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ON A MODEL FOR BARYONS BASED ON A DIRAC EQUATION WITH CONFINING POTENTIALS *

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On a Model for Baryons Based on a Dirac Equation with Confining Potentials*

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Summary

An independent particle model for baryons is studied in which the quarks obey a Dirac equation with an average potential of the form $V(r) = \frac{1}{2}(1+\beta)(V_0 + \lambda r - \frac{\gamma}{r})$. A numerical solution is obtained for S-waves. Several properties of the $\frac{1}{2}^+$ baryons such as the ratio $(\frac{G_A}{G_V})_N$ for nucleons and baryon magnetic moments are analysed in terms of the model. A comparison with the case of a pure linear potential [1] and with a pure harmonic oscillator is made, showing that it is possible to obtain a better agreement with the data in the present case.

It was recently shown [1] that the properties of the low-lying S-wave baryons can be derived from an independent quark model in which the individual quarks in a baryon (qqq) obey a Dirac type equation with an "average" linear potential, defined in the baryon center of mass, of the form

$$V(r) = \frac{1}{2}(1+\beta)(V_0 + \lambda r), \tag{1}$$

where β is the usual Dirac matrix.

The peculiar features of the Dirac equation in consideration , for a generic $V(r)$, namely,

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$$\left[\underset{\sim}{\alpha}\underset{\sim}{p} + \beta m + \frac{1}{2}(1+\beta)V(r) - E\right]\psi(\underset{\sim}{x}) = 0, \qquad (2)$$

have been noticed before [2]. In particular, we mention the absence of spin-orbit splitting in the energy spectrum of Eq. (2). This is a salient aspect of the model, since that sort of splitting is known to be small for baryons.[3]

In view of appealing simplicity of the model and the interesting results obtained from it for the linear confining case, it appeared to us worthwhile to investigate how the results depend on the form of the potential $V(r)$. Having this is mind, we formulated explicitly the model for a generic potential and then specialized it for various cases, in particular for the potential

$$V(r) = V_0 + \lambda r - \frac{\gamma}{r}, \qquad (3)$$

recently suggested in the context of non-abelian colour gauge theory of strong interactions[4]. For the sake of comparison with the pure linear potential,[1] the harmonic oscillator case was also considered.

1. If we write the four-component ψ as $\psi = \begin{pmatrix}\psi_A\\ \psi_B\end{pmatrix}$, Eq.(1) reads

$$\begin{aligned}(E - m - V(r))\,\psi_A - \underset{\sim}{\sigma}\cdot\underset{\sim}{p}\,\psi_B &= 0\\ \underset{\sim}{\sigma}\cdot\underset{\sim}{p}\,\psi_A - (E-m)\,\psi_B &= 0\end{aligned} \qquad (4)$$

Than

$$\psi_B = (E+m)^{-1}\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p}\,\psi_A \quad . \qquad (5)$$

Substituting this equation in the first of Eqs.(4), one gets the Schrödinger-like equation

$$\left[\underset{\sim}{p}^2 - (E + m)(E - m - V(r)) \right] \psi_A = 0 \ . \qquad (6)$$

We confine ourselves to S-wave solution of Eq.(2): $\psi_A = \phi(r)\chi$, where ϕ is a normalized solution of Eq.(6) and χ is a Pauli spinor. Hence

$$\psi = N \begin{pmatrix} \phi \ \chi \\ (E+m)^{-1} \underset{\sim}{\sigma} \cdot \underset{\sim}{p} \ \phi \ \chi \end{pmatrix} \qquad (7)$$

where the normalization constant N is determined from

$$\int \psi^+(\underset{\sim}{x}) \psi(\underset{\sim}{x}) d^3x = 1 \ .$$

One gets

$$N^2 = \left[1 + \frac{\langle \underset{\sim}{p}^2 \rangle}{(E+m)^2} \right]^{-1} \qquad (8)$$

$$= \left[1 + \frac{1}{E+m} \ (E - m - \langle V(r) \rangle) \right]^{-1}$$

where the angular brackets mean expectation values with respect to the states $\phi(r)$.

2. Following the lines of Ref.1, we now develop the consequences of the model for the $\frac{1}{2}^+$ baryon octet. In particular, the ratio $(\frac{G_A}{G_V})_N$ for the β-decay of the neutron[5], the r.m.s radius of the proton, magnetic moments and the mass of the first excitation in S-wave above the fundamental level, will be considered.

The expression for the ratio of the axial and vector constants in the leptonic decay of the neutron has been already analysed in Ref.1. One has the result

$$\left(\frac{G_A}{G_V}\right)_N = \frac{5}{3} N^2 \left[1 - \frac{1}{3} \frac{1}{(E+m)^2} \langle p^2 \rangle\right]$$

$$= \frac{5}{9} (4N^2 - 1) \qquad (9)$$

It is remarkable that this ratio is expressed as a simple function of N^2 only. On the other hand, the dependence of N^2 on the form of $V(r)$ is explicitly seen in Eq.(8).

For the magnetic of the quark magnetic moment one gets[1] ($e_q = eQ_q$ being the charge of the quark)

$$\mu = e_q \frac{N^2}{E + m} \quad . \qquad (10)$$

Consequently, assuming additivity, one obtains for the magnetic moment of the baryon B, in nuclear magneton (n.m) units, the expression

$$\mu_B = 2 M_p \langle B\uparrow \left| \sum_{i=1}^{3} \frac{N_i^2}{E_i + m_i} Q_i \sigma_{3i} \right| B\uparrow \rangle . \qquad (11)$$

Assuming also that the S-wave baryon states are described by the well known states of the <u>56</u> irrep of SU(6), one gets for the proton magnetic moment

$$\mu_p = \frac{2M_p N^2}{E + m} \ n.m \quad . \qquad (12)$$

We notice that Eq.(11) may contain some amount of SU(6) breaking as far as one takes $m_\lambda \neq m$. Estimations of the λ-quark mass will also be given below.

For the proton r.m.s radius, defined by the integral

$$\langle r^2 \rangle_p = \int \psi^+ r^2 \psi \, d^3x \qquad (13)$$

one arrives at the expression

$$\langle r^2\rangle_P = N^2 \; \{\langle r^2\rangle + \frac{1}{E+m}\Big[(E-m)\langle r^2\rangle - \langle r^2 V\rangle + \frac{3}{E+m}\Big]\} \quad (14)$$

with

$$\langle r^2\rangle = \int \phi^* r^2 \, \phi \, d^3x$$
$$\langle r^2 V\rangle = \int \phi^* \, r^2 \, V(r) \, \phi \, d^3x \quad . \quad (15)$$

4. We now specialize the above results for the case of the confining potential Eq.(3).

<u>Normalization</u>:

$$N^2 = \Big[1 + \frac{\beta^2}{x^2} f(a)\Big]^{-1} \quad (16)$$

where

$$f(a) = 0.77937 + 0.24998a + 0.19652a^2 \quad . \quad (17)$$

The constant a in Eq.(17) is a dimensionless constant defined by

$$a = \frac{x\gamma}{\beta} \; , \; \beta = (x\lambda)^{\frac{1}{3}} \quad , \; x = E + m \; . \quad (18)$$

<u>Proton r.m.s radius</u>:

$$\langle r^2\rangle_P = \frac{N^2}{x^2} \frac{1}{\beta^2}\Big[x^2 + F(a)\beta^2\Big] \; , \quad (19)$$

where

$$f(a) = 3.54535 - 0.37212a + 0.53333a^2 \; . \quad (20)$$

Quark energy formula:

$$E = m + V_0 + E(a)\,\frac{\beta^2}{x} \qquad (21)$$

where

$$E(a) = 2.33811 - 0.86286\,a\,. \qquad (22)$$

The above results were obtained from a numerical integration of the non-relativistic wave-equation with a linear plus Coulomb-like potential.[5]

The parameters involved in the present case were determined as follows . We fixed N^2 and x, Eq.(18), by the fitting of $(\frac{G_A}{G_V})_N$=1.25 and μ_p = 2.79 n.m. The value of the parameter γ, instead, was taken γ = 0.1 . This value is nearly that expected from the non--abelian colour gauge theory of strong interactions for the baryon case ($\gamma = \frac{2}{3}\alpha_s$), being half of the corresponding value for mesons ($\frac{4}{3}\,\alpha_s$ = 0.2). In this way, the following results were obtained:

Potential $V(r) = V_0 + \lambda r - \frac{\gamma}{r}$

Input: $(\frac{G_A}{G_V})_N = 1.25$, $\mu_p = 2.79$ n.m., $\gamma = 0.1$. (23)

$N^2 = 0.8125$	$x = 0.582\ M_p$	(24)
$V_0 = -0.279\ M_p$	$\lambda = 0.0519\ M_p^2$	(25)
$m = 0.249\ M_p$	$\langle r^2\rangle_p = 0.37\ M_\pi^{-2}$	(26)
$m_\lambda = 0.559\ M_p$	$M_R = 1.219\ GeV$	(27)

The mass of the strange quark m_λ was determined by using the quark energy formula Eq.(21) for E_λ given by $E_\lambda = M_A - \frac{2}{3} M_P$, where M_A is the average of the masses of the baryons Λ^0 and Σ. This corresponds to an average violation of SU(3) such that to the masses of the Λ^0 and Σ are given the value M_A.

The mass M_R appearing in Eq.(27) represents the mass of excited configuration $(2S)^1$ $(1S)^2$, in which one of the S-wave quarks is excited to the second S-wave level, of energy E_2:

$$M_R = \frac{2}{3} M_P + E_2 \ . \tag{28}$$

It was conjectured in Ref.1 that this excited configuration is a possible assignment for the Roper resonance, which has the same quantum numbers of the nucleon and mass $M \approx 1.4 GeV$.

For the take comparison, we also give the corresponding results for the linear case of Ref.1 and also for the case of a pure harmonic oscillator potential.

<u>Linear potential[1]</u>: $V = V_0 + \lambda r$.

Input : $N^2 = 0.75$, $\quad x = \frac{M_P}{3} + m = 0.58 \ M_P$ (29)

$(\frac{G_A}{G_V})_N = 1.11 \quad \mu_P = 2.59 \ n.m$ (30)

$V_0 = -0.5 \ M_P \quad \lambda = 0.0964 \ M_P^2$ (31)

$m = 0.25 \ M_P \quad \langle r^2 \rangle_P = 0.48 \ M_\pi^{-2}$ (32)

$m_\lambda = 0.62 \ M_P \quad M_R = 1.2 \ GeV$ (33)

<u>Harmonic oscillator potential</u> $V(r) = V_0 + \frac{r^2}{2mR^4}$

Input : the same as in Eq.(29) above.

$(\frac{G_A}{G_V})_N = 1.11 \quad \mu_P = 2.59 \ n.m$ (34)

$V_0 = -0.31 \ M_P \quad R = 3.66 \ M_P^{-1}$ (35)

$$m = 0.25\ M_P \qquad \langle r^2\rangle_P = 0.70\ M_\pi^{-2} \qquad (36)$$

$$m_\lambda = 0.79\ M_P \qquad M_R = 1.47\ GeV \qquad (37)$$

Finally, in the following Table, are given the results for the magnetic moments of the $\frac{1}{2}^+$ octet baryons for the confining potential given by Eq.(3). They are to be compared with the corresponding results for the linear case of Ref.1 and for the harmonic oscillator potential.

Table: Magnetic moments of the $\frac{1}{2}^+$ baryons (in *n.m*)

B	μ_{th} Linear Pot.[1]	μ_{th} Harm. Oscill.	μ_{th} Pot. Eq.(3)	μ_{exp}
P	2.59	2.59	2.79 (input)	2.79
N	-1.73	-1.73	-1.86	-1.913
Λ^0	-0.49	-0.47	-0.54	-0.67 ± 0.06
Σ^+	2.46	2.45	2.66	2.62 ± 0.41
Σ^0	0.74	0.73	0.80	-
Σ^-	-0.99	-0.99	-1.06	-1.48 ± 0.37
Ξ^0	-1.23	-1.20	-1.33	-
Ξ^-	-0.37	-0.33	-0.41	-1.93 ± 0.75

In conclusion, we remark that reasonable results are obtained in all the three cases discussed. However, a better overall agreement with the data was achieved with the potential of Eq. (3).

The present model, when implemented by the introduction of quark-quark residual interactions, may constitute an interesting alternative in the direction of a more complete relativistic treatment of baryons as (qqq) composites.

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