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THEORETICAL CONSIDERATIONS FOR THE SELECTION OF ELECTRO-OPTIC CRYSTALS FOR THE GRATING-BASED LASER BEAM MODULATORS/DEFLECTORS *

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ABSTRACT

The optical properties of a crystal can be altered by the application of an external electric field. The electrically induced changes of the crystal properties may, in some cases, be utilized to modulate an optical beam. The nature of propagation of electromagnetic waves inside an optically anisotropic crystal and the variation of the optical properties with an applied electric field are discussed in this paper. The general criteria that must be satisfied by the crystals for efficient modulation and deflection are analysed. In particular, the properties and suitable orientations of a $Li\,Nb\,O_3$ crystal are analysed in some detail for using as an efficient modulator.

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I. INTRODUCTION

The passage of an electromagnetic wave, having a frequency in the microwave region, inside an electro-optic crystal produces a periodic variation in the refractive index. Thus a beam of light passing through the perturbed medium in a correctly chosen direction, experiences a phase grating and will be diffracted into different orders. The decrease in the energy of the zero-order is proportional to the microwave power. In general, the power required for zero-order extinction is very high (~ 5 W).[1] Further, difficulty arises in matching microwaves into high dielectric constant electro-optic materials. Hence, producing a phase grating inside electro-optic crystals by using an interdigital electrode structure is considered as an alternative approach.

When an interdigital electrode structure is deposited onto a suitably chosen face of an electro-optic crystal and a voltage is applied, a fringing field penetrates into the crystal. This fringing field periodically changes the index of refraction of the crystal, thus causing a phase grating to appear inside the medium. When an optical beam experiences this diffraction grating, the incident beam will be diffracted to the higher orders resulting in the modulation of the zero-order beam. This approach renders a high speed of operation and easy fabrication of the modulator.

II. BIREFRINGENCE IN ELECTRO-OPTIC CRYSTALS

The propagation characteristics of an electromagnetic wave passing through an anisotropic crystal will depend on the direction of propagation of this wave. An optical beam travelling inside a crystal in any arbitrary direction is resolved into two components, the polarizations of the two components are mutually perpendicular to each other. In order to find the indices of refraction associated with these two polarizations, it is convenient to analyse the equation of the index of ellipsoid or the optical indicatrix. The equation of the ellipsoid in general may be written as

$$\frac{X^2}{n_x^2} + \frac{Y^2}{n_y^2} + \frac{Z^2}{n_z^2} = 1 \quad , \qquad (1)$$

where n_x , n_y and n_z are the refractive indices in the direction of the principal axes X , Y and Z respectively.

The two displacement vectors $\underset{\sim}{D}$ and the two refractive indices associated with these $\underset{\sim}{D}$ vectors are geometrically found by considering the plane of intersection of the index ellipsoid and the plane normal to the incident beam which passes through the origin of the ellipsoid. The intersection curve is in general elliptical and half the values of the major and minor axes correspond to the two refractive indices. The directions of the $\underset{\sim}{D}$ vectors are parallel to these major and minor axes and the phase velocities of the two components are determined by the respective indices of refraction. The application of an external electric field changes the transmission properties of a crystal.

The refractive index of a crystal in the presence of an external electric field may be written as

$$n = n^0 + aE + bE^2 + \cdots , \qquad (2)$$

where n^0 is the refractive index at zero applied field, and a,b,... are constants.

In the above equation the linear term (aE) contributes to the linear electro-optic effect (Pockels effect) and the quadratic term (bE^2) contributes to the quadratic electro-optic effect (Kerr effect). In crystals which possess inversion symmetry, the refractive index is unaltered by a reversal of the applied electric field. This occurs only when the constants in Eq.(2) with odd powers are zero. Therefore, in these crystals the linear electro-optic effect cannot exist. However, quadratic effects can occur in crystals with inversion symmetry. It is to be remembered that Eq.(2) was derived under the assumption that $\underset{\sim}{D}$ and $\underset{\sim}{E}$ are parallel. When this does not occur and $\underset{\sim}{D}$ and $\underset{\sim}{E}$ are not parallel, an external electric field applied to the crystal may again change the dimensions of the axes of the ellipsoid (Eq.(1)) but the ellipsoid may rotate. In the Voigt notation,[2] the deformed index ellipsoid may then be represented in general terms as

$$a_1X^2 + a_2Y^2 + a_3Z^2 + 2a_4YZ + 2a_5ZX + 2a_6XY = 1 , \qquad (3)$$

where a_i ($i = 1,2,\ldots,6$) are coefficients related to the indices of refraction.

III. PRIMARY AND SECONDARY EFFECTS

In general, when an electric field is applied to a crystal, the shape of the crystal is altered. This effect is known as the converse piezoelectric effect.[3] When the crystal is deformed, the index of refraction of the crystal will be changed.

The strain in the crystal is related to the change in the refractive index by the elasto-optic coefficients of the crystal and the total change of the index of refraction is conveniently represented as

$$\left(\frac{1}{n^2}\right)_i = \sum_{k=1}^{6} r_{ik} E_k + \sum_{j=1}^{6} p_{ij} e_j , \qquad (4)$$

where $i = 1,2,\ldots,6$ and r, p and e are the electro-optic, elasto-optic and strain tensor coefficients respectively. E is the component of the field intensity and n is the refractive index.

The electro-optic effect that would be obtained if the crystals were not strained is called the primary (or true) effect and that due to the piezoelectricity and photoelasticity is called the secondary (or false) effect. In the above Eq.(4) the two terms on the right-hand side do not necessarily have the same sign.

If the crystal is free from external forces, the observed effect is the sum of the primary and the secondary effects and, since the algebraic signs of the two contributions are not necessarily equal, the combined effect may be greater or smaller than the primary effect.[4] If the applied electric field is at a frequency corresponding to an acoustic resonant frequency of the device, then the magnitude of the secondary effect may be considerably larger than that of the primary effect. The resonant frequencies of the devices are determined by the dimensions of the crystal and for surface waves, the geometry of the electrodes.

The application of an alternating electric field to a crystal, for example, in the shape of a rectangular parallelpiped causes, in general, nine acoustic waves to travel inside the bulk of the crystal. These are one quasi-longitudinal wave and two quasi-shear waves along the normals to the faces of the crystal.[5] However, the symmetry of the crystal causes a reduction in the number of waves capable of being generated inside the medium by an electric field.

Restricting the analyses to the primary linear electro-optic effect, the coefficients a_i $(i = 1,2,\ldots,6)$ of Eq.(3) may be written as

$$a_i = \frac{1}{n_i^2} + \sum_{k=1}^{3} r_{ik} E_k \quad , \qquad (5)$$

where $i = 1,2,\ldots,6$, r_{ik} is a third-rank electro-optic tensor. When $i = 4,5$ or 6, from Eq.(1), $1/n_i$ is zero as no terms of the forms YZ, ZX or XY appear in it. These are the terms in Eq.(3), which gives rise to the rotation of the index ellipsoid.

Substituting Eqs.(1) and (5) in (3) gives

$$\left[\frac{1}{n_X^2} + r_{11}E_X + r_{12}E_Y + r_{13}E_Z\right]X^2 + \left[\frac{1}{n_Y^2} + r_{21}E_X + r_{22}E_Y + r_{23}E_Z\right]Y^2 +$$
$$\left[\frac{1}{n_Z^2} + r_{31}E_X + r_{32}E_Y + r_{33}E_Z\right]Z^2 + 2(r_{41}E_X + r_{42}E_Y + r_{43}E_Z)YZ +$$
$$2(r_{51}E_X + r_{52}E_Y + r_{53}E_Z)ZX + 2(r_{61}E_X + r_{62}E_Y + r_{63}E_Z)XY = 1 \quad . \qquad (6)$$

In Eq.(6) the non-vanishing electro-optic coefficients can be determined by the crystal symmetry.

IV. SELECTION OF ELECTRO-OPTIC CRYSTALS

Eq.(6) shows that the application of an external electric field to the crystal causes:

1) A change in the lengths of the ellipsoid axes (change in n).
2) A rotation of the ellipsoid giving rise to a rotation of the planes of polarization of the optical beam.

This second factor is not desirable in the present consideration of the grating-based modulators as the voltages required for the diffraction effect are dependent on the plane of polarization of the incident beam and, therefore, suitable crystals must be selected which will give a large change in the length of the ellipsoid axes but with zero or negligible contribution to the rotation of the ellipsoid.

A close study of the available electro-optic crystals with large electro-optic coefficients suggests that the triclinic, monoclinic and orthorhombic classes of crystals are not suitable for this purpose (a few of the last two classes of the crystals have reasonably large electro-optic coefficients but they are difficult to produce as large crystals with consistent optical quality).[6]

Consider an interdigital electrode system deposited onto a polished face of a crystal. The reference co-ordinate exes are taken to be along the principal axes of the crystal. The electrodes are in the XY plane and are symmetrically and periodically spaced along the Y direction. The electrode lengths are along the X axis. When alternate polarities are applied to the adjacent electrodes, a fringing field, which reduces rapidly with an increase of Z , penetrates into the crystal. In the arrangement considered in Fig.1, the components of the field are

$$E_X = 0 \quad , \quad E_Y = E_Y \quad \text{and} \quad E_Z = E_Z \quad .$$

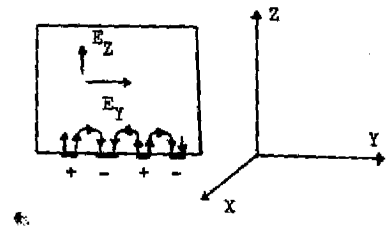

Fig.1

In order to ensure that the incident beam experiences a periodically varying refractive index, the optical beam should be directed in the direction of the electrode lengths. For simplicity, assume that the incident light is travelling along the X direction.

By considering the intersection of the index ellipsoid and the ZY plane, the ellipse which determines the two refractive indices is

$$\left(\frac{1}{n_Y^2} + r_{22}E_Y + r_{23}E_Z\right)Y^2 + \left(\frac{1}{n_Z^2} + r_{32}E_Y + r_{33}E_Z\right)Z^2 + 2(r_{42}E_Y + r_{43}E_Z)YZ = 1 \quad . \qquad (7)$$

The first two terms determine the lengths of the axes of the ellipse and the third term corresponds to the rotation of the ellipse. An identical equation can be obtained for the case when the electrodes are deposited on the

ZX plane with the electrode lengths along the X axis. For the cases when the electrodes are deposited on the XY or YZ planes and the light propagating parallel to the Y axis, the equation of the ellipse reduces to

$$\left(\frac{1}{n_x^2} + r_{11}E_X + r_{13}E_Z\right)X^2 + \left(\frac{1}{n_z^2} + r_{31}E_X + r_{33}E_Z\right)Z^2 + 2(r_{51}E_X + r_{53}E_Z)ZX = 1 \quad . \tag{8}$$

A similar equation can be obtained for electrodes deposited on the ZX or ZY planes with the incident beam traversing in the Z direction.

It can be seen from these equations that the electro-optic coefficients r_{41}, r_{52} and r_{63} do not contribute to either the change in length of the axes of the ellipsoid or to the rotation of the ellipsoid. If the geometries of the device are as chosen, then crystal classes having these three non-vanishing r_{ij} coefficients, but with all other coefficients equal to zero, are not suitable for this application. These are the 422, $\bar{4}2m$ tetragonal, $\bar{4}3m$ cubic and 622 hexagonal classes of crystals. However, these crystals (e.g. KDP) may be used when the crystal is not cut along the principal axes. In the remaining classes of crystals, materials with relatively large linear electro-optic coefficients are $LiTaO_3$, $BaTiO_3$, $Ba_XSr_{1-X}TaO_3$ and $LiNbO_3$.[7]

$LiNbO_3$ crystals are relatively cheap and readily available with consistent optical quality. $LiNbO_3$ is a ferroelectric (Curié temperature 1483°K) uniaxial crystal. The point group symmetry is 3m, and by convention the reference Z axis is chosen parallel to the optic axis. In this convention, the electro-optic matrix for $LiNbO_3$ is

$$r_{ij} = \begin{bmatrix} 0 & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & 0 \\ r_{61} & 0 & 0 \end{bmatrix} \quad \text{where} \quad \begin{aligned} r_{22} = -r_{12} = -r_{61} &= 3.4 \times 10^{-12}\ \text{m/V}, \\ r_{51} = r_{42} &= 28.0 \times 10^{-12}\ \text{m/V}, \\ r_{13} = r_{23} &= 8.6 \times 10^{-12}\ \text{m/V}, \\ \text{and} \quad r_{33} &= 30.8 \times 10^{-12}\ \text{m/V}. \end{aligned}$$

The dielectric constants along and perpendicular to the optic axis are 28.6 and 84.6 respectively.

The refractive index

$$n_x = n_y = n_{ordinary} = 2.28$$

$$\text{and} \quad n_z = n_{extraordinary} = 2.20 \quad .$$

In designing an efficient modulator/deflector the direction of propagation of light and the plane of the interdigital electrodes must be selected so that the coefficient r_{33} is retained in the equation of the final ellipse. Even though r_{51} ($= r_{42}$) is large, these terms only contribute to the rotation of the index ellipsoid.

In the configuration given above, from Eq.(7), the angle of rotation (α) of the ellipse is given by

$$\tan 2\alpha = \frac{2(r_{42}E_Y)}{\left(\frac{1}{n_y^2} + r_{22}E_Y + r_{23}E_Z\right) \sim \left(\frac{1}{n_z^2} + r_{33}E_Z\right)} \quad . \tag{9}$$

Substituting the values for r_{ij} and assuming a reasonable value for E_Y and E_Z ($\sim$ 200 KV/m) it can be shown that the rotation of the ellipse is negligible. This value for E_Y and E_Z was obtained by assuming a voltage of $\sim$ 40 V applied to the symmetrical interdigital electrode structure of electrode pitch 0.1 mm.

V. CONCLUSION

From the considerations of the electro-optic properties, it may be seen that crystals belonging to triclinic, monoclinic and orthorhombic classes are not suitable as modulator materials. Among the other classes of crystals, those belonging to groups 422, $\bar{4}2m$ tetragonal, 432, $\bar{4}3m$, 23 cubic and 622 hexagonal are not suitable to the grating-based modulators when designed to operate in the principal cut modes. The electro-optic coefficients in these crystals do not contribute to either a change in the lengths of the axes of the index ellipsoid or to the rotation of the ellipsoid. Theoretical calculations have shown that when $LiNbO_3$ (class 3m trigonal) is used in the principal cut mode, the rotation of the index ellipsoid caused by the periodically varying electric field is negligible ($\sim$ 0.16 m rad.).

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REFERENCES

1) M.G. Cohen and E.I. Gordon, Appl. Phys. Letters 5, 181 (1964).

2) Amon Yariv, Quantum Electronics (J. Wiley and Sons, Inc., 1968).

3) J.F. Nye, Physical Properties of Crystals (Oxford Univ. Press, 1967).

4) I.P. Kaminow and E.H. Turner, Proc. IEEE, 54, 1374 (1966).

5) W.P. Mason, Physical Acoustics and the Properties of Solids (New York, 1958).

6) I.S. Rez, Soviet Phys.-Usp. 10, 759 (1968).

7) P. De Barras and F. Wilson, Proc. IEEE 7, 807 (1972).