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The Imaginary Part of The Nucleus - Nucleus Optical Potential.

S.C. Phatak and Bikash Sinha

Bhabha Atomic Research Centre
Nuclear Physics Division, Bombay 400 085, INDIA

Abstract

The contribution to the imaginary nucleus - nucleus optical potential has estimated by evaluating the energy - conserving second-order term in the perturbation series. The incoming nuclear field is supposed to excite nucleons in a nucleus in this calculation and the nuclear excitations are approximated by particle-hole excitations in a Fermi gas. The resulting imaginary potential compares favourably with phenomenological potentials.

For several years now, the concept of a complex optical potential has been invoked to analyse elastic scattering and, in general, direct reaction data for two colliding nuclei. Both the real and the imaginary potentials are usually represented by Woods-Saxon form factors. The well-depth, radius and diffuseness parameters of these are adjusted to fit the elastic scattering data. For the analysis of non-elastic direct reactions, these potentials are used to compute the distorted waves, a practice well-founded by now for the case of single nucleon or light-ion scattering.

However, it has been observed [1] that the elastic scattering data does not determine these parameters uniquely and the literature is too frequently plagued with lengthy discussions on "shallow" and "deep" potentials. To overcome the ambiguity problem, it is important to determine both the potentials microscopically. A substantial amount of work has been done to obtain the real part of the nucleus-nucleus optical potential. The constrained Hartree-Fock method, the so called folding models and the proximity potentials have been reasonably successful in predicting elastic scattering data [2]. For the imaginary part of the optical potential, however, little effort has been invested to compute it microscopically; the very many excited states involved in a nucleus-nucleus collision make such a calculation somewhat difficult. The earlier attempts, to date, are mostly based on the assumption of forward scattering amplitude approximation [3] ; the semi-classical approximation, sometimes referred to as the frivolous model has also been used [4] . Recently, the contribution to the imaginary potential due to Coulomb excitations has been estimated [5], [6] .

In this paper we attempt to calculate the imaginary potential by considering the nucleus to be a Fermi gas and compute the contribution of particle-hole excitations to the energy-conserving second - order term (pole term) in the perturbation series. These excitations are assumed to be caused by the incoming single-particle field operating on nucleons in the target — the nucleons are excited above the Fermi level so as not to violate the Pauli principle. The method used here is essentially similar to the one used by Shaw to compute the imaginary part of the nucleon-nucleus optical potential [7] ; the two-body interaction of ref. [7] is being replaced by the nucleon-nucleus single-particle field.

Thus the contribution to the imaginary potential due to particle-hole excitations of (say) nucleus 1 produced by the field of nucleus 2 is

$$W_1 = -\pi \sum_{\substack{j \leq k_F \\ m > k_F}} |\langle jK|V_1|K'm\rangle|^2 \, \delta(jK, K'm) \tag{1}$$

$$= \frac{-\pi}{(2\pi)^6} \int d^3k_j \, d^3k_m \left| \int e^{i(\underline{k}_m - \underline{k}_j)\cdot\underline{\xi}} \, V_1(|\underline{R}-\underline{\xi}|) \, d^3\xi \right|^2 \cdot \delta\left(\frac{K^2}{2\alpha m} + \frac{k_j^2}{2m} - \frac{K'^2}{2\alpha m} - \frac{k_m^2}{2m}\right) \tag{2}$$

with $V_1(|\underline{R} - \underline{\xi}|)$ being the single-particle field due to nucleus 2 acting on a nucleon in nucleus 1 , $|j\rangle$ and $|m\rangle$ being the states of the nucleon before and after excitation;

$$K = \left[2\alpha m\left(E_{rel} - U_R(R) - V_c(R)\right]^{1/2}/\hbar$$

and K' being the local relative momenta before and after excitation, and $\alpha = \frac{A_1 \; A_2}{A_1 + A_2}$ where A_1 and A_2 are the atomic numbers of two nuclei. In obtaining equation 2 from equation 1 we have assumed that the nucleus 1 can be approximated by a Fermi gas of radius R_1. Hence the integration variable spans the volume of nucleus 1 and the excitation of the nucleus can be considered to be a particle in state $|m\rangle$ above the Fermi sea and a hole in state $|j\rangle$ below the Fermi sea. We have also neglected the antisymmetrization between nucleons in nucleus 1 and those in nucleus 2. These approximations are expected to be reasonable when R, the relative distance between two nuclei is larger than the sum of their radii (i.e. when the collisions are peripheral). Indeed the concept of a potential, it has been argued, is relevant only for such peripheral collisons [1] .

Using the momentum transfer, $\underline{q} = \underline{k}_M - \underline{k}_i = \underline{K} - \underline{K}'$ as the integration variable, equation (2) becomes

$$W_1(R) = \frac{-\pi}{(2\pi)^6} \int d^3q\, d^3k_i\, \delta\left(-\frac{q^2}{2m}\left(1+\frac{1}{\alpha}\right) + \frac{\underline{K}\cdot\underline{q}}{\alpha m} - \frac{\underline{k}_i\cdot\underline{q}}{m}\right) |V_1(q)|^2 \quad (3)$$

where

$$V_1(q) = \int e^{i\underline{q}\cdot\underline{\xi}}\, V_1(|\underline{R}-\underline{\xi}|)\, d^3\xi \quad (4)$$

In equation (3) we can first perform the integral over d^3k_i by choosing Z-axis along $\underline{q}$. Then $d^3k_i = k_i^2\, dk_i\, dx_i\, d\varphi_i$ where $x_i = (\underline{k}_i\cdot\underline{q})/(k_i q)$. Later while integrating over d^3q we choose the Z - axis along $\underline{K}$ so that $d^3q = q^2 dq\, dx\, d\varphi$ where $x = (\underline{q}\cdot\underline{K})/(qK)$. We can now use the δ- function to integrate over x_i . The result is,

$$W_1(R) = \frac{-\pi m}{(2\pi)^5} \int q\, dq\, dx\, d\varphi_q\, k_i\, dk_i\, |V_1(q)|^2 \quad (5)$$

The integrals in equation (5) are restricted by the condition $|x_i| \le 1$ or $k_i^2 \geqslant \left(\frac{Kx}{\alpha} - \left(1+\frac{1}{\alpha}\right)\frac{q}{2}\right)^2$ the Pauli conditions $k_i^2 \le k_F^2$ and $k_M^2 = q^2 + k_i^2 + 2qk_ix_i \geqslant k_F^2$ or $k_i^2 \geqslant k_F^2 - \frac{2qKx}{\alpha} + \frac{q^2}{\alpha}$ and the condition $K'^2 \le K^2$ or $q \le 2Kx$

These conditions give the limits for k_i - integral and finally we get $W_1(R) = W_1^{I}(R) + W_1^{II}(R)$

where

$$W_1^{I}(R) = \frac{-\pi m}{2(2\pi)^5} \int q\, dq\, dx\, d\varphi_q \left(\frac{2qKx}{\alpha} - \frac{q^2}{\alpha} \right) \left| V_1(q) \right|^2 \tag{6a}$$

with $\frac{1}{\alpha^2}\left(Kx - \frac{\alpha+1}{2} q\right)^2 \leq k_F^2 - \frac{2qKx}{\alpha} + \frac{q^2}{\alpha}$

and $\frac{q}{2K} \leq x \leq 1$

and

$$W_1^{II}(R) = \frac{-\pi m}{2(2\pi)^5} \int q\, dq\, dx\, d\varphi_q\, k_i\, dk_i \left| V_1(q) \right|^2 \tag{6b}$$

with $\frac{1}{\alpha^2}\left(Kx - \frac{\alpha+1}{2} q\right)^2 \geqslant k_F^2 - \frac{2qKx}{\alpha} + \frac{q^2}{\alpha}$

$$k_F^2 \geqslant \frac{1}{\alpha^2}\left(Kx - \frac{\alpha+1}{2} q\right)^2$$

and $q/2K \leq x \leq 1$

In principle, the integrals for $W_1^{I}(R)$ and $W_1^{II}(R)$ can be performed numerically. For this we need to specify $V_1(|\underline{R} - \underline{\xi}|)$, the field experienced by a nucleon due to nucleus 2. We have chosen it to be a standard Woods - Saxon form :

$$V_1(|\underline{R} - \underline{\xi}|) = V_0 \left[1 + \exp\left(\left\{ (|\underline{R} - \underline{\xi}|) - R_2 \right\} / a_2 \right) \right]^{-1} \tag{7}$$

with $V_0 = -40\,\mathrm{MeV}$, $a = 0{\cdot}75\,\mathrm{fm}$ and $R_2 = 1{\cdot}2\, A_2^{1/3}\,\mathrm{fm}$

However, since we are interested in peripheral collisons, R is greater than the sum of the radii of two nuclei and in this region V_1 falls off rather fast. Hence the Fourier transform of such a function is a fairly smooth function. Therefore, as a first approximation, we have replaced $V_1(q)$ in $W_1(R)$ by $\bar{V}_1 = V_1(q = 0)$. With this approximation, the integrations can be done easily and we can get an analytic form for W

$$W_1(R) = \frac{\pi \bar{V}_1^2}{(2\pi)^3 K\alpha}\left[\frac{2}{3}K^2 k_F^3 - K k_F^4 + \frac{2}{5}k_F^5 + \frac{5k_F^2}{\alpha}(K-k_F)^3\right] + O\left(\frac{1}{\alpha^3}\right) \tag{8a}$$

The terms with higher powers of $1/\alpha$ can be neglected since $\alpha \gg 1$ (e.g. for ^{16}O on ^{16}O, $\alpha = 8$). In addition, we have assumed that $K < \alpha k_F$ (again for ^{16}O on ^{16}O, $K = \alpha k_F$ corresponds to the relative kinetic energy of about 300 MeV; for heavier nuclei, this number is still larger).

So far we have considered the contribution to the imaginary potential due to particle-hole excitations produced in nucleus 1 by the field of nucleus 2. To this we must add the contribution due to the excitations produced in nucleus 2 by the field of nucleus 1 to get the total imaginary potential. Thus, in our model, the imaginary potential is,

$$W(R) = \frac{m(\bar{V}_1^2 + \bar{V}_2^2)}{(2\pi)^3 K\alpha}\left[\frac{2}{3}K^2 k_F^3 - K k_F^4 + \frac{2}{5}k_F^5 + \frac{5k_F^2}{\alpha}(K-k_F)^3\right] \tag{8b}$$

where

$$\bar{V}_{1,2} = \frac{2\pi V_0}{2R}\int_{R-R_{1,2}}^{R+R_{1,2}} r\,dr\,\frac{\{R_{1,2}^2 - (R-r)\}^2}{[1+\exp\{(r-R_{2,1})/a\}]} \tag{9}$$

and R_1 and R_2 are the radii of nucleus 1 and 2 respectively. For $R > R_1 + R_2$, $\bar{V}_1$ and $\bar{V}_2$ can be approximated as,

$$\bar{V}_{1,2} \sim \frac{2\pi V_0 a^2}{R}\left[R\left(R - R_{1,2}\right) + a\left(3R_{1,2} - R\right)\right] e^{-\left(R - R_{1,2} - R_{2,1}\right)/a} \tag{10}$$

In figure 1 we have plotted the imaginary potential given by equation (8) for ^{16}O - ^{16}O elastic scattering at 40 MeV as a function of the R, the distance between the centeres of two nuclei. On the same graph we have also plotted two phenomenological potentials [8]. It is interesting to note that all the three potentials have roughly same strength around 7 fm, which is close to the strong-absorption radius; the two phenomenological potentials however differ considerably as one goes away from the strong - absorption radius. It is apparent that our result agrees well with the so called "deep" energy independent potential but deviates from the "shallow" energy dependent potential. It would be best to compute scattering cross sections with our potential and compare directly with experimental data, a scheme postponed for future publication.

Finally, from equation (8) one can see that the imaginary potential is energy-dependent. This is demonstrated in figure 2 where we have plotted the strength of the imaginary potential at the touching radius ($R = R_1 + R_2$) as a function of the relative kinetic energy between two ions for various target-projectile combinations. The strength appears to change linearly with the relative kinetic energy and the slope seems to be fairly independent of target-projectile combination (0.3 for ^{16}O - ^{16}O and 0.35 for ^{16}O - ^{208}Pb). In conclusion therefore we would like to suggest that a simple microscopic description for the imaginary potential, as presented above, in conjunction with the real part of the optical potential could be used for analysis of the now exhaustive heavy ion scattering data. The authors would like to thank B.K. Jain for many useful discussions.

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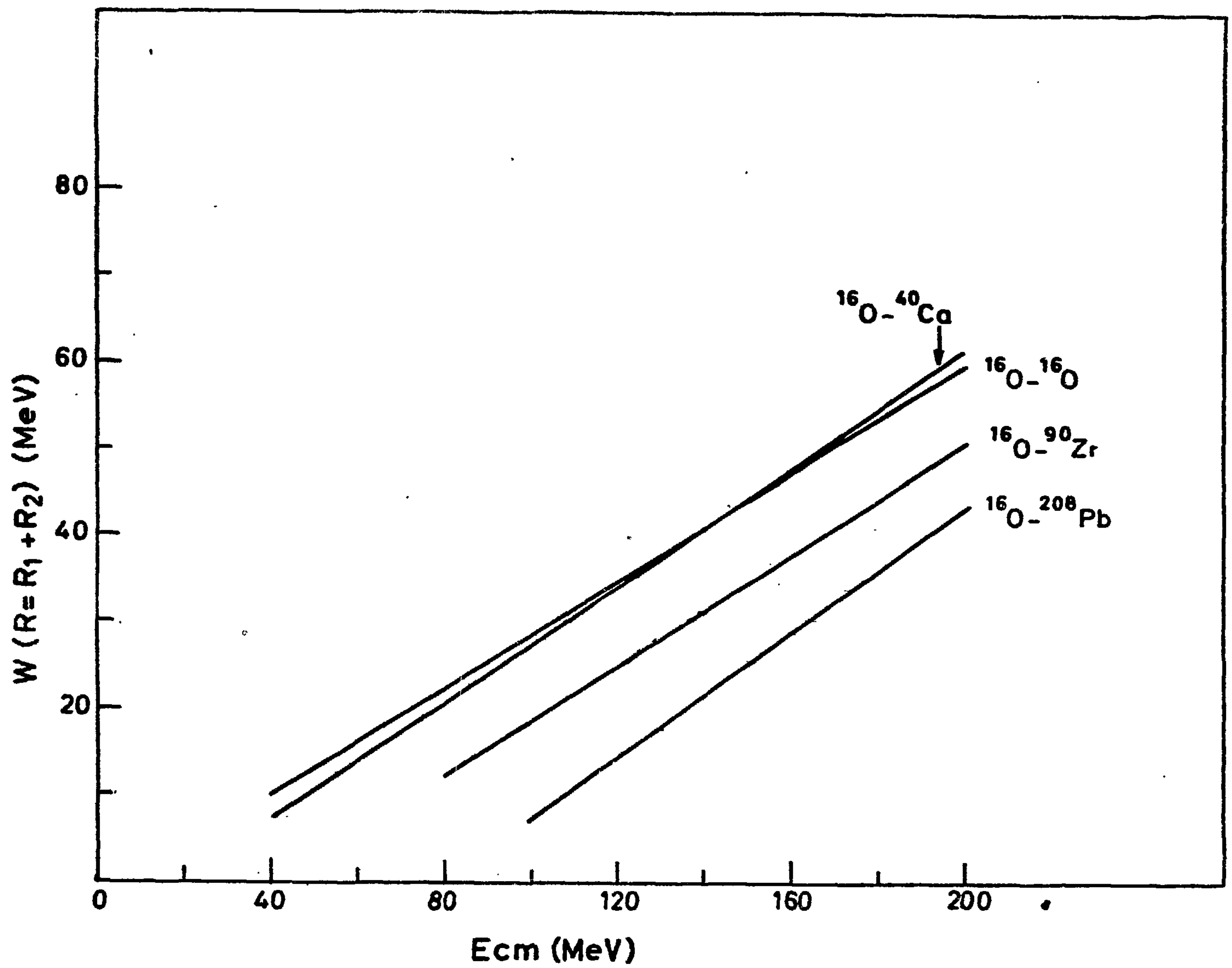
Figure captions

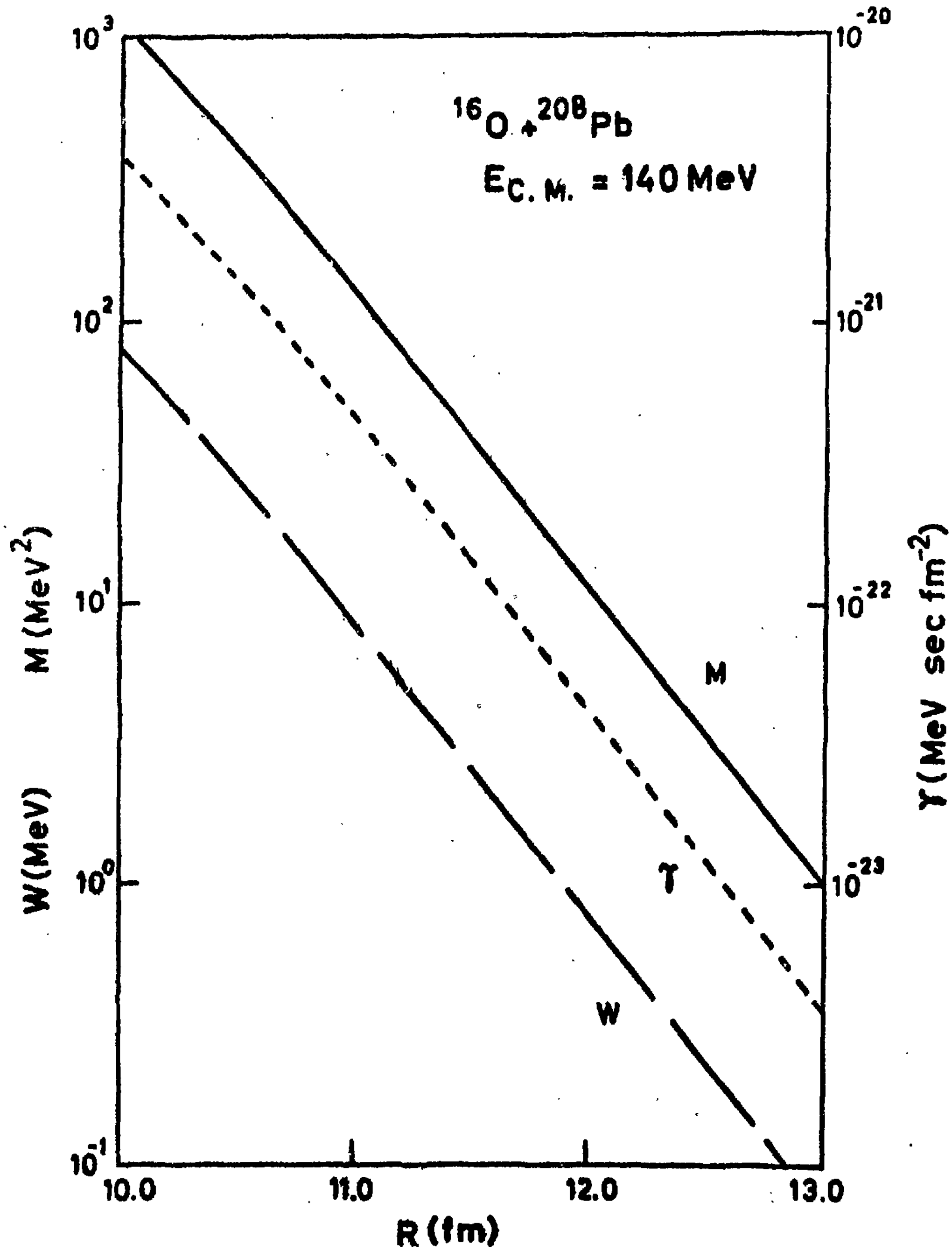
<u>Figure 1</u> : The plot of imaginary potential ($W(R)$) vs. nucleus - nucleus separation (R).

————————	results of equations (8) and (9)
- - - - - - -	phenomenological (shallow, energy - dependent) potential
—— ——	phenomenological (deep, energy - dependent) potential
—— — ——	results of equations (8) and (10)

<u>Figure 2</u> : Imaginary potential depth at the touching distance ($R = R_1 + R_2$) vs. nucleus - nucleus c.m. energy for various target - projectile combinations.



W (R=R1+R2) (MeV)
80
60
40
20
0
40
80
120
160
200
Ecm (MeV)
16O-40Ca
16O-16O
16O-90Zr
16O-208Pb



$^{16}O + ^{208}Pb$
$E_{C.M.} = 140$ MeV
10^3
10^2
10^1
10^0
10^{-1}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
M
T
W
M (MeV^2)
W (MeV)
Υ (MeV sec fm^{-2})
10.0
11.0
12.0
13.0
R (fm)