Abstract:

The isovector monopole state is calculated using the RPA and TDA and the Skyrme III interaction. Calculations are performed for the parent and analog nucleus. Isospin impurities in the parent ground state and Coulomb shifts as well as escape widths of the isobaric analog resonance are evaluated.

1. Introduction

In the recent years it has become clear that in addition to the giant dipole state other giant states appear as well defined resonances in nuclei (1). Giant isoscalar quadrupole states were observed and there is mounting evidence concerning the isovector quadrupole, isoscalar monopole (2), octupole and other states (1).

In this paper we are concerned with the giant isovector monopole \((J^P=0^+, T=1)\) state. As pointed in the past \((3,4)\) this state plays a very important role in a variety of effects connected with isospin mixing or more generally Coulomb mixing \((3,4)\). Since the isovector part of the one-body Coulomb interaction in nuclei is well approximated by the quadratic form:

\[
V_C^{(1)}(r) = \frac{2e^2}{2R^3} \sum_{i=1}^{A} (3R^2 - r_i^2) t_2(1)
\]  

it is obvious that second order Coulomb effects will be dominated by the excitation of an intermediate state which carries a large portion of the total isovector monopole strength, i.e. the strength of the \(\sum i r_i^2 t_2(1)\) operator. Such a state is by definition the giant isovector monopole. Various Coulomb effects were evaluated using the closure (doorway state) approximation, i.e. assuming that there is a single \(J=0^+, T=1\) doorway which exhausts most of the total strength \((3-5)\).

The isovector monopole resonance (IMR) has not yet been identified experimentally. However, based on the recent theoretical progress \((6-8)\) in describing other giant resonances it is important to calculate the IMR and reexamine more thoroughly some Coulomb mixing effects, such as isospin impurities in ground states \((3-6)\), widths of isobaric analog resonances \((3,4)\), second order Coulomb...
2. Method of Calculation

The basis in our calculation are the Hartree-Fock (HF) states calculated using a density dependent interaction of Skyrme type \(^{(9)}\). The above HF basis provides a good description of the ground state properties of nuclei \(^{(10-11)}\). Among the several versions, the SIII force used here, provides probably the best overall HF description of the single-particle spectra in nuclei \(^{(11)}\). In this basis, RPA or TDA calculations are performed. The calculations incorporate the entire p-h space including the particle continuum.

For the particle-hole interaction \(-V_{ph}\) we use a force derived from the same SIII interaction by taking the derivative of the HF potential with respect to the density. One of the advantages of having a zero-range force is the possibility of using the Green's function method in order to calculate the p-h excitations. In this method the perturbed p-h Green's function is given by the integral equation:

\[
G(\omega) = G^{(0)}(\omega) + \sum_{\text{ph}} G(\omega) \frac{1}{H_0 - E_n + \omega} \frac{1}{H_0 - E_n - \omega} G^{(0)}(\omega)
\]

and solved in coordinate space.

The unperturbed p-h Green's function \(G^{(0)}(\omega)\) in the RPA approximation is given by:

\[
G^{(0)}(\vec{r}_1, \vec{r}_2; \omega) = \sum_{\text{h}} \frac{\phi_h^*(\vec{r}_1)}{H_0 - E_h - \omega} \frac{\phi_h(\vec{r}_2)}{H_0 - E_h + \omega} G^{(0)}(\omega)
\]

The sum over h includes all the occupied one-particle states of the HF hamiltonian \(H_0\). The expression in the bra and ket is written in terms of the regular and irregular solutions of \(H_0\) if the potential is local \(^{(7,12)}\).

In order to extract information from the Green's function determined by eq. (2) one calculates the response function of a single-particle operator \(Q\). This function is given by:

\[
S(\omega) = \sum_n \langle 0 | Q | n \rangle^2 \delta(\omega - E_n) = \frac{1}{(2\pi)^3} \int Q^*(\vec{r}) \overline{G(\vec{r}, \vec{r}', \omega)} Q(\vec{r}') d\vec{r} d\vec{r}'
\]

Using eq. (4) we compute the strength distribution of one-body transition operators.

3. The Isovector Monopole Resonance in the Parent Nucleus

We first discuss the results obtained for the IMR in the parent nucleus, i.e. in the nucleus with \(T = T_s\).

The \(J=0^+, T=1\) p-h Green's function was calculated for several nuclei in different regions of the periodic table. Using eq. (4) we evaluate the distribution of strength of the third component of the isovector monopole operator.
Results for $^{40}\text{Ca}$, $^{90}\text{Zr}$ and $^{208}\text{Pb}$ are shown in Fig.1.

In general the isovector monopole strength is concentrated in a more or less limited energy region. However in lighter nuclei the strength is considerably fragmented and part of the strength is found at quite low energies. With the increase in the mass number the strength is more concentrated and the distribution is dominated by one peak of a width at half maximum $\Gamma = 7-8 \text{ MeV}$.

In order to provide a more quantitative measure of the distribution we compute its various moments

$$M_\lambda = \sum_{i=1}^{\lambda} r_i^2 t_2(i)$$  \hspace{1cm} (5)

In the case of a distribution one has several possibilities to define its average energy position by using various combinations of the moments \(\lambda\). In table I we show the $E_2=M_1/M_0$ and $E_1=\sqrt{M_1/M_2}$ and $E_2=\sqrt{M_1/M_1}$ averages. (For a discussion on the significance of these different averages see ref. (13)).

In the region of lighter nuclei neither of the defined averages follow the $A^{-1/3}$ dependence, and all are far below the hydrodynamical value \(E = 170/A^{1/3}\) MeV. In heavier nuclei \(A > 100\) the calculated average positions do show the $A^{-1/3}$ behaviour and $E = 4\hbar\omega = 160/A^{1/3}$ MeV. To characterize the spread of the distribution we calculate the rms deviation about the mean energy $-E_2$, $\rho = (M_2/M_0-E_2^2)^{1/2}$, these are shown in the last column of table I.

In nuclei with $N > Z$ the isovector monopole has two isospin components with $T^v = T$ and $T^v = T+1$. The present calculation does not project the two isospin components and certain amount of spurious isospin mixing is present. The
Table 1 Calculated average energy positions and rms deviations about the mean energy $E_0$ (in MeV) of the $T=1, J=0^+$ strength in several nuclei.

<table>
<thead>
<tr>
<th></th>
<th>$E_0$</th>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}O$</td>
<td>34.2</td>
<td>32.1</td>
<td>40.6</td>
<td>12.2</td>
</tr>
<tr>
<td>$^{40}Ca$</td>
<td>33.6</td>
<td>31.8</td>
<td>38.6</td>
<td>10.8</td>
</tr>
<tr>
<td>$^{90}Zr$</td>
<td>33.1</td>
<td>32.1</td>
<td>35.8</td>
<td>7.7</td>
</tr>
<tr>
<td>$^{208}Pb$</td>
<td>27.8</td>
<td>26.8</td>
<td>30.5</td>
<td>7.0</td>
</tr>
</tbody>
</table>

transition strength to the $T' = T$ component of the IMR is more than $T$ times larger than to the $T' = T+1$ component. Therefore in heavier nuclei with a large excess of neutrons the transition strength calculated is mainly due to the $T'=T$ state. (The $T' = T$ component is also the one which has the lower excitation energy).

Isospin Mixing in the Parent Nucleus

The Coulomb matrix elements are dominated by the one-body part of the Coulomb interaction. In order to calculate isospin mixing we need the distribution of strength of the operator given in eq. (1) and this is of course the distribution of the isovector monopole strength. To be more rigorous we calculated the strength distribution of the isovector part of the Coulomb interaction as determined from the actual calculated charge distribution and not from a homogeneous sphere as in eq. (1). The distribution for the case of $^{208}Pb$ is shown in Fig. 2.

Comparing this figure with the distribution of the isovector monopole strength in $^{208}Pb$ as shown in Fig. 1 we see that the distribution of the $\frac{1}{r_i} r_i^2 t_z (1)$ strength and the isovector Coulomb strength are very similar. The energy position and widths of both distributions are very close. This result justifies the earlier calculations of Coulomb effects (9-5) in which the Coulomb strength was assumed to be exhausted by a simple doorway. In lighter nuclei this approximation is poorer.

The amount of isospin mixing is easily obtained by computing the $M_{-2}$ moment of the isovector Coulomb strength distribution. We obtain:

![Fig. 2 The distribution of the isovector Coulomb strength in $^{208}Pb$.](image)
As mentioned already in $N > Z$ nuclei the calculation does not separate the two isospin components and therefore one cannot calculate the precise amount of isospin mixing. It is possible to provide only an upper bound by multiplying the calculated $N_2$ moment with the Clebsch-Gordan coefficient $(T+1)^{-1}$. We get $\epsilon^1 < 0.5 \%$ for $^{90}Zr$ and $\epsilon^2 < 0.7 \%$ for $^{208}Pb$.

5. The Isovector Monopole and Isobaric Analog Resonances in the Analog Nucleus

We now extend the above method of calculation to the case of $T=1$, $J=0^+$ proton particle-neutron hole configurations ($pn^-$) describing the isovector monopole excitations in the analog nucleus ($\Lambda$). The calculations are performed in the TDA. The important consequence of the calculation is the emergence of the isobaric analog resonance (IAR) and the IMR. The coupling between the analog state $^*(J=3,4)$ and the IMR is inherent in this approach.

The ($pn^-$) $J=0^+$, $T=1$ spectrum is calculated relatively to the HF ground state of the parent. The particle space includes all unoccupied proton HF orbitals while the hole states are the occupied HF neutron states.

We "probed" the resulting p-h Green's function with the two operators:

$$W_\alpha = \sum_\alpha W_\alpha (i) = \sum_\alpha P_\alpha^+ n_\alpha$$

and

$$M_\alpha = \sum_\alpha W_\alpha (i) r_\alpha$$

where $n_\alpha$ denotes an annihilation operator of a neutron in a neutron orbit-$\alpha$ and $P_\alpha^+$ a creation of a proton in the corresponding proton orbit. The $W_\alpha$ is the "analog spin" lowering operator and $M_\alpha$ is the analogous "analog-vector" monopole operator. The distributions of the $W_\alpha$ and $M_\alpha$ strength for $^{208}Pb$, $^{96}Zr$ and $^{48}Ca$ are shown in Fig.3, 4 and 5 respectively. In each analog nucleus we find at lower energies the $J=0^+$ IAR.

(In $^{48}Ca$ the analog is bound). The positions of the IAR's are in satisfactory agreement with the measured Coulomb displacement energies. We point out that in the present calculation the second order Coulomb shift ($\Lambda=3,4,15$) and core polarization corrections ($\Lambda=3,4$) are included. The calculated widths of the IAR's correspond to

$$W_{\text{sum}} \leq \frac{\text{sum of proton escape widths (1$\frac{1}{2}$)}}{\text{all channels (3,4,15)}} \text{ in the case of}$$

We define the analog state as:

$$| \Lambda \geq T_\alpha \rangle \equiv \frac{| \pi \rangle}{\sqrt{\sum}}$$

where $| \pi \rangle$ is the parent state and $T_\alpha$ is the isospin lowering operator.
where there is an experimental number for $\Gamma_A$, the agreement is good.

The broader peak at higher energies ("probed" with $M_-$) is the IMR in the analog nucleus. (The $M_-$ strength of the IAR is not shown since it is identical with the $W_-$ distribution). The IMR is about 25-30 MeV above the IAR. The calculated width corresponds to its escape widths.

To summarize section 5 of this work we remark that:

(a) We performed a fully microscopic calculation of the isovector monopole excitations in the analog nucleus. As a result a narrow peak emerged which is the IAR. The resonance at higher energies is the IMR. (b) The energy and the escape width of the IAR are in agreement with experiment. (c) The coupling between the analog state and the IMR is inherent in the results obtained. As a consequence higher order Coulomb and core polarization terms are included in this approach.

References


