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MASSIVE VECTOR FIELDS AND BLACK HOLES *

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ABSTRACT

A massive vector field inside the event horizon created by the static sources located outside the black hole is investigated. It it shown that the back reaction of such a field on the metric near $r = 0$ cannot be neglected. The possibility of the space-time structure changing near $r = 0$ due to the external massive field is discussed.

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It is currently believed that stationary black holes in vacuum are completely characterized by their mass, angular momentum and electric charge [1)] ("no hair theorem"). Bekenstein [1] proved that a stationary bare black hole cannot be endowed with any exterior classical massive vector field (MVF). Thus baryons which are sources of ρ , φ and ω meson MVF after falling into a black hole cannot interact with any particles outside the event horizon.

The purpose of this note is to demonstrate the fact that static MVF sources distributed outside a black hole can influence the space-time structure under the event horizon. To show this we consider a black hole of mass m which appeared as a result of a spherically symmetric massive body collapse. Not taking into account a back reaction of the MVF on a metric, we shall describe the gravitational field of the black hole by a Schwarzschild metric in Eddington-Finkelstein co-ordinates

$$ds^2 = \Phi dv^2 - 2\,dv\,dr - r^2\,(d\theta^2 + \sin^2\theta d\varphi^2),\ \Phi = 1 - \frac{2m}{r} \ , \quad (1)$$

where v is the advanced time. This metric describes the gravitational field in the region ABCD of the Kruskal diagram (Fig.1).The line AB represents the surface motion of a collapsing body. We consider a thin massless shell possessing the MVF charge g , which was slowly brought to the black hole (curve AE) and then rests at a constant radius $r_0 > r_g = 2Gm/c^2$ (curve EC). In this case the MVF in the region FCDG does not depend on the co-ordinate v .

The MVF equation $* d * F + \mu^2 A = 4\pi I$ under our assumptions gives

$$A = (A_v(r),\ A_r(r),0,0),\ A_v = \frac{f(r)}{r} \ ,\ A_r = -\frac{f(r)}{\Phi\cdot r} \ , \quad (2)$$

$$\frac{d^2f}{dr^2} - \frac{\mu^2}{\Phi}\, f = g\,\frac{\delta(r-r_0)}{r} \ . \quad (3)$$

Passing to dimensionless variables [2)] $\xi = 2\mu(r-2m)$, $\kappa = -m\mu$, $\xi_0 = 2\mu(r_0-2m)$ we can write (3) in the form:

1) The possible existence of stationary black holes with external massless scalar field was discussed in papers [2-4].

2) We put $G = c = h = 1$, and so $\mu = 1/\lambda$, where λ is a Compton wavelength of the MVF and κ is equal to $r_g/2\lambda$.

$$\frac{d^2f}{d\xi^2} + \left(-\frac{1}{4} - \frac{\kappa}{\xi}\right) f = g\,\frac{\delta(\xi-\xi_0)}{\xi_0 + 4\kappa} \qquad (4)$$

The general solution of this equation outside the source is

$$f = \alpha\, W(\xi) + \beta\, M(\xi) \quad , \qquad (5)$$

where $W = W_{-\kappa,\frac{1}{2}}(\xi)$ and $M = M_{-\kappa,\frac{1}{2}}(\xi)$ are the Whittaker functions [5]. The differential Eq.(4) has two singular points, $\xi = 0$ and $\xi = \infty$; the first being regular and the second being irregular. At these points the Whittaker functions have the following behaviour:

$$W_{-\kappa,\frac{1}{2}}(\xi) = \begin{cases} \frac{1}{\Gamma(1+\kappa)} + \frac{1}{\Gamma(\kappa)}\,\xi \ln \xi + O(\xi) \quad , & |\xi| << 1 \;; \\ \xi^{-\kappa}\, e^{-\xi/2}\,(1 + O(\xi^{-1})) \quad , & |\xi| >> 1 \;; \end{cases}$$

$$M_{-\kappa,\frac{1}{2}}(\xi) = \begin{cases} \xi + O(\xi^2) & |\xi| << 1 \;; \\ \frac{1}{\Gamma(1+\kappa)}\,\xi^{\kappa}\, e^{\xi/2}\,(1 + O(\xi^{-1})) \quad , & |\xi| >> 1 \;. \end{cases}$$

If we require the solution to be decreasing at spatial infinity and to have a finite value of the invariant $F_{\alpha\beta}F^{\alpha\beta}$ at the event horizon, we unambiguously have

$$f(\xi;\xi_0) = \begin{cases} g\,\frac{\Gamma(1+\kappa)}{\xi_0 + 4\kappa}\, M(\xi_0)\, W(\xi) \quad , & \xi > \xi_0 \;; \\ g\,\frac{\Gamma(1+\kappa)}{\xi_0 + 4\kappa}\, W(\xi_0)\, M(\xi) \quad , & \xi < \xi_0 \;. \end{cases} \qquad (6)$$

This expression shows that the field at any fixed point outside the black hole tends to zero as the source of the MVF approaches the event horizon, while the field strength F_{vr} at the event horizon tends to g/r_g^2 .

The total flux $P(\Sigma)$ of the MVF force lines through the two-dimensional surface Σ is equal to $P(\Sigma) = \int_\Sigma * F$. If Σ is a boundary of the black hole (v = const, $r = r_g$) and the charge g of the MVF approaches the event horizon, then the total flux $P(\Sigma)$ tends to $4\pi g$. The generalized Gauss theorem [3],

$$\int_{\delta V^3 = \Sigma_1 - \Sigma_2} * F = 4\pi g - \int_{V^3} \mu^2 * A \;,$$

makes it possible to conclude that when the sources of the MVF are very close to the boundary of the black hole almost all of the MVF force lines are drawn into the black hole, and this is the reason why the field strength vanishes in the outside region.

The obtained solution (6) also describes the MVF under the event horizon ($-4\kappa < \xi < 0$). The behaviour of the field near the $r = 0$ singularity is determined by a value $M(-4\kappa)$. It is natural to consider the following three cases [3]: i) microscopic black holes ($\kappa << 1$); ii) small black holes ($\kappa \sim 1$); and iii) macroscopic black holes ($\kappa >> 1$). We have, correspondingly,

$$M(-4\kappa) = \begin{cases} -4\kappa \quad , & \kappa << 1 \;; \\ -4\,\exp(-2) \quad , & \kappa = 1 \;; \\ -\frac{4\Gamma(1/3)}{\sqrt{3}\,\pi}\left(\frac{16}{3}\kappa\right)^{-1/3} \cos[\pi(\kappa + \frac{1}{3})] \;, & \kappa >> 1 \;. \end{cases}$$

It is not difficult to see that near $r = 0$ the energy momentum tensor of the MVF generally diverges and its leading part is of the form:

$$T^{\mu}_{\nu} \simeq \mathrm{diag}\left(\frac{G}{8\pi r^4} \;, \frac{G}{8\pi r^4} \;, -\frac{G}{8\pi r^4} \;, -\frac{G}{8\pi r^4}\right) \;, \quad G \equiv f(-4\kappa;\xi_0) \;,$$

which is similar to the expression for the energy momentum tensor of an electric field created by a point charge G . This behaviour of the energy momentum tensor shows that the MVF of the external sources can change the structure of the $r = 0$ singularity (for example, to make it similar to the Reissner-Nordström singularity). It should be noted that in the case of macroscopic black holes this effect is small (but not exponentially small). But for small primordial black holes ($m \sim 10^{15}$ g) surrounded by baryons at the early stage of the Universe it may be essential.

3) It should be noted that for the ρ-meson MVF λ is 10^{-13} cm and a black hole with $\kappa \sim 1$ ($r_g \sim 2\lambda$) has a mass 10^{15} g . The primordial black holes of about this mass are of particular interest in connection with the phenomenon of a black hole quantum evaporation discussed by Hawking [6].

Using the solution (6) we can find the MVF of an arbitrary spherically-symmetric distribution of the MVF charge $g\rho(\xi)$ around the black hole

$$f(\xi) = \int_{\xi_0}^{\xi_1} f(\xi;\zeta)\ \rho(\zeta)\ d\zeta\ ,\ A_v = \frac{f}{r}\ ,\ A_r = \frac{-f}{\phi \cdot r}\ .$$

The existence of the MVF strength near the boundary of a black hole provides the energy difference between baryons and antibaryons near or under the event horizon. It may possibly result in a difference between the number of particles and antiparticles emitted due to the Hawking process of the quantum evaporation of small black holes if such black holes are surrounded by the MVF sources.

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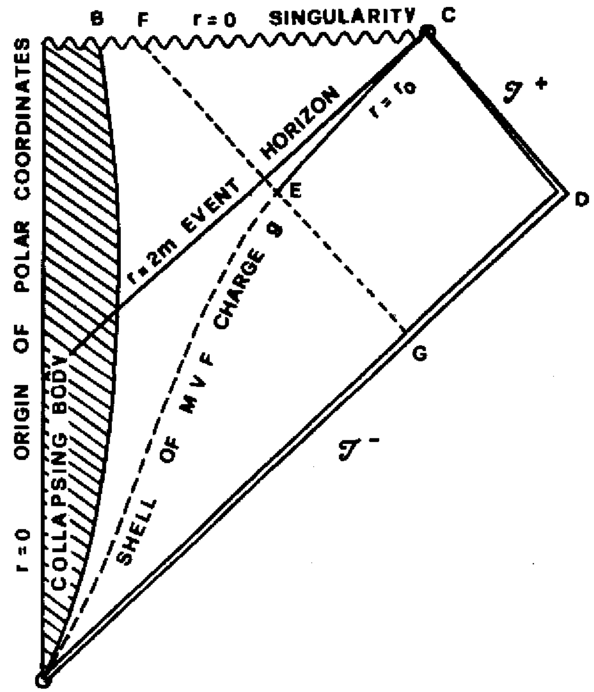

Fig.1

The Penrose picture of the collapsing body space-time.