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Application of the First Approximation of the K-Harmonics Method to the 0^+ States of ^{16}O*

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Abstract: The energy levels of the 0^+ states, the charge form factor and the root mean square charge radius of the ^{16}O were calculated in the first approximation of the K-harmonics method. The calculation were done for six different potentials. The results obtained for the ground state energy, charge form factor and rms charge radius are in agreement with the experimental results, but this is not the case for the energies of the 0^+ excited states.

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1. Introduction

A method to solve the non-relativistic quantum mechanical A-body problem starting only from the explicit knowledge of the nucleon - -nucleon potential was developed by Simonov and Badalyan[1-3] in 1966. This method is nowadays know as the K-harmonics method. Its starting point consists in expanding the wave function of the system, in the C.M. frame, in terms of the so-called K-harmonics which constitute a complete set of totally antisymmetric angular functions, depending also on the spin and isospin variables. The coefficients of such an expansion are functions of the radial variable of the vector space $E_{3(A-1)}$ spanned by the (A-1) relative vectors of the A nucleons. The determination of these radial functions is obtained by integrating an eigenvalue system of second order differential equations. These equations have the same structure of the radial Schrödinger equation in which the interaction term appears as an effective potential, resulting from an angular mean of the two-body interactions.

Concerning the K-harmonics, they are totally antisymmetric under the exchange of the coordinates of any pair of nucleons, what implies that the Pauli-Principle be buit-in in the method. The spatial parts of the K-harmonics are the angular parts of harmonic and homogeneous polynomials of degree K in the space relative variables of the A nucleons. Due to the antisymmetric of the K-harmonics there exists for each nucleus a minimum value which K can assume. The successive approximations

of the method are obtained by considering, in the wave function expansion and in the eigenvalue system of differential equations which determines the radial functions and the energy eigenvalue of the bound states, only the K-harmonics with $K=K_{min}, K_{min}+1, \ldots$.

The $K=K_{min}$ K-harmonics can be obtained by filling a Slater determinant with A orbitals whose space parts are harmonic and homogeneous polynomials in such a way that the sum of the degrees of homogeneity of the orbitals be equal to K_{min}. Then the resulting polynomial is divided by $\rho^{K_{min}}$, where ρ is the radial variable of $E_{3(A-1)}$.

The K-harmonics method has been applied with great success to light nuclei such as ^{3}H, ^{3}He and ^{4}He (Refs 1-10) to all orders of approximations with an excellent convergence rate. It was also applied to less light nuclei such as ^{10}He, $^{15-17}O$ and ^{40}Ca (Refs 11-13) in first approximation using Baz and Zukov[14] formulas. Since Baz and Zukov formulas are approximate, we prefered to use Gorbatov[15] formulas which, for the first approximation of the K-harmonics method, are exact.

In the present paper we applied the first (also called basic) approximation of the K-harmonics method to the nucleus of ^{16}O taken as a system of 8 protons and 8 neutrons interacting through nuclear and Coulomb two-body potentials. The aim of this paper is to obtain the spectrum of the 0^{+} states of ^{16}O and also the charge form factor and the root mean square charge radius. Calculations were done for six

different two body nuclear potentials.

In Section 2 the general formulas of the two-body matrix elements and some other ones relevant to this paper are presented. In Section 3 we determine the radial functions of the 0^+ states of ^{16}O. In Section 4 expressions for the charge form factor and the rms charge radius are obtained. The results obtained are tabulated in Table 2, plotted in Figs 1 to 3 and discussed in Section 5.

2 - Two-Body Matrix Elements

The K_{min} K-harmonics are given by

$$U_{K_{min}}(\Omega) = \frac{B}{\rho^{K_{min}}} \det \left| \Phi_i(\underset{\sim}{\rho}_k) \right| \qquad (2.1)$$

where $\underset{\sim}{\rho}_k$ is the relative vector of the $k^{\underline{th}}$ nucleon and i is a set of 5 indices, three of them accounting for the spatial degrees of freedom and the other two refer to the spin and isospin. Following Gorbatov[15], we will take as orbitals in (2.1) the polynomials

$$\Phi_i(\underset{\sim}{\rho}_j) = (\rho_{xj})^{a_i} (\rho_{yj})^{b_i} (\rho_{zj})^{c_i} \alpha_{\mu_i \tau_i(j)}, \qquad (2.2)$$

where ρ_{xj}, ρ_{yj} and ρ_{zj} are the cartesian coordinates of $\underset{\sim}{\rho}_j$, $\alpha_{\mu_i \tau_i(j)}$ accounts for the spin-isospin degree of freedom of particle j, and a_i, b_i, c_i are non-negative integers.

The orbitals (2.2) used to fill the Slater Determinant in (2.1) are such that

$$\sum_{i=1}^{A} (a_i + b_i + c_i) = K_{min}. \tag{2.3}$$

For the ^{16}O we have $K_{min}=12$ and the only values allowed for the triple (a_i, b_i, c_i) are

$$(0,0,0),\ (1,0,0),\ (0,1,0),\ (0,0,1). \tag{2.4}$$

In order that $U_{K_{min}}(\Omega)$ be normalized in the unit sphere of $E_{3(A-1)}$, the constant B must have its square given by[15]

$$B^2 = \frac{2^{K_{min}-1}\,\Gamma(K_{min}+3(A-1)/2)}{A!\prod_{i=1}^{A} a_i!\,b_i!\,c_i!} \tag{2.5}$$

We will consider in this paper two-body nuclear potentials of the general form

$$V_{ij}(r) = v_{33}(r)\,P_{\mu}^{(+)}P_{\tau}^{(+)} + v_{31}(r)\,P_{\mu}^{(+)}P_{\tau}^{(-)} + v_{13}(r)\,P_{\mu}^{(-)}P_{\tau}^{(+)} + v_{11}(r)\,P_{\mu}^{(-)}P_{\tau}^{(-)}, \tag{2.6}$$

where $r = |\underset{m}{\rho}_i - \underset{m}{\rho}_j|$ and $P_{\mu}^{(+)}$ and $P_{\mu}^{(-)}$ are projection operators of the triplet and singlet states of the spin of the pair (i,j) of nucleons and similirarly for $P_{\tau}^{(\pm)}$ concerning to the isospin. Using Eq.26 of Ref.15 one obtains, for the effective potential due to the interaction (2.6), for the ^{16}O,

$$W(\rho) = \int U^{\dagger}_{K_{min}}(\Omega)\Big[\sum_{i<j=1}^{A} V_{ij}(|\underset{m}{\rho_i} - \underset{m}{\rho_j}|)\Big] U_{K_{min}}(\Omega)\, d\Omega =$$

$$= G\{23808\,[Q^{(30)}_{13}(\rho) + Q^{(30)}_{31}(\rho)] + 1024\,[9\,Q^{(31)}_{33}(\rho) + Q^{(31)}_{11}(\rho)] -$$

$$- 48384\,[Q^{(31)}_{13}(\rho) + Q^{(31)}_{31}(\rho)] + 24762\,[Q^{(32)}_{13}(\rho) + Q^{(32)}_{31}(\rho)] - \qquad (2.7)$$

$$- 1024\,[9\,Q^{(32)}_{33}(\rho) + Q^{(32)}_{11}(\rho)]\},$$

where
$$G = 67!!/(32!\,2^{35}) \approx 54\,, \qquad (2.8)$$

$$Q^{(n)}_{rs}(\rho) = \int_0^1 y^2(1-y^2)^n v_{rs}(\sqrt{2}\,\rho y)\, dy\,, \quad r,s = 1,3\,. \qquad (2.9)$$

For the Coulomb interaction

$$V_{ij}(|\underset{m}{\rho_i} - \underset{m}{\rho_j}|) = \frac{e^2 Q_i Q_j}{|\underset{m}{\rho_i} - \underset{m}{\rho_j}|}\,, \qquad (2.10)$$

where eQ_i is the charge of the $i^{\underline{th}}$ nucleon, one obtains[16], for the ^{16}O,

$$W(\rho) = \int U^{\dagger}_{K_{min}}(\Omega)\Big[\sum_{i<j=1}^{A} V_{ij}(|\underset{m}{\rho_i} - \underset{m}{\rho_j}|)\Big] U_{K_{min}}(\Omega)\, d\Omega = \frac{67!!}{33!\,2^{34}}\,\frac{e^2}{2\sqrt{2}\,\rho}\,. \qquad (2.11)$$

We used in this paper three kinds of two-body nuclear potentials:

i) $$V_{ij}(r) = \alpha_1 e^{-(r/\beta_1)^2}(1 - m_1 + m_1 P_x) + \alpha_2 e^{-(r/\beta_2)^2}(1 - m_2 + m_2 P_x)\,, \qquad (2.12)$$

ii) $$V_{ij}(r) = \alpha\, e^{-(r/\beta)^2}(1 - m + m P_x) + A\,\delta(r)\,, \qquad (2.13)$$

iii) $$V_{ij}(r) = \sum_k \frac{\alpha_k \beta_k}{r} e^{r/\beta_k}\,, \qquad (2.14)$$

where P_x is the Majorana exchange operator and $r = |\underset{m}{\rho_i} - \underset{m}{\rho_j}|$. The para-

meters of these potential are given in Tables 1.1-1.3. For Gaussian, Yukawa and delta potentials, respectively, the integrals (2.9) assume, the forms

$$Q^{(n)}(\rho) = \int_0^1 e^{-2\rho^2 y^2/\beta^2} y^2 (1-y^2)^n \, dy, \tag{2.15}$$

$$Q^{(n)}(\rho) = \int_0^1 e^{-\rho\sqrt{2}\, y/\beta} y (1-y^2)^n \, dy, \tag{2.16}$$

$$Q^{(n)}(\rho) = \int_0^1 y^2 (1-y^2)^n \delta(\rho\sqrt{2}\, y) \, dy. \tag{2.17}$$

We calculated $W(\rho)$ for all potentials (2.12)-(2.14) whose coefficients are given in Tables (1.1)-(1.3). Six of them are plotted in Fig.1.

3. Determination of the Radial Function

For double magic nuclei, such as ^{16}O, there is only one K-harmonic with $K=K_{min}$. Then, the wave function of the ^{16}O, in the basic approximation of the K-harmonics method, is given by

$$\Psi(\underset{m}{\rho_1}, \underset{m}{\rho_2}, \cdots, \underset{m}{\rho_A}) = \rho^{-22} \chi(\rho) U_{K_{min}}(\Omega), \tag{3.1}$$

where $U_{K_{min}}(\Omega)$ is given by (2.1) and $\chi(\rho)$ is obtained by solving the differential equation

$$\left\{ -\frac{\hbar^2}{2m} \left[\frac{d^2}{d\rho^2} - \frac{1122}{\rho^2} \right] + W_{total}(\rho) \right\} \chi(\rho) = E \chi(\rho). \tag{3.2}$$

The effective potential $W_{total}(\rho)$ is the sum of (2.7) and (2.11). Using the results of Ref.17, it is easy to see that the wave function (3.1) with $U_{\zeta_{min}}(\Omega)$ given by (2.1) has L,S,T and J all equal to zero. Since it is even, Eq.(3.2) gives us the energy levels and the radial parts of the wave functions of the bound states of ^{16}O with $J^{\pi}=0^{+}$. Equation (3.2) was put in a dimensionless form by changing to the variable $\eta=\sqrt{\frac{2m_{N}c^{2}}{\hbar^{2}}}\,\rho \approx 0.22\,\rho/fm$. Then, it was solved numericaly by the matching method with Numerov's integration formula. The results obtained for the energies of the 0^{+} levels of ^{16}O are given in Table 2 and plotted in Figs.3.

4. Charge Form Factor and RMS Charge Radius

The nuclear charge form factor is the Fourier transform of the charge density $\rho_{ch}(r)$, i.e.,

$$F_{ch}(q)=\frac{1}{Z}\int e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}}\,\rho_{ch}(\underset{\sim}{r})\,d\underset{\sim}{r} \qquad (4.1)$$

where $\hbar\underset{\sim}{q}$ is the momentum transfer and eZ is the charge of the nucleus. Introducing the one-body operators,

$$\hat{\Pi}_{\pm}(\underset{\sim}{r})=\sum_{i=1}^{A}\delta(\underset{\sim}{r}-\underset{\sim}{\rho}_{i})\left[\frac{1}{2}\pm\tau_{3}(i)\right], \qquad (4.2)$$

whose expectation values relative to the ground state of the nucleus,

$$\Pi_{\pm}(\underset{\sim}{r}) = \int \Psi^{\dagger}_{G.S.}(\underset{\sim}{\rho}_1, \cdots \underset{\sim}{\rho}_A)\, \Pi_{\pm}(\underset{\sim}{r})\, \Psi_{G.S.}(\underset{\sim}{\rho}_1, \cdots, \underset{\sim}{\rho}_A)\, dV_{3(A-1)} \qquad (4.3)$$

give the probability of finding a neutron or a proton in the point $\underset{\sim}{r}$ of space, one may write the charge density as

$$\rho(\underset{\sim}{r}) = \int \left[\rho_+(\underset{\sim}{r} - \underset{\sim}{r}')\, \Pi_+(\underset{\sim}{r}') + \rho_-(\underset{\sim}{r} - \underset{\sim}{r}')\, \Pi_-(\underset{\sim}{r}') \right] d\underset{\sim}{r}'. \qquad (4.4)$$

Substitution of (4.4) in (4.1) and the use of the convolution theorem yield to

$$F_{ch}(\underset{\sim}{q}) = f_+(\underset{\sim}{q})\, F_+(\underset{\sim}{q}) + f_-(\underset{\sim}{q})\, F_-(\underset{\sim}{q}),$$

where

$$f_{\pm}(\underset{\sim}{q}) = \int e^{i\,\underset{\sim}{q}\cdot\underset{\sim}{r}}\, \rho_{\pm}(\underset{\sim}{r})\, d\underset{\sim}{r}, \qquad (4.6)$$

$$F_{\pm}(\underset{\sim}{q}) = \frac{1}{Z} \int e^{i\,\underset{\sim}{q}\cdot\underset{\sim}{r}}\, \Pi_{\pm}(\underset{\sim}{r})\, d\underset{\sim}{r}. \qquad (4.7)$$

For the neutron and the proton charge form factors (4.6), closed formulas obtained by fitting experimental results on electron-proton scattering are available. We use those of Janssens et al.[18], i.e.

$$f_+(\underset{\sim}{q}) = \frac{1.25}{1 + q^2 fm^2/15.7} - \frac{0.8}{1 + q^2 fm^2/26.7} - \frac{0.58}{1 + q^2 fm^2/8.19} + 0.13, \qquad (4.8)$$

$$f_-(\underset{\sim}{q}) = \frac{1.25}{1 + q^2 fm^2/15.7} - \frac{0.8}{1 + q^2 fm^2/26.7} + \frac{0.58}{0 + q^2 fm^2/8.19} - 0.03. \qquad (4.9)$$

Concerning the body form factors (4.7), their explicit expression for the ^{16}O, in the first approximation of the K-harmonics method, is[19]

$$F_+(q) = F_-(q) = 67!! \int_0^\infty \chi^2_{G.S.}(\rho) \left[\frac{j_{33}(\gamma)}{\gamma^{33}} - \frac{4}{15} \gamma^2 \frac{j_{34}(\gamma)}{\gamma^{34}} \right] d\rho, \qquad (4.10)$$

with $\gamma = q\rho\sqrt{15}/4$.

Using the numerical values of $\chi_{G.S.}(\rho)$ obtained in the numerical integration of (3.2) one can evaluate (4.10) and then use (4.5), (4.8) and (4.9) to obtain $F_{ch}(q)$. The values obtained for the six selected potentials are plotted in Fig.2, together with the experimental curve given by Sick and McCarthy[20].

The root means square charge radius

$$a = \left[\frac{1}{Z} \int r^2 \rho_{ch}(r) dr \right]^{1/2} \qquad (4.11)$$

can be obtained from (4.1) through

$$a^2 = -\left[\nabla_q^2 F_{ch}(q) \right]_{q=0}. \qquad (4.12)$$

Since $F_{ch}(q)$ depend on q only through $q = |q|$, Eq.(4.12) can be rewritten as

$$a = -6 \left[\frac{dF_{ch}(q)}{dq^2} \right]_{q=0}. \qquad (4.13)$$

Using (4.8)-(4.10) one obtains for the ^{16}O, in the first approximation of the K-harmonics method,

$$a = \frac{1}{16} \int_0^\infty \chi^2_{G.S.}(\rho)\, \rho^2 \, d\rho + 0.6\, fm^2. \qquad (4.14)$$

The values of a obtained for the six selected potentials are given in Table 1.

5. Discussion of the Results

When we calculated the effective potential $W(\rho)$ for the potentials (2.12)-(2.14), whose parameters are given in Table (1.1)-(1.3), we noticed that only the potentials of kind i), with nonzero coefficient of the Majorana term, yielded a depth compatible with the experimental value of the ^{16}O binding energy. We also noticed that the presence of the Majorana term was essential in order to have a potential well with a reasonable depth.

A possible explanation for the relative success of Brink and Volkov potentials and the failure of the remainding ones that we have studied could be the fact that the parameters of Brink and Volkov potentials have been obtained in Hartree-Fock calculations taking into account the behavior of the nucleons in confinement inside real nuclei, while the other potentials refer to free nucleons.

Among the Brink and Volkov potentials of kind i) we selected three of each whose depth could give a ground state energy close to the expe-

rimental value of ^{16}O binding energy. The selected potentials were : $V_5, V_7, V_8, V_{10}, V_{16}$ and V_{17}.

Looking at Table 2 one sees, at first glance, that potentials V_5, V_7 and V_8 give excellent results for the ^{16}O binding energy while V_{10}, V_{16} and V_{17} give only reasonable results. This situation is, however illusory since the calculations were done in first approximation and it is reasonable to expect that, when the next approximation be taken into account, V_5, V_7 and V_8 will overbind the system while V_{10}, V_{16} and V_{17} may give good results. This conclusion is partially based in a recent paper by Sadovoi and Simonov[21] where the contribution of the $K_{min}+2$ approximation to the ground state energy of ^{16}O was estimated in 25% for a certain kind of two-body nuclear potentials.

Concerning to the charge form factor, one sees from Figs.2 that Volkov potentials V_5, V_7 and V_8 give bad results while those of Brink (V_{10}, V_{16} and V_{17}) give very good results. This enforces our claim that the potentials V_{10}, V_{16} and V_{17} represent better the nuclear interactions inside ^{16}O.

For the root mean square charge radius all of the six selected potentials gave good results.

Concerning the spectrum of the remaining 0^+ states of ^{16}O, none of the six potentials was able to reproduce low-lying energy levels. For all of them the energy gap between the ground state and the first exci-

ted state was around 22 and 28 MeV, very close to the 30 MeV value estimated, in the shell model, for the excitation of 2 nucleons from the 1p to the 2s-1d shell[22]. Strobel[13], using the approximate formulas of Baz and Zhukov[14] for the two-body matrix elements in the K_{min} approximation of the K-harmonics method for the ^{16}O, obtained 27 MeV for the excitation energy. This agreement of results leads us to believe that the plain use of two-body local potentials are impotent to generate 0^+ states with low excitation energy.

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Table Captions

Table 1.1 - Parameters for the potentials of kind i) [Eq.2.12] .

Table 1.2 - Parameters for the potentials of kind ii) [Eq.2.13] given in Ref.24.

Table 1.3 - Parameters for the potentials of kind iii) [Eq.2.14] given in Ref.26. For all of them $\beta = 1.58$ fm.

Table 2 - Energy (in MeV) levels of the 0^+ states of ^{16}O and rms charge radius (in fm) for the six selected potentials. The experimental values were taken from Refs.13, 27, and 28. In order to have a better visualization, the results of this table were plotted in Fig.3.

Figures Captions

Fig.1-a) Effective potential produced by V_5.

Fig.1-b) Effective potential produced by V_7.

Fig.1-c) Effective potential produced by V_8.

Fig.1-d) Effective potential produced by V_{17}.

Fig.1-e) Effective potential produced by V_{10}.

Fig.1-f) Effective potential produced by V_{16}.

Fig.2-a) Charge form factor obtained with V_5.

Fig.2-b) Charge form factor obtained with V_7.

Fig.2-c) Charge form factor obtained with V_8.

Fig.2-d) Charge form factor obtained with V_{17}.

Fig.2-e) Charge form factor obtained with V_{10}.

Fig.2-f) Charge form factor obtained with V_{16}.

Fig.3- Plot of the numerical data do Table 2.

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Potential	Ref.23	α_1 (MeV)	β_1 (fm)	m_1	α_2 (MeV)	β_2 (fm)	m_2
V_1		-60.65	1.3	0.6	61.14	1.01	0.6
V_2		-106.67	1.5	0.6	106.67	1.05	0.6
V_3		-73.23	1.7	0.6	81.33	1.01	0.6
V_4		-54.30	1.8	0.6	74.49	0.81	0.6
V_5		-83.34	1.6	0.6	144.86	0.82	0.6
V_6		-53.98	1.8	0.6	100.67	0.71	0.6
V_7		-76.69	1.5	0.6	408.27	0.45	0.6
V_8		-70.64	1.7	0.6	75.01	1.01	0.6
	Ref.24						
V_9		-117.4	1.4	0.3815	271.0	0.7	-1.635
V_{10}		-247.7	1.2	0.4781	895.9	0.6	-0.309
V_{11}		-177.4	1.4	0.3815	271.0	0.7	-1.635
V_{12}		-203.9	1.2	0.3315	649.4	0.6	-1.42
V_{13}		-412.2	1.0	0.2565	1719.0	0.5	-1.448
V_{14}		-1042.0	0.8	0.1235	5444.0	0.4	-1.795
V_{15}		-508.9	1.0	0.4738	2315.0	0.5	-0.141
V_{16}		-1307.9	0.8	0.4769	7228.0	0.4	0.009
V_{17}		-140.6	1.4	0.4864	389.5	0.7	-0.529
	Ref.25						
V_{18}		-31.50	1.60				

Table 1.1

Potential	α(MeV)	β(fm)	m	A(MeV fm^2)
V_{19}	-24532	0.4	0.5735	7687
V_{20}	-6632	0.5	0.5800	3667
V_{21}	-497.6	0.9	0.6082	982.5
V_{22}	-301.0	1.12	0.6169	1011

Table 1.2

Potential	α (MeV)
V_{23}	-45.8636
V_{24}	-78.6233
V_{25}	-101.5551
V_{26}	-111.3830
V_{27}	-144.1427
V_{28}	-183.4544

Table 1.3

	V_5	V_7	V_8	V_{10}	V_{16}	V_{17}	Experimental
E_0	-129.8	-124.4	-131.1	-99.3	-108.9	-93.9	-127.40
E_1	-104.6	-98.8	-106.5	-75.0	-80.6	-71.8	
E_2	-82.5	-73.0	-84.8	-54.6	-56.9	-53.0	
E_3	-63.3	-52.8	-65.8	-37.6	-37.6	-37.2	
E_4	-46.8	-35.9	-49.5	-23.7	-22.1	-24.3	-121.35
E_5	-32.9	-22.1	-35.6	-12.8	-10.1	-13.9	
E_6	-21.4	-11.1	-23.9	-4.4	-1.2	-5.8	-115.35
E_7	-12.1	-2.7	-14.4				
E_8	-4.9		-6.9				-113.40
E_9			-1.0				
$\langle r^2_{ch} \rangle^{1/2}$	2.33	2.21	2.29	2.66	2.54	2.74	2.75

Table 2

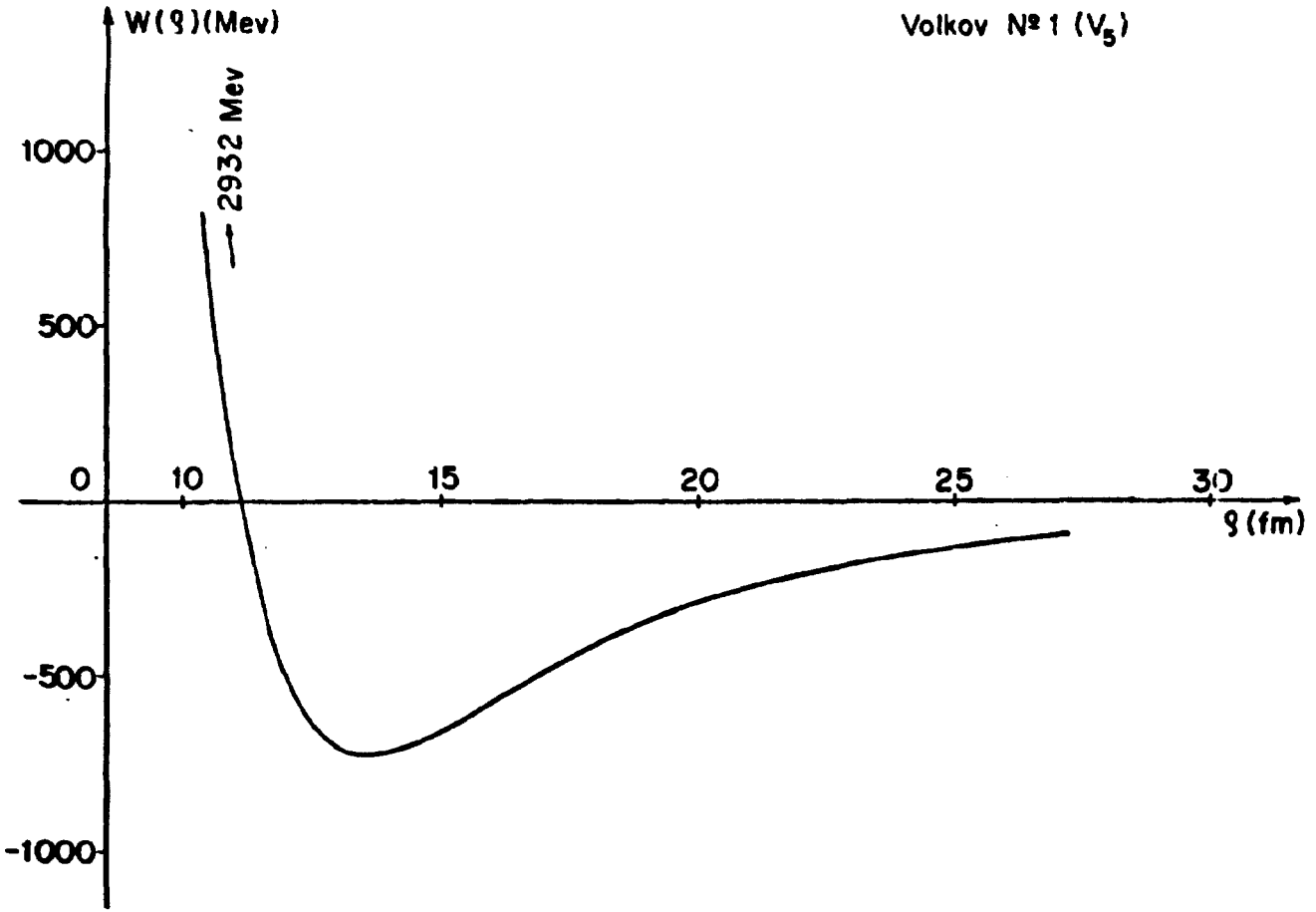

W(ϱ)(Mev)
Volkov Nº 1 (V_5)
2932 Mev
1000
500
0
10
15
20
25
30
ϱ (fm)
-500
-1000

Fig. 1 - a)

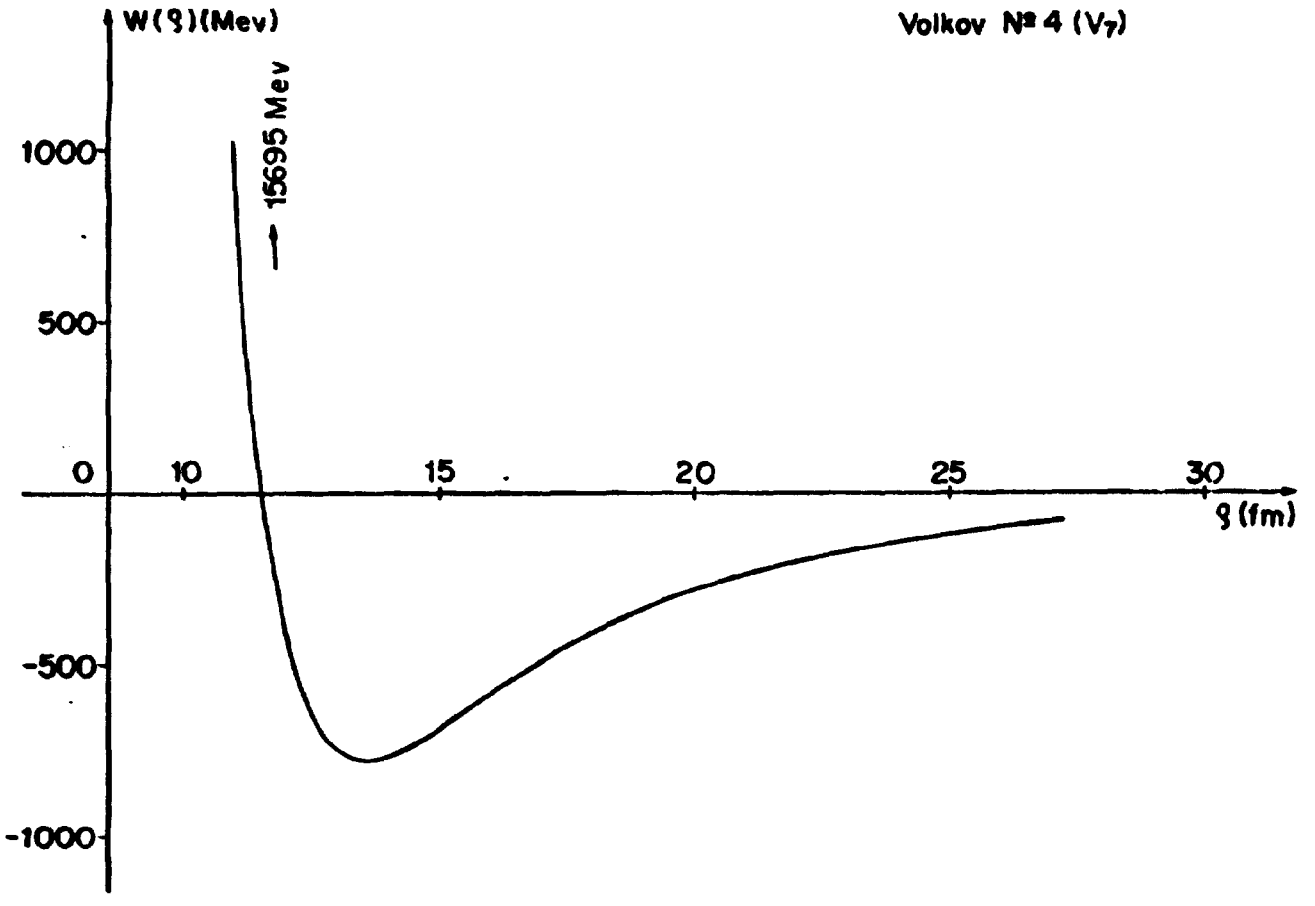

W(ϱ)(Mev)
Volkov Nº 4 (V7)
15695 Mev
1000
500
0
10
15
20
25
30
ϱ (fm)
-500
-1000

Fig. 1 - b)

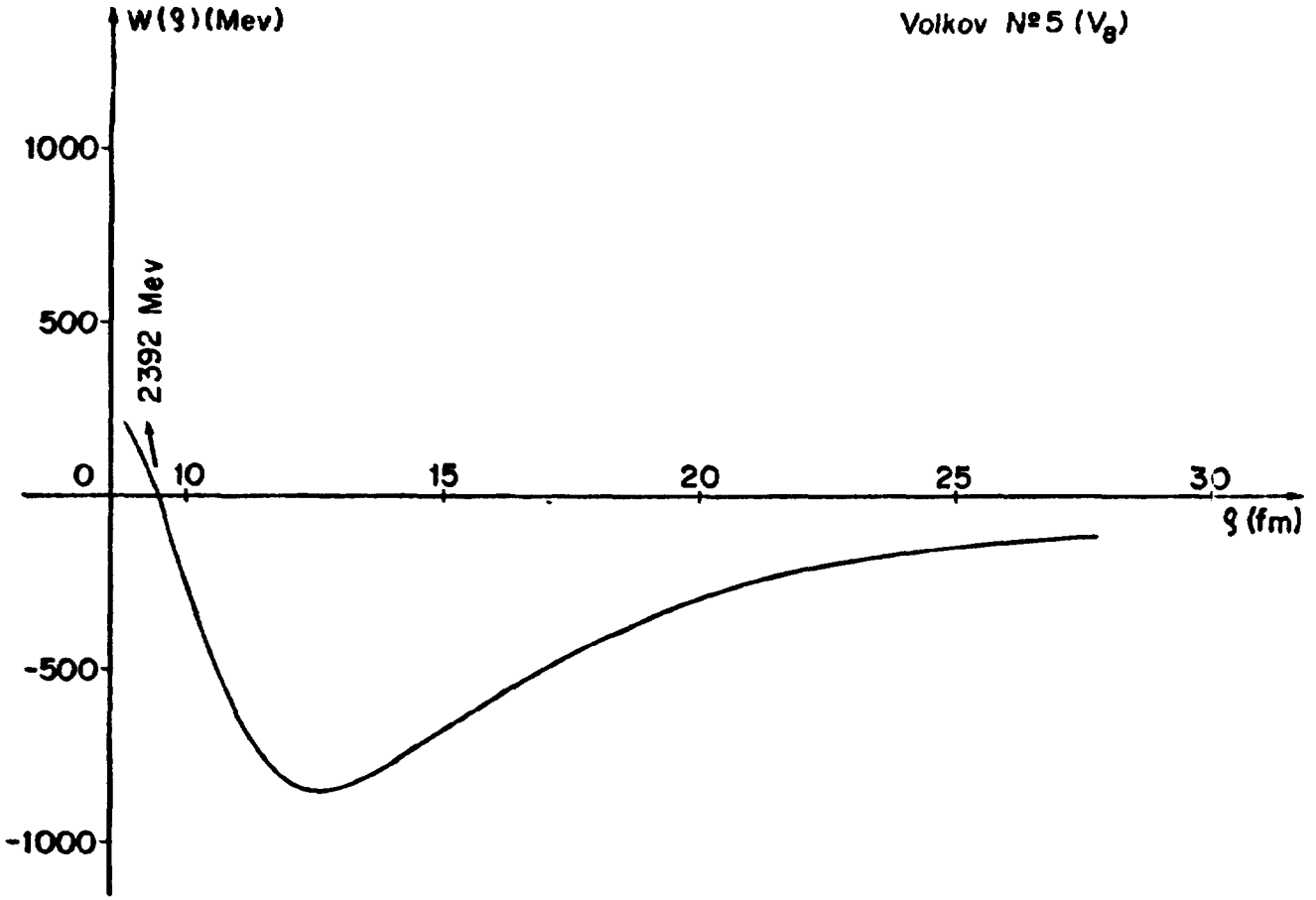

W(ϱ)(Mev)
Volkov Nº5 (V_g)
1000
500
0
-500
-1000
2392 Mev
10
15
20
25
30
ϱ (fm)

Fig. 1 - c)

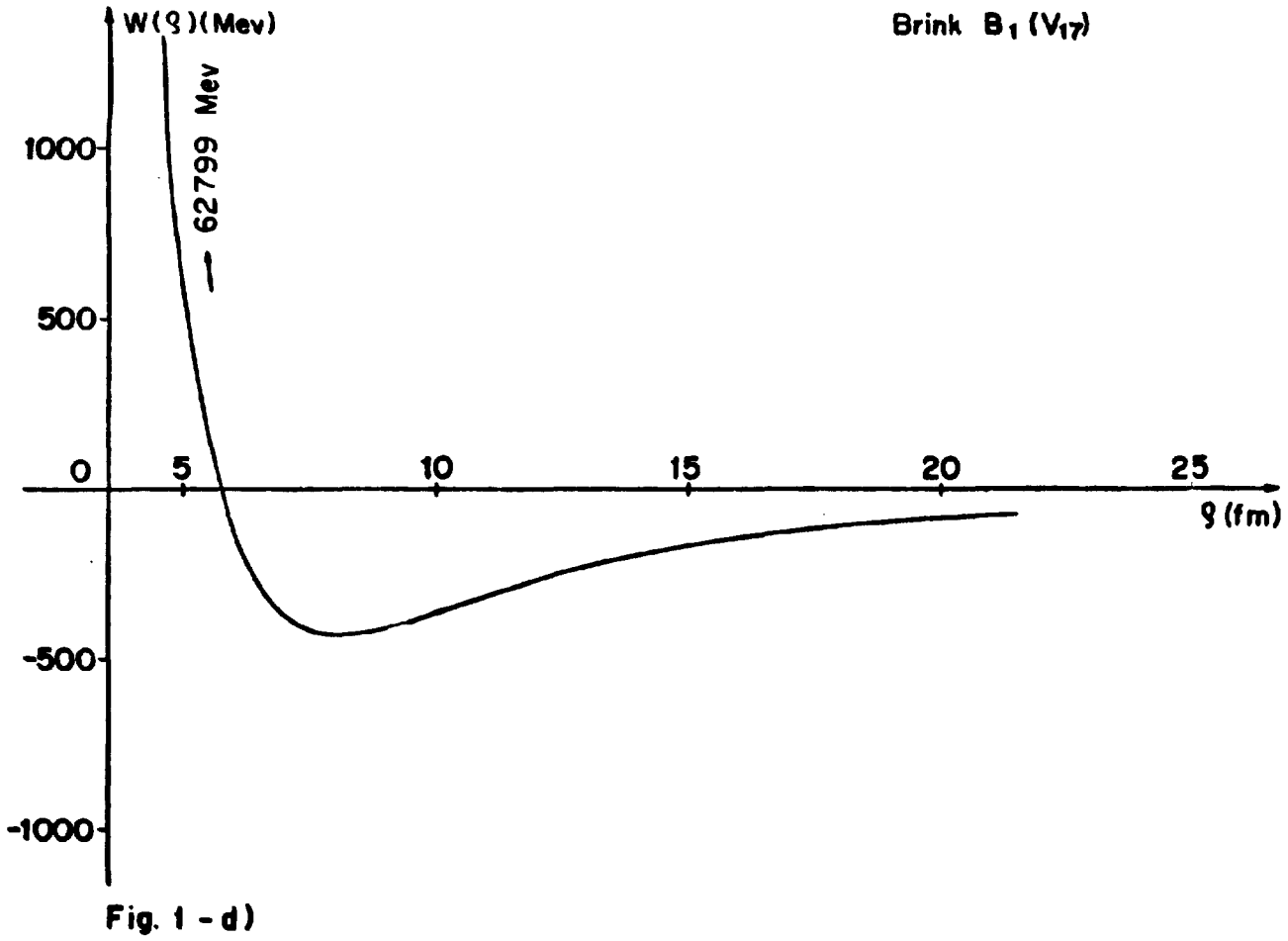

W(ϱ)(Mev)
Brink B_1 (V_{17})
62799 Mev
1000
500
0
5
10
15
20
25
ϱ (fm)
-500
-1000

Fig. 1 - d)

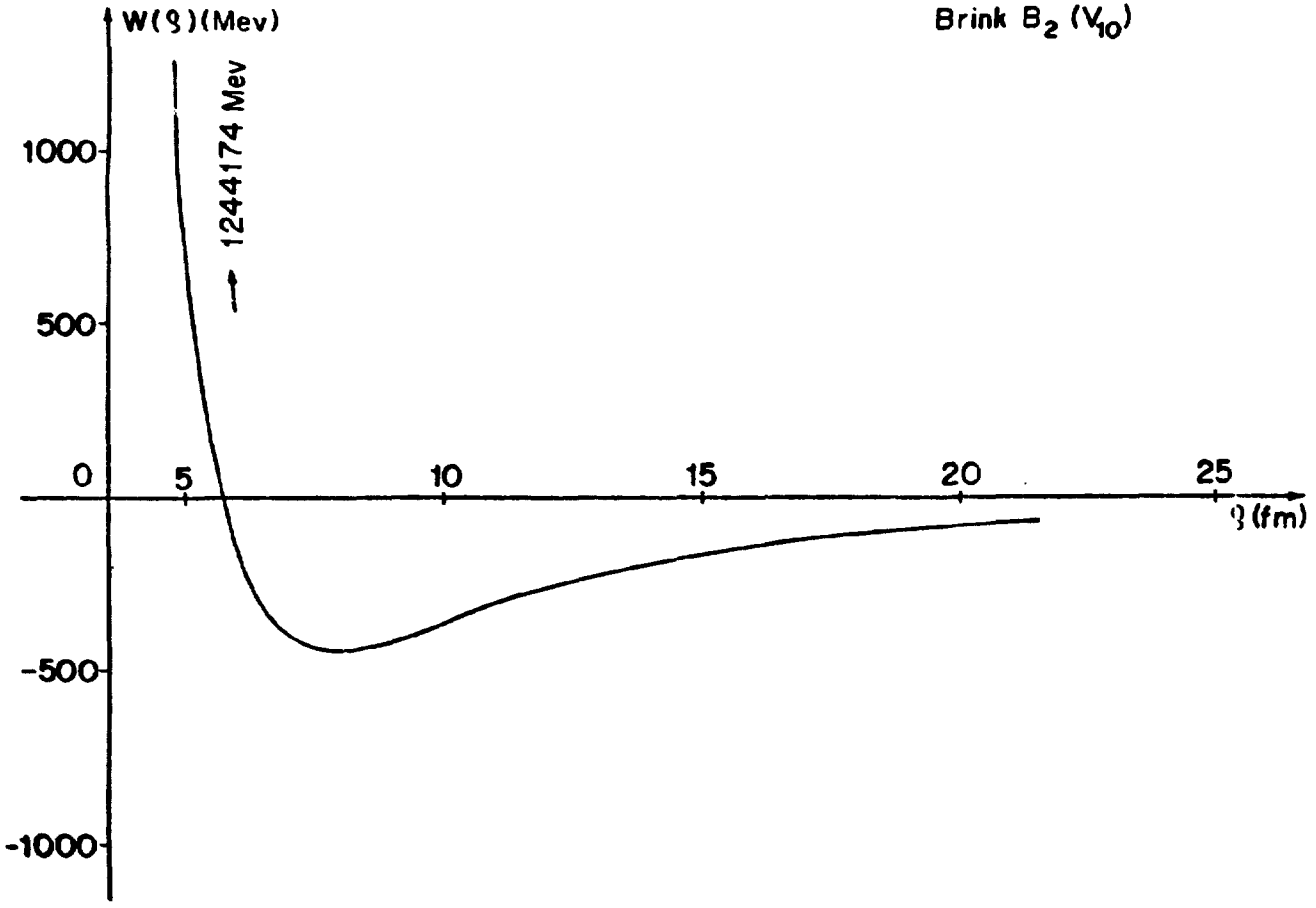

W(ϑ)(Mev)
Brink B_2 (V_{10})
1244174 Mev
1000
500
0
-500
-1000
5
10
15
20
25
ϑ (fm)

Fig. 1 - e)

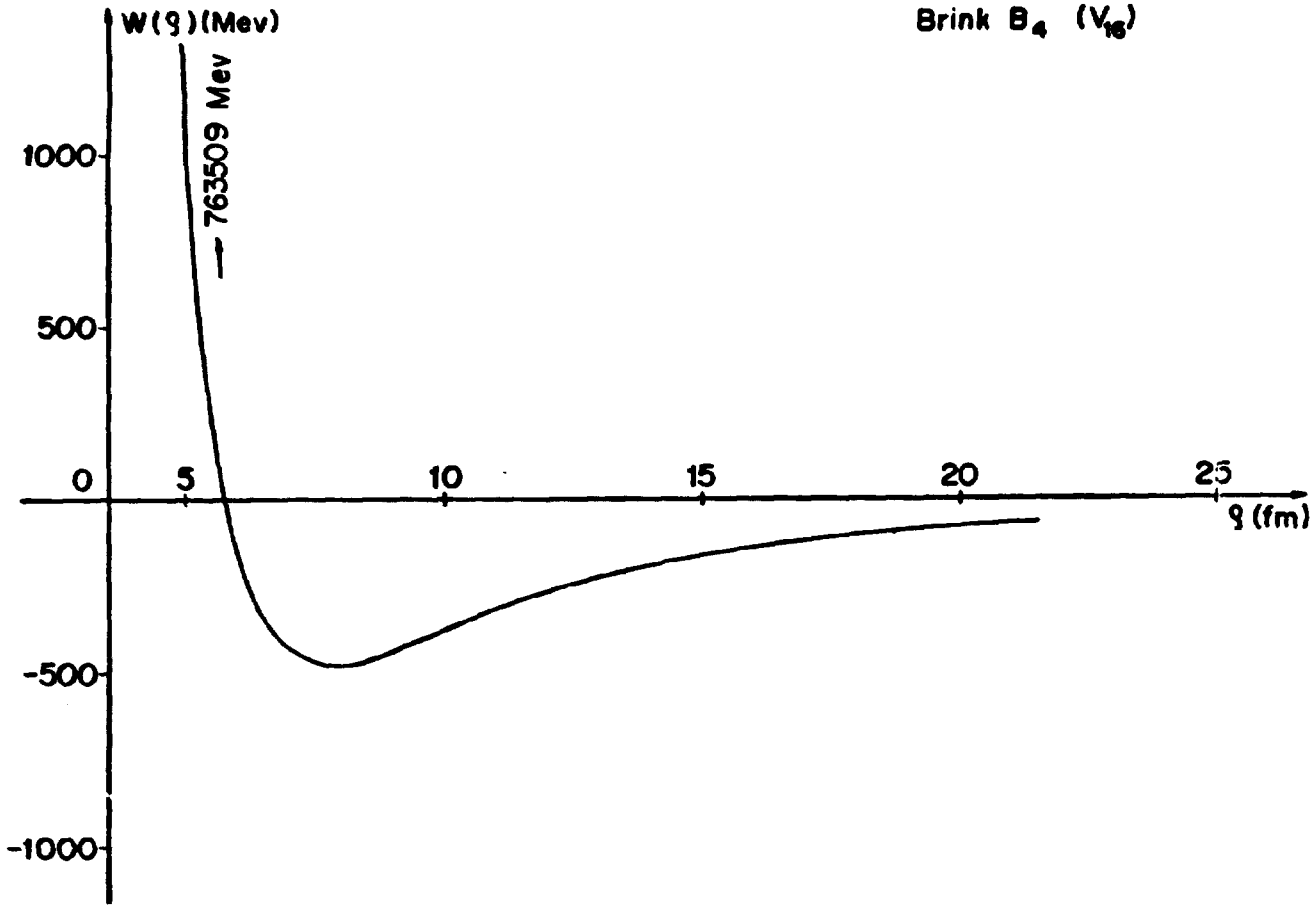
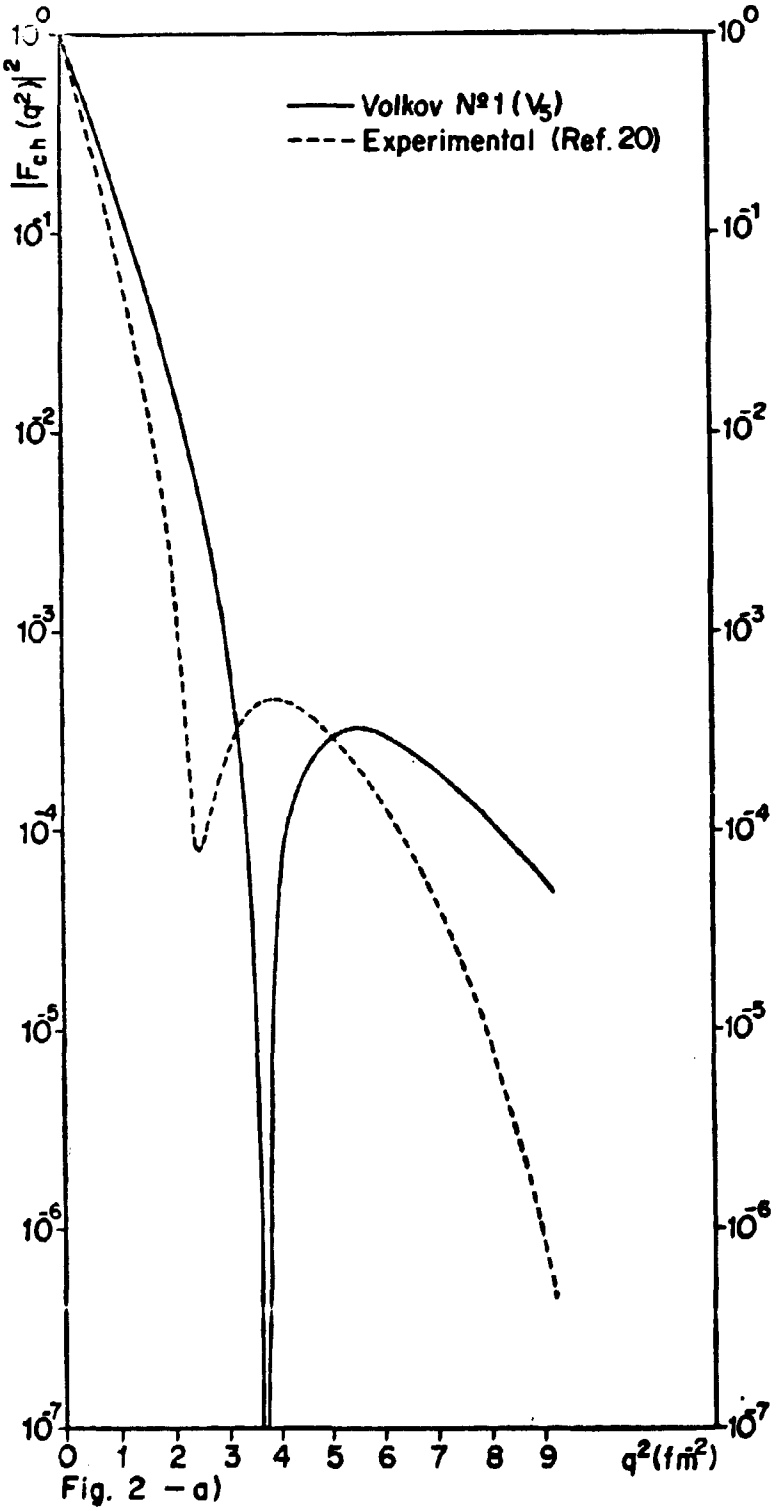

W(ϱ)(Mev)
Brink B_4 (V_{16})
— 763509 Mev
1000
500
0
5
10
15
20
25
-500
-1000
ϱ (fm)

Fig. 1 — f)


Volkov Nº1 (V_5)
Experimental (Ref. 20)
$|F_{ch}(q^2)|^2$
$q^2(fm^2)$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0 1 2 3 4 5 6 7 8 9
Fig. 2 – a)

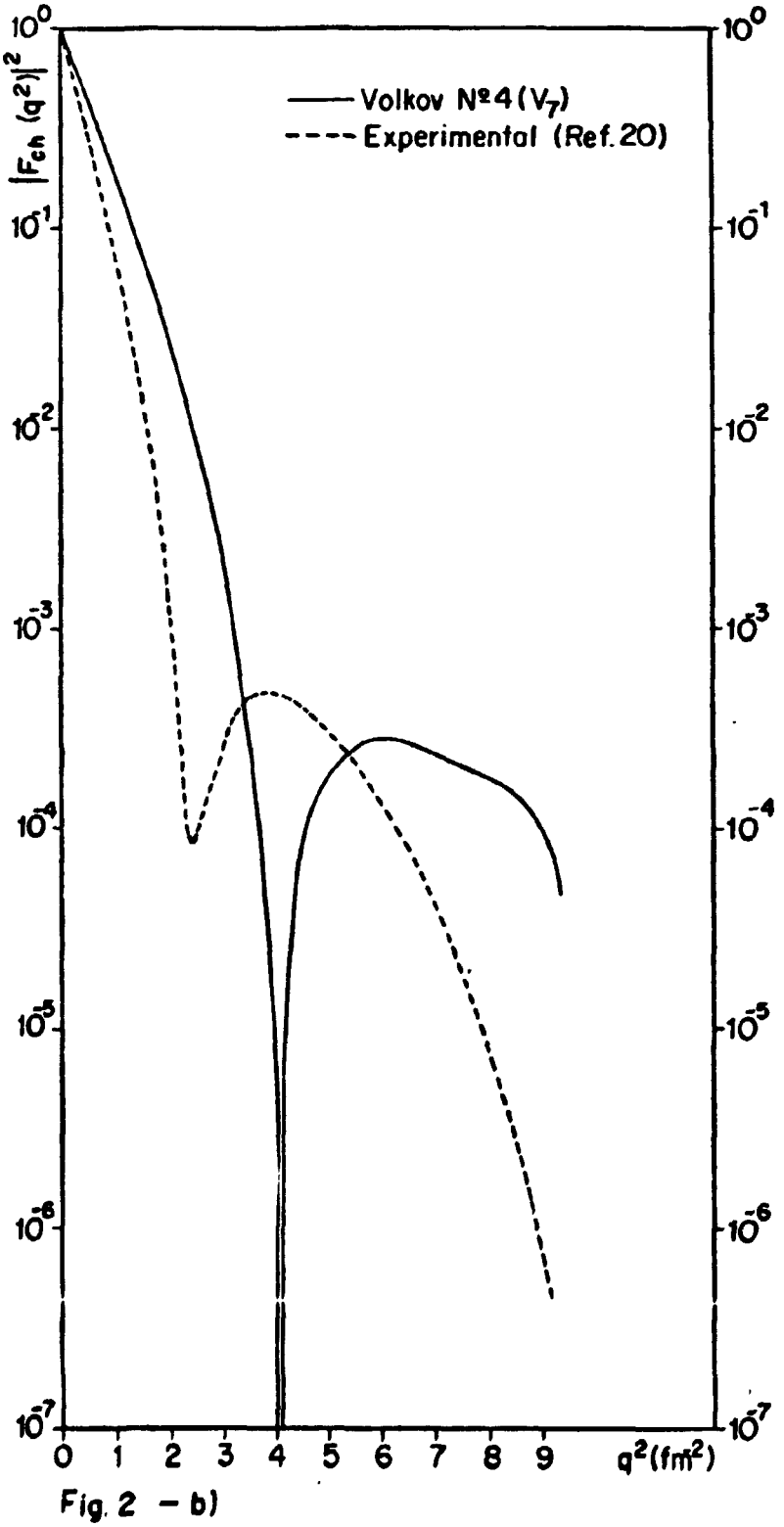

Volkov Nº4 (V_7)
Experimental (Ref. 20)
$|F_{ch}(q^2)|^2$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0 1 2 3 4 5 6 7 8 9
$q^2(fm^2)$

Fig. 2 – b)

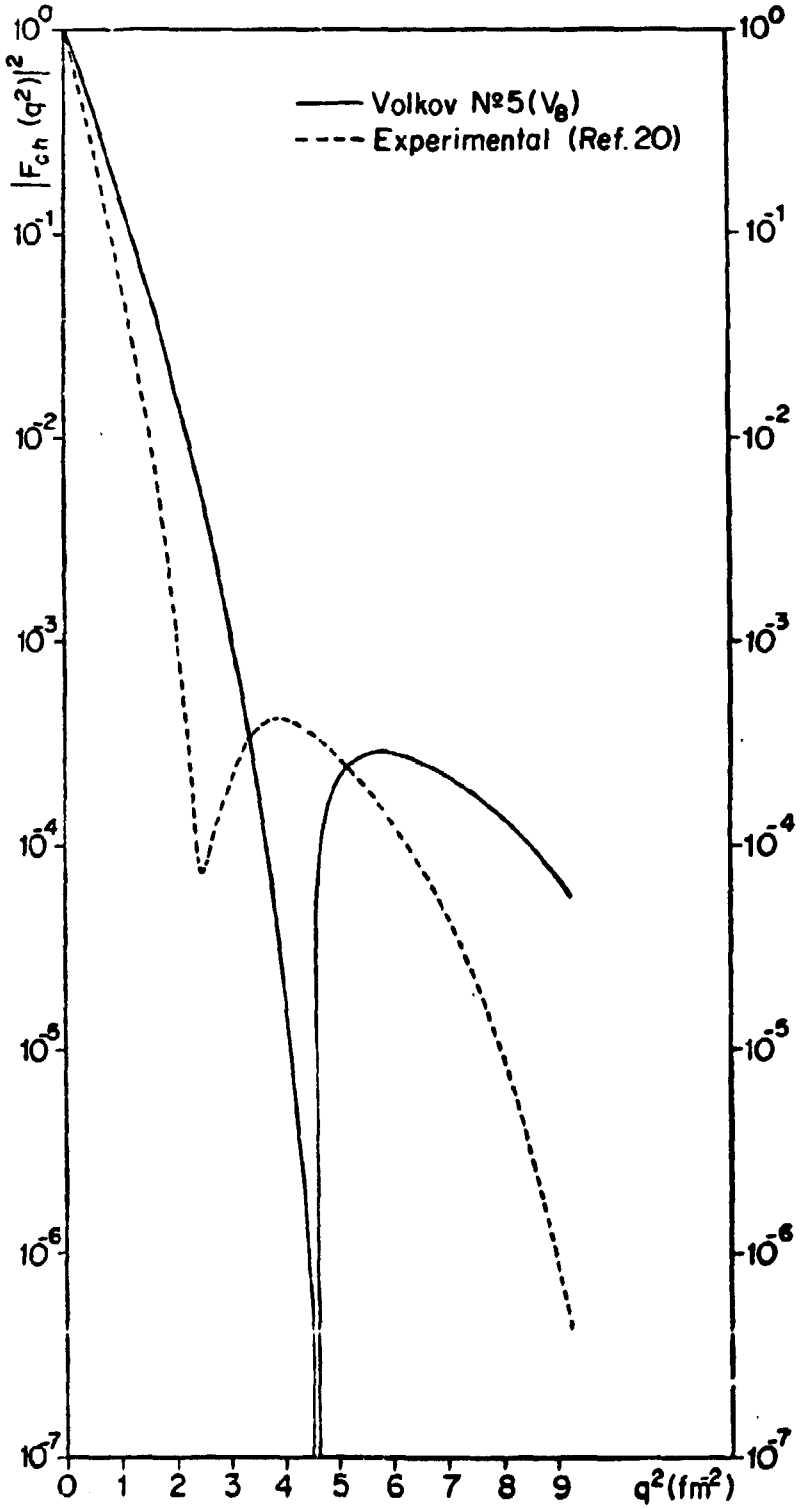

Volkov №5 (V_8)
Experimental (Ref. 20)
$|F_{ch}(q^2)|^2$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0 1 2 3 4 5 6 7 8 9
$q^2(fm^2)$

Fig. 2 – c)

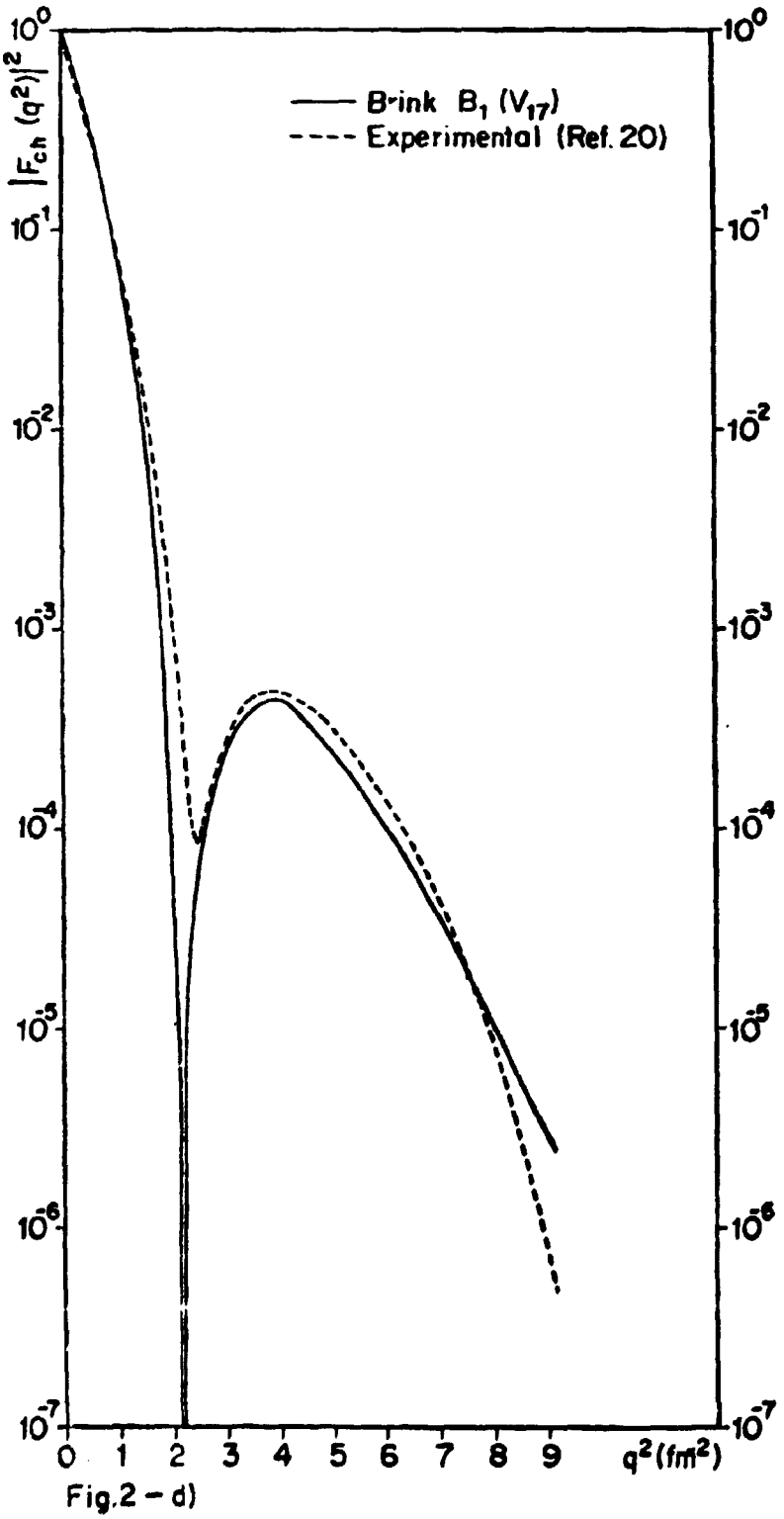

Brink B_1 (V_{17})
Experimental (Ref. 20)
$|F_{ch}(q^2)|^2$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0 1 2 3 4 5 6 7 8 9
q^2 (fm^2)

Fig.2 – d)

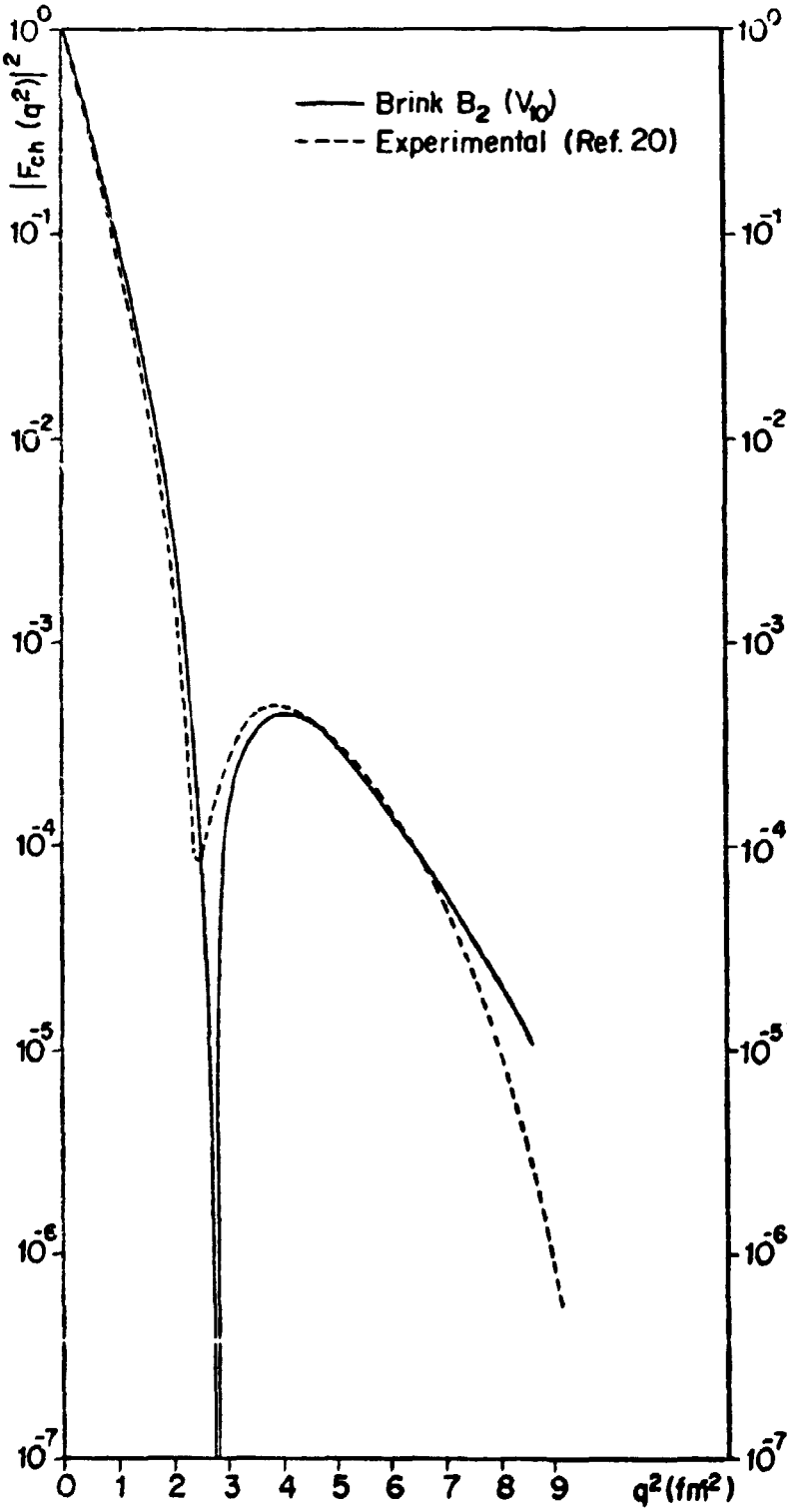

Brink B_2 (V_{10})
Experimental (Ref. 20)
$|F_{ch}(q^2)|^2$
$q^2(fm^2)$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0 1 2 3 4 5 6 7 8 9

Fig. 2 — e)

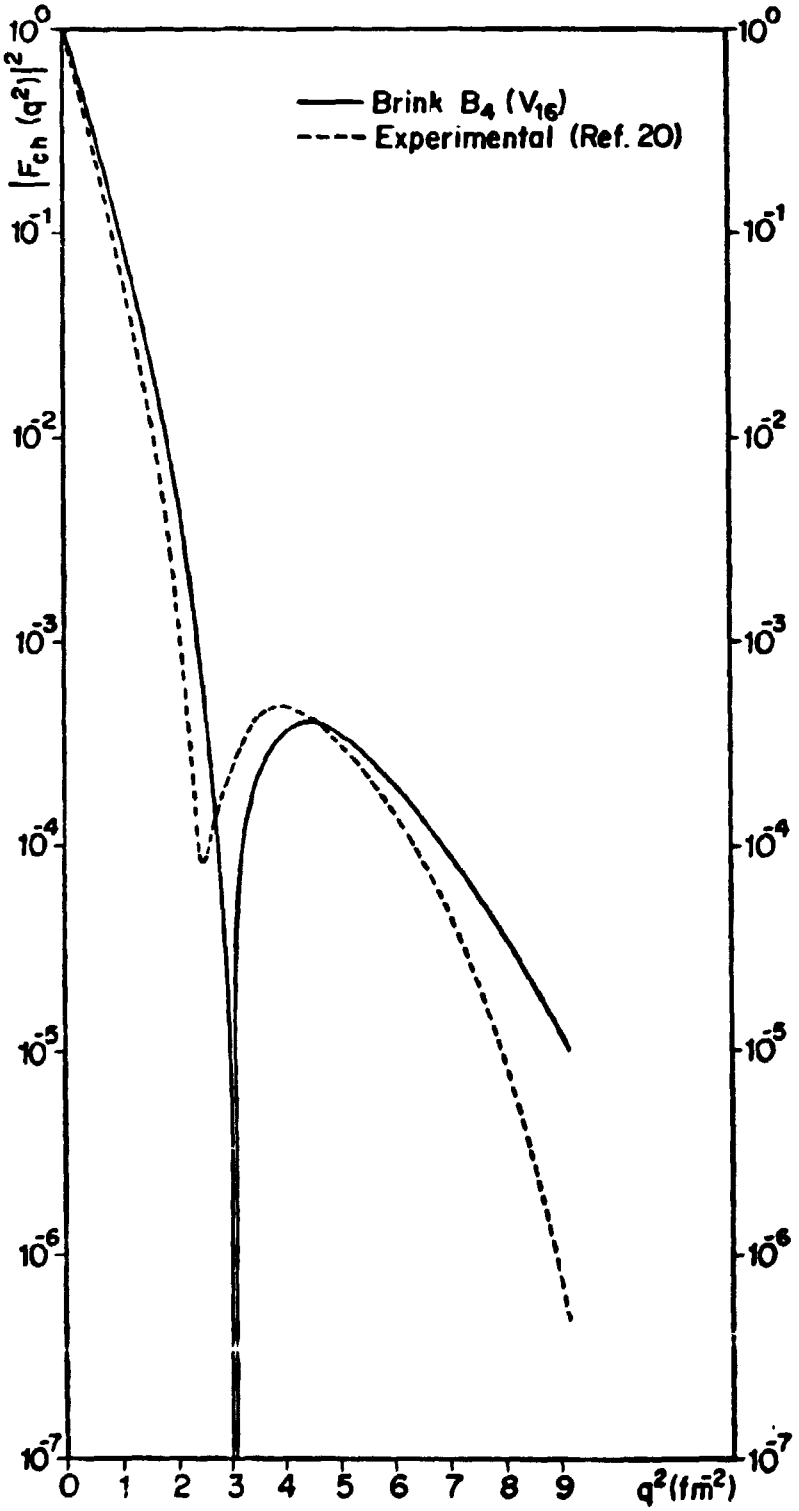

Brink B_4 (V_{16})
Experimental (Ref. 20)
$|F_{ch}(q^2)|^2$
$q^2(fm^2)$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0 1 2 3 4 5 6 7 8 9

Fig. 2 – f)

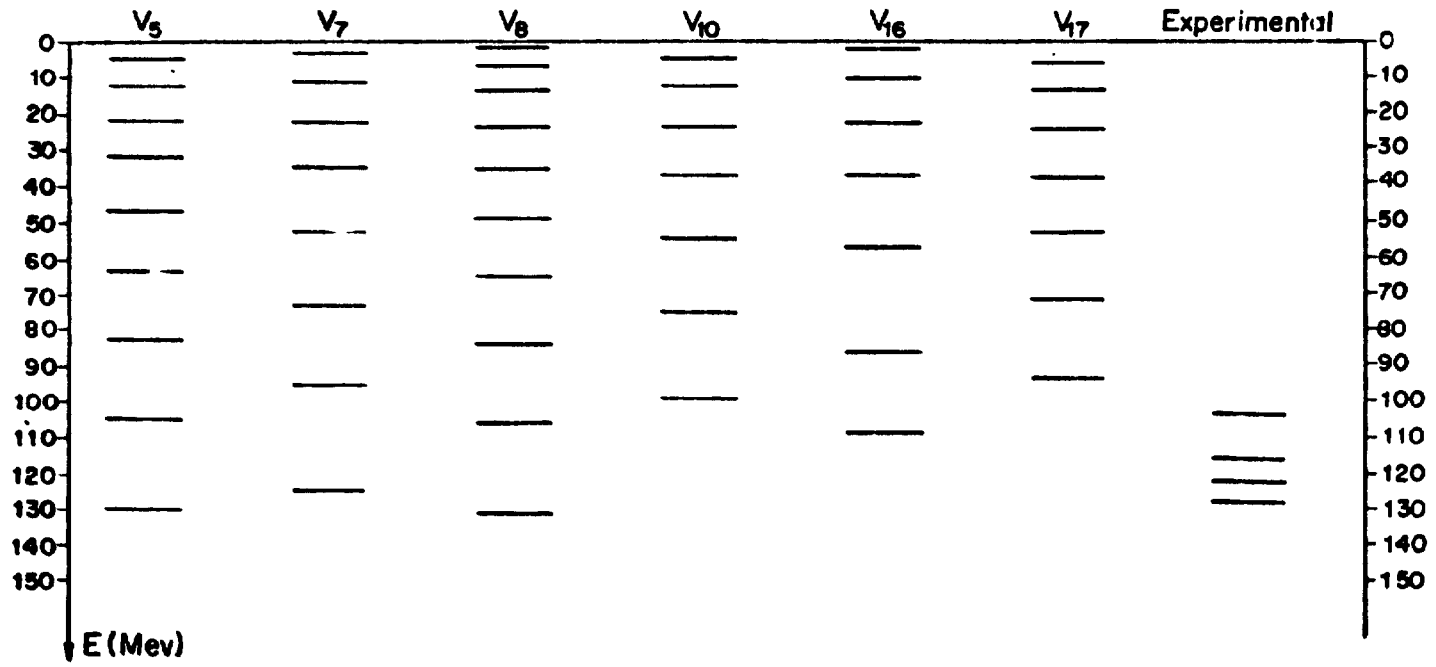

V5
V7
V8
V10
V16
V17
Experimental
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
E (Mev)

Fig. 3 –