IC/77/31
INTERNAL REPORT
(Limited distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

COLLISIONAL ABSORPTION OF TWO LASER BEAMS IN PLASMA *

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ABSTRACT

In this paper the collisional absorption of two laser beams is considered by solving the kinetic equation for the plasma electron. Results show that the simultaneous effect of two laser beams on the heating rate is greater as compared with the individual contribution of each laser beam when the two laser beams have a difference of frequencies equal to the plasma frequency.

MIRAMARE - TRIESTE
April 1977

* To be submitted for publication.

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I. INTRODUCTION

In recent years, due to the development of powerful quantum optical devices, e.g. lasers, it has become possible to study the interaction of intense electromagnetic field with matter. At such high intensity many new physical phenomena have come into existence, e.g. generation of harmonics, multiphoton absorption, self-focussing of wave beams in a medium, cascade ionization, inverse bremmstrahlung process, etc. In the process of interaction of a laser with plasma it may happen that the plasma electrons gain energy from the e.m. field and may be raised to thermonuclear temperature. One of the chief mechanisms for absorption of the e.m. field by plasma may be the inverse multiphoton bremmstrahlung process (Raizer 1965, Bunkin et al. 1966, 1973, Seely and Harris 1973, Mohan 1974). In this process an electron absorbs energy from the laser beam during a collision with the nucleus. From a classical point of view, the electron oscillates in the electromagnetic field of the laser beam. During a collision with a nucleus, the electron is knocked out of phase with the electric field and the oscillatory energy of the electron is converted into random thermal energy. From a quantum point of view, the electron can gain energy only in units of $\hbar\omega$, where ω is the frequency of the laser radiation and it is not clear that the classical viewpoint is valid. In this paper we have studied the collisional absorption of two laser beams in plasma due to the inverse multiphoton bremmstrahlung process when the two laser beams have a difference frequency nearly equal to the plasma frequency (Kroll et al. 1964, Cohen et al. 1972, Rosenbluth 1972).

Here we develop the theory by describing the electrons by the second quantized theory and treat the e.m. field in a classical manner. The transition probability for the inverse multiphoton bremmstrahlung process is derived using first-order perturbation theory. A kinetic equation is derived for plasma electrons and the rate of change of kinetic energy (Seely and Harris 1973) of the electrons is calculated.

II. THEORY

The total hamiltonian of the system in the presence of two laser beams denoted by vector potentials $\vec{A}_1(t)$ (with frequency ω) and $\vec{A}_2(t)$ (with frequency ω') can be written as (Davydov 1965, Harris 1972)

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \ , \tag{1}$$

where

$$\mathcal{H}_0 = \int d^3r\ \psi^\dagger(\vec{r},t)\frac{1}{2m}\left[\frac{\hbar}{i}\vec{\nabla} - \frac{e\vec{A}_1(t)}{c} - \frac{e\vec{A}_2(t)}{c}\right]^2\psi(\vec{r},t). \tag{2}$$

$$\mathcal{H}_I = \int d^3r\ \psi^\dagger(\vec{r},t)\ V(\vec{r})\ \psi(\vec{r},t). \tag{3}$$

Here $\mathcal{H}_0$ is the unperturbed hamiltonian, $\mathcal{H}_I$ is the interaction hamiltonian and $V(\vec{r})$ is the screened Coulomb potential (Wyld and Pines 1962). We further assume that the difference in frequencies of two laser beams is nearly equal to the plasma frequency, i.e. $\omega - \omega' \cong \omega_p + \Delta_L$ with mismatch Δ_L small (say, $10^{-2}\ \omega_p$ and ω or $\omega' \gg \omega_p$ (Cohen <u>et al</u>. 1972, Rosenbluth 1972)). Using second quantized theory (Davydov 1965, Harris 1972) we can expand $\psi(\vec{r},t)$ and $\psi^\dagger(\vec{r},t)$ in terms of a complete set of functions $\varphi_{\vec{p}}(\vec{r},t)$, as

$$\psi(\vec{r},t) = \sum_{\vec{p}} b_{\vec{p}}(t)\ \varphi_{\vec{p}}(\vec{r},t) \tag{4}$$

and

$$\psi^\dagger(\vec{r},t) = \sum_{\vec{p}} b^\dagger_{\vec{p}}(t)\ \varphi^*_{\vec{p}}(\vec{r},t)\ , \tag{5}$$

where $b_{\vec{p}}$, $b^\dagger_{\vec{p}}$ are the fermion annihilation and creation operators and $\varphi_{\vec{p}}(\vec{r},t)$ is the eigenfunction of the Schrödinger equation with free hamiltonian corresponding to electron with momentum $\vec{p}$ and is given by

$$\varphi_{\vec{p}}(\vec{r},t) = \frac{1}{(2\pi\hbar)^{3/2}}\exp\left\{\frac{i}{\hbar}\left(\vec{p}\cdot\vec{r} - \int_0^t \left(\vec{p} - \frac{e\vec{A}_1(\tau)}{c} - \frac{e\vec{A}_2(\tau)}{c}\right)^2 \Big/ 2m\ d\tau\right)\right\}. \tag{6}$$

The commutation relations satisfied by these operators are:

$$\left[b_{\vec{p}}(t),\ b^\dagger_{\vec{p}'}(t)\right]_+ = \delta_{\vec{p},\vec{p}'}\ .$$

$$\left[b_{\vec{p}}(t),\ b_{\vec{p}'}(t)\right]_+ = \left[b^\dagger_{\vec{p}}(t),\ b^\dagger_{\vec{p}'}(t)\right] = 0\ . \tag{7}$$

As shown by Davydov, if $N_S(a)$ is the number of particles of species S in the state S then the state vectors of the particle system can be represented by

$$|\cdots N_S(a)\cdots\rangle\ . \tag{8}$$

Similarly we can represent the state vectors of the electrons with momentum $\vec{p}$ by $|\cdots N_e(\vec{p})\cdots\rangle$. Now using the commutation relation given by Eq.(7) (Davydov 1965), the following properties of creation and annihilation operators are obtained:

$$\left.\begin{aligned} b_{\vec{p}}\,|\cdots N_e(\vec{p})\cdots\rangle &= \sqrt{N_e(\vec{p})}\,|\cdots 1 - N_e(\vec{p})\rangle \\ b^\dagger_{\vec{p}}\,|\cdots N_e(\vec{p})\cdots\rangle &= \sqrt{1 - N_e(\vec{p})}\,|\cdots 1 - N_e(\vec{p})\cdots\rangle \end{aligned}\right\} \tag{9}$$

Substituting the values of Eqs.(4) and (5) in Eqs.(2) and (3) we obtain

$$\mathcal{H}_0 = \sum_{\vec{p}} \frac{1}{2m}\left|\vec{p} - \frac{e}{c}\vec{A}_1(t) - \frac{e}{c}\vec{A}_2(t)\right|^2 b^\dagger_{\vec{p}}\ b_{\vec{p}}\ , \tag{10}$$

$$\mathcal{H}_I = \sum_{\vec{p}_1}\sum_{\vec{p}_2} \langle \vec{p}_2 | V(\vec{r}) | \vec{p}_1 \rangle\ b^\dagger_{\vec{p}_2}\ b_{\vec{p}_1}\ . \tag{11}$$

where

$$\langle \vec{p}_2 | V(\vec{r}) | \vec{p}_1 \rangle = \int d^3r\ \varphi^*_{\vec{p}_2}(\vec{r},t)\ V(\vec{r})\ \varphi_{\vec{p}_1}(\vec{r},t).$$

Let us assume that initially there are $N_e(\vec{p}_1)$ electrons with momentum $\vec{p}_1$ and $N_e(\vec{p}_2)$ electrons with momentum $\vec{p}_2$. Then using Eqs.(9) and (11) we get that the transition matrix element between the transition from the initial state $|i\rangle$, representing the electrons with momentum $\vec{p}_1$, to final state $|f\rangle$, representing the electrons with momentum $\vec{p}_2$, due to the screened Coulomb potential $V(\vec{r})$ is given by

$$\langle f | \mathcal{H}_I | i \rangle = [N_e(\vec{p}_1)]^{1/2} [1 - N_e(\vec{p}_2)]^{1/2} \langle \vec{p}_2 | V(\vec{r}) | \vec{p}_1 \rangle . \tag{12}$$

From the first-order perturbation theory, the transition probability amplitude is

$$a(i \to f) = \frac{-i}{\hbar} \int_{-T/2}^{T/2} \langle f | \mathcal{H}_I | i \rangle \, dt . \tag{13}$$

Using Eqs. (11) and (12) in Eq.(13) we obtain the transition probability per unit time for the absorption (or emission) of n photons of frequency ω and absorption (or emission) of one photon of frequency ω' as (Seely and Harris 1973, Mohan 1974, Thareja and Mohan 1976)

$$\frac{|a(i \to f)|^2}{T} = \sum_{n=-\infty}^{+\infty} T(n\omega, \pm\omega', \vec{p}_1 - \vec{p}_2) , \tag{14}$$

where

$$T(n\omega, \pm\omega', \vec{p}_1 - \vec{p}_2) = \frac{2\pi}{\hbar} N_e(\vec{p}_1) [1 - N_e(\vec{p}_2)] \frac{|B(\vec{q})|^2}{(2\pi\hbar)^3} \times \left(\frac{e \vec{A}_2 \cdot \vec{q}}{2mc\hbar\omega'} \right) J_n \left(\frac{e \vec{A}_1 \cdot \vec{q}}{mc\hbar\omega} \right) \delta(\Delta E - n\hbar\omega \pm \hbar\omega') , \tag{15}$$

$$\vec{q} = \vec{p}_2 - \vec{p}_1 \; ; \quad \Delta E = \frac{p_2^2 - p_1^2}{2m} , \tag{16}$$

$$|B(\vec{q})|^2 = \frac{4\pi Z e^2 \hbar^2}{|\vec{q}|^2 \, \varepsilon(\vec{q}, \omega_0)} \tag{17}$$

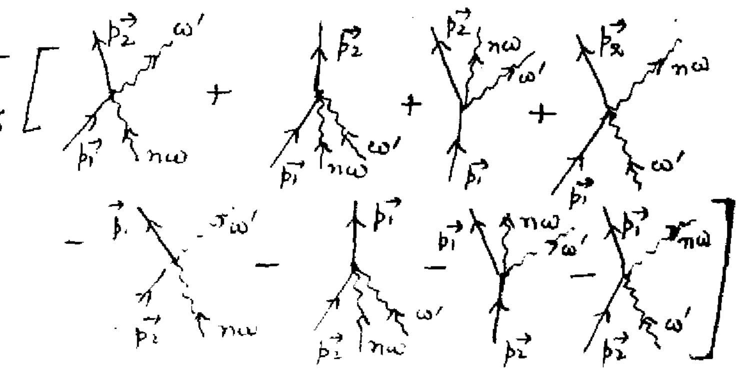
$$\omega_0 = \omega_1 - \omega_2 . \tag{18}$$

Here ΔE is the change in the kinetic energy of the electron, J_n is the Bessel function of the order n, $|B(\vec{q})|$ is the Fourier transform of the Coulomb screened potential as the system is in plasma, $\varepsilon(\vec{q}, \omega_0)$ is the dielectric function of the plasma, $\hbar q$ is the momentum transfer, and $\hbar\omega_0$ is the energy transfer in the collision. According to Wyld and Pines (1962) and Harris (1969), the matrix element of Coulomb collisions, i.e. $-\frac{4\pi Z e^2 \hbar^2}{|\vec{q}|^2}$ must be modified by the factor $\varepsilon^{-1}(\vec{q}, \vec{\omega}_0)$, as the whole system is considered in the plasma. From the δ function of Eq.(15) it is clear that $T(n, \pm\omega', \vec{p}_1 - \vec{p}_2)$ is the transition probability per unit time for the transition in which an electron with momentum $\vec{p}_1$ is destroyed, an electron with momentum $\vec{p}_2$ is created, n laser photons of frequencies ω are absorbed ($n > 0$) or emitted ($n < 0$) and 1 photon of frequency ω' is absorbed (+1) or emitted (−1). It is clear from expression (15) that

$$T(\mp n\omega, \mp\omega', \vec{p}_1 - \vec{p}_2) = T(\pm n\omega, \pm\omega', \vec{p}_2 - \vec{p}_1) . \tag{19}$$

Now as $N_e(\vec{p}_2)$ and $N_e(\vec{p}_1)$ represent the number of electrons with momentum $\vec{p}_2$ and $\vec{p}_1$, respectively, therefore change in $N_e(\vec{p}_2)$ can be written schematically as (Harris, 1969 and 1972)

$$\frac{\partial N_e(\vec{p}_2)}{\partial t} = \sum_{n=1}^{\infty} \sum_{\vec{p}_1}$$



(20)

where the process in which the electron with momentum $\vec{p}_1$ is destroyed is subtracted from the process in which an electron with momentum $\vec{p}_2$ is created. This difference gives the net increase in $N_e(\vec{p}_2)$. The above schematic equation can be converted to a mathematical equation by replacing the diagrams in Eq.(20) with the transition probabilities per unit time for the process given by Eq.(16). The mathematical equation is:

$$\frac{\partial N_e(\vec{p}_2)}{\partial t} = \sum_{n=1}^{\infty} \sum_{\vec{p}_1} \Big[T(n\omega, -\omega', \vec{p}_1 - \vec{p}_2) + T(n\omega, \omega', \vec{p}_1 - \vec{p}_2) + T(-n\omega, -\omega', \vec{p}_1 - \vec{p}_2) + T(-n\omega, \omega', \vec{p}_1 - \vec{p}_2)$$
$$- T(n\omega, -\omega', \vec{p}_2 - \vec{p}_1) - T(n\omega, \omega', \vec{p}_2 - \vec{p}_1) - T(-n\omega, -\omega', \vec{p}_2 - \vec{p}_1) - T(-n\omega, \omega', \vec{p}_2 - \vec{p}_1) \Big] \tag{21}$$

We now take the classical limit of Eq.(21). We assume that the electrons are non-degenerate so that $N_e(\vec{p}) \ll 1$, and assume the volume of the box in which the system is normalized becomes infinite so that the sum over $\vec{p}_1$ becomes an integral over $\vec{V}_1$ (Harris 1969, p.188). The classical limit of Eq.(21) under such a prescription is:

$$\frac{\partial f_e(\vec{v}_2)}{\partial t} = \frac{8 Z^2 e^4 N}{m\, \varepsilon(\vec{q}, \omega_0)} \left(\frac{m}{2\pi kT}\right)^{3/2} \times \int d^3 v_1 \, |\vec{v}_2 - \vec{v}_1|^{-4} \left\{ \exp\left(\frac{-m v_1^2}{2kT}\right) - \exp\left(\frac{-m v_2^2}{2kT}\right) \right\}$$
$$\times \left(\frac{e \vec{A}_2 \cdot \vec{q}}{2mc\hbar\omega'}\right) \cdot \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} J_n^2\left(\frac{e \vec{A} \cdot \vec{q}}{mc\hbar\omega}\right) \delta(\Delta E - n\hbar\omega \pm \hbar\omega'), \tag{22}$$

where $f_e(\vec{V})$ is the electron distribution function assumed to be Maxwellian and N is the electron density.

Now assuming the laser field with frequency ω is also weak, we can expand the Bessel function as

$$J_n\left(\frac{e \vec{A} \cdot \vec{q}}{mc\hbar\omega}\right) \simeq \frac{1}{n!} \left(\frac{e \vec{A} \cdot \vec{q}}{mc\hbar\omega}\right)^{|n|}. \tag{23}$$

Substituting Eq.(23) in Eq.(22) and using the first two terms of the sum, we obtain the kinetic equation (Seely and Harris 1973)

$$\frac{\partial f_e(\vec{v}_2)}{\partial t} = \frac{8 Z^2 e^4 N}{m} \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m v_2^2}{2kT}}$$
$$\times \int d^3 v_1 \, |\vec{v}_2 - \vec{v}_1|^{-4} \left(\frac{e \vec{A}_2 \cdot \vec{q}}{2mc\hbar\omega'}\right)^2 \left(\frac{e \vec{A}_1 \cdot \vec{q}}{2mc\hbar\omega}\right)^2$$
$$\times \Big[\left(e^{\hbar(\omega \mp \omega')/kT} - 1\right) \delta(\Delta E - \hbar\omega \mp \hbar\omega')$$
$$+ \left(e^{-\hbar(\omega \pm \omega')/kT} - 1\right) \delta(\Delta E + \hbar\omega \pm \hbar\omega') \Big]. \tag{24}$$

The first δ function corresponds to the absorption of a photon of frequency ω and absorption or emission of photon of frequency ω'. Similarly the second δ function represents the emission of photon of frequency ω and emission or absorption of photon of frequency ω'.

Finally, in the limit $\hbar \to 0$ and after the evaluation of the integral over $\vec{V}_1$, the kinetic equation (24) becomes

$$\frac{\partial f_e(\vec{v}_2)}{\partial t} = \frac{4\pi Z^2 e^4 N v_2 \omega_0^4}{3\, \varepsilon(\vec{q}, \omega_0)\, m^2 (kT)^4} \left(\frac{m}{2\pi kT}\right)^{3/2} \left(\frac{e a_2}{2c\omega'}\right)^2 \left(\frac{e a_1}{2c\omega}\right)^2 \times e^{-m v_2^2/2kT}, \tag{25}$$

where a_1 and a_2 are the amplitudes of the vector potentials $\vec{A}_1(t)$ and $\vec{A}_2(t)$, respectively, and we have assumed that the beams are propagating in the same direction.

We can now obtain the change in the average kinetic energy of the electron by substituting Eq.(25) in the following equation:

$$\frac{\partial \langle \epsilon \rangle}{\partial t} = \int d^3 v_2 \, \frac{m v_2^2}{2} \, \frac{\partial f_e(\vec{v}_2)}{\partial t}. \tag{26}$$

Thus putting (25) in (26) gives the change in the average kinetic energy of the electron as

$$\frac{\partial\langle\epsilon\rangle}{\partial t} = \frac{Z^2 e^4 N \omega_p^4}{3\sqrt{2m\pi}\;\varepsilon(\vec{q},\omega_0)} \frac{1}{(kT)^{5/2}} \left(\frac{e a_2}{2c\omega_2}\right)^2 \left(\frac{e a_1}{2c\omega_1}\right)^2 , \tag{27}$$

which is clearly proportional to the intensities of the two beams, thus showing the direct influence of intensities on the change of the average kinetic energy of the electrons

III. CONCLUSION

It is clear from Eq.(27) that the change in kinetic energy of the electron $\frac{\partial\langle t\rangle}{\partial t}$ is positive for the Maxwellian distribution. It follows that the mixing of two laser beams leads to a decrease of the radiation energy and to heating (and not cooling) of the plasma electrons. The appearance of $\varepsilon(\vec{q},\omega_0) \simeq [1-(\omega_p/\omega_0)^2]^2$ in the denominator of Eq.(27) indicates the possibility of the resonant process when the two laser beams have a difference frequency nearly equal to the plasma frequency, i.e. $\omega_0 = \omega_1 - \omega_2 \cong \omega_p$ the plasma frequency. Thus we conclude that by making the condition $\omega_0 \simeq \omega_p$, the heating due to the inverse bremmstrahlung process by mixing two laser beams may be one of the efficient processes for the heating of plasma by radiation.

ACKNOWLEDGMENTS

One of the authors (MM) would like to thank Professor M. Rosenbluth for useful discussions and taking interest in this investigation. Thanks are also due to Professors S.N. Biswas and M. Key for their interest in this work. He is also grateful to Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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