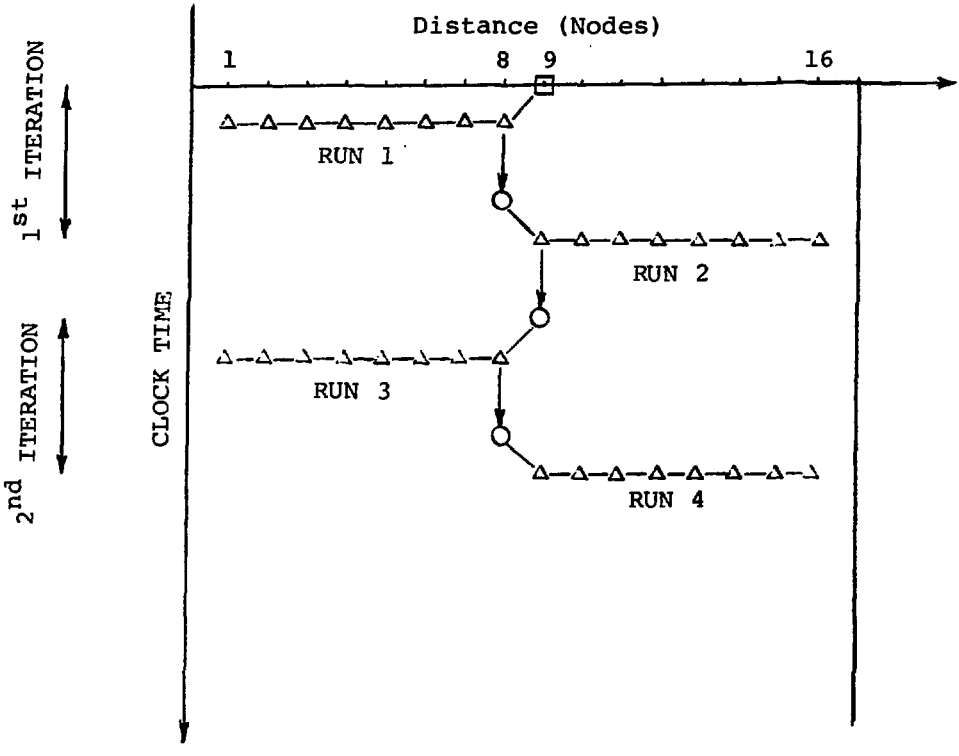
We have sufficient components on one AD/FIVE analog console to patch 8 nodes, each with two active delayed-neutron groups. A third inactive delayed group may also be included if, for the problem time of interest, its contribution remains practically constant (i.e. $1/\lambda_i$ is large compared to the problem time). One of the major sources of error in an 8-node model was thought to be the rather coarse mesh size. To increase the number of nodes to 16, it is necessary to partition the equation set into two identical (in form) halves, each representing 8 nodes of the reactor core. The Hsu-Howe multiplexing scheme [2] is used to switch back and forth between the two halves of the core. This is explained more fully with the aid of Figure 1, as follows:

(1) The model is set up to represent the left half of the core, for which it requires the flux at node 9 as a function of time. The latest estimate of this function is supplied by the digital computer, normally from the results of the previous run, but for the first run it is chosen to be zero. A run is made, and the flux at node 8, as a function of time, is recorded digitally.

(2) The model is set up to represent the right half of the core, for which it requires the flux at node 8, as a function of time. An estimate of this was just obtained in the previous run, and is supplied by the digital computer. A run is made, and the flux at node 9, as a function of time, is recorded digitally.

Runs (1) and (2) constitute an iteration. After many iterations the estimates of the fluxes will not change and the solution is obtained.

In our case, we use 100 points over 2 seconds to store the flux functions in the digital computer; linear interpolation is used during the playback.



△ Function $\phi_j(t)$ obtained from the run

□ Assumed function $\phi_9(t) = 0$

○ Function $\phi_j(t)$ obtained from a previous run

FIGURE 1 - MULTIPLEXING SCHEME FOR 16-NODE REACTOR

4. RESULTS

The model described above was tested against a LOCA-type benchmark problem originally specified with two energy groups and two delayed-neutron groups. The geometrical arrangement is shown in Figure 2, and the other data are given in Tables I and II. The rates of change of thermal neutron absorption are given in Table III. Whenever two perturbations overlap (region 3), the rates of change are additive; in other regions and at other times, there is no change in absorption.

The problem was solved using FORSIM [3], with 26 nodes, and the thermal flux shapes at three times (0.7, 1.0 and 1.5 seconds) are plotted as continuous curves in Figure 3.

The problem was converted to one energy group, having the properties shown in Table IV; note that we do not require neutron absorption cross-sections. The flux shape is chosen to be the same as the thermal flux shape in the two energy-group case. The changes in neutron absorption required to obtain the same static reactivity effect as in the original problem were also computed and these are shown as the adjusted values in Table III.

The problem was then run on the hybrid computer and the results are shown, for the same three times, as points on Figure 3. They are also available in continuous graphical form, as shown in Figure 4.

The problem specification was then altered to consider the case of the shutoff rods entering from one end of the reactor. (The end remote from the voiding region was chosen, as it is the worst case.) The final position and worth of the shutoff rods are the same as in the original problem; they also start and stop moving at the same times (0.6 and 1.5 s, respectively). The results for this case are shown in Figure 5, with the 26-node FORSIM results at 1.1 and 1.5 s plotted as solid curves; the hybrid model results are plotted as points. Continuous plotting against time instead of space gives the results shown in Figures 6 and 7.

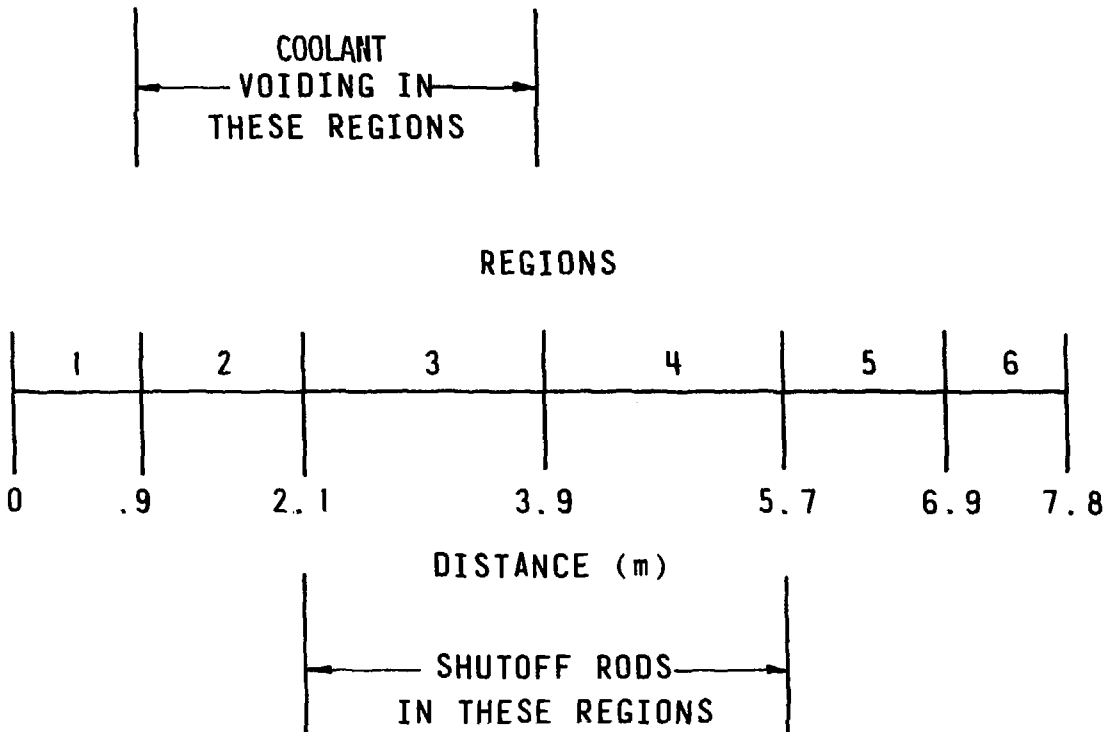


FIGURE 2 REACTOR GEOMETRY

TABLE I

PARAMETERS FOR 2 ENERGY GROUPS

v_1 = 100 000 $m \cdot s^{-1}$

v_2 = 3 000 $m \cdot s^{-1}$

Regions	Group	D_i (m)	Σ_{ai} (m^{-1})	Σ_{fi} (m^{-1})	$\Sigma_{1\to2}$ (m^{-1})
1,6	1	0.013 100	1.021	0.0	1.018
	2	0.008 695	0.023 35	0.0	
2,5	1	0.012 640	0.817 7	0.009 363	0.736 8
	2	0.009 328	0.403 1	0.188 46	
3,4	1	0.012 640	0.816 3	0.009 238	0.736 8
	2	0.009 328	0.410 6	0.185 92	

TABLE II

DELAYED-NEUTRON DATA

Group i	Fraction β_i	Decay Constant λ_i (s^{-1})
1	3.213×10^{-3}	0.062 97
2	4.556×10^{-3}	0.687 1

TABLE III

CHANGES IN NEUTRON ABSORPTION

Description		Voiding	Voiding	Shutoff rods
Time (s)		0<t<0.4	0.4<t<1.5	0.6<t<1.5
Region		2,3	2,3	3,4
Final Static Reactivity (mk)		6.08	7.70	-12.7
Rate of Change of Absorption ($m^{-1} \cdot s^{-1}$)	Specified	-0.010 000	-0.000 800	+0.010 000
	Adjusted 8-nodes	-0.009 448	-0.000 747	+0.010 954
	Adjusted 16-nodes	-0.009 580	-0.000 761	+0.009 820

TABLE IV

PARAMETERS FOR 1 ENERGY GROUP

$v = 3000\ m \cdot s^{-1}$

Regions	D (m)	Σ_f (m^{-1})
1,6	0.008 995	0.0
2,5	0.015 559	0.1698
3,4	0.015 686	0.1678

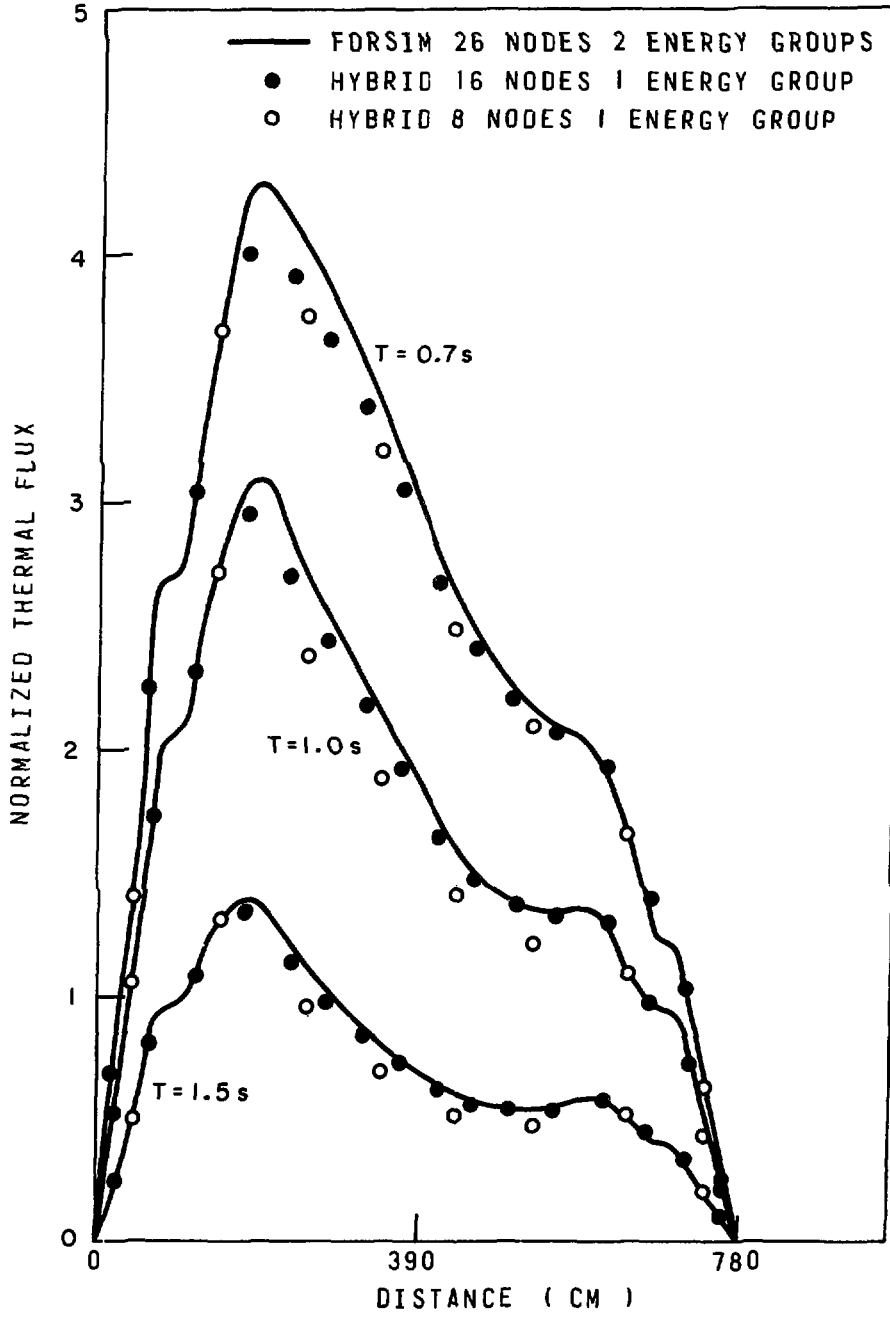


FIGURE 3 COMPARISON OF REACTOR MODELS
(ORIGINAL LOCA PROBLEM)

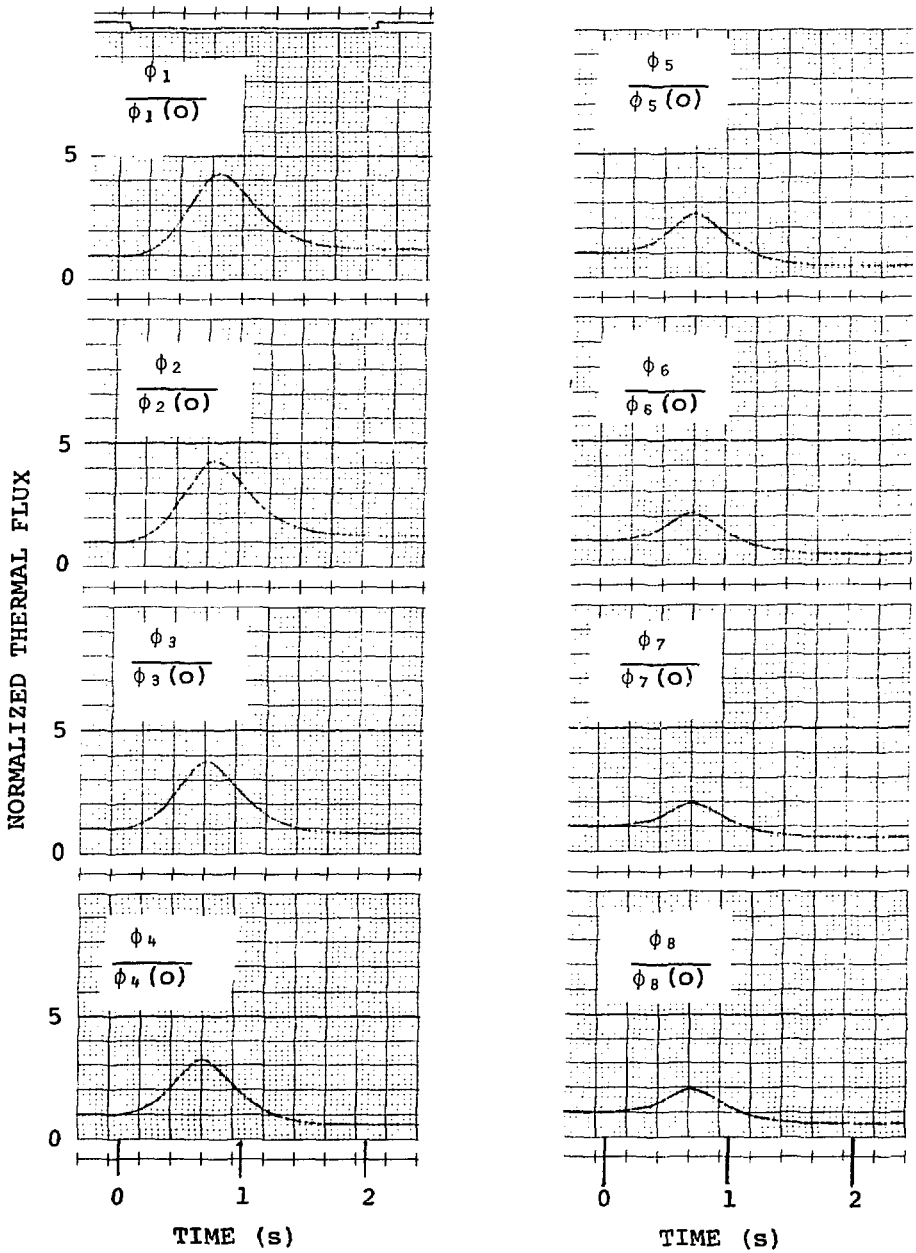


FIGURE 4 - NODAL FLUX TRANSIENTS FOR ORIGINAL LOCA PROBLEM - 8-NODE HYBRID SIMULATION

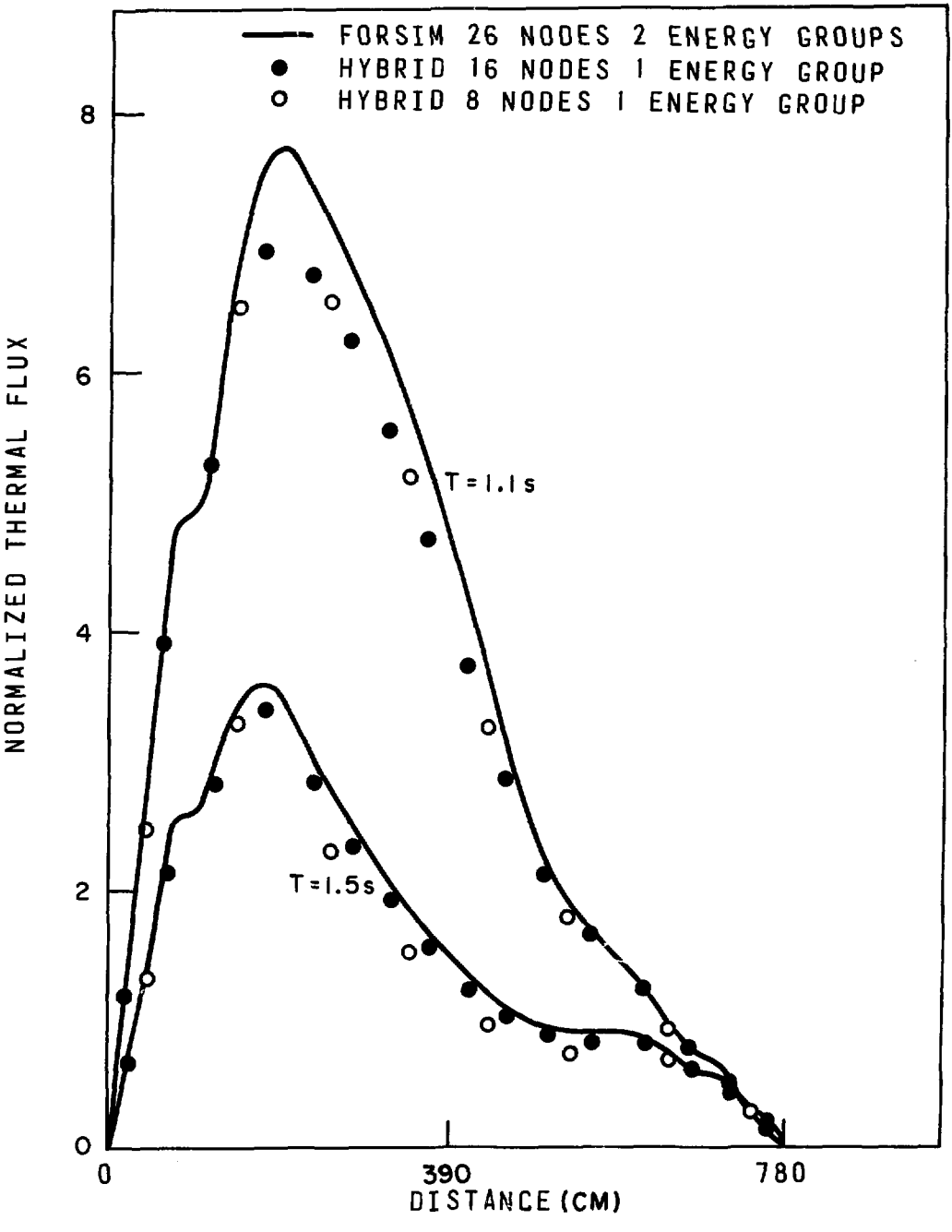


FIGURE 5 COMPARISON OF REACTOR MODELS
(MODIFIED LOCA PROBLEM)

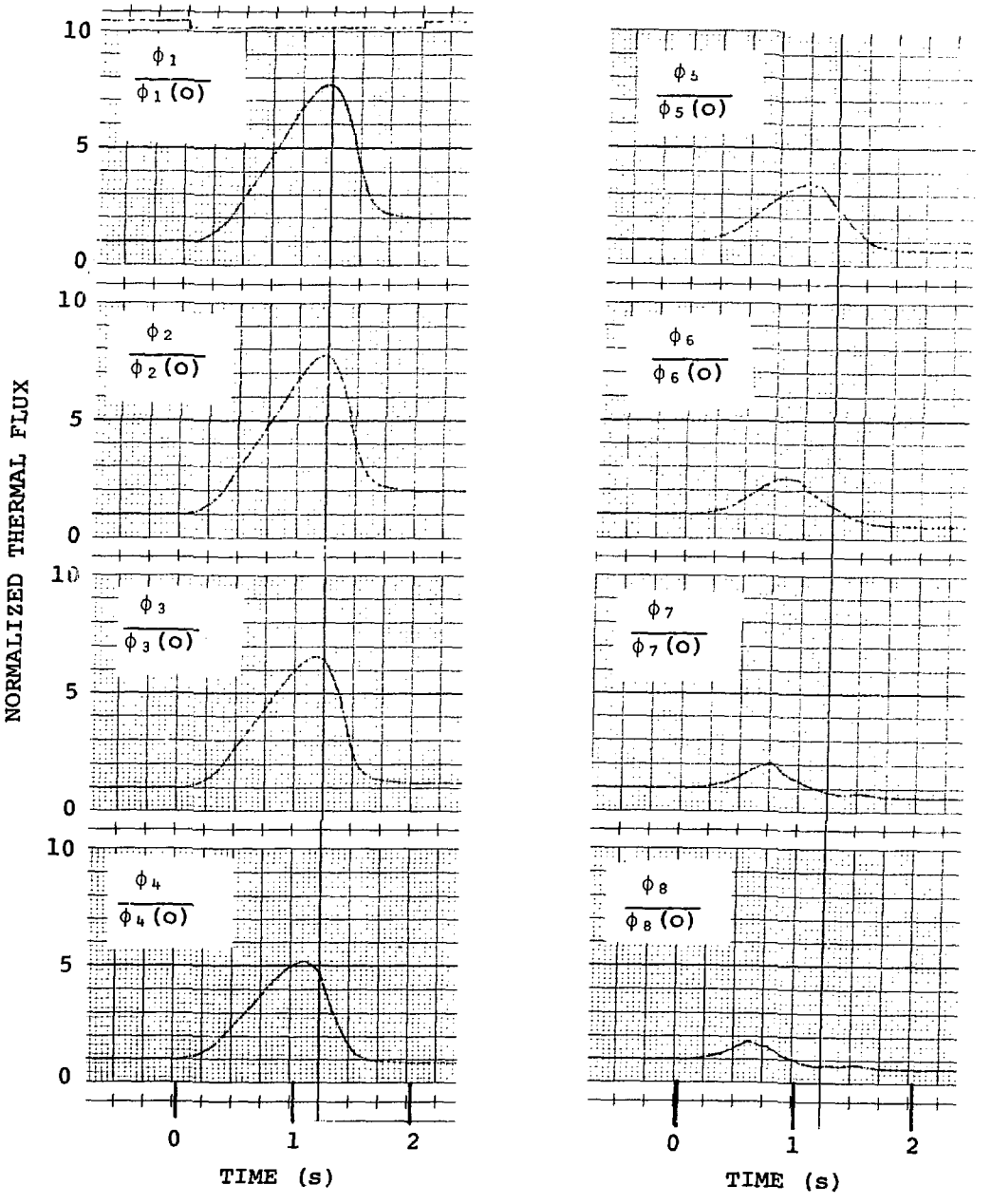


FIGURE 6 - NODAL FLUX TRANSIENTS FOR MODIFIED LOCA PROBLEM - 8-NODE HYBRID SIMULATION

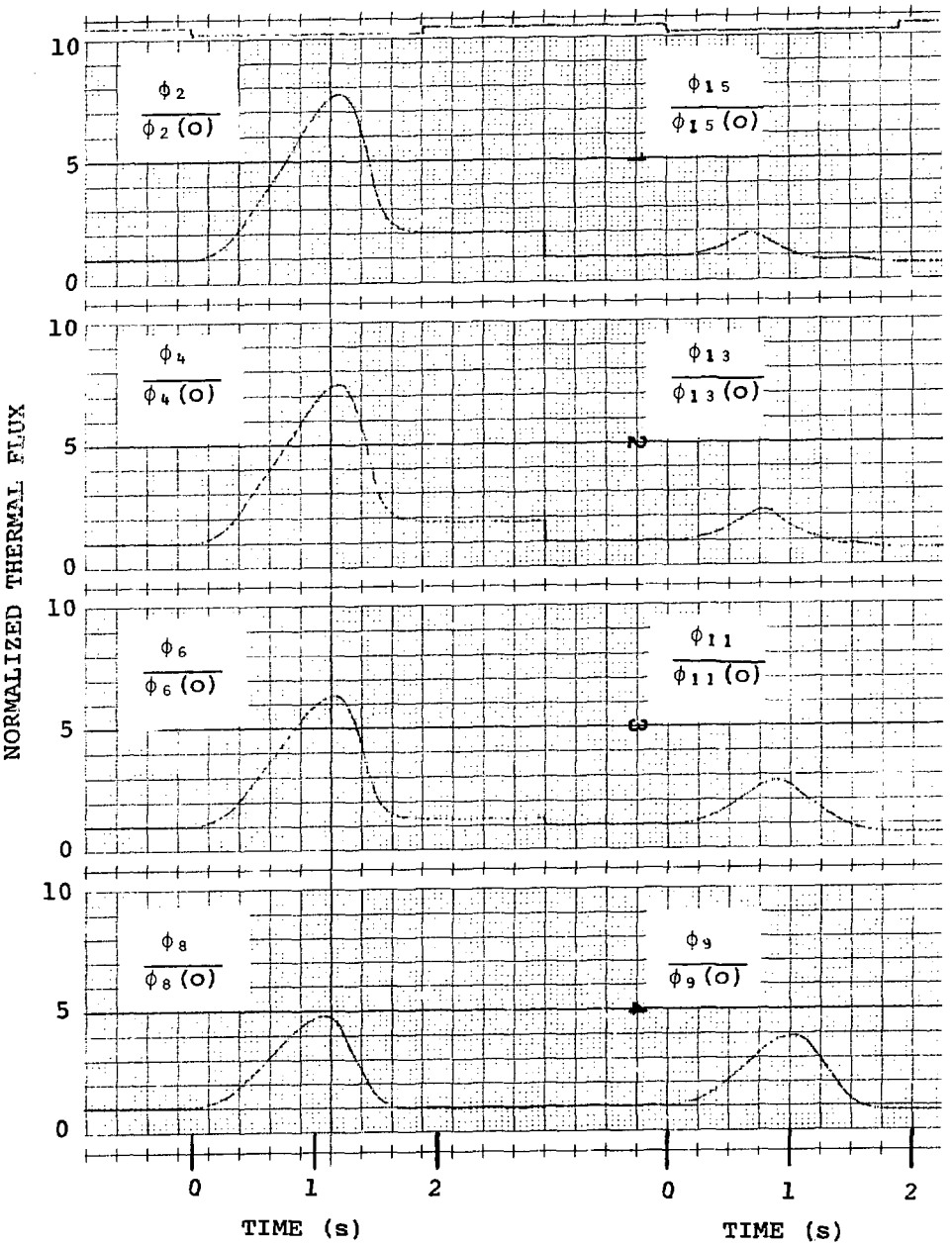


FIGURE 7 - NODAL FLUX TRANSIENTS FOR MODIFIED LOCA PROBLEM - 16-NODE HYBRID SIMULATION (SELECTED NODES)

Compared to the original problem, the flux peak is approximately twice as high and the flux tilts are also worse, as expected.

The disagreement between the relatively accurate FORSIM model and the hybrid simulation is about 10% for the 8-node version and 5% for the 16-node version. We believe there are three major sources of error in the hybrid model:

(1) the use of the one-speed approximation
(2) spatial discretization
(3) machine tolerances.

For the 8-node version, it is felt that spatial discretization is the major source of error. This error is practically eliminated in the 16-node version; however, the machine-produced errors are greater.

In comparing Figures 6 and 7, we see that the 8-node results exhibit slight discontinuities of slope ("scallops") which are not evident in the 16-node results. These scallops are the result of the shutoff rods passing node boundaries, i.e. a partial absorber acting over a large area gives results different from a strong absorber acting over a small area.

5. CONCLUSIONS

The finite-difference approximation to the one-speed neutron diffusion equation has been recast in a novel form, particularly well-suited for solving dynamics problems using a hybrid computer, as all variables appear in a normalized form, i.e. they are prescaled. The approach is also applicable to digital computation although the benefits may not be as great.

Normally, several preliminary steps are taken when solving a reactor dynamics problem:

(1) the value of ν is adjusted to make the reactor critical,
(2) the static flux shape is calculated,
(3) the properties (usually absorption cross-sections) are adjusted to obtain a "reasonable" flux shape, and the whole process is repeated.

This process ensures a set of parameters that yields a stable reactor with the required flux shape. However, this set is valid only for the particular nodal arrangement and finite-difference scheme used; the parameters are not necessarily universal constants of the reactor.

Using the derivation given in this report, the parameters required for a dynamic simulation are the "given" flux shape and a subset of the physics parameters (i.e. excluding the neutron absorption cross-section). The neutron absorption is implied, as that cross-section which makes the reactor stable with the desired flux shape. The fission cross-section enters the equations only via the delayed-neutron term, which is minor relative to the diffusion term in most cases. The key parameter, as far as the dynamics are concerned, is the combined change in absorption/fission cross-sections from the reference condition.

Two versions of a one-dimensional model based on the above approach have been implemented and tested against two LOCA-type benchmark problems. The results obtained agree to within $\sim$ 5% for the 16-node version, and $\sim$ 10% for the 8-node version, with those for a two-energy group, 26-node model run using FORSIM. However, these test cases are thought to be severe, as the flux tilts are large, and the power changes are rapid; better agreement would be expected for less severe transients. The good performance of the hybrid-computer model, compared to a considerably more complex digital model, is due to two factors:

(1) the use of the correct initial flux shape, obtained from an accurate static model
(2) the use of changes in neutron absorption which yield the correct reactivity.

The hybrid computer is readily available to the analysis group as a hands-on facility. This, coupled with the fact that the model is relatively fast, leads to a rapid solution turnaround, thus making the hybrid-computer model described here particularly useful for parametric or survey-type, calculations.

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