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STATISTICAL ANALYSIS OF RANDOM PULSE TRAINS *

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ABSTRACT

We present here some experimental and theoretical results concerning the statistical properties of optical beams formed by a finite number of independent pulses. The considered waves (corresponding to each pulse) present important spatial variations of the illumination distribution in a cross-section of the beam, due to the time-varying random refractive index distribution in the active medium. Some examples of this kind of emission are: a) free-running ruby laser emission, b) mode-locked pulse trains, c) randomly excited nonlinear media.

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In some cases, Van Cittert-Zernike's theorem1) allows one to establish the concept of a thermal source "equivalent" (with respect to second-order statistical properties) to a given laser.2) In other words, if one is able to measure the second-order coherence function of the laser wave in some particular plane region, it is possible to determine (via the VC-Z theorem) a thermal source whose associated radiation field has the same second-order coherence properties as the laser field, far from the source. It should be noticed, nevertheless, that the random process associated with the radiation field of a thermal source is gaussian and stationary. That means, in particular, that the degree of coherence and the intensity of the radiation field are approximately constant near the optical axis of the system. We shall see that these conditions considerably limit the kind of laser sources which can be represented by means of an equivalent thermal source (ETS).

Consider, for example, the spiking emission proceeding from a free-running ruby laser. Indeed, the illuminance distribution in a cross-section of the beam is rather uniform if one records the superposition of the whole pulse train. This is always true when the pumping energy is well above the oscillation threshold, and no mode selection is performed.3) On the other hand, when the whole pulse train is processed by any of the interferometric devices currently used to study second-order coherence properties 2), one records in the far field a fringe system whose visibility is approximately constant near the optical axis of the system. Also, the fringe system appears superimposed on an almost uniform background. The degree of coherence and the illuminance being nearly constant, we conclude that an ETS can be assigned to the laser source emitting the whole train of spikes. In addition, it is well known that an image of the ETS can be directly obtained by Fourier transforming the illuminance distribution of the interferogram. The latter behaves, in fact, as an incoherent hologram.2),3)

The same kind of interferogram is obtained when one uses incoherent holographic techniques to analyse the statistical properties of the Stokes radiation generated in a stimulated Raman scattering experiment.4) The reason for the "incoherent" behaviour of the integrated emission is the same in both cases. In the SRS experiment, Stokes radiation is formed from δ-correlated seeds (the spontaneously decaying atoms of the amplifying medium). If the exciting wave (coming from a Q-switched ruby laser in Ref.4) is not smooth, the time-varying intensity dependent refractive index of the nonlinear medium introduces random variations in the phase of the parametrically generated waves, thus diminishing the coherence of the time-integrated emission.

In the free-running strongly excited ruby laser, the inhomogeneity in the active medium is produced by the pumping field, which gives rise to a space-time varying refractive index and population inversion distribution. It is reasonable to infer that the same kind of phenomenon should appear also in other situations involving the interaction of trains of high power optical pulses with matter (for example, in experiments with mode-locked lasers).

Let us now look separately at each one of the waves corresponding to the different pulses. Experiments performed with a high-speed, rotating mirror camera in combination with a Hopkins double prism interferometer have shown 3),5) that, in the case of a free-running ruby laser:

a) The illuminance distribution corresponding to each spike is rather non-uniform. Indeed, when the laser works well above the excitation threshold each wave has the grainy structure characteristic of speckle patterns. This grainy illuminance distribution changes randomly from one spike to the other.

b) The correlation radius of the field amplitude is much greater for each individual wave than for the integrated emission. Also, the correlation radius changes randomly all over the interference field.

In this case one clearly cannot assign an ETS to each spike of the time-resolved emission. In other words, no thermal source would give rise to the results (a) and (b). On the contrary, all happens as if, due to the non-uniform structure of the active medium, the latter behaves as a random diffuser illuminated (a finite number N of times) by a coherent wave. This fact leads to a great deal of confusion when incoherent holographic methods are used to study these "speckle-emitting sources" (SES) because in the general case one really does not know which is the relation between the reconstituted image and the actual laser source.

In order to be able to give a correct interpretation to this kind of experiments, we have calculated the first two moments of the statistical distribution of the illuminance in the reconstituted image, as a function of:

a) the number N of pulses forming the emission,

b) the moments of the random distribution of the field amplitude in each of the N waves,

c) the geometrical form of the actual source.

As a main result, we have proved that only in the limit when the mean dimension g of the speckles tends to zero and the number N of spikes tends to infinity, the illuminance distribution in the integrated reconstituted image tends to the mean value of the illumination distribution in the actual source. Only in that case $(g \to 0 , N \to \infty)$ the notion of a thermal source equivalent to a given SES has a precise sense. If $g \neq 0 , N \neq \infty$, all that can be calculated is the probability for the illuminance in the integrated reconstituted image to resemble the actual illuminance distribution in the SES within certain given limits.

Finally, the theory has been applied to the study of a time-resolved interferometric record of the ruby laser free-running emission. The N images required by the theory are reconstituted separately, then they are superposed to form a unique integrated image. This is compared with the actual form of the source, obtained by processing a much greater number of interferograms $(N \to \infty)$.

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