FIRST WALL [illegible] *

[illegible]

[illegible]

The failure [illegible] is investigated. [illegible] given. [illegible] and [illegible] related to either the [illegible] tensile stress [illegible] compressive failure [illegible] figure of merit, $k(1-\nu)\sigma/\alpha E^{1/2}$, is identified [illegible] silicon carbide [illegible].

## INTRODUCTION

[illegible] the major concerns in the design of a first wall is the thermal stress induced by [illegible] fluxes. In the case of inertial [illegible] high intensity, [illegible] are applied to the first wall. These [illegible] light, [illegible], and pellet debris. The [illegible] thermal stress may ultimately determine [illegible] of the reactor [illegible] the first wall. In the [illegible] the same analysis [illegible] design [illegible] of a plasma [illegible].

The stresses induced in a first wall plate which is subjected to [illegible] pulses of energy [illegible] have been investigated in detail. It is shown that the general thermoelastic equations may be uncoupled and that stresses due to thermal effects can be considered independently of those arising from inertial effects which are produced, for example, by the change of momentum of the pellet debris, or sputtering of the first wall. Stresses due to thermal effects only are considered in this investigation.

## [illegible]

[illegible]

The problem [illegible] which [illegible] design which [illegible] of the first wall, and [illegible] particle debris [illegible] It will be assumed that the debris [illegible] a prescribed surface heat flux. The [illegible] wall loading due to particle debris [illegible] approximately 700 MW/m$^2$ for a 10 m radius [illegible].

The first wall material will be assumed to behave linear elastically, which is a good [illegible] for brittle materials such as graphite and silicon carbide. Failure is assumed [illegible]

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the maximum normal stress (or strain) reaches the ultimate strength (or strain limit). The effect of rapid rate of loading on material strength is typically not a strong function for brittle materials.

Uncoupling of Thermoelastic Equation

In general, the equations of thermoelastic theory are coupled in that the transient theories of elasticity and heat conduction are combined. Transient mechanical loads which produce strains may also produce changes in temperatures and, conversely, transient thermal loads produce strains which may also alter thermal behavior. It is normally assumed that this coupling is small. In fact, according to Ref. 1, these coupling effects can be neglected if it can be shown that the effects of inertia are small. The latter will be demonstrated.

If $t_o$ represents a measure of the rate of applied heat flux and $t_M$ and $t_T$ characteristic mechanical and thermal response times, respectively, defined as:

$$t_M = \frac{L}{c} \tag{1a}$$

$$t_T = \frac{L^2}{\kappa} \tag{1b}$$

where L = characteristic length, i.e., plate thickness

c = wave propagation speed = $\sqrt{\frac{E}{\rho}}$

$\kappa$ = thermal diffusivity,

inertia effects, and hence coupling, may be *neglected if*

$$t_T >> t_M \tag{2a}$$

and

$$t_o >> t_M \tag{2b}$$

or if the parameter H is much less than one, where H is defined as:

$$H = \sqrt{\frac{t_M}{t_T} \cdot \frac{t_M}{t_o}} \tag{3}$$

For a 5 mm thick graphite plate, c = 2160 m/s, $\kappa$ = 0.18 cm$^2$/sec, $t_M$ = 2.3 μs, $t_T$ = 1.4 s, and for $t_o$ = 500 μs, H equals $8.7 \times 10^{-5}$, which is much less than unity. (For H ≈ 1.0, $t_o$ must be of the order of pico-seconds.) The mechanical response of the material to the transient temperature is much faster than the imposed temperature changes so that the material can be assumed to have negligible mass density and not respond inertially. Therefore, the transient thermal stresses may be treated separately and independently of the wave propagation problem.

Thermally induced vibration of a plate which is exposed on one face to a step change in heat input is treated in Ref. 1, and it is shown that the ratio of maximum dynamic to maximum static deflection is a function of the nondimensional parameter $B_1$ where

$$B_1 = \frac{h}{a\sqrt{\kappa}}\left(\frac{D}{h\rho}\right)^{1/4} \tag{4}$$

where h = plate thickness

a = plate size

D = plate bending rigidity = $\frac{E\,h^3}{12\,(1-\nu^2)}$

$\nu$ = Poisson's ratio

$\rho$ = mass density.

For a 50 cm square, 5 mm thick graphite plate, $B_1$ = 7.5 which indicates that the dynamic deflection is approximately 1% greater than the static deflection, indicating the plate responds essentially quasi-statically.

Semi-Infinite Solid

One approach that may be chosen to analyze the transient thermal stresses in the liner plate is to use an infinitely thick plate model. This approach is normally valid for problems of very short time duration since the maximum stresses will be localized near the surface boundary. However, only compressive stresses can be developed using this model, which is a disadvantage if failure due to tension is expected. An evaluation of the conditions under which this model is justified is developed in the next section where the liner plate is modeled as a freely supported flat plate.

For the case of a plate or body which has a temperature variation through the thickness (z-direction) only, the uncoupled thermoelastic equations of equilibrium, compatibility and boundary conditions can all be satisfied[1] if:

$$\sigma_x = \sigma_y = -\frac{E\,\alpha}{1-\nu}\Delta T(z,t) + C_1 + C_2 z \qquad (5)$$

where E = Young's modulus

α = coefficient of thermal expansion

and all other stresses are identically zero. Constants $C_1$ and $C_2$ are evaluated from the boundary conditions. For a semi-infinite solid, the boundary conditions are:

$$\sigma_x = \sigma_y = -\frac{E\,\alpha}{1-\nu}\Delta T(z,t)\Big| \quad \text{at } z = 0 \qquad (6a)$$

$$\sigma_x = \sigma_y = 0 \quad \text{at } z = \infty \quad . \qquad (6b)$$

Equation (6a) results from the complete restraint of the thermal expansion in two directions. Note that expansion normal to the surface is allowed. The second boundary condition, Eq. (6b), simply states that the stresses in the solid infinitely away from the surface are unaffected. The boundary conditions yield that both $C_1$ and $C_2$ are zero, so that a temperature variation normal to the surface of an infinite solid produces normal in-plane stresses which are proportional to the temperature variation through the depth. All other stresses are zero.

The surface temperature increase $T - T_o$ produced by a suddenly applied heat flux q is given by[2]

$$T - T_o = \frac{2}{\sqrt{\pi}}\frac{q}{k}(\kappa t)^{1/2} \qquad (7)$$

where k = thermal conductivity, so that the compressive surface stress is simply:

$$\sigma_s = \frac{-2}{\sqrt{\pi}}\frac{q}{k}\frac{E\,\alpha}{1-\nu}(\kappa t)^{1/2} \quad . \qquad (8)$$

There are never any tensile stresses produced which would result in a tensile failure. In order to develop tensile stresses, a different structural model must be used by assuming that the first wall is a thin plate which cannot develop a net in-plane force or a net out-of-plane bending moment.

## THERMOELASTIC PLATE ANALYSIS

### Step Response

An assessment of thermoelastic stresses in a freely supported plate where the in-plane force and the out-of-plane bending moment are both zero can be made without resorting to computer stress analysis codes. Both the temperatures and stresses are available as explicit functions of depth and time. A one-dimensional thermal analysis for the case where a suddenly applied heat flux is prescribed on one surface (z/h = 1.0) of a plate which is initially at a uniform temperature $T_o$ and whose back face (z/h = 0.0) is maintained at the initial temperature for all time yields[3]

$$T - T_o = \frac{qh}{k}\left[\frac{z}{h} - \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\, e^{-(2n+1)^2\frac{\pi^2}{4}\left(\frac{\kappa t}{h^2}\right)} \sin\frac{(2n+1)\,\pi z}{2h}\right] \qquad (9)$$

where nondimensional forms of plate depth and time have been used.

The constant temperature boundary is particularly appropriate for the case of forced cooling, however, the results at small times will be representative and relatively insensitive to the back surface boundary condition. For example, the surface temperature given by Eq. (9) is essentially equal to that for a semi-infinite solid, given by Eq. (7) for Fourier numbers $(\kappa t/h^2)$ up to about 0.3, which, for a 5 mm thick graphite plate represents about 0.7 s.

The expression for the in-plane stresses is[3]:

$$\sigma = \frac{8}{\pi^2}\frac{qh}{k}\frac{E\,\alpha}{(1-\nu)}\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\, e^{-(2n+1)^2\frac{\pi^2}{4}\left(\frac{\kappa t}{h^2}\right)}\left\{\sin\left[\frac{(2n+1)\,\pi z}{2h}\right] - \frac{2}{(2n+1)\pi} - \frac{12\left(\frac{z}{h}-\frac{1}{2}\right)}{(2n+1)\pi}\left[\frac{4(-1)^n}{(2n+1)\pi} - 1\right]\right\}\right) \qquad (10)$$

and the stresses normal to the plate ($\sigma_z$) and all shear stresses are zero, neglecting edge effects. The stress distribution in the plate was evaluated using a double precision computer program and is plotted at various times in Fig. 1. As expected, the stress at the surface is compressive, and in fact, is never tensile. The surface stress reaches a maximum value of -0.103 at $\kappa t/h^2$ = 0.053. In order to maintain zero in-plane force, a tensile stress must be developed, and in order to maintain zero bending moment about the midplane, the back surface stress must be compressive. The back surface compressive stress reaches a maximum at $\kappa t/h^2$ = 0.115. In the interior, at z/h = 0.56, a maximum tensile stress of 0.0415 is developed at $\kappa t/h^2$ = 0.089. The ratio of maximum compressive stress to maximum tensile stress is 2.5. With the assumptions of this model and assuming a maximum stress theory of failure, a compressive failure will occur at the surface if the compressive strength is less than 2.5 times the tensile strength. If the material is more than 2.5 times weaker in tension, a crack in the interior of the plate will be produced, redistributing the stresses in the plate, so that even greater tensile stress is developed, thereby producing a failure.

A comparison of the surface stress with that given by a model of a semi-infinite solid where the thermal expansion is completely restrained is shown in Fig. 2. [The semi-infinite solid curve is plotted on the same axes; variable h can be a dummy parameter in Eq. (8).] It is immediately evident that substantial reduction of the surface stress is realized. The semi-infinite solid model for stress is accurate to within 12% for $\kappa t/h^2$ up to 0.001. (Remember that the surface temperatures agreed well up to about 0.3.)

Pulse Response

Response to a surface heat flux in the form of a square pulse may be obtained simply by

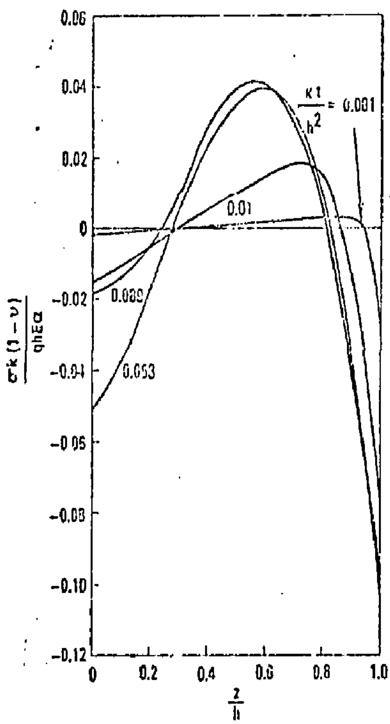


FIGURE 1. Stress distributions through the plate thickness at various times following a suddenly applied heat flux

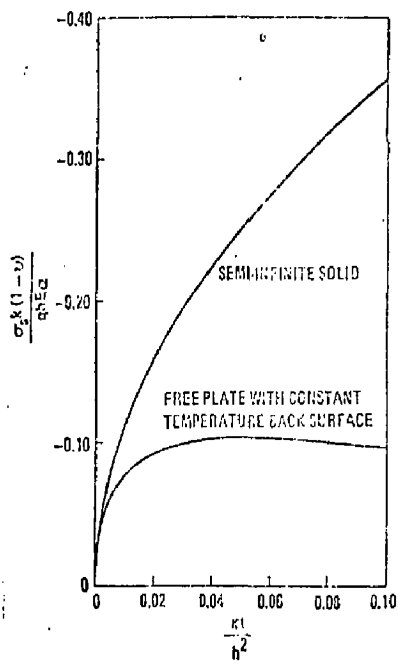


FIGURE 2. Comparison of surface stress in a free plate with that of a semi-infinite solid

superposing a negative flux step at a time $t_0$ after the beginning of the pulse to yield a zero net applied flux. In this way the equilibrating of the temperatures and stresses may be studied.

A pulse width of 0.001 is arbitrarily taken to demonstrate the pulse response. For graphite, where $\kappa$ = 0.18 $cm^2/s$ at 1300°K ($\kappa$ = 0.06 $cm^2/s$ for SiC) and for h = 5 mm, $\kappa t_0/h^2$ = 0.001 represents a pulse of 1.4 ms. Alternatively, a pulse width of 500 μs is equivalent to $\kappa t_0/h^2$ = $0.4 \times 10^{-3}$.

Figure 3 shows the stresses induced in the plate by a pulse of 0.001 duration. It is immediately observed that a large surface compressive stress is produced which drops off rapidly at the end of the pulse. Similar behavior is noted at the back surface except the dropoff is not as rapid, as is to be expected. Just below the surface, at z/h = 0.95, the stress is first tensile, then compressive at the end of the pulse, and after the pulse is over, greater compressive stress is induced before the stress equilibrates. Interestingly, the maximum tensile stress of 0.0031 which is developed at z/h = 0.87 does not behave similarly. That is, the maximum tensile stress produced by a pulse will occur at the end of the pulse, and not afterward.

Figure 4 gives the location of the maximum tensile stress for varying values of pulse width. It is observed that the longer the pulse, the greater the depth at which the maximum tensile stress occurs.

Figure 5 gives an indication of whether a tensile or compressive failure can be expected. It plots the ratio of maximum compressive stress to maximum tensile stress as a function of pulse width. For very short pulses it indicates that the maximum compressive stress may be over 10 times the maximum tensile stress, and unless the material is excessively (greater than 10 times) weaker in tension, compressive failure is to be expected.

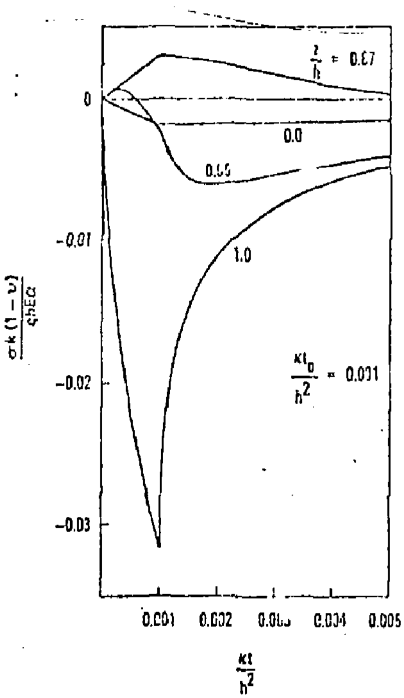


FIGURE 3. Stresses in a free plate for a heat flux pulse of short duration

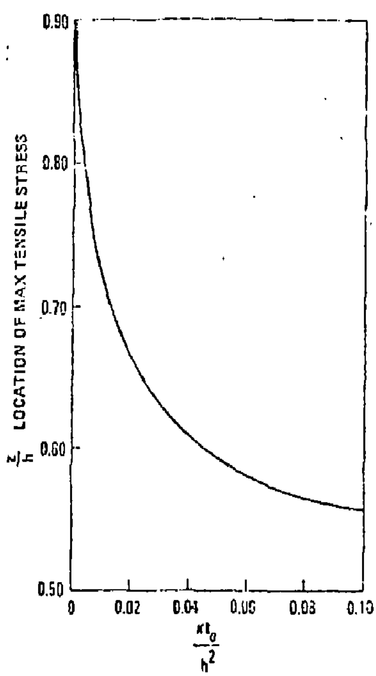


FIGURE 4. Location of maximum tensile stress as a function of pulse length

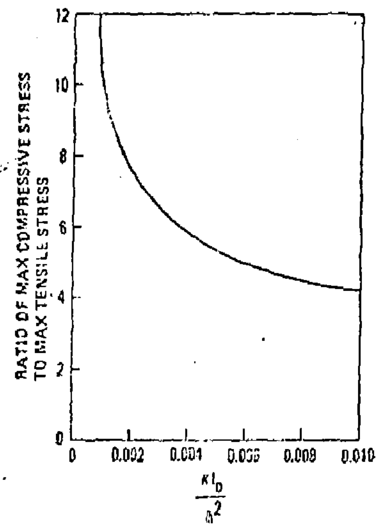


*FIGURE 5. Ratio of maximum compressive stress to maximum tensile stress as a function of pulse length*

The ratio of compressive strength to tensile strength is about 5 for graphite and about 8 for silicon carbide. Therefore, for pulse widths of less than 0.001, the stress criterion is simply that the surface stresses should not exceed the compressive strength of the material.

## DISCUSSION

### Design Implications

It may be concluded from the previous discussion that the stresses produced by very short pulses, i.e., $\kappa t_0/h^2$ is less than 0.001, are approximated very well by simply $E\,\alpha\,\Delta T/(1-\nu)$ where $\Delta T$ is the rise in surface temperature given by the semi-infinite solid model, Eq. (7). This indicates that the failure will be essentially independent of the way the first wall is constrained, that is, whether it is freely supported or whether it is attached to a forced cooled structure.

For the case where the plate is freely supported and the maximum tensile stress produces failure, it is possible to predict the location of the failure. Near the center of the plate, where it will be assumed the plate is supported, substantial inertia forces may exist, at small times, if the plate is sufficiently thick, which will prevent the plate from responding quasi-statically. That is, a non-zero in-plane force may temporarily exist, in which case the maximum tensile stress will be reduced. Near the edges of the plate, where the solution of the previous section is not valid, it is clear that not only are the net forces and moments zero, but the stresses themselves are nearly zero because of the close proximity to the stress-free edge. A short distance from the edge, typically two or three plate thicknesses, the edge effect is negligible, and at the same time small inertia resistance to bending exists. Therefore the present solution is valid. This implies that, all other things being equal, the maximum tensile stress and hence a tensile fracture is most likely to occur near a stress free edge. For the case of a compressive failure the location of failure will be near the center of the plate where inertia forces preventing thermal expansion are largest.

Because of the large temperature variations expected, the temperature dependence of the material properties should be included. Ideally, material properties such as E, $\alpha$ and $\kappa$ which affect the thermal stress should be integrated over the temperature swing. However, for the results that follow, the material properties have not been integrated but, instead, the properties of an intermediate temperature (1800°K for C and 1300°K for SiC) were used. The ultimate strength of the material should be that at the temperature of maximum stress, i.e., the maximum temperature. It was discovered very early in the investigation that temperatures exceeding the allowable limits were quickly reached indicating an erosion design limit. Therefore, it is assumed in the following analysis that the initial plate temperature after steady state cycling conditions have been reached is 800°K. This implies an actively cooled first wall which is constrained from bending.

If R, the reactor cavity radius, is chosen as the independent variable, Eq. (8) for maximum compressive surface stress and Eq. (7) for surface temperature can be written as a function of R since q is a function of R. (A pulse length of

500 με is assumed.) If the unirradiated material properties listed in Table 1[4] are used, curves can be drawn which plot maximum compressive stress against R. Figure 6 indicates that at R = 16.3 m the maximum compressive strength of graphite (74.9 MPa or 10.9 ksi) is reached when the temperature reaches 2220°K, which is probably acceptable from the point of view of maximum temperature. For silicon carbide, Fig. 7 indicates approximately the same minimum radius (16.1 m) for considerably higher compressive strength (4480 MPa or 650 ksi), however the maximum surface temperature is 2650°K which is unacceptably high. Therefore, the minimum reactor radius should be something in the neighborhood of 20 m. Only preliminary conclusions can be drawn from this theoretical

TABLE 1. Material properties of graphite and silicon carbide

| Material Property | Units | H-451 Graphite |  | CVD β-Silicon Carbide |  |
|---|---|---|---|---|---|
|  |  | Unirradiated | Irradiated | Unirradiated | Irradiated |
| Density | $kg/m^3$ | 1700 | 1700 | 3150 | 3150 |
| Young's modulus | GPa | 8.5 | 17.0 | 370.0 | 370.0 |
| Thermal expansivity | $10^{-6}$ $°K^{-1}$ | 5.5 | 2.8 | 4.9 | 4.9 |
| Thermal diffusivity | $10^{-6}$ $m^2/s$ | 14.2 | 12.0 | 6.3 | 3.0 |
| Thermal conductivity | W/m - °K | 48 | 41 | 25 | 12 |
| Poisson's ratio | -- | 0.11 | 0.11 | 0.24 | 0.24 |
| Ultimate compressive strength | MPa | 74.9 | 96.7 | 4480 | 4480 |

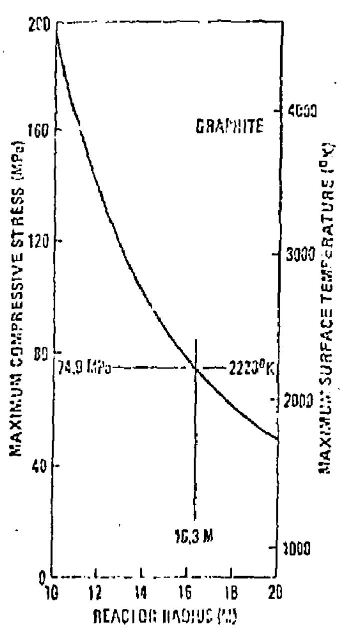


FIGURE 6. Maximum compressive stress and temperature as a function of reactor radius for graphite

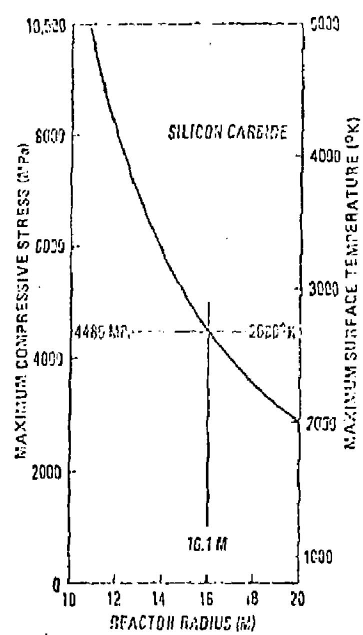


FIGURE 7. Maximum compressive stress and temperature as a function of reactor radius for silicon carbide

exercise with the understanding that many important aspects other than temperature and thermal stress have not been included.

The effect of irradiation may be assessed simply by substituting irradiated ($10^{26}$ neutrons/$m^2$) material properties in the previous calculations. For graphite, even though the modulus of elasticity drops substantially, changes in other properties, such as compressive strength, temper the effect and the minimum reactor radius drops to 15.0 m indicating that irradiation does not aggravate thermal stresses. However, the temperature calculation indicates that the maximum surface temperature will increase about 110°K with irradiation. For silicon carbide the minimum reactor radius increases to 19.3 m and the maximum temperature is 840°K hotter. Therefore irradiated material property data should be used for both the stress and temperature calculations for silicon carbide.

The effect of fatigue is difficult to assess because no fatigue data exist for the case where the minimum and maximum stresses are both compressive. Uniaxial tension-compression tests indicate an endurance limit of about one-half the tensile strength at about $10^5$ to $10^6$ cycles for graphite, which is equivalent to 1 to 12 days of reactor operation. No conclusion concerning compressive stress fatigue can be drawn from this data. However, since all stresses remain compressive, crack initiation and growth may not occur and fatigue may not be life limiting.

## Figure of Merit

For the case of a suddenly applied heat flux which is maintained for a relatively long time, say greater than $\kappa t/h^2 = 0.05$ (see Fig. 2), the relative ability of materials to resist thermal shock may be estimated by comparing the figure of merit $k(1-\nu)\sigma/E\alpha$. However, for pulses of short duration, say $\kappa t_o/h^2 < 0.001$, the applicable figure of merit is $k(1-\nu)\sigma/E\alpha\kappa^{1/2}$ which includes the effect of the rate of change of surface temperature due to thermal diffusion for a given applied heat flux. On the basis of the unirradiated data given in Table 1, graphite and silicon carbide rank very comparably.

## Recommendations

Only a limited amount of information can be obtained by surveying published material property data to assess the potential of candidate first wall materials. Once a limited number of materials can be identified as having promise, the suitability of these materials can be more accurately assessed only through the collection of more pertinent material data. For instance, to assess first wall lifetime, biaxial fatigue data are required where the stresses always remain compressive. Similarly, the thermal shock resistance of the material and the mode of failure can be more accurately determined only by a testing program which closely simulates the expected loading produced by the heat flux pulses.

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