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# PION-INDUCED KNOCK-OUT REACTIONS

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Abstract: A strong absorption model for pion-induced Knock-out reactions is proposed. The distortion of the in-coming and out-going pions has been included by (1) computing pion wave number in nuclear medium (dispersive effect) and (2) excluding the central region of the nucleus where the real pion-absorption is dominant (absorption effect). In order to study the dependence of the $(\pi^+, \pi^+ p)$ reaction on the off-shell pion-nucleon t-matrix, different off-shell extrapolations are used. The magnitude of the cross-sections seems to be sensitive to the type of off-shell extrapolation; their shapes, however, are similar. The theoretical results are compared with experimental data. The agreement between the theoretical results for separable off-shell extrapolation and the data is good.

## 1. INTRODUCTION

Several activation experiments have been done for $A(\pi,\pi N)B$ reactions on various targets and in these experiments, an 'anomalous' isospin ratio for $\pi^+$– and $\pi^-$– induced knock-out reactions has been observed[1]. Therefore most of the theoretical work so far has been directed towards explaining this anomaly[2]. In these calculations, the "impulse approximation" framework is used and the cross section is written as,

$$\frac{d^3\sigma}{d\Omega_\pi dE_\pi d\Omega_N} = F_{kin} \frac{d\sigma^{\pi N}}{d\Omega} P(Q) \qquad (1)$$

where $F_{kin}$ is a kinematic factor, P(Q) is the momentum distribution of the nucleon in the nucleus and $d\sigma^{\pi N}/d\Omega$ is the free $\pi N$ cross-section. In the knock-out formalism the off-shell $\pi N$ t-matrix is required, and the factorised form that appears in equation (1) is valid only if off-shell effects are not very important. In proton-induced knock-out reactions, this is probably true since the energy-variation of the free nucleon-nucleon cross section is small[3]. However in case of )$(\pi,\pi N)$ reactions in the 3-3 resonance region, these off-shell effects could be very important. One of the aims of this article is to investigate these off-shell effects in the $(\pi^+,\pi^+ p)$ reaction.

For the theoretical description of the $(\pi^+,\pi^+ p)$ reaction we are guided by the observation that there are three important channels in the pion-nucleus interaction: a) elastic scattering, b) pion absorption reactions (i.e. reactions in which pion disappears) and c) knock-out and inelastic reactions[4]. For example, in $\pi$-$^{12}C$ interaction, $\sigma_{tot}$ of about 700 mb at 130 MeV consist of: $\sigma_{el} \sim 270$mb and $\sigma_{re} \sim 430$ mb. The $\sigma_{re}$ is further split into $\sigma_{abs} \sim 215$ mb, $\sigma_{inel} \sim 12$ mb, and $\sigma_{knonk\text{-}out} \sim 148$ mb.

This suggests that in the theoretical formalism for the knock-out reaction, besides the elastic scattering, the real pion-absorption channels must be properly included. Since the two-nucleon mechanism is the dominant mechanism in real pion absorption, it mainly takes place in the nuclear interior where the nuclear density is high. This means that the region in the nuclear interior does not contribute much to the $(\pi,\pi N)$ reaction. This effect of the removal of the pion flux due to real pion absorption can be accounted for by excluding the central region of the nucleus of radius $R(E_\pi)$ for the knock-out reaction. A similar procedure is advocated by Bethe[5]. In the sharp cut-off model the radius $R(E_\pi)$ may be chosen by the relation $\sigma_{abs}(E_\pi) = \pi R^2(E_\pi)$. (One may also choose a smooth cut off).

Based on this idea, we have developed a formalism for $(\pi^+,\pi^+ p)$ reaction and applied it to pion-induced proton knock-out reaction on $^{12}C$. In section 2 we present this formalism. The results are discussed in Section 3.

## 2. FORMALISM

In the distorted wave impulse approximation (DWIA), the scattering matrix for $A(\pi,\pi N)B$ reaction is,

$$T_{\pi,\pi N} = \langle \chi^-_{\underset{\sim}{K}'_\pi}, \chi^-_{\underset{\sim}{K}'_N} \mid \tau_{\pi N}(E) \mid \chi^+_{\underset{\sim}{K}_\pi}, \qquad (2)$$

where $\chi^-_{\underset{\sim}{K}'_\pi}$ $(\chi^-_{\underset{\sim}{K}'_N})$ is the distorted wave for out-going pion (nucleon) due to the residual nucleus B, $\chi^+_{\underset{\sim}{K}_\pi}$ is the distorted wave for incoming pion due to the target nucleus A, and $\phi_N$ is the bound state wave function of the knocked-out nucleon. $\phi_N$ includes the spectroscopic amplitude. The energy E at which the $\pi N$ scattering matrix, $\tau_{\pi N}$ is evaluated is defined later. In coordinate space this matrix element becomes,

$$T_{\pi,\pi N} = (2\pi)^{-6} \int d\underset{\sim}{r}'_{\pi B}\, d\underset{\sim}{r}'_{NB}\, d\underset{\sim}{r}_{\pi A}\, d\underset{\sim}{r}_{NB}\, \chi^{-*}_{\underset{\sim}{K}'_{\pi}}(\underset{\sim}{r}'_{\pi B})\, \chi^{-*}_{\underset{\sim}{K}'_{N}}(\underset{\sim}{r}'_{NB})$$

$$\chi^{+}_{\underset{\sim}{K}_{\pi}}(\underset{\sim}{r}_{\pi A})\, \phi_N(\underset{\sim}{r}_{NB}) \int d\underset{\sim}{k}_{\pi}\, d\underset{\sim}{k}_{N}\, d\underset{\sim}{k}'_{\pi}\, d\underset{\sim}{k}'_{N}\, \tau(\underset{\sim}{k}_{\pi}, \underset{\sim}{k}_{N};$$

$$\underset{\sim}{k}'_{\pi}, \underset{\sim}{k}'_{N}; E) \exp\{i(\underset{\sim}{k}'_{\pi}\cdot\underset{\sim}{r}'_{\pi} + \underset{\sim}{k}'_{N}\cdot\underset{\sim}{r}'_{N} - \underset{\sim}{k}_{\pi}\cdot\underset{\sim}{r}_{\pi} - \underset{\sim}{k}_{N}\cdot\underset{\sim}{r}_{N})\}, \quad (3)$$

The coordinates ($r$'s) in equation (3) are defined in figure 1. Since the range of $\pi N$ interaction is much smaller than the nuclear size, we can approximately factor out the dependence of $\chi$ on $\underset{\sim}{r}_{\pi N}$ as,

$$\chi^{+}_{\underset{\sim}{K}_{\pi}}(\underset{\sim}{r}_{\pi A}) = \chi^{+}_{\underset{\sim}{K}_{\pi}}(\underset{\sim}{r}_{\pi N} + a\,\underset{\sim}{r}_{NB}) \simeq e^{i\underset{\sim}{K}_{\pi}(\underset{\sim}{r}_{NB})\cdot\underset{\sim}{r}_{\pi N}}\, \chi^{+}_{\underset{\sim}{K}_{\pi}}(a\,\underset{\sim}{r}_{NB})$$

and

$$\chi^{-*}_{\underset{\sim}{K}'_{\pi}}(\underset{\sim}{r}'_{\pi B}) \simeq e^{-i\underset{\sim}{K}'_{\pi}(\underset{\sim}{r}'_{NB})\cdot\underset{\sim}{r}'_{\pi N}}\, \chi^{-*}_{\underset{\sim}{K}'_{\pi}}(\underset{\sim}{r}'_{NB}) \quad (4)$$

where $a = (A-1)/A$; $K_{\pi}(a r_{NB})$ and $\underset{\sim}{K}'_{\pi}(\underset{\sim}{r}_{NB})$ are the local pion momenta of incident and outgoing pions near $\underset{\sim}{r}_{NB}$ and $\underset{\sim}{r}'_{NB}$ respectively. Further, since the total energy of the pion is much smaller than nuclear mass, we can approximate the $\pi$-nucleus frame-of-reference by the lab. frame. With these approximations,

$$T_{\pi,\pi N} \simeq \int d\underset{\sim}{r}'_{NB}\, d\underset{\sim}{r}_{NB}\, d\underset{\sim}{k}'_{N}\, e^{ia(\underset{\sim}{K}'_{\pi} + \underset{\sim}{K}'_{N})\cdot(\underset{\sim}{r}'_{NB} - \underset{\sim}{r}_{NB})}$$

$$\tau_{\pi N}(\underset{\sim}{k}_{\pi N}, \underset{\sim}{k}'_{\pi N}; E)\, \chi^{-*}_{\underset{\sim}{K}'_{\pi}}(\underset{\sim}{r}'_{NB})\, \chi^{-*}_{\underset{\sim}{K}'_{N}}(\underset{\sim}{r}'_{NB})\, \chi^{+}_{\underset{\sim}{K}_{\pi}}(a\underset{\sim}{r}_{NB})\, \phi_N(\underset{\sim}{r}_{NB})$$

$$\simeq (2\pi/a)^3 \int d\underset{\sim}{r}_{NB}\, \chi^{-*}_{\underset{\sim}{K}'_{\pi}}(\underset{\sim}{r}_{NB})\, \chi^{-*}_{\underset{\sim}{K}'_{N}}(a\underset{\sim}{r}_{NB})\, \chi^{+}_{\underset{\sim}{K}_{\pi}}(a\underset{\sim}{r}_{NB})$$

$$\phi(\underset{\sim}{r}_{NB})\, \tau(\underset{\sim}{k}_{\pi N}, \underset{\sim}{k}'_{\pi N}; E) \quad (5)$$

In the last step we have assumed that the distorted wave of the nucleon peaks around its asymptotic momentum with little momentum spread and the $\pi N$ t-matrix does not depend strongly on this momentum spread. $\underset{\sim}{k}_{\pi N}$ and $\underset{\sim}{k}'_{\pi N}$ in equation (5) are $\pi N$ relative momenta and we approximate them to be,

$$\underset{\sim}{k}_{\pi N} \simeq \underset{\sim}{K}_{\pi} - \epsilon_{\pi} ( \underset{\sim}{K}'_{\pi} + \underset{\sim}{K}'_{N} ) / ( m + \epsilon_{\pi} ) \tag{6}$$

and

$$\underset{\sim}{k}'_{\pi N} \simeq ( m \underset{\sim}{K}'_{\pi} - \epsilon'_{\pi} \underset{\sim}{K}'_{N} ) / ( m + \epsilon'_{\pi} ) \tag{7}$$

Here $\underset{\sim}{K}'_N$ is the asymptotic momentum of the outgoing nucleon in lab., $\epsilon_{\pi} = ( p_0^2 + m_{\pi}^2 )^{1/2}$, $\epsilon'_{\pi} = ( q_0^2 + m_{\pi}^2 )^{1/2}$ and $p_0\,(q_0)$, $m_{\pi}$ and $m$ are incoming (outgoing) pion momentum in lab., pion mass and nucleon mass respectively. The on-shell $\pi$ N energy, E, is still not defined; the best choice seems to be

$$E = E_{\pi} ( p_0 ) + E_B - ( q_0 + q_N )^2 / 2 ( m + E_{\pi} ( p_0 ) ), \tag{8}$$

where $E_B$ is the binding energy of the knocked-out nucleon.

Thus, in order to compute the scattering matrix for knock-out reaction, we have to evaluate the integral inequation (5). The details of the calculation and some further approximations that are used in evaluating this integral are discussed below.

a) The $\pi$ N scattering matrix: Only $s$ and $p$-waves of the $\pi$ N scattering matrix have been included in this calculation. This is a good approximation since other partial waves do not contribute significantly in the 3-3- resonance energies. Explicitely,

$$\begin{aligned} t_{\pi N} ( \underset{\sim}{K}, \underset{\sim}{K}'; E ) = \sum_{\substack{\ell=0,1 \\ I = 3/2}} \Big[ & \left\{ (\ell+1)\, t^{\ell}_{I,\ell+1/2} (K, K'; E) + \ell\, t^{\ell}_{I,\ell-1/2} (K, K'; E) \right\} \times \\ & \times P_{\ell} ( \hat{K} \cdot \hat{K}' ) + \left\{ t^{\ell}_{I,\ell+1/2} (K, K'; E) - t^{\ell}_{I,\ell-1/2} (K, K'; E) \right\} \times \\ & \times \underset{\sim}{\sigma} \cdot ( \underset{\sim}{K} \times \underset{\sim}{K}' ) / | \underset{\sim}{K} \times \underset{\sim}{K}' |\; P'_{\ell} ( \hat{K} \cdot \hat{K}' ) \Big] \end{aligned} \tag{9}$$

where, in separable form,

$$t^{\ell}_{I,j} (K, K'; E) = v^{\ell}_{Ij} (K)\, v^{\ell}_{Ij} (K') / \{ v^{\ell}_{Ij} (K_0) \}^2\, t^{\ell}_{Ij} (K_0, K_0; E(K_0)) \tag{10}$$

(All symbols have the usual meaning). The off-shell form factor $v$ is chosen to be $v(K) = K(K^2+\beta^2)^{-1}$ for p-wave and $v(K) = (K^2+\beta^2)^{-1}$ for s-wave. Parameter $\beta$ is chosen to be 1.8fm$^{-1}$. This value is consistent with the $\pi N$ scattering data.

b) Pion distorted waves: One of the important effect of pion distortion arises from pion absorption. As explained in section I, we approximately account for this effect by excluding the central region of the nucleus from the integration in equation (5). In order to account for the dispersive effects, we approximate the pion distorted waves by,

$$\chi^{+}_{\underset{\sim}{K}_\pi}(a\underset{\sim}{r}_{NB}) = (2\pi)^{-3/2} \exp(ia\underset{\sim}{K}(\underset{\sim}{r}_{NB})\cdot\underset{\sim}{r}_{NB})$$

and

$$\chi^{-*}_{\underset{\sim}{K}'_\pi}(\underset{\sim}{r}_{NB}) = (2\pi)^{-3/2} \exp(-i\underset{\sim}{K}'(\underset{\sim}{r}_{NB})\cdot\underset{\sim}{r}_{NB}) \qquad (11)$$

where momenta $\underset{\sim}{K}$ and $\underset{\sim}{K}'$ are real and are parallel to $\hat{p}_0$ and $\hat{q}_0$ respectively. Their magnitudes are obtained by solving the dispersion relation

$$K^2 = p_0^2 + 4\pi\rho f(K,0)$$
$$K'^2 = q_0^2 + 4\pi\rho f(K',0) \qquad (12)$$

with $\rho$ being the nuclear density at $\underset{\sim}{r}_{NB}$ and $f$ being the $\pi N$ forward scattering amplitude at the pion momentum in the medium.

c) Nucleon distorted wave: We have neglected the distortion of nuclear wavefunction. It has been shown that the charge-exchange effect of final state interaction between the nucleon and the residual nucleus are small in $(\pi^+,\pi^+p)$ reaction[2]. Also, since in our model the reaction takes place in the low-density region of the nucleus, refraction effect for the nucleon s are also expected to be small.

d) Nucleon wave functions: Harmonic oscillator wavefunctions with oscillator parameter $r_0 =$ 1.66fm have been used to compute bound-nucleon wavefunctions. This oscillator parameter reproduce the electron scattering form factor on $^{12}C$ (ref.6). Since the knock-out reactions takes place on the surface of the nucleus, we realize that it would be better to use nucleon wave functions which have correct asymptotic behaviour. However, as we will be calculating the integrated cross-section the use of harmonic oscillator wavefunctions would not change the results of this paper much. The binding energies for 1S and 1P orbitals for $^{12}C$ are taken to be 34 MeV and 16 MeV respectively. These are consistent with the values obtained from (p,2p) reaction[7]. With this, equation 5 becomes,

$$T_{\pi,\pi N} = (2\pi a^2)^{-3/2} \int_{r > R(E_\pi)} d\underline{r}\; e^{i\underline{Q}\cdot\underline{r}}\, \phi_N(\underline{r})\, \tau_{\pi N}(\underline{k}_{\pi N}, \underline{k}'_{\pi N}; E) \qquad (13)$$

Here $\underline{Q} = \underline{K} - \underline{K}' - \underline{q}_N$ . In terms of this scattering matrix, the cross-section for $(\pi^+, \pi^+ p)$ reaction is,

$$\frac{d^3\sigma}{dT_N\, d\Omega_\pi\, d\Omega_N} = (2\pi)^4 \frac{q_N\, q_0\, m\, E_\pi(P_0)\, E_\pi(q_0)}{P_0\left[1 - \underline{q}_0\cdot(\underline{P}_0 - \underline{q}_0 - \underline{q}_N)\, E_\pi(q_0)/q_0^2 M_R\right]} \left|T_{\pi,\pi N}\right|^2 \qquad (14)$$

where $M_R$ is the mass of recoiling nucleus, $T_N(q_N)$ is the kinetic energy of the outgoing nucleon, and $q_0$ is determined by solving the energy equation,

$$E_\pi(q_0) + T_N + (\underline{P}_0 - \underline{q}_0 - \underline{q}_N)^2/(2M_R) = E_\pi(P_0) + E_B$$

For heavy nuclei, the phase space factor may be approximated by

$$m\, E_\pi(q_0)\, E_\pi(P_0)\, q_0\, q_N / P_0$$

## 3. RESULTS AND DISCUSSION

The quasi-elastic proton knock-out reaction has been done on $^{12}C$ in the immulsion experiments at pion incident energies of 60 and 112 MeV[8].

In these experiments three particles, the pion, the proton and the residual nucleus in the final state are observed; and the energy spectrum of the proton and the angular distribution of the pion are measured. The typical characteristics of these distributions are: (i) the proton energy distribution peaks. The peak is narrow at 60 MeV and becomes broad (possibly having two peaks close to each other) at 112 MeV. The position of the peak shifts to higher energy, (ii) the pion angular distribution has a shallow minimum around 70°. The distribution is also asymmetric about this angle. This asymmetry increases with the incident energy.

In figures 2-5 we have plotted the calculated results together with the experimental data. Since the experiments do not separate the contributions from the protons in 1S and 1P shells of $^{12}C$, the calculated results are the summed cross-section from these two shells. As we see from the figures approximately all the features of the proton energy spectrum are reproduced. In the pion angular distributions, the position of the minima and the cross-section beyond this angle is well reproduced. The cross-sections in the forward direction are however, not reproduced. Contrary to the experiment, the calculated results show a flatter distribution at lower energy. This disagreement of the theory and the experiment is also reflected in the slow energy (< 20 MeV) part of the proton energy distribution. Since the trend of the disagreement in this part of the spectrum has opposite direction at two energies (i.e. $\sigma_{exp} > \sigma_{th}$ at $E_\pi = 60$ MeV and $\sigma_{exp} < \sigma_{th}$ at $E_\pi = 112$ MeV), the explanation for this discrepancy may either lie in some uncertainty in the experimental data or some new reaction dynamics which shows up in going from 60 to 112 MeV. However, before efforts are put to investigate the latter part it may be necessary to have better data.

It may be mentioned that to achieve the kind of agreement with the experiments, as shown in figures 2-5, it is necessary to include the spin-flip term in the pion-nucleon t-matrix (eq.9). To demonstrate its importance, in figures 2-5 we have also plotted the contribution of the spin-flip and non spin-flip terms separately.

With a view to investigate the reason for the broadening of the proton energy spectrum in going from 60 to 112 MeV, we calculated this spectrum at a few more energies. All these results are shown in figure 6. The inspection of these curves suggests that, in fact, the proton energy spectrum comprises of two peaks. These two peaks arise essentially due to the folding of the pion-nucleon scattering amplitude (which has dominant p-wave character) and the phase space (which is zero at $E_p = 0$ and $E_p = E_{max}$). Since with the lowering of the incident pion energy the range of $E_p$ gets reduced, and consequently the $\theta_{\pi}$ (ranging from 0 to 180°) corresponding to various $E_p$ appear on a compressed scale, two peaks start merging into each other. First this effect reduces two distinct peaks to one broad peak and then the width of this peak gets reduced.

As mentioned in section 1, normally factorized form (equation 1) for the calculation of the knock-out reaction is used. This form, in addition to overlooking the off-shell effect, suffers from the ambiguity of defining the energy at which the free pion-nucleon cross-section is to be evaluated. In order to see the sensitivity of the cross-section to the off-shell behaviour of $t_{\pi N}$, in figures 7-8 we have plotted the proton energy spectrum and the pion angular distribution for four choices of the off-shell behaviour. Two curves correspond to the form of equation (11) with $\beta$ = 1.8 and 3.6 $fm^{-1}$. The other two curves are for the Kisslinger form[9] and the multi-channel separable model of Londergen et al (LMM)[10]. In all these curves the shape of the cross-section is similar, the magnitude of the cross-section, however, changes considerably. Even the LMM and the separable forms(equation 11) with $\beta$ = 1.8 both of which fit the scattering data in the 3-3 resonance region, give substantially different results for the knock-out reactions. We feel that the sensitivity of the kinematically complete ($\pi, \pi N$) data may be even more to the off-shell behaviour of $t_{\pi N}$. This then suggests that the ($\pi, \pi N$) reaction may serve as a tool to differentiate between different off-shell forms of $t_{\pi N}$.

In figures 7-8 results are also plotted for the factorized form (equation 11). This shows that the factorized form can underestimate the cross-sections considerably.

The results shown in this section so far assume a sharp cut off of the nuclear matter to account for the absorption of pions (equation 14). In order to see the effect of the sharpness of the cut off we have also calculated the pion angular distribution and the proton energy spectrum at $T_\pi$ = 112 MeV for a diffuse cut off where the cut-off function, $f(r) = 1-(1+\exp[(r-R_0)/a])^{-1}$ with $R_0$ = 2.43 fm and $a$ = 0.5 fm. The parameters $R_0$ and $a$ are chosen so that $\int(1-f(r))\,r\,dr = R^2(E_\pi)/2$. The results with these two forms of cut-off are same within 5-10%.

In conclusion the results of this paper can be summarized as follows: (i) Strong absorption model, as suggested in section 2, described well the pion-induced knock-out reactions. In order to reproduce the typical features of the experimental cross-section, it is necessary to include the spin-flip part of the t-matrix. (ii) In the description of ($\pi, \pi N$) reactions it is very important to include the off-shell nature of the pion-nucleon scattering matrix. The use of factorized form for the cross-section can overestimate it by a considerable factor.

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## Figure Captions

Fig.1. Vector diagram.

Fig.2. Angular distribution of quasi-elastically scattered pions on $^{12}C$. Dashed (----), dot-dashed (-.-) and solid (——) curves represent non-spin-flip, spin-flip and summed cross-sections respectively.

Fig.3. Energy spectrum of the knocked-out protons on $^{12}C$. Dashed (---), dot-dashed (-.-) and solid (——) curves correspond to the non-spin-flip, spin-flip and summed cross-sections respectively.

Fig.4. Same as figure 2 at $T_\pi$ = 112 MeV.

Fig.5. Same as figure 3 at $T_\pi$ = 112 MeV.

Fig.6. Proton energy spectrum on $^{12}C$ at various incident pion energies ( $T_\pi$ ).

Fig.7. Sensitivity of the angular distribution of the quasi-elastically scattered pions on $^{12}C$ at $T_\pi$ = 112 MeV to various prescriptions of the pion-nucleon scattering matrix.

Fig.8. Sensitivity of the energy spectrum of the knocked-out proton from $^{12}C$ at $T_\pi$ = 112 MeV to various prescriptions of the pion-nucleon scattering matrix.

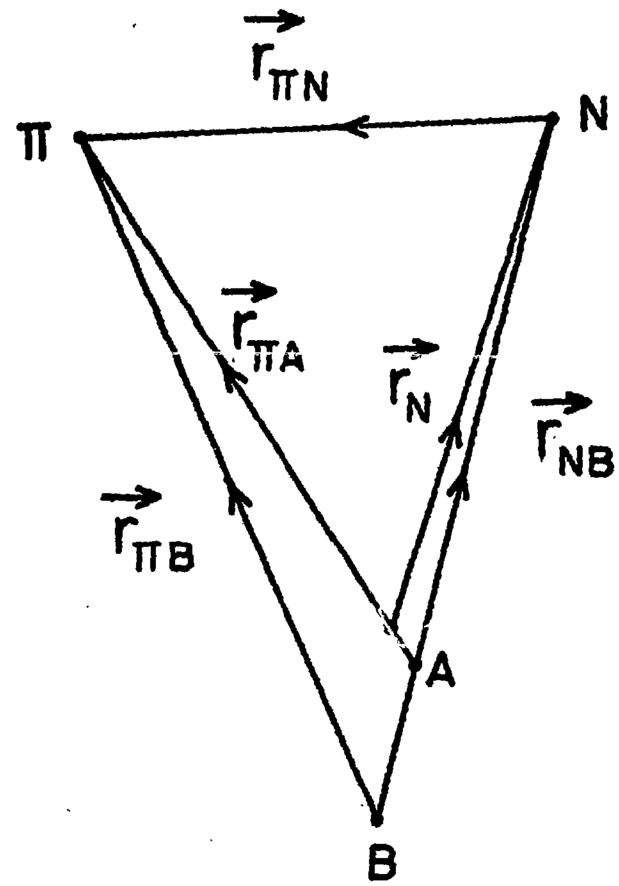
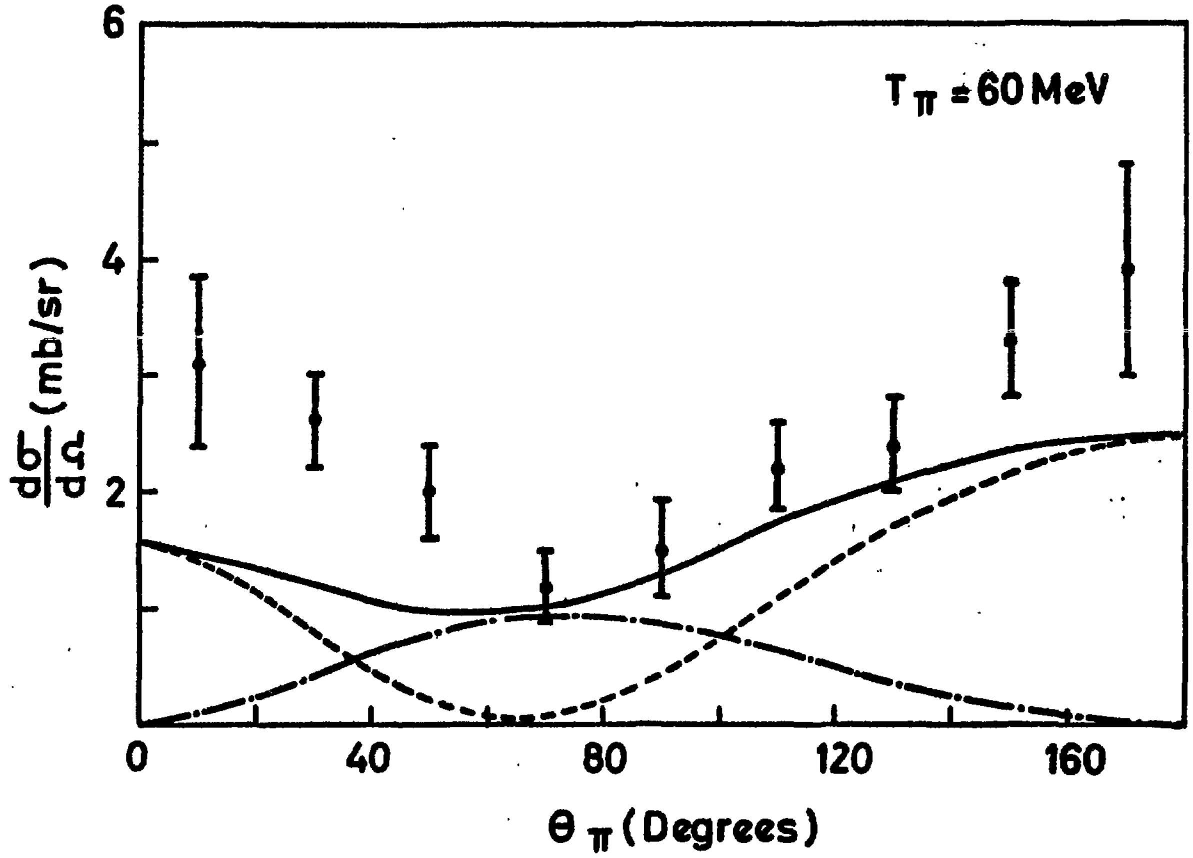
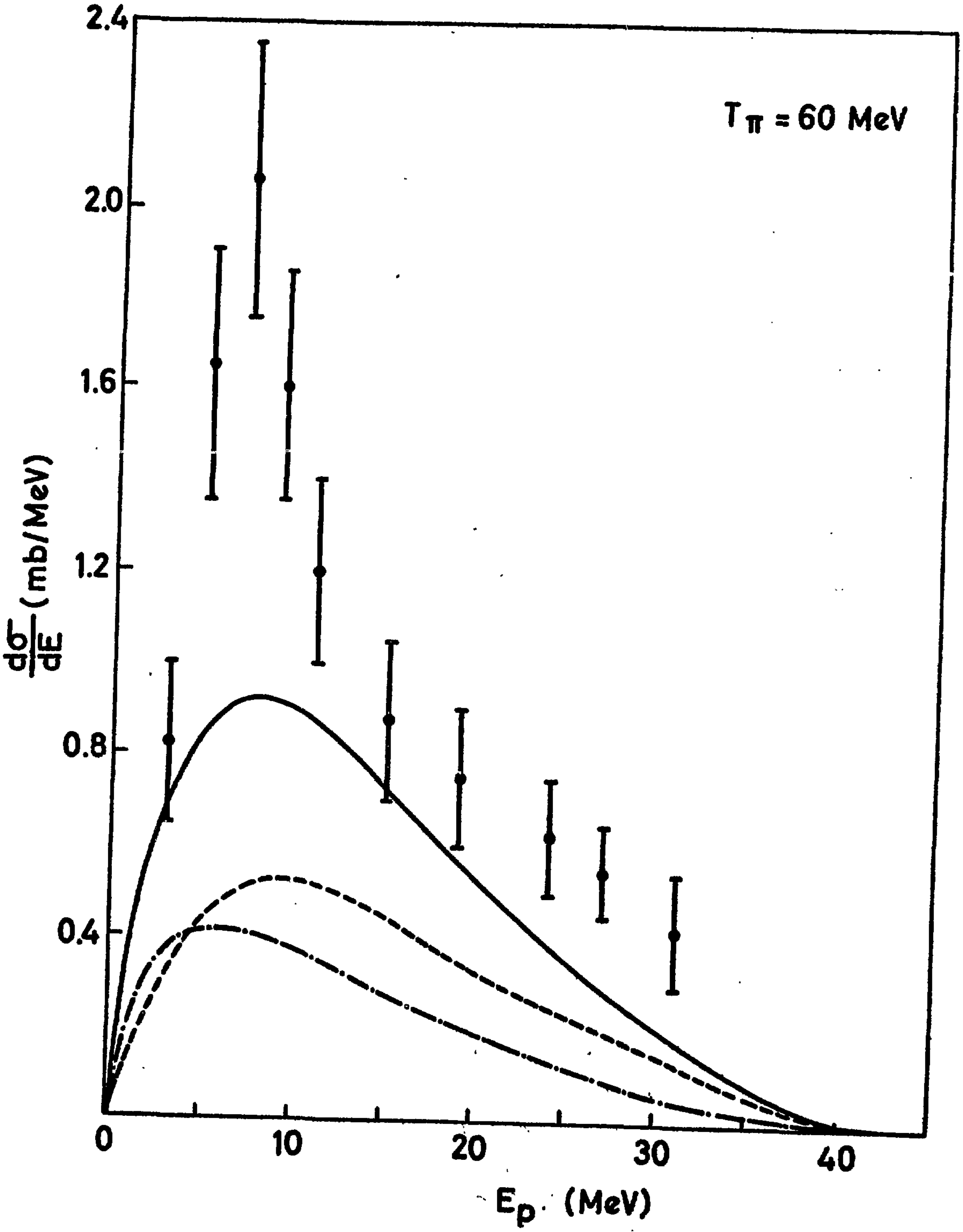



Fig. 1



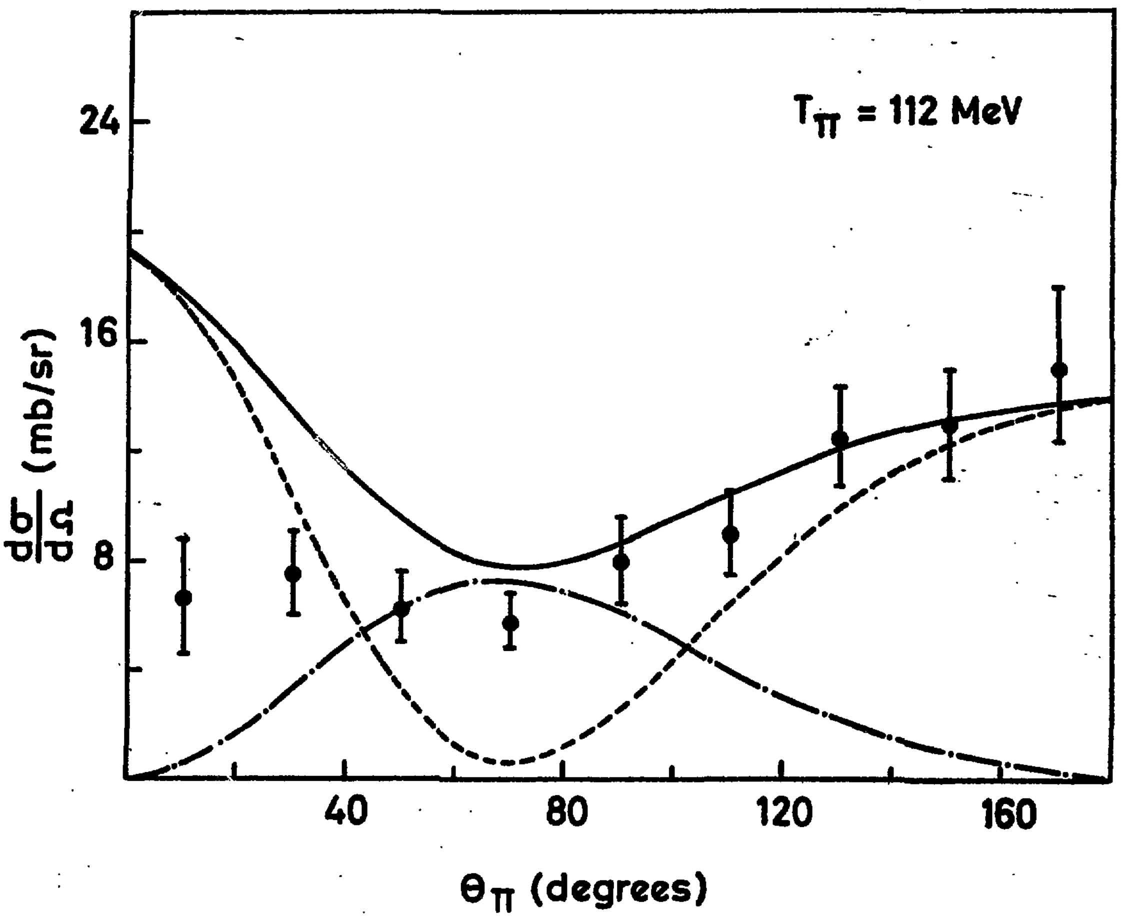
6
4
2
$T_\pi = 60$ MeV
$\frac{d\sigma}{d\Omega}$ (mb/sr)
0
40
80
120
160
$\theta_\pi$ (Degrees)


Fig. 2



2.4
2.0
1.6
1.2
0.8
0.4
$T_\pi$ = 60 MeV
$\frac{d\sigma}{dE}$ (mb/MeV)
0
10
20
30
40
$E_p$ (MeV)


Fig. 3



$T_\pi$ = 112 MeV
24
16
8
40
80
120
160
$\frac{d\sigma}{d\Omega}$ (mb/sr)
$\Theta_\pi$ (degrees)


Fig. 4

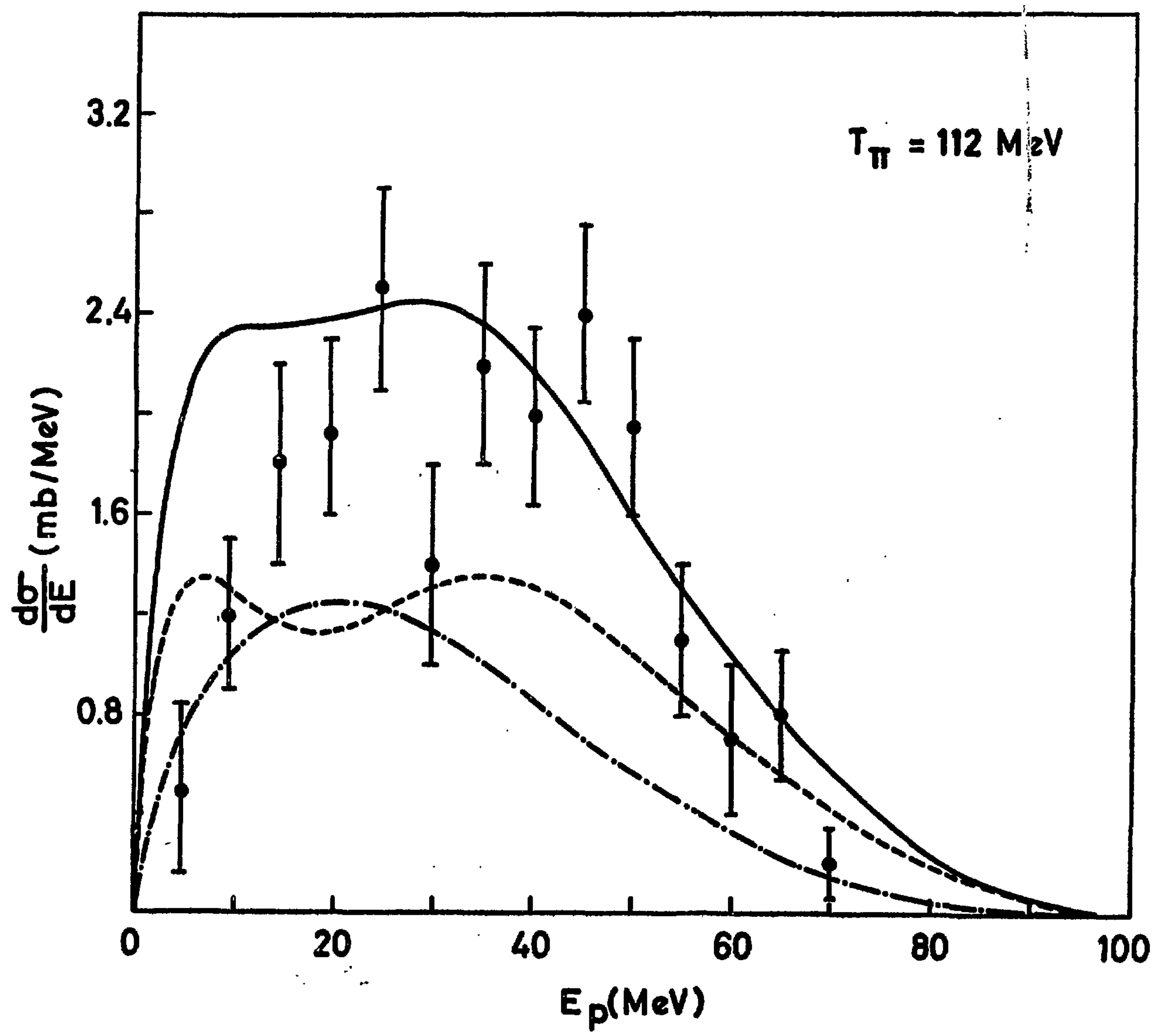
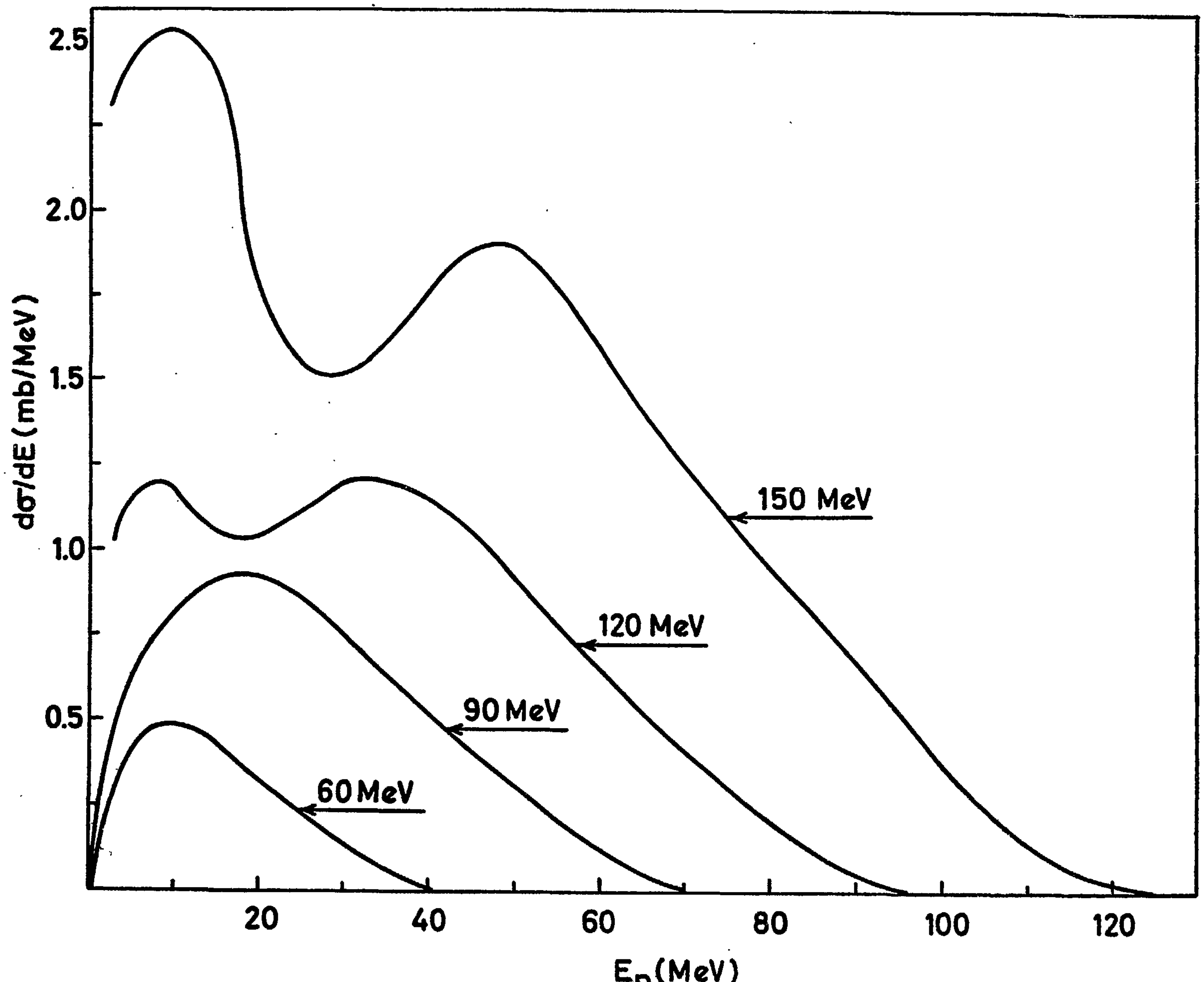



Fig. 5



2.5
2.0
1.5
1.0
0.5
dσ/dE (mb/MeV)
150 MeV
120 MeV
90 MeV
60 MeV
20
40
60
80
100
120
Ep(MeV)


Fig. 6

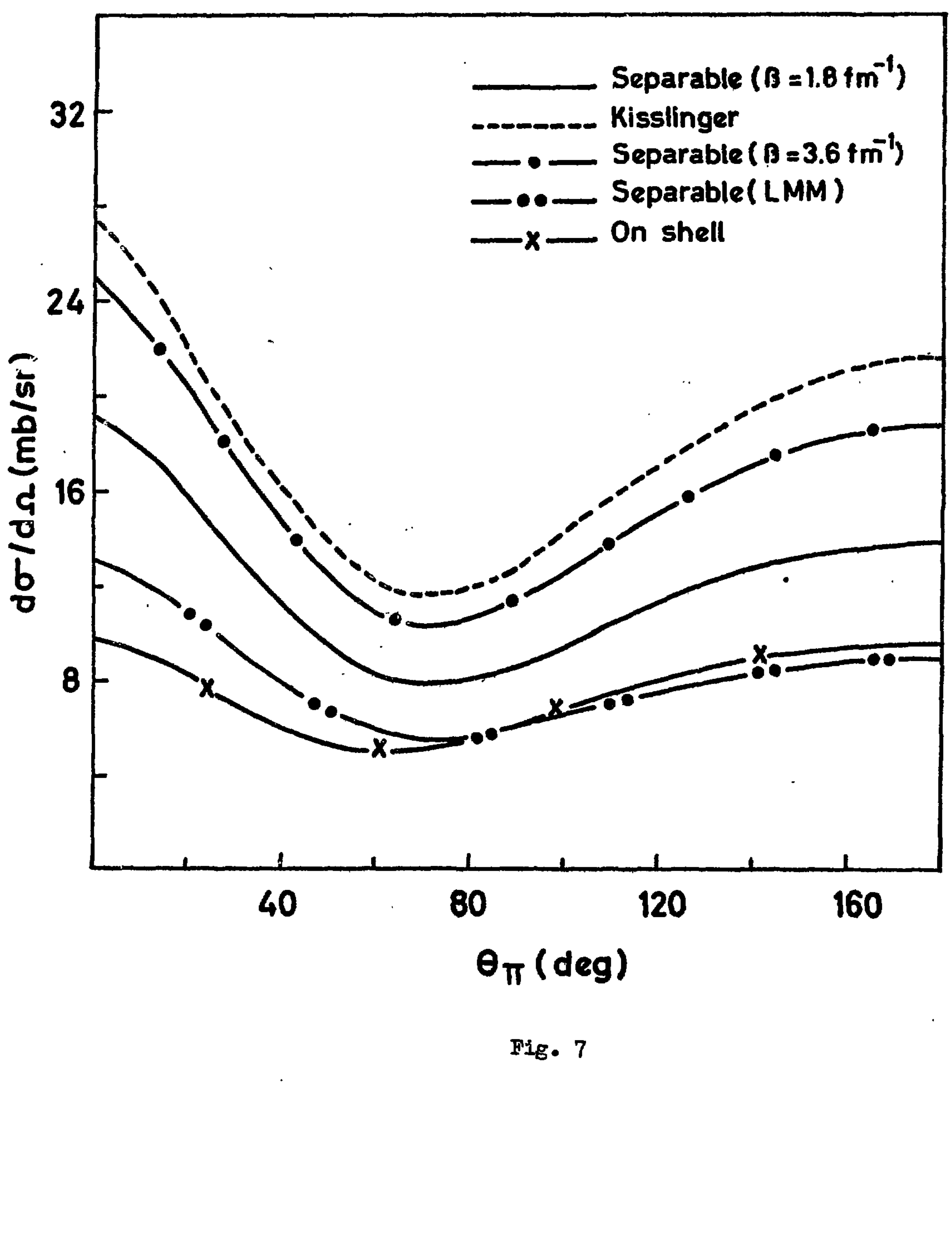

Separable (β = 1.8 fm⁻¹)
Kisslinger
Separable (β = 3.6 fm⁻¹)
Separable (LMM)
On shell
32
24
16
8
dσ/dΩ (mb/sr)
40
80
120
160
θπ (deg)

Fig. 7

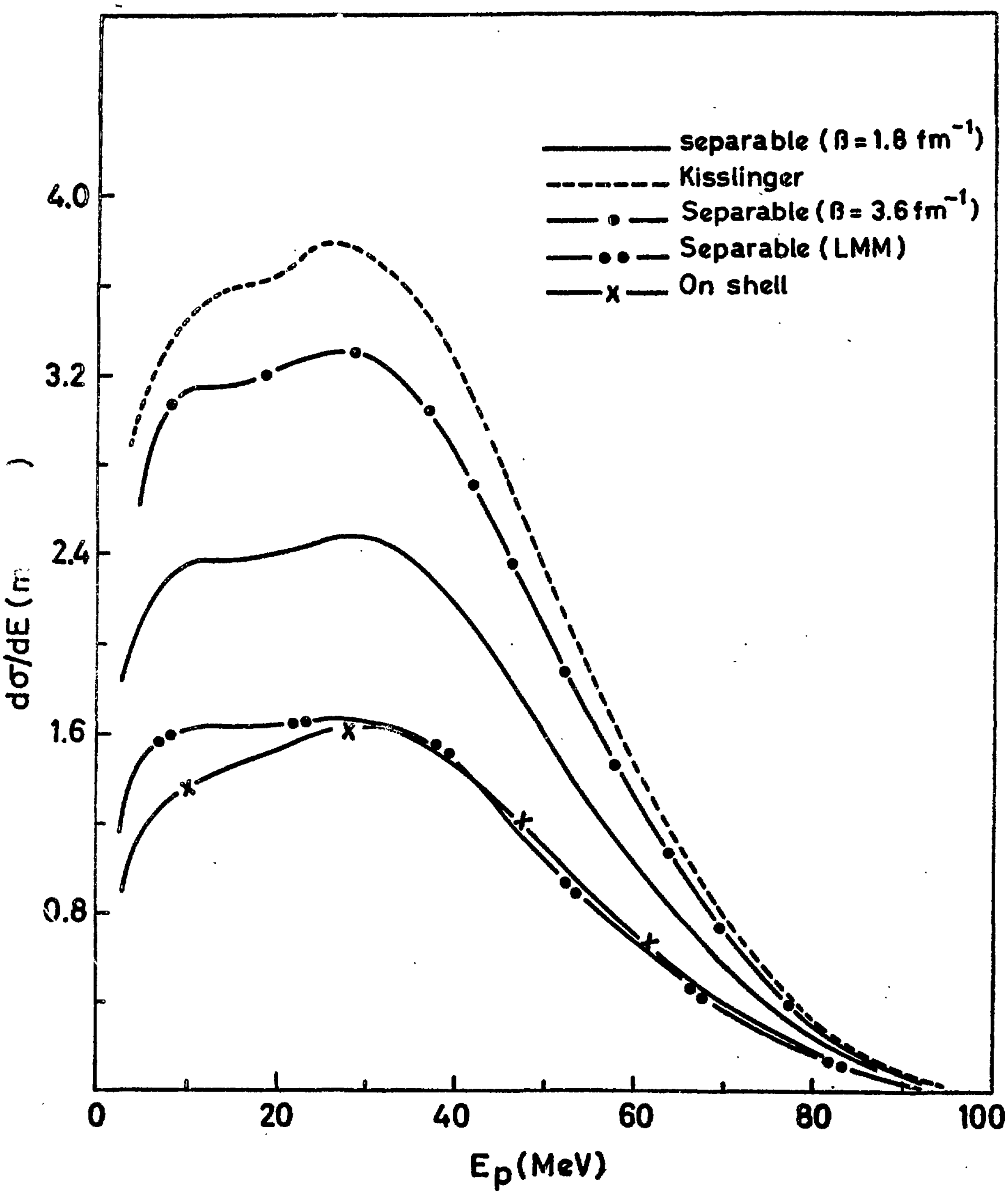

separable (β = 1.8 fm$^{-1}$)
Kisslinger
Separable (β = 3.6 fm$^{-1}$)
Separable (LMM)
On shell
dσ/dE
4.0
3.2
2.4
1.6
0.8
0
20
40
60
80
100
Ep(MeV)

Fig. 8