THE UNIVERSITY OF TEXAS AT AUSTIN

Center for Particle Theory

ORO-3992-320
October 1977

An Approach to Measurement in Quantum Mechanics*†

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*Work supported in part by the U. S. Energy Research and Development Authority.
†Talk presented by T. N. Sherry at the 1977 Banff Summer Institute on Particles and Fields.

Abstract

We consider an unconventional approach to the measurement problem in quantum mechanics - we treat the apparatus as a classical system, belonging to the macro-world.

In order to have a measurement the apparatus must interact with the quantum system. As a first step, we embed the classical apparatus into a larger quantum mechanical structure, making use of Superselection rules. We can project back to the classical system. We now couple the apparatus and system such that the apparatus remains classical (principle of integrity), and unambiguous information of the values of a quantum observable are transferred to the variables of the apparatus. Finally, we project back to the classical formulation.

Further measurement of the classical apparatus can be done, causing no problems of principle. Thus interactions causing pointers to move (which we do not treat) can be added.

We examine the restrictions placed by the principle of integrity on the form of the interaction between classical and quantum systems.

I. Introduction

The work I am going to discuss concerns a rather unusual and unconventional approach to the description of what we mean by measurement.[1,2] We do not claim to have worked out a complete theory which realistically describes a measurement process--rather we have concentrated on points of principle, illustrating by means of some simple examples.

The unusual feature of our work is that we treat the apparatus as a system belonging to the regime of classical mechanics. Our reasons for considering such an unusual possibility are philosophical--since a piece of apparatus belongs to the macro-physical world it would seem more reasonable to treat it in a macrophysical way. This is not of its own accord too convincing; nevertheless, we feel that it is worthwhile to pursue such thoughts. Useful ideas could be unconventional.

We shall not be dealing directly with the problem of pointer readings or irreversible processes. Rather, we will consider a measurement complete if unambiguous information has been transferred from the quantum system being examined to the classical apparatus system. We can list the chief requirements for what we mean by measurement as follows:

(1) the apparatus is classical

(2) the apparatus interacts with the quantum system

(3) the interaction is such that the apparatus retains its classical identity--Principle of Integrity

(4) the interaction results in unambiguous information concerning the values of the relevant system observables being transferred to the observables of the classical system.

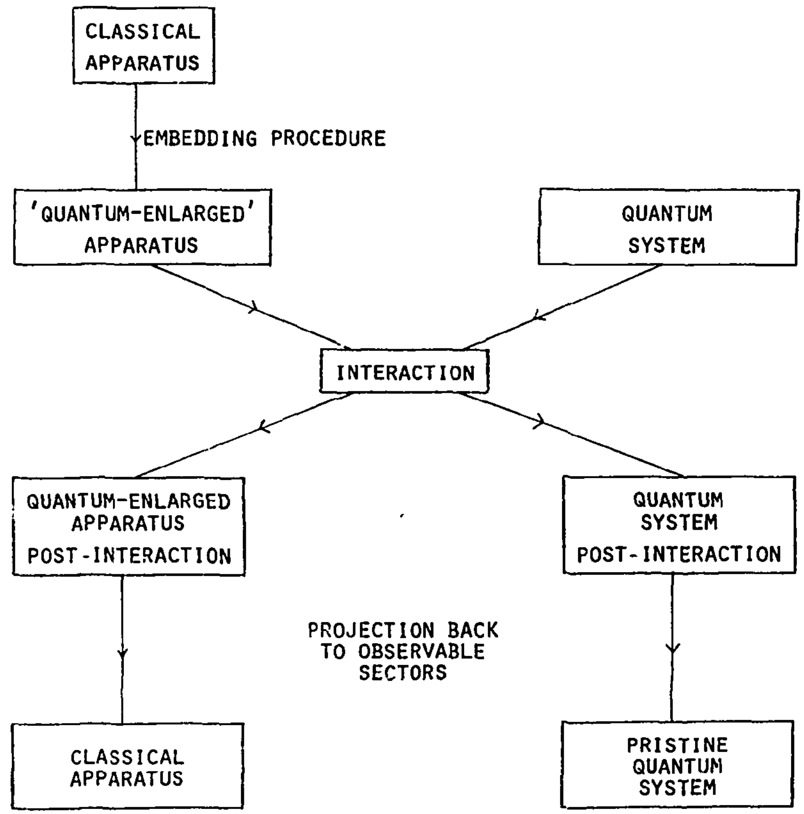
An important point to notice about the above outlined procedure is the following: in classical mechanics the measurement of dynamical variables is possible <u>in principle</u> without causing a disturbance on the system. [The technological problem is another matter.] Once we have successfully transferred the information into the classical system, the transferring of this information to an observer, or another system, is straightforward.

Clearly, the first problem we must examine is how to cause classical and quantum systems to interact. To achieve this we introduce an unorthodox embedding of classical mechanical systems into quantum mechanical systems. This procedure makes use of a superselection principle.[3] Once this is understood, we can go on to make use of it to couple together the classical and quantum mechanical systems. The requirements we listed above will place certain restrictions on the allowed couplings --and this we shall examine in some detail.

The proof of any pudding is in the eating. So, I shall then discuss some questions relating to an example. This example is a form of the Stern-Gerlach[4] experiment.

Many of the ideas I will present are at the level at which quantum mechanics is usually discussed. In particular I refer to the discussion of Superselection, where manipulations valid for bounded operators are assumed to hold in some sense for unbounded self-adjoint operators on infinite dimensional Hilbert space. Also we assume that unbounded operators can be treated as having eigenvectors corresponding to points belonging to their continuous spectra. We are presently carrying out a more careful analysis to see if these assumptions can be made more precise.

SCHEMATIC DIAGRAM



II. Superselection and Non-observables

Within the context of a Hilbert space of states (to which all physical states should belong), there is defined the concept of a selection rule. This has to do with the dynamics of an isolated quantum system. Given two distinct subspaces of the total Hilbert space, a selection rule operates between them if the vectors of one remain orthogonal to the vectors of the other in time--no spontaneous transitions can occur if the system is left to evolve on its own.

The concept of a Superselection rule[4] is a generalization of this concept--a superselection rule operates if a selection rule operates and further, no observable can connect states belonging to the different subspaces.

In many cases the selection rule can be understood to arise as a result of a symmetry of the dynamics of the system. The dynamical symmetry gives rise to invariant charge operators. It is between eigenspaces of the charge operators that the selection rule operates. Similarly, the superselection rule can be understood to arise from a symmetry, not restricted just to the dynamics however. There is an associated charge operator--the superselecting operator--and it is between its eigenspaces that the superselection rule operates.

The existence of a superselection rule has certain consequences. The selection rule means that a state which belongs to a particular eigenspace h_i of the charge operators, initially, will always belong to h_i. The superselection part has the following consequences: in a superposition over distinct subspaces of the superselecting operators the relative phases are not measurable. No observable can connect the subspaces, and so the relative phases cannot be measured by any observable. Such a state we call 'an effective mixture state.' The subspaces h_i of the Hilbert space are then said to be superselected. Furthermore, those operators which fail to commute with the superselecting operators are not observable.

Within this conventional scheme, the set of all observables is included within the commutant of the set of superselecting operators, and those operators which fail to commute with the superselecting operators form a subset of the set of non-observable operators.

We found it necessary in the course of our investigations to use a different notion of a superselecting operator.[2] The essential properties listed above are

(1) it commutes with the time evolution operator (selection rule)

(2) it commutes with all observables

(3) it is an observable.

In conventional quantum mechanical systems the time evolution operator is observable, and so the first condition is a special case of the second. We actually gain in generality by dropping (1) in its explicit form. We furthermore extend (2) to the status of an 'if and only if' condition and delete (3), but

but these are not relevant for our present considerations.

As motivation for these changes, we see that now an operator will be unobservable if and only if it fails to commute with some superselecting operator--a reasonable definition. Thus given an arbitrary algebra of operators one can check the consistency of a particular choice of unobservable operators.

The extended concept clearly allows a superselecting operator which is not a constant of the motion. Then in the evolution of time states can spontaneously move from one superselected subspace to another. Thus, in a superposition over distinct subspaces h_i, the relative phases are not observable--but may be measureable. At a later time the superposition may evolve into a state belonging to one of the subspaces h_i. In that case the different superpositions would lead to different states, which could be distinguished. But this difference can only be seen after the state is altered. In addition, the Hamiltonian operator is not observable, since it does not commute with the relevant superselecting operator.

Clearly it will only be in a highly non-conventional theory that such a concept could have a use. I will now describe such a theory.

III. The Embedding Procedure

If we are to describe the apparatus in a measurement experiment as a classical system, even when measuring the attributes of a quantum system, then we must needs be able to couple together classical and quantum systems. It is not a priori obvious how this should be done. On the face of it, it appears that the structure of these theories are so different, that a coupling cannot be achieved. The differences result from

(1) in quantum physics the dynamical variables are non-commuting while in classical physics they are commuting

(2) a quantum state is specified by the eigenvalues of a complete commuting set of operators, a proper subset of the set of dynamical variables. A classical state, on the other hand, is specified by values for all the dynamical variables

(3) a linear superposition of 'pure' quantum states is also a pure quantum stae, but a linear superposition of pure classical states is an incoherent state.

Despite these inherent differences, I will now describe a procedure for embedding a classical system into a quantum system. When this is done we shall use the procedure in the description of a measurement expt.

Consider a classical mechanical system in the Hamiltonian formulation. The canonical co-ordinates are, e.g., $(q^1,...,q^n)$ and $(p^1,...,p^n)$. The Hamiltonian is a function of these variables H(q,p). The time development is given by

$$\dot{q} = \frac{\partial H(q,p)}{\partial p} \qquad \dot{p} = -\frac{\partial H(q,p)}{\partial q}$$

Such a system is a simple one--with no complexities to describe the action of pointers for example.

Let us now consider these variables q and p as operators acting on a Hilbert space, in much the same way as λ is an operator in Schrödinger's wave mechanics. We denote them by $\omega^\mu = (q^1 \cdots q^n, p^1 \cdots p^n)$. We introduce operators which are conjugate to ω with respect to commutation

$$[\omega^\mu, \pi^\nu] \equiv \omega^\mu \pi^\nu - \pi^\nu \omega^\mu = i\delta^{\mu\nu}$$

In the ω-*representation we would have* $\pi^\nu = -i \frac{\partial}{\partial \omega^\nu}$. Then, if we define the time evolution, or Hamiltonian, operator to be

$$H_{op} = -\frac{\partial H(\omega)}{\partial \omega^\mu} \varepsilon^{\mu\nu} \pi^\nu$$

where

$$\varepsilon^{\mu\nu} = \{\omega^\mu, \omega^\nu\}_{p.b.}$$

the Heisenberg picture equations of motion for $\omega^\mu(t)$ exactly mimic the classical *equations given earlier.*

The operators $\{\omega^\mu\}$ and $\{\pi^\mu\}$ have continuous spectra.

Strictly speaking they do not have eigenvectors within the Hilbert space. Following conventional practice we shall include in the Hilbert space *the non-normalizable states* $|\omega'\rangle$ where

$$\omega|\omega'\rangle = \omega'|\omega'\rangle \quad .$$

Then, as in the Hamiltonian formulation where one could specify a precise initial value for $\omega(0) = (q(0),p(0))$, so also one can choose a precise initial state

$$|\omega^0\rangle = |q^0,p^0\rangle$$

which is an eigenvector of the operators q(0) and p(0).

We now supplement this structure by the Superselection principle

(A) ω^μ are superselecting operators, for all times

or equivalently

(B) π^ν are unobservable operators for all times

in the sense that we saw earlier.

Notice here that there is no selection rule involved--the superselecting operators are not constants of the motion. Thus if one chooses as initial state an eigenvector of the superselecting operators, at later times the system will not be in the same eigenspace. Also the Hamiltonian operator is not itself an observable.

The observable sector of the quantum *theory mimics* exactly the classical theory. We have thus embedded the classical theory in a larger quantum mechanical framework.

The embedding procedure outlined above can be understood as a mapping from the classical phase space to a set of quantum mechanical operators and the space upon which they act. This map can be written as follows

$$Q: \{\omega\} \longrightarrow \{\omega_{op}\} \times \{\pi_{op}\} \times h$$

where

$$Q(\omega) = (\omega_{op}, \pi_{op}, |\omega\rangle)$$

and

$$\omega_{op}|\omega\rangle = \omega|\omega\rangle$$

$$[\omega_{op}, \pi_{op}] = +i \quad .$$

Here we explicitly put a subscript 'op' to denote an operator. Then with the definition of H_{op} given earlier, we have at a later time t

$$Q(\omega(t)) = (\omega_{op}(t), \pi_{op}(t), |\omega\rangle)$$

where

$$\omega_{op}(t)|\omega\rangle = \omega(t)|\omega\rangle$$

$$[\omega_{op}, \pi_{op}] = i$$

and

$$\pi_{op}(t) = e^{iH_{op}t}\pi_{op}(0)e^{-iH_{op}t} \quad .$$

The mapping Q can be inverted, that is, we can map back from the quantum system to the original classical system

$$Q^{-1}: \{\omega_{op}\} \times \{\pi_{op}\} \times h \to \{\omega\}$$

where

$$Q^{-1}(\omega_{op}(t), \pi_{op}(t), |\omega\rangle) = \omega(t)$$

and

$$\omega_{op}(t)|\omega\rangle = \omega(t)|\omega\rangle \quad .$$

By means of these mappings we can achieve steps (1) and (3) in the schematic description of our work. We now consider step 2.

Example:

A simple example will help to illustrate these ideas. Consider a classical free particle in one dimension. Its Hamiltonian formulation is:

canonical co-ordinates	(q,p)
Hamiltonian	$H(q,p) = \frac{1}{2m} p^2$

In the quantum-enlarged formulation we have

canonical co-ordinates	$\omega = (q,p)$ and $\pi = (\pi^q, \pi^p)$
and Hamiltonian operator	$H = \frac{1}{m} p\pi^q$

The Heisenberg equations of motion for the observables ω are

$$\dot{p}(t) = 0$$
$$\dot{q}(t) = \frac{1}{m}\, p(t)$$

with solutions

$$p(t) = p(0)$$
$$q(t) = q(0) + \frac{t}{m}\, p(0)$$

We choose as the state of the system

$$|q_0, p_0\rangle$$

then at the later time t

$$q(t)|q_0,p_0\rangle = (q_0 + \frac{t}{m}\, p_0)|q_0,p_0\rangle$$

and

$$p(t)|q_0,p_0\rangle = p_0|q_0,p_0\rangle \quad .$$

IV. Interaction between Classical System and Quantum System

Since classical systems and quantum systems have such different structures, the manner in which they could interact is not clear. However, the interaction of systems of like structure is a straightforward exercise. The naturalness of the embedding procedure described suggests that this way to cause the interaction may be the correct way.

Consider a quantum mechanical system with dynamical variables $\{\xi\}$. The theory is specified when the energy operator of the isolated quantum system, $X(\eta)$, and the commutation relations among the ξ are known. The Hamiltonian operator for the uncoupled apparatus and system is

$$-\frac{\partial H(\omega)}{\partial \omega^\mu}\, \varepsilon^{\mu\nu}\pi^\nu + X(\eta) \quad .$$

A coupling term is $\Phi(\omega,\pi;\xi')$. This coupling will affect both classical and quantum systems.

However, the coupling is restrained by the requirement that after interacting the apparatus remain classical. The classical nature of the apparatus system is characterized by

(1) $\omega^\mu(t)$ observable for all t.

(2) $\omega^\mu(t)$ and $\omega^\nu(t')$ compatible for all t and t',

which are equivalent conditions.

Immediately these require that the coupling term be at most linear in the non-observables π. Thus we concentrate on couplings

$$\phi^\mu(\omega,\eta')\pi^\mu + h(\omega,\hat{\varepsilon}) \quad .$$

However this alone does not guarantee that (1) and (2) are satisfied for all times--because the primary and secondary coupling functions depend on unspecified sets of quantum variables $\{\eta'\}$ and $\{\hat{\varepsilon}\}$ respectively.

We now will see what further restrictions we can deduce for these couplings. Let us first restrict our attention to couplings which are analytic everywhere. In this case an equivalent condition to examine is

$$(3)\quad [\frac{d^m}{dt^m}\,\omega^\mu(t),\frac{d^n}{dt^n}\,\omega^\nu(t)] = 0 \qquad m,n \geq 0,\ \text{all } t$$

From this condition we can derive a set of conditions which must be satisfied by the coupling functions so that the apparatus remain classical--the Integrity Criteria. In the analytic case they are both necessary and sufficient, whereas in the non-analytic case they are merely necessary.

From the Hamiltonian operator

$$H_{op} = \underbrace{F^\mu(\omega,\eta')}_{(\frac{\partial H}{\partial\omega}\varepsilon + \phi)}\pi^\mu + \underbrace{G(\omega,\hat{\varepsilon})}_{X(\eta) + h(\omega,\hat{c})}$$

we see that the time derivatives of ω are

$$\dot{\omega}^\mu(t) = -F^\mu(\omega,\eta')$$
$$\ddot{\omega}^\mu(t) = i[F^\mu,F^\nu\pi^\nu + G] \quad .$$
$$\vdots$$

The first criterion that we have follows at once.

A. $\{\phi^\mu\}$ forms a commuting set.

Let $\{\rho\}$ be the maximal algebraically independent subset of $\{\phi^\mu\}$, allowing ω-dependent coefficients.

We consider extensions of this set to a maximal commuting algebraically independent subset of the algebra of dynamical variables $\{\rho\} \cup \{\rho'\}$, and we denote the algebra generated by this set allowing ω-dependent coefficients by $A_\omega(\rho,\rho')$.

The remaining criteria we derive by ensuring that the time derivatives of $\omega^\mu(t)$ satisfy the condition (3) are

B. $F^\nu[\rho_m,\pi^\nu] + [\rho_m,G] \in A_\omega(\rho,\rho')\ \forall\, m$

C. $F^\nu[\rho'_m,\pi^\nu] + [\rho'_m,G] \in A_\omega(\rho,\rho')$

for those ρ'_m which occur in the expansions of

$$[\rho_m,H_{op}],[[\rho_m,H_{op}],H_{op}],\ldots \quad .$$

These criteria do not force ϕ^μ to depend on a commuting set of quantum variables. If the ϕ^μ are such, then B and C reduce to $[\rho_m,G]$ and $[\rho'_m,G]$ belonging to the algebra. These are conditions on h, the secondary coupling function. On the other hand, if the ϕ^μ depend in a non-trivial manner on non-commuting operators, these criteria restrict ϕ^μ also.

The primary coupling functions depending on non-commuting quantum variables will lead to a problem when we wish to read the information stored in the classical variables. It will be of the ϕ^μ, not the underlying non-commuting quantum operators. It is only when ϕ^μ depend on commuting quantum variables that information on the ϕ^μ can be seen to give us similar information about the quantum variables.

Thus there may be certain interactions between the classical apparatus and the quantum system which allow the classical apparatus to remain classical, but fail to yield a measurement.

In the cases that a measurement can result from the interaction, there are more steps to be taken before one can consider it accomplished. The quantum-enlarged apparatus has interacted with the quantum system, and information has been transferred to the apparatus observables.

However, in its present form we cannot 'measure' the apparatus variables without disturbing the system. A measurement of the $\omega^\mu(t)$ will cause a disturbance to the $\pi^\mu(t)$. This in turn, if more interactions with the quantum system are envisaged, may have an effect on the quantum system. It is only when the apparatus is described in the classical Hamiltonian formulation that its observables can be measured at will.

Thus we must project back, by means of the map Q^{-1} to the classical phase space. Because this mapping is achieved via the eigenvalue, the classical variables will now possess the information about the eigenvalues of the relevant system observables.

At the same time we must also project the quantum system back to a pristine form--projecting out all π-dependence in operators which do not determine the state of the system.

V. Application of Measurement Model

I will now briefly discuss a particularly simple example to illustrate a measurement example, and the use of the integrity criteria.

The quantum system to be examined is an inert quantum spin system. The operators are

$$S^2 = S_1^2 + S_2^2 + S_3^2 \quad \text{and} \quad S_1, S_2, S_3$$

$$[S_i, S_j] = i\varepsilon_{ijk} S_k$$

$$[S^2, S_i] = 0$$

The Hamiltonian of this quantum system is zero.

The apparatus I use is also the simplest I can imagine, with no claim to realism, a freely moving classical particle. In quantum enlarged form, the operators are

$$\omega = (q_i, p_i) \ , \qquad \pi = (\pi_i^q, \pi_i^p)$$

and the Hamiltonian of the apparatus is $\frac{1}{m}\, \underset{\sim}{p} \cdot \underset{\sim}{\pi}^q$.

In order to describe an interaction between the apparatus and system we envisage the quantum spin system being carried along as internal degrees freedom by the (electrically neutral) particle (classical)--it gives the magnetic moment of the classical particle.

The interaction will be induced by sending the particle through an inhomogeneous magnetic field. Thus our experiment can be considered as a Stern-Gerlach experiment--the heavy atoms are usually treated as classical.

The Hamiltonian operator will take the form

$$H_{op} = \frac{1}{m} \underset{\sim}{p}\cdot\underset{\sim}{\pi}^q - \gamma B(q)\cdot S - \gamma\left[\frac{\partial}{\partial q_j} \underset{\sim}{B}(q)\cdot\underset{\sim}{S}\right]\pi_j^p \quad .$$

Here,

$\underset{\sim}{\mu} = \gamma\underset{\sim}{S}$ is the magnetic moment caused by the internal degrees of freedom,

$\gamma \frac{\partial}{\partial q_j} \underset{\sim}{B}(q)\cdot\underset{\sim}{S} = \phi_j^p$ is chosen to give the correct classical equations of motion for a particle of magnetic moment $\underset{\sim}{\mu}$ moving in an inhomogeneous magnetic field, and $\underset{\sim}{\mu}\cdot\underset{\sim}{B}(q)$ is the classical potential energy for this motion.

We allow for the most general (analytic) coupling between the apparatus and system by allowing $B_i(q)$ to be arbitrary analytic functions. The coupling functions ϕ_j^p will now depend in a non-trivial manner on non-commuting quantum operators.

We must check the integrity criteria, however.

(A) $\{\phi^j\}$ commuting set

if an only if $\frac{\partial}{\partial q_i} \underset{\sim}{B}(q) \times \frac{\partial}{\partial q_j} \underset{\sim}{B}(q) = 0$

i.e., they are parallel.

(B) $[\phi_i, H_{op}]$ commutes with ϕ_j

if and only if the inhomogeneous magnetic field takes the form

$B(q)\underset{\sim}{n}$, $\underset{\sim}{n}$ a constant unit vector .

(C) All the other criteria are then satisfied.

In this case the primary coupling functions depend on the single quantum variable $S_n \equiv \underset{\sim}{S}\cdot\underset{\sim}{n}$ and the interaction leads to a measurement.

If we allow non-analytic couplings, (B) restricts the external magnetic field to the form

$$\sum_m B^m(q)\underset{\sim}{n}_m$$

where $\underset{\sim}{n}_m$ are constant unit vectors, and the functions $B^m(q)$ have disjoint regions of support. In this case, however, the criteria are necessary, but not sufficient. The other criteria one must check reduce the magnetic field to the form $B(q)\underset{\sim}{n}$.

Result: The only interaction which preserves the classical nature of the apparatus is one where the magnetic field is unidirectional in regions through which the particle passes.

Let us now see how the measurement proceeds in this example. Consider a magnetic field

$$B(q) = \begin{cases} 0 & q_2 \le y_0 \\ aq_3 + b & y_0 < q_2 < y_1 \\ 0 & y_1 \le q_2 \end{cases}$$

where $\underset{\sim}{n}$ is along the q_3-direction, and $a < 0$. We shall ignore the edge effects caused by this choice of $B(q)$. Elsewhere we have shown that the edge effects can be treated without altering substantially the solution.[5]

We must specify the state of the system initially

$$|\psi\rangle \otimes |\phi\rangle$$

where the apparatus state is $|\hat{q}_2,\hat{p}_2;\hat{q}_3,\hat{p}_3\rangle = |\psi\rangle$

$$\hat{q}_2 < y_0 \qquad \hat{q}_3 = \hat{p}_3 = 0$$

$$\hat{p}_2 > 0$$

and the quantum state is $|\phi\rangle = |s,S_1 = s'\rangle$. The Hamiltonian is

$$\frac{1}{m}\,\underset{\sim}{p}\cdot\underset{\sim}{\pi}^{q} - \gamma B(q)\cdot\underset{\sim}{S} + \phi_3\pi_3^p$$

$$\phi_3 = \begin{cases} 0 \\ -a\gamma S_3 \\ 0 \end{cases}$$

The equations of motion are

$$\dot{p}_2(t) = 0 \qquad \dot{p}_3(t) = -\phi_3$$

$$\dot{q}_2(t) = \frac{1}{m}\,p_2(t) \qquad \dot{q}_3(t) = \frac{1}{m}\,p_3(t)$$

$$\dot{S}_3(t) = 0$$

The solutions in the region to the right of the magnetic field are

$$p_3(t) = p_3(0) - a\gamma S_3(0)(t_1 - t_0)$$

$$q_3(t) = q_3(0) + \frac{1}{m}\,p_3(0)t - \frac{a\gamma}{2m}\,S_3(0)(t_1 - t_0)^2 - \frac{a\gamma}{m}\,S_3(0)(t_1 - t_0)(t - t_1)$$

where $t_0 = \frac{m}{\hat{p}_2}\,(y_0 - \hat{q}_2)$, $t_1 = t_0 + (y_1 - y_0)\,\frac{m}{\hat{p}_2}$.

Now we see that the apparatus observables $q_3(t)$ and $p_3(t)$ have become functionally dependent on the quantum operator S_3. To see that this will result in a measurement we need to examine the action of $p_3(t)$ on $|\psi\rangle \otimes |\phi\rangle$

$$p_3(t)(|\psi\rangle \otimes |\phi\rangle) = p_3(t)|\psi\rangle \otimes \sum_{s''}\langle s,S_3 = s''|s,S_1 = s'\rangle|s,S_3 = s''\rangle$$

$$= p_3(t)\sum_{s''}a_{s''s'}|\psi\rangle \otimes |s,S_3 = s''\rangle$$

$$= \sum_{s''}a_{s''s'}[-a\gamma(t_1 - t_0)s'']|\psi\rangle \otimes |s,S_3 = s''\rangle$$

--the single trajectory has split into $(2s+1)$ beams of particles, each with state

$$|\psi\rangle \otimes |s,S_3 = s''\rangle$$

Now, in the projection back to the classical phase space, each beam will have values for $q_3(t)$ and $p_3(t)$ dictated by their action on $|\psi> \otimes |\phi>$. Thus we see that here also the beams will be split into many trajectories.

As it is now in purely classical form, a measurement of which beam the particle belongs to will yield a measurement (for the observer) of the value of S_3 for the internal quantum spin system.

VI. Conclusion

I have presented an unusual approach to the measurement problem in which the strictly classical apparatus interacts directly with the quantum system. I have examined this interaction, in particular whether or not the classical system might 'become quantized' as a result of it. Finally I have discussed a simple example to illustrate the procedures.

We have elsewhere[5] examined more complicated arrangements of the systems in section V, for example with crossed magnetic fields. The model is seen to be able to describe the outcomes.

We are currently examining approximate methods of solution, the rigorous treatment of superselection, and other experiments which can be described by the model.

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