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ON THE DISTINCTION BETWEEN DENSITY AND LUMINOSITY EVOLUTION

by

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ABSTRACT

It is shown that the assumptions of pure density evolution and pure luminosity evolution lead to observable differences in the distribution of sources for all convergent luminosity functions. The proof given is valid for sources with an arbitrary number of intrinsic luminosities (e.g., optical, infrared, and radio) and also holds in the special cases of mixed evolution that are considered.

1. INTRODUCTION

Is the apparent evolution of quasars the result of their being more sources at earlier epochs (with the shape of the luminosity function remaining constant)? Or is the luminosity of each source an increasing universal function of redshift (with the total number of sources remaining constant)? These two extreme possibilities are known, respectively, as pure density and pure luminosity evolution. It is obviously of great interest to know which possibility, if either, correctly describes the observed evolution. However, there has been considerable discussion in the literature concerning whether or not it is possible, even in principle, to distinguish observationally between the two forms of evolution (cf. Longair and Scheuer 1970 and Scheuer 1975). The purpose of the present note is to clarify the theoretical situation by providing a mathematical proof that pure density and pure luminosity evolution can always be distinguished with the aid of sufficiently accurate and numerous observations.

It seems intuitively clear that one can generally distinguish between pure density and pure luminosity evolution. The following is an in-principle operational method for distinguishing between the two forms of evolution. Suppose, for simplicity, that we consider only one kind of luminosity (e.g., optical or radio luminosity) and do not explicitly consider variability. (For variable sources, luminosities are to be interpreted as average luminosities for archivial identifications and instantaneous luminosities for contemporary identifications see Bahcall, 1977). With sufficiently many and accurate observations, one can determine at different redshifts the empirical luminosity function in the luminosity range permitted by the sensitivity of the available equipment. The appropriate luminosity functions for different redshifts can then be compared directly in the regions in which they overlap. If pure density evolution obtains, the shape of the luminosity functions will be found to be independent of redshift. If pure luminosity evolution is occurring, the same number of sources will be shifted to successively larger (or smaller) luminosities as the redshift considered increases.

There are, in fact, discussions in the literature that limit the amount of pure optical luminosity evolution for quasars. For example, Schmidt (1972) showed that two specific assumed forms of pure luminosity evolution were in conflict with the available data on the QSO redshift distribution and number-magnitude relation. Bahcall and Turner (1976) showed that the intrinsic luminosities of the brightest quasars observed in different redshift bins are approximately constant. However, Longair and Scheuer (1970) (in an important paper that elucidated several aspects of the V/V_{MAX}-test) gave an apparent mathematical proof that pure density and pure luminosity evolution were indistinguishable. Our results contradict this conclusion of Longair and Scheuer.

For simplicity, we consider first the case where all sources are characterized by only one luminosity. In §IIA we introduce the relevant definitions and restate the Longair-Scheuer criterion for indistinguishability. In §IIB, we give some simple examples of assumed present-epoch luminosity functions for which the distinction between density and luminosity evolution leads to observable differences. In §IIC, a brief mathematical proof is given which shows that pure density and pure luminosity evolution always predict, for convergent luminosity functions, differences in the distribution of sources as a function of redshift and luminosity that are in principle observable.

We generalize, in §III, the discussion to the case in which each source is characterized by more than one luminosity (e.g., by radio, infrared and optical luminosities). We also consider examples in which sources evolve by pure density evolution with respect to one luminosity and by pure luminosity evolution with respect to another luminosity. In all the cases we consider, there are observable differences between the predictions based on the different assumptions for the nature of the evolution.

II. SOURCES CHARACTERIZED BY A SINGLE LUMINOSITY

A. Definition

Let $\rho(L,Z)$ be the density in comoving coordinates of sources with luminosity between L and L+dL in the volume element between V(Z) and V(Z)+dV(Z). Here Z is the redshift that would be measured by an observer at rest on the earth. Pure density evolution is defined by the relation

$$\rho(L,Z) = \rho_0(L)\psi(Z) \ , \qquad (1)$$

where $\rho_0(L)$ is the local (i.e., Z=0) luminosity function and ψ is a function only of Z (and not L). Pure luminosity evolution implies that the total number of sources remains constant with Z but that all the sources with luminosity L at the present epoch had luminosity $L\phi(Z)$ at redshift Z. Therefore the number density of sources with redshift Z and luminosity L is $dn(Z) \equiv \rho(L,Z)dL = \rho_0(L')dL'$, where $L' = (L/\phi(Z))$. Thus for pure luminosity evolution

$$dn(Z) = \rho_0(L/\phi(Z))\frac{dL}{\phi(Z)} \ . \qquad (2)$$

Comparing eqs. (1) and (2), one finds that pure density evolution and pure luminosity evolution are equivalent if and only if the function

$$\Psi \equiv \frac{1}{\rho_0(L)} \ \frac{\rho_0(L/\phi(Z))}{\phi(Z)} \qquad (3)$$

depends only on Z (i.e., independent of L). Equation (3) was derived by Longair and Scheuer (1970).

B. Examples

Longair and Scheuer (1970) noted that if the luminosity function is a power law, $\rho_0(L) \equiv AL^{-m}$,

$$\Psi \equiv \phi^{m-1}(Z) \ , \qquad (4)$$

i.e., Ψ depends only on Z. However, no luminosity function that actually occurs in nature can be a pure power law since any power law diverges at either very large or very small luminosities.

It is easy to see that Ψ does depend on L in specific cases. For example, if $\rho_o(L) = CL/(1+\alpha L^2)$, then

$$\Psi = \frac{1+\alpha(L/\phi)^2}{1+\alpha L^2} \quad . \tag{5}$$

If $\rho_o(L) = C \exp L/L_*$, then

$$\Psi = \phi^{-1}(Z) \exp\left[\frac{L}{L_*}(\phi^{-1}(Z)-1)\right] \quad . \tag{6}$$

C. Proof

We now show that the function Ψ defined by eq.(3) is independent of L if and only if $\rho_o(L)$ is a pure power law. The first part of this lemma is established by eq.(4). Suppose then that Ψ depends only on Z. We want to prove that ρ_o is a power law. From eq.(3) we have by assumption

$$\rho_o(\lambda L) = h(\lambda)\rho_o(L) \quad , \tag{7}$$

where h is some function of the parameter λ and we have used the fact that $\phi(Z)$ is a single-valued function of Z.

Equation (7) implies that the function $h(\lambda)$ satisfies

$$h(\lambda\lambda') = h(\lambda)h(\lambda') \quad . \tag{8}$$

Let

$$K(\lambda) \equiv \ln h(e^{+\lambda}) \quad . \tag{9}$$

Then K satisfies

$$K(\lambda+\lambda') = K(\lambda)+K(\lambda') \quad . \tag{10}$$

The <u>unique</u> solution to eq.(10) is

$$K(\lambda) = a\lambda \quad , \tag{11}$$

where a is some constant. Equation (11) is easily established by comparing the derivatives with respect to λ and λ' of $K(\lambda+\lambda')$, as computed from eq.(10). Combining eqs.(7), (9) and (11), one finds

$$\rho_o(\lambda L) = \lambda^a \rho_o(L) \quad . \tag{12}$$

Equation (12) implies that ρ_o is a power-law in L, q.e.d.

III. GENERALIZATION TO SEVERAL INTRINSIC LUMINOSITIES

It is easy to generalize the previous discussion to the case where the individual sources are characterized by several intrinsic luminosities, L_1, L_2, ..., L_N. The condition for pure density evolution is then

$$\rho(L_1,L_2,\ldots,L_N,Z) = \rho_o(L_1,L_2,\ldots,L_N)\psi(Z) \quad . \tag{13}$$

The condition for pure luminosity evolution may be written

$$dn(Z) = \rho_o\left(L_1/\phi_1(Z),\ L_2/\phi_2(Z),\ \ldots,\ L_N/\phi_N(Z)\right)\prod_{i=1}^{N} \frac{dL_i}{\phi_i(Z)} \ , \tag{14}$$

where dn(Z) is the number of sources at redshift Z with L, between L_1 and L_I+dL_1, etc. Pure density and luminosity evolution are indistinguishable if and only if the function

$$\Psi \equiv \frac{\rho_o(L_1/\phi_1(Z),\ L_2/\phi_2(Z),\ldots)}{\rho_o(L_1,L_2,\ldots)\prod_{i=1}^{N}\phi_i(Z)} \tag{15}$$

depends only on Z. Equation (15) is the generalization of eq.(3) of §II (eq.(4) of Longair and Scheuer 1970). Note that if the luminosity function is a power-law in all of the luminosities,

$$\rho_0(L_1, L_2, \ldots, L_N) = A\prod_{i=1}^{N} L_i^{m_i} \quad , \tag{16}$$

then Ψ is indeed dependent only on Z (cf. eq.(4)). However, this solution is unphysical because the function under consideration diverges, for each luminosity, at either very large or very small luminosities.

The only functional form for ρ_0 that makes the right-hand side of eq.(15) independent of the luminositites is the unphysical power-law solution given in eq.(16). The proof is similar to that given for the case of luminosity in §IIC and proceeds by considering each luminosity separately, holding all the others fixed.

It is also possible to distinguish pure density evolution from various combinations of density and luminosity evolution. Suppose, for example, that the evolution in one luminosity occurs by an increase in density (conserving the shape of the luminosity function) and in another luminosity occurs by increasing all luminosities as a function of redshift. Then, if the two luminosity functions are separable

$$dn(Z) = \rho_0(L_1)\Psi_1(Z)\chi_0\big(L_2/\Phi_2(Z)\big)dL_1 dL_2/\Phi_2(Z) \quad . \tag{17}$$

Equating the number densities from equations (13) and (17) one finds that equivalence between pure density evolution and the partial-density evolution, partial-luminosity evolution defined by eq.(17) requires that

$$\Psi_{1,2} = \frac{\psi_1(Z)\ \chi_o\left(L_2/\phi_2(Z)\right)}{\phi_2(Z)\chi_o(L_2)} \tag{18}$$

be a function only of Z. Exactly the same proof as given in §IIC shows that there are no physically acceptable solutions of eq.(18) for $\Psi_{1,2}$ only a function of Z.

If the luminosity evolution has the form (cf. Schmidt 1970)

$$dn = \rho_o(L_1)\psi_1(Z)\chi(L_2/\phi_2(Z)L_1)dL_1dL_2/\phi_2(Z)\ , \tag{19}$$

one obtains an equation exactly the same as (18) with, however, L_2 replaced by L_2/L_1. Once again, a similar proof to those discussed above shows that the evolutionary forms (13) and (19) can be distinguished with sufficient data.

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