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ON GEOMETRIZED GRAVITATION THEORIES

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Abstract

Logunov A.A., Folomeshkin V.N.
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General properties of the geometrized gravitation theories have been considered. Geometrization of the theory is realized only to the extent that by necessity follows from experiment (geometrization only of the density of the matter Lagrangian). For a general case the gravitation field equations and the equations of motion for matter are formulated in different Riemann spaces. A covariant formulation of the energy-momentum conservation laws is given in an arbitrary geometrized theory. The noncovariant notion of "pseudotensor" is not required in formulating the conservation laws. In the completely geometrized theory (e.g., the Einstein theory) free gravitational waves do not carry any energy. If by analogy with other physical fields one demands that gravitational waves should carry energy-momentum, then the gravitational field Lagrangian should be nongeometrized. It is shown that in the general case(i.e.,when there is an explicit dependence of the matter Lagrangian density on the covariant derivatives)a symmetric energy-momentum tensor of the matter is explicitly dependent on the curvature tensor. There are enlisted different geometrized theories that describe a known set of the experimental facts. The properties of one of the versions of the quasilinear geometrized theory that describes the experimental facts are considered. In such a theory the fundamental static spherically symmetrical solution has a singularity only in the coordinate origin. The theory allows one to construct a satisfactory model of homogeneous nonstationary Universe.

Аннотация

Логунов А.А., Фоломешкин В.Н.
О геометризованных теориях гравитации. Серпухов, 1977.
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Рассмотрены общие свойства геометризованных теорий гравитации. Геометризация теории проводится только в том объеме, который с необходимостью следует из эксперимента (геометризация только плотности лагранжиана вещества). В общем случае уравнения гравитационного поля и уравнения движения вещества формулируются в различных римановых пространствах. Дана ковариантная формулировка законов сохранения энергии-импульса в произвольной геометризованной теории. Формулировка законов сохранения не требует введения нековариантного понятия "псевдотензора". В полностью геометризованной теории (например, в теории Эйнштейна) свободные гравитационные волны не переносят энергии. Если по аналогии с другими физическими полями потребовать, чтобы гравитационные волны переносили энергию-импульс, то лагранжиан гравитационного поля должен быть негеометризован. Показано, что в общем случае (при наличии явной зависимости плотности лагранжиана вещества от ковариантных производных) симметричный тензор энергии-импульса вещества явно зависит от тензора кривизны. Приведены примеры различных геометризованных теорий, описывающих известную совокупность экспериментальных фактов. Рассмотрены свойства одного из вариантов геометризованной квазилинейной теории, описывающей экспериментальные факты. В такой теории фундаментальное статическое сферически симметричное решение имеет особенность только в начале координат. Теория позволяет построить удовлетворительную модель однородной нестационарной Вселенной.

INTRODUCTION

Einstein's general relativity is one of the fundamental theories in modern physics. GR has had a great influence on the character of physical thinking and has cleared a new way for the development of the space and time theory. It has incarnated the ideas of Lobachevsky /1/ and Riemann /2/ on possible connection between the geometric properties of the space-time and physical interactions. In the framework of the Einstein's theory it turned out to be possible to construct nonstationary models of Universe, that describe satisfactorily the effects of cosmological red shift and are free from divergences of the Newton infinite Universe.

However in spite of the achievements of the theory and attractiveness of its main equations

$$R_{ik} - \frac{1}{2} g_{ik} R = 8\pi T_{ik} \qquad (1)$$

experimental evidences for GR remain very scanty. The experimentally confirmed predictions of the theory (drift of Mercury perihelion, light deflection, delay of radiosignals within the solar field) are essentially described by a linear approximation of the fundamental solution

$$ds^2 = [(1 - \frac{m}{2r})/(1 + \frac{m}{2r})]^2 dt^2 - (1 + \frac{m}{2r})^4 (dx^2 + dy^2 + dz^2). \qquad (2)$$

A nonlinear term (quadratic term in the expansion $g_{oo} \simeq 1 - \frac{2m}{r} + 2\frac{m^2}{r^2}$, defines only a small part of Mercury perihelion drift(if we neglect this term we will obtain 4/3 of the drift value, given by the Einstein's theory). The following nonlinear terms of the expansion g_{ik} are practically unessential for the description of the body motion within the Solar system.

In connection with this there arises a question whether the gravitation theory formulated by Einstein is the only one. A consideration of possible gravitation theories compatible with experimental facts is useful as it helps in understanding what protерties of the GR are common for all the possible gravitation theories and which of them are dependent on specific details of the theory (e.g., a choice of Lagrangian).

We shall deal with the formulation of the theory of symmetric tensor field of the second rank - a gravitation field ϕ_{ik} in the Euclidean space-time (Sec. 2). The Euclidean space-time gives a possibility to formulate quite satisfactorily the conservation laws, in a similar way which is true for all the other physical fields.

A universal character of the gravitational field interaction with the matter (equility of the inertial and gravitational masses) leads to the fact that the motion of the matter interacting with the external gravitational field is locally undistinguishable from the free motion of the matter in the corresponding accelerated coordinate system. As a consequence of this universality there arises a necessity to geometrize the Lagrangian density of matter.

Though we will consider the formulation of the gravitational field equations in the Euclidean space-time with a metric tensor γ_{ik}, the theory will be geometrized in the sense that the matter motion is completely described in terms of the Riemann space-time with a metric-tensor $g_{ik} = g_{ik}(\gamma_{ik}, \phi_{ik})$. Therefore the class of theories treated here differs from the standard constructions of the tensor field theories in flat space-time. Our approach differs from the Einstein's theory in the point that the theory is geometrized only to the extent imposed by experiments, i.e., no additional geometrization is assumed for the gravitational part of the Lagrangian.

Treatment of the energy-momentum problem in the arbitrary geometrized theory (Sec. 2) makes it clear that the known equality to zero of the covariant derivative of the matter energy-momentum density $\nabla_k T_i^k = 0$ is just another form of writing down of the usual conservation law(matter plus gravitational field) in the Euclidean space-time. Therefore the statement that the equation $\nabla_k T_i^k = 0$ does not express the conservation law, is wrong. Noncovariant "pseudotensors" are not necessary in covariant formulation of the energy-momentum conservation laws. From the general covariant formalism it follows that in a completely geometrized theory the gravitational waves do not carry energy-momentum. If in analogy with other physical fields one demands that gravitational waves had to carry energy-momentum, then the gravitational part of the Lagrangian density should be ungeometrized.

Section 3 is devoted to discussions of the general structure of the matter energy-momentum tensor in the framework of the geometrized theory. It is shown, that in the case the Lagrangian density of matter is explicitly dependent on the covariant derivatives of the corresponding fields, the energy-momentum tensor of matter will explicitly be dependent on the curvature tensor. Consequently, even in the locally inertial coordinate system the density of the matter energy-momentum tensor does not have a form, characteristic for special relativity in the flat space-time. Therefore in the general case in non-Euclidean space the special relativity laws do not take place.

We will present some examples of various geometrized theories that explain the known experimental facts. However these theories differ from the Einstein gravitation theory in the case of strong gravitational fields (point particle field, collapse problem, problem of singularity, gravitational waves, Universe models). All the considered teories are logically uncontradictory, generally-covariant and locally Lorentz-invariant (under condition that the term with the curvature tensor are neglected in the energy-momentum tensor of matter). Hence in such theories no local effects connected, e.g., with the relative motion of the Sun in the galaxy may arise (see refs./3,4/).

One of the simplest geometrized theories of gravity, compatible with the known experimental facts, is a geometrized quasilinear theory of gravity with linear equations for the free gravitational field (Sec. 5). A fundamental spherically symmetric static solution in this theory has a singularity only at r=0 and no problem with the Schwarzschild radius arises. The star collapses to a singular point r=0 in infinite time of the external observer as well as in infinite proper time of a freely falling observer, if the pressure is neglected.

Possibilities to construct nonstationary homogeneous Universe models in the framework of the quasilinear theory are considered in Sec. 6. The theory describes satisfactorilly the effect of cosmclogical red shift. Our versions of the model are "flat", nevertheless they are cyclic in contrary with the flat and open models of the Einstein theory. No matter whether the "expansion" is monotoneous or not, the models are characterised by the finite proper time of the evolution cycle. In non-monotoneous models of the quasilinear theory first there comes "shrinkage" and then "expansion" of the Universe. In such models in observation of very remote objects the red shift should be replaced with violet one.

2. GENERAL PROPERTIES OF GRAVITATIONAL FIELD. CONSERVATION LAWS. GEOMETRIZATION

In analogy with other physical fields we will consider the gravitational field (the field of a symmetric second rank tensor ϕ_{ik}) in the framework of a standard approach of the classical Lorentz-invariant field theory. We also assume and it is quite natural that all the equations of the theory may be written covariantly in arbitrary curvilinear (generalized) coordinates. (The curvilinear coordinates should be, e.g., used when one derives an expression for a symmetric energy-momentum tensor).

Let us consider some closed system of fields (gravitational field ϕ_{ik} and the remaining fields ϕ_A) with the Lagrangian density

$$L = L(\gamma_{ik}, \phi_{ik}, \phi_A), \tag{3}$$

in an arbitrary curvilinear coordinate system in the Euclidean space-time with a metric tensor γ_{ik}. In Eq. (3) $L(\phi)$ stands for the dependence on ϕ, on $\partial_i \phi$, and $\partial_i \partial_k \phi$. From the total Lagrangian density we will select the gravitational part (the part of the total Lagrangian that contains only the gravitational field variables[6]). We will call the remaining part the matter Lagrangian density,

$$L = L_g(\gamma_{ik}, \phi_{ik}) + L_M(\gamma_{ik}, \phi_{ik}, \phi_A). \qquad (4)$$

The equations for the gravitational field and for the matter motion are

$$\delta(L_g + L_M)/\delta\phi_{ik} = 0, \qquad (5)$$

$$\delta L_M / \delta\phi_A = 0, \qquad (6)$$

where

$$\delta L/\delta\phi = \partial L/\partial\phi - \partial_i(\partial L/\partial(\partial_i \phi)) + \partial_i \partial_k (\partial L/\partial(\partial_i \partial_k \phi)).$$

From general covariancy of the theory it follows that for arbitrary infinitesimal transformation of the coordinates

$$x^i \rightarrow x^i + \xi^i(x) \qquad (7)$$

the variations of the scalar functionals, e.g., the action integrals

$$J_g = \int L_g d^4x, \qquad J_M = \int L_M d^4x \qquad (8)$$

are equal to zero. The variation δJ_M has the form

$$\delta J_M = \int\left(\frac{\delta L_M}{\delta\gamma_{ik}} \delta\gamma_{ik} + \frac{\delta L_M}{\delta\phi_{ik}} \delta\phi_{ik} + \frac{\delta L_M}{\delta\phi_A}\delta\phi_A + \mathrm{Div}\right)d^4x = 0, \qquad (9)$$

where Div designates divergential terms. The variations $\delta\gamma_{ik}$, $\delta\phi_{ik}$, and $\delta\phi_A$ are presented in the form

$$\delta\gamma_{ik} = -\gamma_{ni}\,D_k\xi^n - \gamma_{nk}D_i\xi^n,$$

$$\delta\phi_{ik} = -\phi_{ni}D_k\xi^n - \phi_{nk}D_i\xi^n - \xi^n D_n\phi_{ik}, \qquad (10)$$

$$\delta\phi_A = -F^{B,i}_{A,k}\,\phi_B D_i\xi^k - \xi^n D_n\phi_A,$$

where D_k stands for a covariant derivative in the space-time with a metric tensor γ_{ik}. The action of the operator D_k e.g., on the density of a symmetric tensor A^{ik} is defined by

$$D_k(\gamma_{in}A^{kn}) = \partial_k(\gamma_{in}A^{kn}) - \frac{1}{2}A^{kn}\partial_i\gamma_{kn}.$$

Taking into account Eq. (10) we may present relation (9) in the following form

$$\delta J_M = \int\{\xi^i[-D_kT^k_i + 2D_k(\phi_{in}\frac{\delta L_M}{\delta\phi_{kn}}) - \frac{\delta L_M}{\delta\phi_{kn}}D_i\phi_{kn} -$$

$$- D_k(\frac{\delta L_M}{\delta\phi_A}F^{B,k}_{A,i}\phi_B) - \frac{\delta L_M}{\delta\phi_A}D_i\phi_A] + \mathrm{Div}\}\,d^4x = 0. \qquad (11)$$

The magnitude T^{ik} is the density of a symmetric energy-momentum tensor of matter in the space-time with a metric tensor γ_{ik}

$$T^{ik} = -2\delta L_M/\delta\gamma_{ik}.$$

From arbitrariness of the drift vector in (11) and under condition that equation of motion (6) is fulfilled there follows an identity

$$D_k T_i^k = 2D_k(\phi_{in}\,\delta L_M/\delta\phi_{kn}) - \frac{\delta L_M}{\delta\phi_{kn}} D_i\,\phi_{kn}. \qquad (12)$$

In a similar way one can obtain a corresponding identity for a gravitational part of the Lagrangian density,

$$D_k t_i^k = 2D_k(\phi_{in}\,\delta L_g/\delta\phi_{kn}) - \frac{\delta L_g}{\delta\phi_{kn}} D_i\,\phi_{kn}, \qquad (13)$$

where t^{ik} is the density of the symmetric energy-momentum tensor of the gravitational field:

$$t^{ik} = -2\delta L_g/\delta\gamma_{ik}, \qquad t_i^k = \gamma_{in}\,t^{kn}.$$

If field equations (5) are valid, from identities (12) and (13) there follows a covariant conservation equation for the density of the total symmetric energy-momentum tensor in the space-time with a metric tensor γ_{ik}

$$D_k(T^{ik} + t^{ik}) = 0. \qquad (14)$$

A presence of a covariant derivative in equality (14) does not prevent one from formulating integral energy-momentum conservation laws.

Let us introduce orthonormal basis $\lambda^i_{(n)}$ with the properties:

$$\gamma^{ik} = \lambda^i_{(n)}\lambda^k_{(n)}, \qquad \lambda_{(n)i} = \gamma_{ik}\lambda^k_{(n)},$$

$$\gamma_{ik} = \lambda_{(n)i}\,\lambda_{(n)k}, \qquad \delta_{kn} = \lambda^i_{(n)}\,\lambda_{(k)i}.$$

A covariant integral of the tensor density $(T_i^k + t_i^k)$

$$P_i(y) = \lambda_{(n)i}(y)\int \lambda^k_{(n)}(x)[T_k^m(x) + t_k^m(x)]dS_m(x) \tag{15}$$

is a total energy-momentum vector of the system. It satisfies the covariant equation of a paralell displacement (the equation of geodesic) $D_k P_i = 0$. In the Cartesian coordinate system expression (15) turns into a usual "arythmetical" integral

$$P_i = \int (T_i^k + t_i^k)dS_k = \text{const} .$$

The density of the total angular energy-momentum tensor

$$M^{kn\ell} = (T^{kn} + t^{kn})x^\ell - (T^{k\ell} + t^{k\ell})x^n$$

also satisfies the covariant conservation equation $D_k M^{kn\ell} = 0$ and the total angular momentum of the system may be presented in the form of the covariant integral of the angular energy-momentum tensor density[6].

As is seen, the gravitational field treated in the terms of the Euclidean space-time has a behaviour similar to all the other physical fields.

We call the theory geometrized, if the matter Lagrangian depends on the gravitational field only via a metric tensor g_{ik}, i. e., if

$$L = L_g(\gamma_{ik}, \phi_{ik}) + L_M(g_{ik}, \phi_A), \tag{16}$$

$$g_{ik} = g_{ik}(\gamma_{ik}, \phi_{ik}). \tag{17}$$

In the theory with an arbitrary coupling equation (17) one can always choose the quantities $\psi_{ik} = g_{ik} - \gamma_{ik}$ as new variables of the gravitational field. Then we will obtain other field equations for the new variables ψ_{ik}, but with simple coupling equations. In other words the choice of a definite coupling equation at a fixed Lagrangian density L_g is equivalent to a choice of a definite Lagrangian density at a fixed coupling equation. Therefore in a general case one can choose as gravitational field variables a difference

$$\phi_{ik} = g_{ik} - \gamma_{ik} \tag{18}$$

without any restrictions of the generality.

From the experiments of Newton/7/, Bessel/8/, Eötvös/9,10/, Dicke/11/, Braginsky/12/ there follows a weak principle of equivalence (equality of inertial and gravitational masses) up to the account of the proper energy of the gravitational interaction, whose contribution is small to influence the experimental results. It indicates that with a quoted accuracy the density of the matter Lagrangian should be geometrized.

However the known experimental facts do not demand the Lagrangian density of gravitational field should also be geometrized. Moreover if one demands that gravitational waves should carry energy-momentum in analogy with other physical fields, then the Lagrangian density of gravitational field should be nongeometrized (see below).

In the theories with nongeometrical Lagrangian density of the gravitational field the space-time with the metric tensor γ_{ik} is just the real space-time as the space-time with the metric tensor g_{ik}. In the geometrized theories with the Lagrangian density(16) the motion of matter is completely described in the terms of space-time with the metric tensor g_{ik}. Still the equations for the gravitational field itself are formulated in the terms of another space-time with a metric tensor γ_{ik}. Such a possibility was pointed out by Lobachevsky: "...there cannot be any contradiction, if we assume that some forces in Nature follow one geometry, and other their own peculiar one"[1].

A choice of the geometry to be used for the description of a physical system is a matter of agreement to a great extent. Formally a mathematical foundation for this arbitrariness lies in the known possibility to perform an Euclidean modeling of non-Euclidean geometry[13]. Within the same experimental facts a choice of geometry to describe the proper gravitational field is equivalent to the choice of definite equations to describe this field[14-18]. Therefore there exist several equivalent possibilities to formulate the theory and they will be dealt with further on in Section 5. In particular, it is always possible to formulate field equations (5) in the terms of Euclidean space-time, though for some problems, e.g., when describing the Universe model, dealing with the corresponding non-Euclidean geometry may result in simpler field equations.

In the geometrized theory the energy-momentum conservation laws have a standard form, i.e., all relations (9)-(15) are valid. Besides in geometrized theory conservation equation (14) may be presented in the form of a covariant derivative from the energy-momentum tensor of the matter only in space-time with a metric tensor g_{ik}. Field equations (5) and equalities (12),(13) are taken into account, equation (14) may be presented in the form

$$D_k[\gamma_{in}\frac{\delta(L_g+L_M)}{\delta\gamma_{kn}}]=D_k(\gamma_{in}\frac{\delta L_M}{\delta\gamma_{kn}}+\phi_{in}\frac{\delta L_M}{\delta\phi_{kn}})-\frac{1}{2}\frac{\delta L_M}{\delta\phi_{kn}}D_i\phi_{kn}=0. \quad (19)$$

With account of the relation*)

$$D_k(\gamma_{in}\frac{\delta L_M}{\delta\gamma_{kn}}+\phi_{in}\frac{\delta L_M}{\delta\phi_{kn}})-\frac{1}{2}\frac{\delta L_M}{\delta\phi_{kn}}D_i\phi_{kn}=\nabla_k(g_{in}\frac{\delta L_M}{\delta g_{kn}}) \quad (20)$$

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*) Relation (20) can easily be checked in the case of a simple linear coupling (18), still it is true for an arbitrary coupling equation (17). One can easily get convinced, having repeated the procedure of deriving identity of type (12) for geometrized Lagrangian density $L_M(g_{ik},\phi_A)$ or having used the identity

$$\gamma_{in}\partial g_{m\ell}/\partial\gamma_{kn}+\phi_{in}\partial g_{m\ell}/\partial\phi_{kn}=\frac{1}{2}(\delta^k_m g_{i\ell}+\delta^k_\ell g_{im}),$$

that follows from equalities

$$\delta g_{ik}=-\gamma_{ni}D_k\xi^n-\gamma_{nk}D_i\xi^n-\xi^n D_n g_{ik}=\frac{\partial g_{ik}}{\partial\gamma_{\ell m}}\delta\gamma_{\ell m}+\frac{\partial g_{ik}}{\partial\phi_{\ell m}}\delta\phi_{\ell m}$$

and equalities (10).

equality (19) takes the form:

$$D_k[\gamma_{in}\delta(L_g+L_M)/\delta\gamma_{kn}] = \nabla_k(g_{in}\delta L_M/\delta g_{kn}) = 0. \qquad (21)$$

Operator ∇_k is an operator of the covariant derivative in space-time with a metric tensor g_{ik} . The action of the operator ∇_k on the density of a symmetric tensor A^{ik} is defined by the expression:

$$\nabla_k(g_{in}A^{kn}) = \partial_k(g_{in}A^{kn}) - \frac{1}{2}A^{kn}\partial_i g_{kn} = D_k(g_{in}A^{kn}) - \frac{1}{2}A^{kn}D_i g_{kn}$$

Thus field equations being fulfilled,the conservation law for total energy-momentum tensor (14) and the conservation law in the form

$$\nabla_k(\delta L_M/\delta g_{ik}) = 0 \qquad (22)$$

are simply different forms of writing down the same conservation law. Therefore the statement that equation (22) does not express the conservation law is incorrect.

Different forms of the energy-momentum conservation law in terms of different spaces results in differencies with interpretations.In terms of the space-time with a metric tensor γ_{ik} a total energy-momentum tensor is conserved (covariantly), and the gravitational field ϕ_{ik} as well as other physical fields makes a contribution to the total energy-momentum tensor. At the same time in terms of the space-time with a metric tensor g_{ik} matter moves freely (along the geodesic), and the role of gravitational field manifests itself only in the changes of the space-time curvature.

As it follows from the above considerations, there is no need to introduce noncovariant "pseudotensors"*) when formulating the energy-momentum conservation laws.

It is often said that in the gravitation theory, equations of motion (22) follow from gravitational field equations, and it is specific for a nonlinear theory only. In reality according to identities (20) and (22) in the most general case in an arbitrarily geometrized theory with arbitrary gravitational field equations, both linear and nonlinear, covariant conservation equa-

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*) One of the main principles of the physical theory, i.e., the principle of general covariance turns out to be violated in the standard formulation of the energy-momentum problem in Einstein's theory, where noncovariant "pseudotensors" τ_i^k, that satisfy noncovariant equation $\partial_k \tau_i^k = 0$ [19-25] in the arbitrary curvilinear coordinate system, are introduced. It is obvious that τ_i^k is a generalization for the case of continuous medium of the known notion of classical mechanics-generalized momentum, that does not have any relation to the usual energy-momentum vector in a general case. The physical sense of τ_i^k depends on the choice of a specific system of curvilinear coordinates. Therefore appearence of paradox due to Lorentz[21]-Bauer[26]-Schredinger[27] is not surprising at all in this case. As a consequence when calculating the energy of gravitational waves some authors obtain negative values, other derive positive or zero values. It may be shown that the four components of τ_o^k may be made equal to zero globally, simultaneously at any point of a definite coordinate system[28-32], chosen in a particular way.

tions (equations of motion) (22) are valid no matter whether the field equations are fulfilled or not. Equations (22) simply stand for the fact of free motion of matter in the space-time with a metric tensor g_{ik} . The gravitational field equations determine the properties only of the space-time in terms of which free motion of matter is described.

In geometrized theories with coupling equation (18) relations

$$\delta L_M/\delta\phi_{ik} = \delta L_M/\delta\gamma_{ik} = \delta L_M/\delta g_{ik}$$

take place and the field equations take the form

$$\delta L_g/\delta\phi_{ik} = -\delta L_M/\delta g_{ik}. \qquad (23)$$

With account of (22) we obtain that in such theories the LH part of field equations (23) should satisfy the equation

$$\nabla_k(\delta L_g/\delta\phi_{ik}) = 0. \qquad (24)$$

Equation (24) is simply another form to present the energy-momentum conservation law (14), thus it does not impose any additional limitations on the field variables ϕ_{ik}.

In the theories with arbitrary coupling equation (17) the gravitational field equations take the form:

$$\delta \mathsf{L}_g / \delta \phi_{ik} = -\frac{\delta \mathsf{L}_M}{\delta g_{\ell m}} \partial g_{\ell m} / \partial \phi_{ik}. \tag{25}$$

As it was mentioned, the choice of the Lagrangian density at a fixed coupling equation is equivalent to the choice of coupling equation at a fixed Lagrangian density. Therefore it is always possible to come to formulation of the theory where the field source in the RHS of equation (25) will be the energy-momentum tensor of matter (it is equivalent to the transition to the theory with simple coupling equation (18) but with another Lagrangian density L_g). However a choice of various theories by choosing different coupling equations at some fixed Lagrangian density (e.g. the one, that leads to linear equations for free field) seems to be practically more convenient than specification of the theory by choosing different nonlinear Lagrangians.

All the known experimental facts are described by weak field approximation for a fundamental solution of the equations. The choice of different coupling equations at a fixed Lagrangian density L_g influences only the structure of an effective source for the gravitational field in (25), and it is of no matter for the form of free field equations. Therefore if some theory with a Lagrangian density L_g and coupling that permits e.g. the representation

$$g_{ik} \approx \gamma_{ik} + \phi_{ik} + \frac{1}{2}\gamma^{m\ell}\phi_{im}\phi_{\ell k}, \tag{26}$$

within the weak field approximation, whould describe the known experimental facts, then a theory with the same Lagrangian density but other coupling equation, that permits representation (26) within the weak field approximation, whould also describe the known experimental facts. However these theories will differ when describing strong gravitational fields and phenomena, for which the structure of the field source is essential.

A subclass of geometrized theories with Lagrangian density (16) is a set of theories with <u>complete geometrization</u> when the Lagrangian density of gravitational field depends only on some combination of the tensors γ_{ik} and ϕ_{ik} :

$$L_g = L_g(f_{ik}), \qquad f_{ik} = f_{ik}(\gamma_{ik}, \phi_{ik}). \tag{27}$$

The possibility to present L_g in form (27) is obviously connected with the choice of a particular form of the Lagrangian density L_g. The field equations in the completely geometrized theories with a corresponding choice of the Lagrangian density L_g differ from the Einstein equations only in the fact, that the effective source for the field f_{ik} is not an energy-momentum tensor. Hence the theory with complete geometrization differs from the

Einstein's theory for the problems where the structure of the gravitational field source is essential. Indeed, as was noted above, it is always possible to formulate the theory where the energy-momentum tensor of matter will be a source of field. But then the LHS of the field equation will differ from the LHS of the Einstein equation.

In the theory with complete geometrization the components of the tensor f_{ik} may be chosen as the field variables. Then the free gravitational field equations will have the form

$$\delta L_g / \delta f_{ik} = 0 \qquad (28)$$

and in fact they state the equality of the symmetric energy-momentum tensor of gravitational fields to zero in the space-time with a metric tensor f_{ik} . The equations for a free tensor field ϕ_{ik}

$$\frac{\delta L_g}{\delta f_{ik}} \partial f_{ik} / \partial \phi_{\ell m} = 0$$

are a consequence of field equations(28). As a consequence of field equations (28), in the theory with complete geometrization the density of the energy-momentum tensor of free gravitational field in the space-time with a metric tensor γ_{ik}

$$t^{ik} = -2 \delta L_g / \delta \gamma_{ik} = -2 \frac{\delta L_g}{\delta f_{\ell m}} \partial f_{\ell m} / \partial \gamma_{ik} \qquad (29)$$

turns out to be identically equal to zero as well. This is a regorous result, which is certainly true in any approximate consideration of the energy-momentum problem in the theory with complete geometrization, e.g., when considering the gravitational waves in weak field approximation in the Einstein's theory.

In the theory with complete geometrization free gravitational waves possess no energy. If one demands in analogy with other physical fields, that free gravitational waves should carry energy-momentum, then the Lagrangian density of free gravitational field should be non-geometrized.

In the Einstein's theory in addition to assumption (16) on geometrization of the Lagrangian density of matter, derived from the experimental facts, two other assumptions are made: 1) hypothesis (27) on complete geometrization is adopted; 2) it is assumed that the metric tensors f_{ik} and g_{ik} coincide, i.e. the simplest possible coupling is chosen

$$f_{ik} = g_{ik} . \qquad (30)$$

Let us call the theories with properties (27), (30) the theories with the Einstein geometrization (the Einstein's theory corresponds to a particular choice of the Lagrangian density $L_g = \sqrt{-g}R$). Whether the hypothesis on the Einstein geometrization is realized in reality is the question of experiments, e.g., the ones on studying the motion of gravitational waves.

The theories with the Einstein geometrization are particular cases of the theories with complete geometrization. Therefore everything said above about the energy of free gravitational waves in the theories with complete geometrization is true for them. In the Einstein's theory energy-momentum tensor of free gravitational waves is equal to zero, and the free gravitational waves carry no energy. This regorous result is valid in the weak field approximation as well, in which the gravitational waves are usually analysed in the Einstein's theory.

There is a stronger result in Einstein's gravitation theory. Not only the density of energy momentum tensor of gravitational waves, but the energy-momentum tensor density of the matter plus gravitational field system

$$T^{ik} + t^{ik} = -2\frac{\delta(L_g + L_M)}{\delta g_{\ell m}} \partial g_{\ell m}/\partial \gamma_{ik}$$

are equal to zero in accordance with the field equation

$$\delta(L_g + L_M)/\delta g_{\ell m} = 0.$$

Thus if one demands that the total energy of the system will be positive, then the gravitation theory should belong to the class of theories where the Einstein geometrization is impossible.

A dual character of the metric tensor in the theories with Einstein geometrization, where the metric tensor is at the same time a dynamic field variable, leads to the fact that in the theories of this class field equations (variations of the Lagrangian dens·ty in the field variables) state the equality to zero of the total symmetric energy-momentum tensor (variations of the Lagrangian density in the metric tensor components). This viewpoint was presented in works[20,33-37] quite independently and with some other arguments. The total canonical energy-momentum tensor in the space-time with the metric tensor g_{ik} is also equal to zero in the Einstein's theory[36].

This situation is similar to that in electrodymanics, where the "current" of free electromagnetic field $\delta L_A/\delta A_\mu$ is equal to zero according to the field equations. In electrodynamics this fact is simply an evidence of the electromagnetic field being neutral. In the gravitation theory mass plays the role of charge, and "gravitational neutrality" of gravitational field in the Einstein's theory ($\delta L_g/\delta g_{ik} = 0$) brings one to a conclusion that such fields possess no energy. The system of free electromagnetic wave has a zero Coulomb charge, similarly the system of free gravitational waves has a zero Schwarzschild mass[32].

Equality to zero of free gravitational wave energy and the total energy in the Einstein's theory does not require any changes of the notions in other field theories. In all other physical theories the matter energy is independently conserved. Therefore saying ergy one usually understands positively definite energy of matr. The gravitation theory is the only one, where matter energy not conserved, but only the total energy, which is equal to ro in Einstein's theory, is conserved.

3. ON STRUCTURE OF MATTER ENERGY-MOMENTUM TENSOR IN GEOMETRIZED THEORY

From the general invariance principles there follows a fact at in geometrized theory the Lagrangian density of matter is a nctional of the form:

$$L_M = L_M(g_{ik}, R_{ik\ell m}, \phi_A, \nabla_i \phi_A, \{\nabla_i \nabla_k\} \phi_A),$$

ere

$$\nabla_i \phi_A = \partial_i \phi_A + F^{B,n}_{A,m} \Gamma^m_{in} \phi_B,$$

$$\{\nabla_i \nabla_k\} \phi_A = \frac{1}{2}(\nabla_i \nabla_k + \nabla_k \nabla_i) \phi_A,$$

$$\Gamma^m_{in} = \frac{1}{2} g^{mk}(\partial_i g_{kn} + \partial_n g_{ik} - \partial_k g_{in}).$$

explicit dependence of L_M on $R_{ik\ell m}$ does not violate the general principles of the theory and does not contradict the experintal facts. In some cases (e.g., scalar field) such a dependence ises by necessity in the Einstein's theory.

Let the curvature tensor be not present in its explicit form in the expression for L_M . A substitution $\gamma_{ik} \to g_{ik}$, $\partial_k \to \nabla_k$ is a standard generalization of the Lagrangian density of special relativity for the case of non-Euclidean space-time. In particular cases a cancellation of different terms is possible, so that L_M may be independent of $\partial_i g_{k\ell}$ *). But in the general case there are no reasons to expect such a cancellation. If L_M is explicitly dependent on the derivatives $\partial_i g_{k\ell}$ (e.g, via the covariant derivative $\nabla_i \phi_A$), then in the density of the symmetric energy-momentum tensor of matter there arise the terms with R_{iklm}.

Let us consider the structure of the energy-momentum tensor of matter in the simplest case, where $L_M = L_M(g_{ik}, \phi_A, \nabla_i \phi_A)$. In the arbitrary infinitesimal transformation of coordinates (7), the variations $\delta(\nabla_i \phi_A)$ and δJ_M have the form:

$$\delta(\nabla_i \phi_A) = \nabla_i \delta \phi_A + F^{B,n}_{A,m} \phi_B \delta \Gamma^m_{in},$$

$$\delta J_M = \int \Big(\frac{\partial L_M}{\partial g_{ik}} \delta g_{ik} + H^{ik}_m \delta \Gamma^m_{ik} + \frac{\delta L_M}{\delta \phi_A} \delta \phi_A + \mathrm{Div}\Big) d^4x = 0,$$

---

*) For example in electrodynamics one can take $L_M = L_M(F_{ik})$, where $F_{ik} = \nabla_i A_k - \nabla_k A_i = \partial_i A_k - \partial_i A_k$. But in this case the field equation for the vector potential A_i is explicitly dependent on the curvature tensor, $\nabla_m \nabla^m A_i - R_{ik} A^k = -J_i$, and consequently as was noted long ago by Eddington /38/, the laws of special relativity do not hold in the non-Euclidean space-time (even in the locally inertial coordinate system).

$$H_m^{ik} = \frac{1}{2}\left[\frac{\partial L_M}{\partial(\nabla_i \phi_A)} F_{A,m}^{B,k} + \frac{\partial L_M}{\partial(\nabla_k \phi_A)} F_{A,m}^{B,i}\right]\phi_B,$$

$$\frac{\delta L_M}{\delta \phi_A} = \frac{\partial L_M}{\partial \phi_A} - \nabla_i \frac{\partial L_M}{\partial(\nabla_i \phi_A)}.$$

With account of the equality

$$\delta \Gamma_{ik}^m = \frac{1}{2} g^{m\ell} (\nabla_i \delta g_{k\ell} + \nabla_k \delta g_{i\ell} - \nabla_\ell \delta g_{ik})$$

we find that

$$\delta J_M = \int (-\frac{1}{2} \tilde{T}^{ik} \delta g_{ik} + \frac{\delta L_M}{\delta \phi_A} \delta \phi_A + \mathrm{Div}) d^4 x = 0,$$

where the density of matter symmetric energy-momentum tensor is

$$\tilde{T}^{ik} = -2\delta L_M / \delta g_{ik} = -2\partial L_M / \partial g_{ik} + \nabla_\ell (g^{im} H_m^{\ell k} + g^{km} H_m^{\ell i} - g^{\ell m} H_m^{ik}).$$

Under condition that the motion equation $\delta L_M / \delta \phi_A = 0$ is fulfilled the quantity $\tilde{T}^{ik}$ may be presented in the form

$$\tilde{T}^{ik} = -2\partial L_M / \partial g_{ik} + \frac{1}{2}(F_{A,m}^{B,k} g^{im} +$$

$$+ F_{A,m}^{B,i} g^{km})\left[\frac{\partial L_M}{\partial \phi_A} \phi_B + \frac{\partial L_M}{\partial(\nabla_\ell \phi_A)} \nabla_\ell \phi_B\right] - \nabla_\ell (U^{\ell ik} + U^{\ell ki}); \qquad (31)$$

$$U^{\ell ik} = \frac{1}{2}(F_{A,m}^{B,\ell} g^{im} - F_{A,m}^{B,i} g^{\ell m}) \phi_B \frac{\partial L_M}{\partial(\nabla_k \phi_A)}.$$

If the system is spacialy bounded then in the Cartesian coordinates of the Euclidean space-time the terms with the superpotentials $U^{\ell ik}$ do not make a contribution to the integral $\int \tilde{T}^{oo} dV$. Thus energy-momentum tensor is defined usually with an accuracy up to the contribution from such type of the addends. The gravitational theory is indeed the only one, where the energy-momentum tensor of matter is defined strictely unambigously. In the non-Euclidean space-time the terms with $U^{\ell ik}$ lead to an explicit appearence of a curvature tensor in the structure of the tensor density $\tilde{T}^{ik}$. For a characteristic case, when L_M is a quadratic function of the first derivatives

$$\partial L_M / \partial(\nabla_k \phi_A) = \Phi^{AC,kn} \nabla_n \phi_C .$$

For $\nabla_\ell U^{\ell ik}$ we obtain the expression

$$\nabla_\ell U^{\ell ik} = \frac{1}{2}(F^{B,\ell}_{A,m} g^{im} - F^{B,i}_{A,m} g^{\ell m})[(\nabla_n \phi_C) \nabla_\ell (\Phi^{AC,kn} \phi_B) +$$

$$+ \phi_B \Phi^{AC,kn} ([\nabla_\ell \nabla_n] \phi_C - \frac{1}{2} F^{D,p}_{C,q} R^q_{pn\ell} \phi_D)].$$

The terms with the curvature tensor arise from the commutator of covariant derivatives

$$[\nabla_\ell \nabla_n] \phi_C = \frac{1}{2}(\nabla_\ell \nabla_n - \nabla_n \nabla_\ell) \phi_C = -\frac{1}{2} F^{D,p}_{C,q} R^q_{pn\ell} \phi_D .$$

Hence if L_M depends explicitly on $\nabla_k \phi_A$ then in the matter energy-momentum tensor there appear the terms with the curvature tensor. For example, for a vector field with the Lagrangian density

$$L_M = \frac{1}{2}\sqrt{-g}\, g^{\ell m}(g^{ik}\nabla_i \phi_\ell \nabla_k \phi_m - \mu^2 \phi_\ell \phi_m)$$

a symmetric energy-momentum tensor has the form

$$\frac{1}{\sqrt{-g}} \tilde{T}^{ik} = -2\nabla_m \phi^i \nabla^m \phi^k + \mu^2 \phi^i \phi^k - \nabla^i \phi^m \nabla^k \phi_m +$$

$$+ \frac{1}{2} g^{ik} (\nabla_\ell \phi_m \nabla^\ell \phi^m - \mu^2 \phi_m \phi^m) - \frac{1}{2}[\nabla_\ell \phi^i \nabla^k \phi^\ell +$$

$$+ \nabla_\ell \phi^k \nabla^i \phi^\ell - \nabla_\ell \phi^\ell (\nabla^k \phi^i + \nabla^i \phi^k) + \phi^i \{\nabla^k \nabla^\ell\} \phi_\ell +$$

$$+ \phi^k \{\nabla^i \nabla^\ell\} \phi_\ell - \phi_\ell \{\nabla^\ell \nabla^k\} \phi^i - \phi_\ell \{\nabla^\ell \nabla^i\} \phi^k] +$$

$$+ \frac{1}{4}\phi_m (\phi^i R^{mk} + \phi^k R^{mi} + \phi_\ell R^{imk\ell} + \phi_\ell R^{kmi\ell}). \qquad (32)$$

An explicit presence of the terms proportional to the components of the curvature tensor, is an evidence of the fact that even in the locally inertial coordinate system the energy-momentum tensor density of matter does not have a form, characteristic for the special relativity in the Euclidean space-time. But even if the curvature tensor does not enter the matter energy-mometum tensor, it enters in its explicit form the motion equation (see nete on p 26). Therefore in the general case in the non-Euclidean space-time there is no coordinate system where physical processes whould go on even locally as in the case without gravitational field. In this the equivalence principle, i.e., the equality of the inertial and gravitational masses for a point prob body may be fulfilled.

4. VARIOUS FORMS OF REPRESENTATION OF GEOMETRIZED THEORIES

The class of geometrized theories in the form

$$L = L_g(\gamma_{ik}, \phi_{ik}) + L_M(g_{ik}, \phi_A) \tag{A}$$

may be presented in two equivalent forms. Expressing the tensor ϕ_{ik} via tensors γ_{ik} and g_{ik} with the help of coupling (17) one can present the formulation (A) in the following form

$$L = L_g(\gamma_{ik}, g_{ik}) + L_M(g_{ik}, \phi_A). \tag{B}$$

To formulate the gravitational field equations one can also choose the same space-time, in terms of which matter motion is described i.e., if we express the tensor γ_{ik} via tensors g_{ik} and ϕ_{ik}, we may present the formulation (A) in the form

$$L = L_g(g_{ik}, \phi_{ik}) + L_M(g_{ik}, \phi_A). \tag{C}$$

The formulations (A), (B) and (C) are equivalent in the sense that for each theory in the form (A) there exist treories (with a corresponding Lagrangian density and coupling) in the forms (B) and (C) that would lead to the same experimental consequences. However a practical construction of such equivalent theories in a closed form may be a complicated problem.

In a set of theories in the form (A) there are theories with linear equations for the free gravitational field. In the presence of a source these field equations become nonlinear, and the character of nonlinearity depends on a particular choice for the coupling. The geometrized theories with linear equations for free gravitational field will be called quasilinear.

Quasilinear equations in the form(A), that describe the known experimental facts are e.g., the theories with the Lagrangian density

$$L_g = \frac{1}{64\pi}\sqrt{-\gamma}\,\gamma^{ik}\gamma^{\ell m}\gamma^{pq}(D_i\,\phi_{q\ell}\,D_k\,\phi_{mp} - \frac{1}{2}D_i\,\phi_{qp}\,D_k\,\phi_{\ell m}) \qquad (33)$$

and with arbitrary coupling, that permit representation (26) within the weak field approximation.

One of the simplest theories with the nonlinear Lagrangian density of gravitational field, that describes the experimental facts, is the theory with the Lagrangian density

$$L_g = \frac{1}{64\pi}\sqrt{-\gamma}\,\gamma^{ik}\gamma^{\ell m}(\gamma^{pq} - 2\phi^{pq})(D_i\,\phi_{q\ell}\,D_k\,\phi_{mp} - \frac{1}{2}D_i\,\phi_{qp}\,D_k\,\phi_{\ell m})$$

and simple coupling (18).

The Rosen theory[39] with the Lagrangian density

$$L_g = \frac{1}{64\pi}\sqrt{-\gamma}\,\gamma^{ik} g^{\ell m} g^{pq}(D_i\,g_{q\ell}\,D_k\,g_{mp} - \frac{1}{2}D_i\,g_{qp}\,D_k\,g_{\ell m}).$$

is an example of geometrized theory in the form (B) that describes the experimental facts.

Another example of the theory in the form (B) is the Ni[40] theory that has the same post-Newtonian limit as the Einstein's theory, but differs from the latter one in describing the motion of gravitational waves[41].

From the point of view of linearity and simplicity one can mark out the quasilinear theory with Lagrangian density (33) and coupling (18). In such a simple theory the field equations have the form

$$D_m D^m (\phi_{;k} - \frac{1}{2}\gamma_{ik}\,\phi^n_n) = -16\pi\gamma_{im}\gamma_{kn}\tilde{T}^{mn}/\sqrt{-\gamma}\,.$$

The example of electrodynamics shows that the principles of simplicity and linearity are very fruitful, and their applicability goes beyond the initial theoretical ideas and experimental facts. It is obvious that many nonlinear theories of electromagnetism, whose equations coincide with the Maxwell equations in the weak field approximation, may be formulated. However the development of quantum electrodynamics does not reveal the necessity to construct such nonlinear theories. A standard classical linear Maxwell theory, added with the quantization and renormalization procedures, gives a complete description of all the known experimental facts. A simplest quasilinear gravitational theory with coupling (18) essentially describes the experimental facts. Therefore it seems reasonable to study the consequences of this version of the theory.

There are no quasilininear theories in the form (C). The theory in the form (C), equivalent to a quasilinear theory in the form (A), seems as a considerably nonlinear theory. It is natural.There cannot exist a quasilinear formulation of the theory if a metric tensor is factually also a field variable. That means that a possibility to formulate a simple quasilinear theory that permits a standard quantization procedure, depends on a particular choice of space-time, used to formulate the theory equations.

5. QUASILINEAR GRAVITATION THEORY

This Section is devoted to a detailed consideration of one of the versions of quasilinear gravitation theory. Coupling (17) may be given not only in its explicit form, but in the form of some functional relation or differential equation similarly as it takes place when coordinate conditions are imposed in the General Relativity. For example coupling(18) may be given by the equation

$$\partial g_{\ell m} / \partial \phi_{ik} = \frac{1}{2}(\delta^i_m \delta^k_\ell + \delta^i_\ell \delta^k_m).$$

Another coupling that allows one to write down the field equation in a simple and compact form is the equation

$$\gamma_{in}\gamma_{kp}\partial g_{\ell m}/\partial\phi_{np} = \frac{1}{4}(\gamma_{\ell i}\, g_{mk} + \gamma_{\ell k}\, g_{mi} + \gamma_{mi}\, g_{\ell k} + \gamma_{mk}\, g_{\ell i}). \quad (34)$$

Here

$$\gamma_{in}\,\partial g_{m\ell}/\partial\gamma_{kn} = (\gamma_{ip} - \phi_{ip})\partial g_{m\ell}/\partial\phi_{kp}\,.$$

The solution of equation (34) has form (26) with an accuracy up to the terms of the order of $(\phi)^2$. In the coordinate system with an orthogonal metric the solution for equation (34) would be

$$g_{ii} = \gamma_{ii}\exp(\gamma^{ii}\phi_{ii}) \text{ (no summing)},$$

i.e., connection (34) corresponds to a simple exponential form for the component of a metric tensor

$$ds^2 = e^{\phi_{oo}}dt^2 - e^{-\phi_{11}}dx_1^2 - e^{-\phi_{22}}dx_2^2 - e^{-\phi_{33}}dx_3^2.$$

As is known the magnitude $\phi_{oo} = \ln g_{oo}$ plays a role of a potential in the case of static gravitational field. Therefore the exponential form for metric tensor corresponds to the formulation of the field equation for the gravitational field potential. Another attractive feature of the coupling of the exponential type is the fact that if L_g depends only on the derivatives $\partial_i\phi_{kn}$, then the Lagrangian density and field equation are invariant with respect to the conformal transformation $g_{ik} \to cg_{ik}$ that corresponds to the transition to another gauge for the potential

$$\phi_{ik} \to \phi_{ik} + \ell n\, c.$$

The gravitational field equations in the theory with coupling (34) has the form

$$\delta L_g / \delta \phi_{ik} = \frac{1}{2}(\gamma^{ni} T^k_n + \gamma^{nk} T^i_n), \tag{35}$$

where

$$T^k_i = -2 g_{in}\, \delta L_M / \delta g_{kn}.$$

In the quasilinear theory with Lagrangian density (33) equation (35) takes the form

$$D_m D^m (\phi_{ik} - \frac{1}{2}\gamma_{ik}\, \phi^n_n) = -8\pi(\gamma_{ni}\, T^n_k + \gamma_{nk} T^n_i)/\sqrt{-\gamma}. \tag{36}$$

A static spherically symmetric field in the theory with coupling (34) is described by the interval

$$ds^2 = e^{\phi} dt^2 - e^{-\psi}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2) \tag{37}$$

with the field equation in the form

$$\Delta\phi = 8\pi(2T^0_0 - T)/\sqrt{-\gamma},$$

$$\Delta\psi = -8\pi(2T^1_1 - T)/\sqrt{-\gamma}.$$

The external static spherically symmetric field in the vacuum is presented in the form:

$$\phi = -2M/r, \qquad M = 4\pi\int(2T^{o}_{o} - T)r^2dr/\sqrt{-\gamma},$$
$$\psi = -2M_1/r, \qquad M_1 = -4\pi\int(2T^{1}_{1} - T)r^2dr/\sqrt{-\gamma}. \tag{38}$$

For the Sun $M \approx M_1$ since $T^{1}_{1} \ll T^{o}_{o}$ and $T \approx T^{o}_{o} - \epsilon$. But in a general case the static field of a spherically symmetric body is dependent on two parameters (masses), M and M_1 that can differ from each other greatly.

The expression for the interval has the form (when $M \approx M_1$):

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{2M^2}{r^2}\right)dt^2 - \left(1 + \frac{2M}{r}\right)\left(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right)$$

with an accuracy up to the terms that practically do not manifest themselves in the observed effects. This solution coincides with a corresponding expansion of the fundamental solution in the Einstein's theory. Thus the predictions of the treated quasilinear gravitational theory for the shift of Mercury perihelion and motion of light in the solar field coincide with the corresponding predictions of the Einstein's theory.

Solution (38) has a singularity only at $r=0$. No problems of the type of the Schwarzschild radius problem arise in interpreting this solution. If pressure is neglected the star collapses to a singular point $r=0$ both for infinite time of an outer observer and infinite proper time of freely falling observer.

For an outer observer the "coordinate" velocity dr/dt of a prob body, that falls down radially towards the point $r = 0$ tends to zero when $r \to 0$. For an observer that is at rest in the point r the instantaneous velocity of a falling down prob particle

$$d\ell/d\tau = \sqrt{-g_{11}}\, dr/(\sqrt{g_{00}}\, dt)$$

at $r \to 0$ tends to velocity of light ($c=1$). As is known similar properties are characteristic for particle motion in the Einstein's theory when approaching the gravitational radius.

In the theory with field equation (36) the gravitational field is not geometrized, consequently gravitational waves carry energy-momentum. Equation (14) allows one to calculate the energy flux of gravitational waves.

6. HOMOGENEOUS MODEL OF UNIVERSE

One of the achievements of the GRT was a possibility to construct satisfactory nonstationary models of the Universe. These models are able to describe the effect of red cosmologic shift and are free of discrepancies of the Newton stationary model.

A homogeneous isotropic Universe is described by the interval

$$ds^2 = d\tau^2 - \frac{\Psi^2(\tau)}{(1+kr^2/4)^2}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2).$$

Let us as usually treat the matter in the Universe as an ideal liquid with the energy-momentum tensor density

$$T_i^k = -2g_{in}\,\delta L_M/\delta g_{kn} = \sqrt{-g}\,[(p+\varepsilon)u_i u^k - p\delta_i^k].$$

In the approximation

$$p = 0 \tag{39}$$

from equation (22) there follows a relation

$$\varepsilon = \varepsilon_0 \Psi^{-3} \tag{40}$$

(we assume that at the present moment of time $\tau = \tau_0$ the normalization $\Psi = \Psi_0 = 1$ takes place) and field equations (1) take the form

$$\frac{8}{3}\pi\varepsilon_0 = \Psi(k + \dot{\Psi}^2), \tag{41}$$

$$0 = 2\ddot{\Psi}\Psi + k + \dot{\Psi}^2. \tag{42}$$

As it follows from (41), the function $\Psi(\tau)$ is defined with the equality

$$\tau = \int \frac{d\Psi}{\sqrt{\dfrac{8\pi\varepsilon_0}{3\Psi} - k}} + \text{const}.$$

Depending on the sign of the magnitude $\varepsilon/\varepsilon_{crit}-1$, where $\varepsilon_{crit}=\frac{3H^2}{8\pi}$, $H=\dot{\Psi}/\Psi$ is the Habble parameter, the Universe curvature is either positive ($k>0$) or negative ($k<0$). When

$$\varepsilon=\varepsilon_{crit}=3H^2/(8\pi) \tag{43}$$

$k=0$ and the function $\Psi(\tau)$ has the form:

$$\Psi(\tau)=[\frac{3}{2}H(\tau-\tau_{min})]^{2/3},$$

i.e. such a Universe "expands" unlimitedly during an infinite period of time in the future.

No matter what the type of the model is $(k(>,<,=)0)$ the acceleration parameter is negative in the Einstein's theory under cona. tion (39)

$$A=\frac{dH}{d\tau}=\frac{\ddot{\Psi}}{\Psi}-\frac{\dot{\Psi}^2}{\Psi^2}=-\frac{4}{3}\pi\varepsilon-H^2<0.$$

The age of the flat Universe (from the moment of time $\tau=\tau_{min}$ till the present moment of time $\tau=\tau_o$) is

$$\Delta\tau=\tau_o-\tau_{min}=\frac{2}{3H}.$$

A quasilinear theory with field equation (36) permits also to construct satisfactory nonstationary models of the Universe. Since field equation (36) are formulated in the terms of the Euclidean space-time, the homogeneous models in this theory correspond to the flat Universe.The models of quasilinear theory differ considerably from the flat model of GRT, for example, in the following features: 1) any values of the energy density of matter are permitted; 2) both monotoneous ($d\Psi/d\tau > 0$) and unmonotoneous behaviours are permitted too, in this in the unmonotoneous models first there goes on a "contraction" (but the singular "point" is not achieved) and then there comes the Universe "expansion"; 3) all the models of the quasilinear theory are cyclic, i.e., they are characterized with the finite proper time of the evolution cycle in the state between two consequent singularities.

We shall consider the properties of nonstationary models of the Universe in the framework of a quasilinear theory with field equations (36). The Universe will be called homogeneous if in the comoving coordinate system

$$\varepsilon = \varepsilon(t), \qquad p = p(t). \tag{44}$$

In the coordinate system with the orthogonal metric

$$ds^2 = \Phi^2 dt^2 - \Psi^2 (dx^2 + dy^2 + dz^2)$$

field equations (36) take the form

$$(\partial^2/\partial t^2 - \Delta)\ln\Phi = -4\pi(\varepsilon + 3p)\Phi\Psi^3, \tag{45}$$

$$(\partial^2/\partial t^2 - \Delta)\ln\Psi = 4\pi(\varepsilon - p)\Phi\Psi^3, \tag{46}$$

and equations of motion (22) take the form

$$\partial p/\partial r = -(p + \varepsilon)\frac{\partial \ln \Phi}{\partial r}, \tag{47}$$

$$\partial \varepsilon/\partial t = -3(p + \varepsilon)\frac{\partial}{\partial t}\ln \Psi. \tag{48}$$

Under conditions (44) from equations (47) and (45) there follows that

$$\Phi = \Phi(t), \quad \Psi = \Psi(t).$$

Let us consider the versions with condition (39). Relation (40) that follows from equation (48) allows one to write field equations (45) and (46) in the form

$$d^2(\ln \Phi)/dt^2 = -4\pi\varepsilon_o\Phi, \tag{49}$$

$$d^2(\ln \Psi)/dt^2 = 4\pi\varepsilon_o\Phi. \tag{50}$$

For the initial conditions in solution of these equations we will choose the conditions in the present moment of time $t = t_o$:

$$\Phi = \Psi = 1, \quad \frac{d}{dt}\ln \Phi = F, \quad \frac{d}{dt}\ln \Psi = H.$$

The first integral of equation (49) is

$$\frac{d}{dt}\ln \Phi = \pm\sqrt{F^2 + \varkappa - \varkappa\Phi}; \quad \varkappa = 8\pi\varepsilon_o.$$

Choosing the zero-time t=0 at the moment, when the function $\Phi(t)$ reaches the maximum, we obtain the solution in the form

$$\Phi(t) = (1 + F^2/\varkappa)[1 - \text{th}^2(\sqrt{\varkappa + F^2}\, t/2)].$$

The solution for the function $\Psi(t)$ has the form:

$$\psi(t) = \Phi^{-1} \exp[(F + H)(t - t_o)].$$

The function $\Phi(t)$ differs from zero within the interval $-\infty < t < +\infty$. Therefore there exists a transformation to the proper time $\tau(t)$ such that $d\tau = \Phi dt$,

$$t = \frac{1}{\sqrt{\varkappa + F^2}} \ln \frac{2\sqrt{\varkappa + F^2} + \varkappa\tau}{2\sqrt{\varkappa + F^2} - \varkappa\tau}.$$

Here the interval takes the form

$$ds^2 = d\tau^2 - \psi^2(\tau)(dx^2 + dy^2 + dz^2),$$

and the functions $\Phi(\tau)$ and $\psi(\tau)$ are presented in the form

$$\Phi(\tau) = 1 + \frac{F^2}{\varkappa} - \frac{\varkappa\tau^2}{4},$$

$$\Psi(\tau) = \left(\frac{\sqrt{\varkappa + F^2} + F}{\sqrt{\varkappa + F^2} - \frac{\varkappa\tau}{2}}\right)^{2(F+H)/\sqrt{\varkappa + F^2}} \Phi^{[(F+H)/\sqrt{\varkappa + F^2} - 1]}.$$

The value $\tau = \tau_o = -2F/\varkappa$ corresponds to the present moment of time. The function Φ achieves the minimum $\Phi = 0$ at the moment of proper time $\tau^{max}_{min} = \pm 2\sqrt{\varkappa + F^2}/\varkappa$. The Universe age is

$$\Delta\tau = \tau_0 - \tau_{min} = \frac{2}{\varkappa}\left(\sqrt{\varkappa + F^2} - F\right).$$

In the neighbourhood of a singular point $\tau = \tau_{min}$

$$\Phi \simeq \sqrt{\varkappa + F^2}(\tau - \tau_{min}) \simeq 4\left(1 + \frac{F^2}{\varkappa}\right)\exp\left(\sqrt{\varkappa + F^2}\, t\right),$$

$$\Psi(\tau) \simeq \left(\frac{\sqrt{\varkappa + F^2} + F}{2\sqrt{\varkappa + F^2}}\right)^{2(F+H)/\sqrt{\varkappa + F^2}} \Phi^{[(F+H)/\sqrt{\varkappa + F^2} - 1]}. \quad (51)$$

In the neighbourhood of the present moment of time $\tau = \tau_0$ the functions $\Phi(\tau)$ and $\Psi(\tau)$ have the form

$$\Phi(\tau) \simeq 1 + F(\tau - \tau_0) - \frac{\varkappa}{4}(\tau - \tau_0)^2,$$

$$\Psi(\tau) \simeq 1 + H(\tau - \tau_0) + \frac{1}{2}\left(\frac{\varkappa}{2} + H^2 - HF\right)(\tau - \tau_0)^2.$$

If ω_1 is light frequency at the moment of its emission $\tau = \tau_1$ and $\Psi = \Psi_1$, then light frequency ω_0 observed at the present moment of time $\tau = \tau_0$ is as is known equal to

$$\omega_0 = \omega_1\Psi_1/\Psi_0 \simeq \omega_1[1 - H(\tau_0 - \tau_1)].$$

The acceleration parameter at the present moment of time equals

$$A = \ddot{\Psi} - H^2 = \frac{\varkappa}{2} - HF.$$

Depending on the value and sign of the parameter F the magnitude A may take both positive and negative values.

As is seen the considered class of cosmological models gives a satisfactory description of the principle observed effect-cosmological red shift. Depending on the value of the parameters F and $\mathscr{X}$ various versions with different acceleration parameters and Universe age are possible.

MODEL A

Let the function $\Psi(\tau)$ be finite and not equal to zero at the starting moment of time. As it follows from (51) it is possible if

$$(F + H)/\sqrt{\mathscr{X} + F^2} - 1 = 0.$$

In this $F = (\mathscr{X} - H^2)/(2H)$, and the function $\Psi(\tau)$ is described with the expression

$$\Psi(\tau) = 4\left(1 + \frac{\varepsilon_{crit}}{3\varepsilon_0} - H\tau\right)^{-2},$$

i.e., the function $\Psi(\tau)$ increases monotoneously starting from the value $\Psi_{min} = \left(1 + \frac{\varepsilon_{crit}}{3\varepsilon_0}\right)^{-2}$ at the moment of time $\tau = \tau_{min} = -\frac{1}{H}\left(1 + \frac{\varepsilon_{crit}}{3\varepsilon_0}\right)$ up to the value $\Psi = +\infty$ at the moment of time $\tau_{max} = -\tau_{min}$. In spite of monotoneous behaviour of the function $\Psi(\tau)$, the model has a finite proper time of the evolution cycle, this is equal to $2\tau_{max}$. The acceleration parameter is positive in the given model,

$$A = \frac{\varkappa}{2} - HF = H^2/2 > 0.$$

(as is known the available data do not allow one to determine the sign of the acceleration parameter quite reliably).

The Universe age is equal to

$$\Delta\tau = \frac{2}{3H}(\varepsilon_{crit}/\varepsilon_o).$$

Under condition (43) $\Delta\tau$ coincides with the age of the flat Universe in the Einstein's theory.

To estimate the Universe age we will use the value

$$\varepsilon_o / \varepsilon_{crit} \approx 0{,}2, \qquad (52)$$

which is in agreement with the data of observation.

At the value of (52) we find that

$$\Delta\tau \approx 3/H,$$

which is quite a tolarable value.

The behaviour of the functions Φ and Ψ has the form, illustrated in fig. 1. Depending on the sign of the magnitude

$$F = \frac{H}{2}\left(\frac{3\varepsilon_o}{\varepsilon_{crit}} - 1\right)$$

either point $\tau_o^{(1)}$ ($F > 0$,"young" Universe) or point $\tau_o^{(2)}$ ($F < 0$,"old" Universe) corresponds to the present moment of time. Value(59) corresponds to the "old" Universe.

MODEL B

In this model the function Ψ equals zero at the starting moment of time $\tau = \tau_{min}$ and the derivative $d\Psi/d\tau$ is finite and not equal to zero. As it follows from (51) it is possible for the case $(F + H)/\sqrt{æ + F^2} - 2 = 0.$

Here

$$F = \frac{H}{3}(1 \pm 2\sqrt{1 - 9\varepsilon_o/\varepsilon_{crit}}).$$

Hence such a model is valid only in the case

$$\varepsilon_o \leqslant \frac{1}{9}\varepsilon_{crit}.$$

The function $\Psi(\tau)$ is defined with the expression

$$\Psi(\tau) = \frac{1}{4æ}(H + 3F)^4 (H + F + æ\tau)(H + F - æ\tau)^{-3}$$

and it monotoneously grows from the value $\Psi = 0$ at $\tau = \tau_{min}$ up to the value $\Psi = +\infty$ at $\tau = \tau_{max}$. The acceleration parameter and the age of the Universe is equal to

$$A = \frac{æ}{2} - HF = H^2[\frac{3}{2}\frac{\varepsilon_o}{\varepsilon_{crit}} - \frac{1}{3}(1 \pm 2\sqrt{1 - 9\varepsilon_o/\varepsilon_{crit}})],$$

$$\Delta\tau = \frac{H - F}{æ} = \frac{2}{9H}\frac{\varepsilon_{crit}}{\varepsilon_o}(1 \mp \sqrt{1 - 9\varepsilon_o/\varepsilon_{crit}}).$$

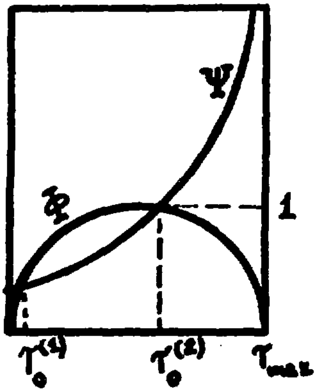

Fig. 1.

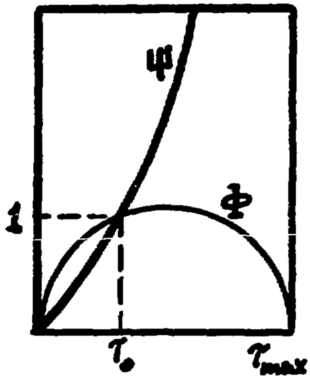

Fig. 2.

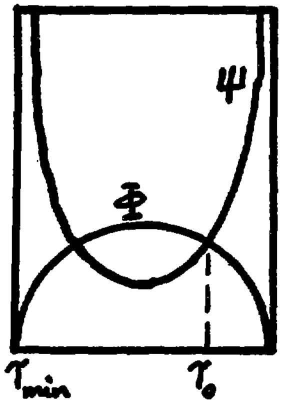

Fig. 3.

At the maximum value of energy density $\epsilon_o = \epsilon_{krit}/9$ that is possible in the given model,

$$F = H/3, \quad A = -H^2/6 < 0, \quad \Delta\tau = 2/H.$$

The behaviour of the function Φ and Ψ is shown in fig. 2.

MODEL C

This model is non-monotoneous, and the relation $F + H = 0$ takes place at any moment of time.

In such a model

$$\Psi(\tau) = \left(1 + \frac{\epsilon_{crit}}{3\epsilon_o} - \frac{3}{4}\frac{\epsilon_o}{\epsilon_{crit}} H^2 \tau^2\right)^{-1}.$$

The behaviour of the functions Φ and Ψ has the form, shown in fig. 3. In the time interval from $\tau = \tau_{min}$ till $\tau = 0$ the function Ψ decreases from $\Psi = +\infty$ down to

$$\Psi = \left(1 + \frac{\epsilon_{crit}}{3\epsilon_o}\right)^{-1}$$

and then in the interval $0 < \tau < \tau_{max}$ it starts increasing again up to $\Psi = +\infty$. The acceleration parameter and the Universe age (till the time moment $\tau = \tau_o$ with $H > 0$) are equal to

$$A = H^2 \left(\frac{3}{2}\frac{\epsilon_o}{\epsilon_{crit}} + 1\right) > 0,$$

$$\Delta\tau = \frac{2}{3H}\,\frac{\varepsilon_{crit}}{\varepsilon_o}\,\left(1 + \sqrt{1 + 3\,\varepsilon_o/\varepsilon_{crit}}\right).$$

At value (52) the magnitude $\Delta\tau$ is about $7{,}6/H$. It is sufficiently high value (about 10^{11} years) that may be considerably reduced if the ratio $\varepsilon_o/\varepsilon_{crit}$ is made larger. For the value $\varepsilon_o/\varepsilon_{crit} = 1$ the age is $\Delta\tau \simeq 2/H$. When observing very far removed objects in the given model the red shift should be replaced by violet one.

These models are peculiar for their cycling no matter whether the expansion is monotoneous or not. Even in the case of monotoneous expansion the theory predicts the finite proper time of the Universe evolution cycle, on the contrary to the open models in the Einstein's theory. Contrary to the cyclic model of GRT in the non-monotoneous models of quasilinear theory first there takes place contraction (a singular state is not achieved) and then comes expansion of the Universe, here both symmetric evolution (model B) and asymmetric one with respect to the point $\tau = 0$ are possible. Under conditions (39) and (43) the flat model is defined unambiguously in the Einstein's theory. In the quasilinear theory different versions of the flat model are possible, that would differ in the acceleration parameter value and the Universe age.

In the quasiliniar theory it is also possible to construct the Universe models with positive and negative curvatures ($k \gtrless 0$)

in spite of the fact that basic equation (36) is formulated in the terms of the Euclidean space-time. In the given formulation of the theory such models will correspond to unhomogeneous distribution of matter, when $\varepsilon = \varepsilon(r, t)$, $p = p(r, t)$.

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