IC/77/58
INTERNAL REPORT
(Limited distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

CLASSICAL SOLUTIONS IN SUPERGRAVITY *

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MIRAMARE - TRIESTE
June 1977

* To be submitted for publication.

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**** Supported by NSF Grant No.PHY-76-15328.

ABSTRACT

Classical solutions of supergravity are obtained by making finite global supersymmetry rotations on known solutions of the field equations of the bosonic sector. The Schwarzschild and the Reissner-Nordström solutions of general relativity are extended to various supergravity systems and the modification to the perihelion precession of planets is discussed.

In the last year, interest has been raised in the theoretical development of gauge theories with a fermionic local symmetry. These theories are called supergravity theories [1),2)] because they contain gravitation and offer hope of unifying gravitation with the other interactions [3)]. Moreover, they are the only theories of gravitation in which first- and second-order quantum corrections are finite [4)].

Classical solutions of supergravity are of interest because they may provide corrections to the classical tests of Einstein's general relativity, and because, in particular, instantons [5)] may explain the nature of the vacuum and of the anomalies in the corresponding quantum theory. Below we obtain non-trivial classical solutions of pure supergravity and of supergravity with sources by applying a finite supersymmetry transformation to certain solutions of the field equations of the bosonic sector. First, the Schwarzschild solution of the sourceless Einstein theory is extended to a solution of pure supergravity, and then the Reissner-Nordström solution [6)] of the bosonic Maxwell-Einstein system is extended twice: once into a solution of the spin $(2, \frac{3}{2}, 1, \frac{1}{2})$ supersymmetric Maxwell-Einstein system, [7)] and once into a solution of the spin $(2, \frac{3}{2}, \frac{3}{2}, 1)$ O_2 model of extended supergravity [8)].

The pure supergravity action [1),2)] is invariant under the following infinitesimal local supersymmetry transformations:

$$\delta e^a{}_\mu = \kappa\bar{\epsilon}\gamma^a\psi_\mu \quad , \quad \delta\psi_\mu = 2\kappa^{-1}\,(\partial_\mu\epsilon + \tfrac{1}{2}\,\omega_\mu{}^{ab}\,\sigma_{ab}\epsilon)\,,$$

$$\delta\omega_\mu{}^{ab} = \tfrac{1}{2}\,(B_\mu{}^{ab} - e^b{}_\mu\, e_\ell{}^\lambda\, B_\lambda{}^{a\ell}) - (a \leftrightarrow b)\,,$$

$$B_\mu{}^{ab} = \kappa\epsilon^{abrs}\,(\bar{\epsilon}\gamma_5\gamma_\mu\; D_r\psi_s)\;. \tag{1}$$

The transformations of ω in first- and second-order formalismsagree if the field equations of e, ψ and ω are satisfied. When the spin-$\frac{3}{2}$ field ψ_μ vanishes, a finite global rotation of a given vierbein field $e_\mu^{a(0)}$ yields

$$e^a{}_\mu = e_\mu^{a(0)} + \bar{\epsilon}\gamma^a\,\Gamma_\mu\epsilon - \tfrac{1}{24}\,(\bar{\epsilon}\epsilon)^2\, R^a{}_\mu(e^0) \quad ,$$

$$\psi_\mu = 0 + 2\kappa^{-1}\,\Gamma_\mu\epsilon - \tfrac{1}{6}\,(\bar{\epsilon}\epsilon)\;(\gamma_\rho\epsilon)\; R^\rho{}_\mu(e^0) \quad , \tag{2}$$

where $\Gamma_\mu = \frac{1}{2}\omega_\mu{}^{cd}\sigma_{cd}$. Since the Bose sector of pure supergravity theory is equal to Einstein's theory, the Ricci tensors $R^a{}_\mu(e^0)$ vanish. Consequently, the modification to the metric is given by

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \bar{\epsilon}\gamma_\mu\Gamma_\nu\epsilon + \bar{\epsilon}\gamma_\nu\Gamma_\mu\epsilon + (\bar{\epsilon}\gamma^a\Gamma_\mu\epsilon)(\bar{\epsilon}\gamma_a\Gamma_\nu\epsilon) \quad . \qquad (3)$$

For the Schwarzschild solution, $e^a{}_\mu = (e^{-\frac{1}{2}\nu}, r, r\sin\theta, e^{\frac{1}{2}\nu})$ and for $\mu = r,\theta,\varphi,t$ with $\exp\nu = 1 - 2GMr^{-1}$, and using [9])

$$\Gamma_1 = 0 \quad , \Gamma_2 = \frac{1}{2}e^{\frac{1}{2}\nu}\gamma_1\gamma_2 \quad , \quad \Gamma_4 = -\frac{1}{4}\nu' e^\nu\gamma_1\gamma_4 \quad ,$$

$$\Gamma_3 = \frac{1}{2}\quad(\sin\theta e^{\frac{1}{2}\nu}\gamma_1\gamma_3 + \cos\theta\,\gamma_2\gamma_3) \quad , \quad \nu' = (d\nu/dr) \; , \qquad (4)$$

one obtains for the line element:

$$\begin{aligned} -1 = {} & (e^{-\nu}\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\varphi}^2 - e^\nu\dot{t}^2) - A_4\cos\theta e^{\frac{1}{2}\nu}\dot{r}\dot{\varphi} + \\ & + A_3 e^\nu(1 - \tfrac{1}{2}\nu' r)\dot{\theta}\dot{t} - A_2 e^\nu(1 - \tfrac{1}{2}r\nu')\sin\theta\dot{\varphi}\dot{t} + \\ & + A_1\cos\theta e^{\frac{1}{2}\nu}\dot{\varphi}\dot{t} + (\bar{\epsilon}\epsilon)^2\Big[\tfrac{1}{2}e^\nu\dot{\theta}^2 + \tfrac{1}{2}(\sin^2\theta\, e^\nu + \cos^2\theta)\dot{\varphi}^2 \\ & - \tfrac{1}{8}(\nu' e^\nu)^2\dot{t}^2\Big] \quad , \; A_\mu = \bar{\epsilon}\gamma_5\gamma_\mu\epsilon \quad . \qquad (5) \end{aligned}$$

Here a dot denotes differentiation with respect to proper time. Note that the spin-$\frac{3}{2}$ field vanishes on the equator of the horizon. This concludes the determination of the Schwarzschild solution in supergravity.

We now consider the motion of a point particle in this modified Schwarzschild background. This is of interest since the short-range effects due to the presence of fermions might modify the short-range effects of the corrections of higher order in G of general relativity. For the equation of motion we take the usual equation of Einstein's theory without the torsion term, and do not take a supersymmetric equation of motion. We are led to this choice because our Universe is not supersymmetric - it is at best spontaneously broken supersymmetric. Only experiment can tell whether this choice is correct. Hence, point particles move along the shortest paths which are not autoparallel [6] but the gravitational field is produced by sources which differ from those in the Einstein theory. This different way of treating the gravitational field and the point particle under super-

symmetry rotations will lead to observable effects. If we had treated them in the same way, we would not have obtained physical effects since we would merely have made a gauge transformation.

Let us now impose certain symmetry requirements on the motion of the point particle, similarly to the procedure in the Einstein theory [6]. Requiring that a point particle keeps moving in the equatorial plane, it follows from (5) that A_1, A_3 and A_4 vanish. Assuming that ϵ is small, A_2 is the only non-zero parameter. Inserting the equations of motion for t and φ into the line element and defining $u(s) = r^{-1}(s)$ one obtains, after differentiating with respect to φ :

$$\underbrace{\frac{d^2u}{d\varphi^2} + u - \frac{4GM}{L^2}}_{\text{Newton}} \quad \underbrace{- \; 3GMu^2}_{\text{Einstein}} = \underbrace{\frac{A_2 EGM}{2L^3}\left[12 - \frac{E^2}{(1-2GMu)^2}\right]}_{\text{supergravity}} \quad , \qquad (6)$$

where L and E are the integration constants which are given in the Einstein theory by $2r^2\dot{\varphi} = L$ and $-2e^\nu\dot{t} = E$. It follows that to order $A_2 G$, the effect of supergravity is merely a renormalization of the constant L . To higher order in G there are effects, but we shall leave these academic questions undiscussed. We only note that, upon explicit evaluation, $R_{\varphi t}(r)$ is non-zero, showing that the metric in (5) is not related to the usual Schwarzschild metric by a general co-ordinate transformation. A similar result holds for a Yang-Mills classical solution in the spin $(1,\frac{1}{2})$ supersymmetry system: there the fermionic energy momentum tensor as well as the Yang-Mills current do not vanish after a supersymmetry rotation if one uses only the field equations, but they do vanish for instanton solutions with $F = \tilde{F}$. [10),11)]

Finally, we consider the bosonic Maxwell-Einstein system which has the exact Reissner-Nordström solution describing an electrically charged black hole [6]. Extending this solution to a solution of the supersymmetric spin $(2, \frac{3}{2}, 1, \frac{1}{2})$ Maxwell-Einstein system, we find to order ϵ^2 the same result for the metric as in (5), but with $\exp\nu = 1 - 2GMr^{-1} + 4\pi e^2 r^{-2}$. This is because the transformation laws for e and ψ are still given by (1) to this order [7]. The complete result for a global supersymmetry rotation is given by:

$$\delta e^a_\mu = e^{a(0)}_\mu + \bar{\epsilon}\gamma^a \Gamma_\mu \epsilon - \frac{1}{24}(\bar{\epsilon}\epsilon)^2 \left[R^a_\mu - \frac{3}{2}\Gamma^a_\mu(\text{phot})\right] ,$$

$$\delta\psi_\mu = 0 + 2\kappa^{-1}\Gamma_\mu \epsilon - \frac{1}{6}(\bar{\epsilon}\epsilon)(\gamma_a \epsilon)\left[R^a_\mu - \frac{3}{2}\Gamma^a_\mu(\text{phot})\right] , \qquad (7)$$

and inserting the Maxwell-Einstein field equations one obtains an order ϵ^4 correction for e^a_μ proportional to the photonic energy momentum tensor. Different results are obtained for the extension of the Reissner-Nordström solution to the O_2 extended supergravity system, whose spin content is $(2, \frac{3}{2}, \frac{3}{2}, 1)$. Due to the presence of two rather than one local supersymmetry invariances, there are many more parameters in the solution. The transformation rules are given by [8)]

$$\delta e^a_\mu = \kappa\bar{\epsilon}^j\gamma^a \psi_\mu^j \ , \ \delta A_\mu = \sqrt{2}\ \bar{\epsilon}^j \psi_\mu^j ,$$

$$\delta\psi_\mu^j = 2\kappa^{-1} D_\mu \epsilon^j - 2^{-1/2} F_{\rho\sigma}(\sigma^{\rho\sigma}\gamma_\mu \epsilon^k)\epsilon^{jk} + O(\psi^2) , \qquad (8)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. For the vierbein component of the supersymmetric Reissner-Nordström solution in the O_2 model we find

$$e_{a\mu} = e_{a\mu}^{(0)} + \bar{\epsilon}^j \gamma_a \Gamma_\mu(e^0)\epsilon^j + \sqrt{2}\epsilon^{jk}\bar{\epsilon}^j \left[\sigma_{a\rho} F^\rho{}_\mu + \sigma_{\mu\rho} F^\rho{}_a + \frac{1}{2}\sigma^{\alpha\beta} F_{\alpha\beta} e_{a\mu}^{(0)}\right]\epsilon^k .$$

A similar result holds for A_μ, so that a modification of the Coulomb law occurs already at the ϵ^2 order. The modification of supergravity to light bending, radar time delay and the red shift will also be unobservable, but it might be interesting to consider cosmological solutions and Big Bang aspects in this light.

ACKNOWLEDGMENTS

The authors would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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