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HADRON-INDUCED SEMI-COHERENT SCATTERING AND THE EXCITATION OF THE $^{12}C(2^+,4.4$ MeV) LEVEL

L. Bertocchi

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C. Troncon

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HADRON-INDUCED SEMI-COHERENT SCATTERING AND THE EXCITATION OF THE $^{12}C(2^+, 4.4\ MeV)$ LEVEL *

L. Bertocchi **
Istituto di Fisica Teorica dell'Università, Trieste, Italy,
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Italy,
and
International Centre for Theoretical Physics, Trieste, Italy,

and

C. Troncon
Istituto di Fisica Teorica dell'Università, Trieste, Italy.

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** Present address: Theory Division, CERN, Geneva, Switzerland.

ABSTRACT

We derive, in the framework of Glauber theory, the expression of the amplitudes for the semicoherent excitation of the 2^+(4.4 MeV) level of ^{12}C with incident pion and proton beams, including the spin-flip amplitudes in the latter case, using the formalism of harmonic solid tensors.
The expressions of the differential cross-section and of the nuclear density matrices are computed and compared with the existing high-energy data. The overall agreement between theory and experimental results for the differential and integrated cross-sections and for the density matrix elements is satisfactory; solving, therefore, a discrepancy of a factor two found by previous authors.

1. Introduction

We discuss in this paper the problem of the theoretical description of the elastic excitation of the 2^+(4.4 MeV) level of ^{12}C with high-energy hadron beams

$$h + {}^{12}C \rightarrow h + {}^{12}C(2^+, 4.4\ \text{MeV})^* . \qquad (1.1)$$

We are concerned in particular with the problem of a simultaneous description of the reaction induced by high-energy protons and pions, for which good experimental data exist[1-4], showing the angular distribution of the scattered hadron for momentum transfers up to 0.6 $(\text{GeV}/c)^2$ for protons and up to 0.18 $(\text{GeV}/c)^2$ for pions.

We want to stress immediately that we are not concerned with the problem of the description of the proton angular distribution for momentum transfers larger than those corresponding to the position of the first minimum in the angular distribution (about 0.16 $(\text{GeV}/c)^2$), where specific nuclear properties, such as for instance correlations or density deformations, are important. Our only concern is , as we have said, a simultaneous fit to pion- and proton-induced data in the region of the first maximum in the angular distribution.

The interest in the correct theoretical description of reaction (1.1) is connected to a quite well known property, which has been extensively discussed in the literature[2,5]. If one considers the semi-coherent production reaction

$$h + {}^{12}C \rightarrow h^* + {}^{12}C(2^+, 4.4\ \text{MeV})^* , \qquad (1.2)$$

where, as in the example, the nucleus is excited to a definite nuclear level together with the production of a state h* (a resonance), it can easily be shown that this category of reactions can be used to filter out definite quantum numbers of the produced state h*.

The actual use of reactions like (1.2) to extract interesting parameters, such as the total cross-section of the state h*, however, has been hindered up to now by the puzzling situation regarding the theoretical description of the simpler reaction (1.1).

In fact, while good theoretical descriptions exist for the proton-induced data, some of the authors who have tried a simultaneous fit to proton- and pion-induced data[4,6] have found that, using the same inelastic form factor for the nuclear transition (extracted from the inelastic electron scattering,

$$e + {}^{12}C \rightarrow e + {}^{12}C(4.4\ \mathrm{MeV})^*,$$

one gets a reasonable fit to proton-induced data, while pion-induced data are overestimated by a factor which ranges from 2 to 1.4. This result is particularly disturbing, as the calculations are parameter-free, since the hadron amplitudes and elastic and inelastic nuclear form factors are taken from scattering on proton targets or from nuclear elastic and inelastic electron scattering.

In other theoretical treatments[7] the spirit of the calculation is quite different, since their main purpose is to test nuclear physics microscopic models for the elastic and inelastic form factors.

We shall see in this paper that the root of the discrepancy between the theories for pion and proton induced data lies either in a kinematical mistreatment of the problem, or in approximations which turn out to be too rough. In the only paper[8] in which the correct theoretical treatment is applied to both the pion and proton initiated reactions (but neglecting spin-flip for the protons), we **suspect a numerical mistake**, since we have not been able to reproduce the numerical results of Ref.8 (in the case of pions, where spin-flip is absent) using the same theoretical formalism and the same parameters for the elementary amplitudes and form factors. We are quite confident of our numerical results, as we have obtained exactly the same results using two different numerical techniques.

In addition, we have considered the effect of spin-flip in the proton initiated reaction.

The paper is organized as follows. In Sec. 2 we shall discuss our theoretical treatment, which is compared with the existing experimental data in Sec. 3.

2. Theoretical analyis

We start with the spinless beam case (pions), and we shall describe the amplitude for the semi-coherent scattering (1.1) using Glauber theory, so that we have

$$T^{fo}(\vec{q}) = \frac{ikA}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}}\,\bar{f}(b)^{A-1}\int d_3x\,\Gamma(|\vec{b}-\vec{s}|)\,\Psi_f^*(\vec{x})\,\Psi_o(\vec{x}), \qquad (2.1)$$

where $\Psi_o(\vec{x})$, $\Psi_f(\vec{x})$ are the wave functions of the ground and excited nuclear levels, $\Gamma(|\vec{b}-\vec{s}|)$ is the profile function of the elementary hadron-nucleon elastic scattering amplitude

$$\Gamma(|\vec{b}-\vec{s}|) = \frac{1}{2\pi i k}\int d_2q'\, e^{-i\vec{q}'\cdot(\vec{b}-\vec{s})} f(q'). \qquad (2.2)$$

and $\bar{f}(b)$ is the absorption factor, related to the hadron-nucleon amplitude (2.2) and the ground state elastic form factor $F_{el}(\vec{q})$ by

$$\bar{f}(b) = 1 - \frac{1}{2\pi i k}\int d_2q\, e^{-i\vec{q}\cdot\vec{b}} f(q)\, F_{el}(q). \qquad (2.3)$$

A denotes the total nucleon number of the target, $\vec{b}$ is the impact parameter orthogonal to the incident momentum $\vec{k}$, $\vec{s}$ the component of the coordinate $\vec{x}$ in the plane perpendicular to $\vec{k}$, and we have neglected every difference between protons and neutrons, as both the ground and excited state of the target have isospin equal to zero.

The ground-state level has zero angular momentum, and we are considering the excitation of the 2^+ level; the angular momentum properties of the excited level are contained in the final wave function $\Psi_f(\vec{x})$. To specify these properties, we shall employ the formalism of the solid harmonic tensors in 3 dimensions, writing[9]

$$T^{fo}_{\mu\nu}(\vec{q}) = \frac{ikA}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}}\,\bar{f}(b)^{A-1}\int d_3x\,\Gamma(|\vec{b}-\vec{s}|)\,\rho_{fo}(x^2)\,T^{(3)}_{\mu\nu}(\hat{x}), \qquad (2.4)$$

where

$$T^{(3)}_{\mu\nu}(\hat{x}) = \sqrt{\tfrac{3}{2}}\left[\hat{x}_\mu\hat{x}_\nu - \tfrac{1}{3}\delta_{\mu\nu}\right] \qquad (2.5)$$

is the solid harmonic tensor of rank 2 in 3 dimensions, and $\rho_{fo}(x^2)$ is a scalar function, connected to the transition form factor measured in inelastic electron scattering by

$$F_{fo}(q) = \int d_3x\, e^{i\vec{q}\cdot\vec{x}}\rho_{fo}(x^2)\,P_2(\cos\theta). \qquad (2.6)$$

Now the important observation is that the integral

$$\int d_3x\,\Gamma(|\vec{b}-\vec{s}|)\,\rho_{fo}(x^2)\,T^{(3)}_{\mu\nu}(\hat{x}) \qquad (2.7)$$

does <u>not</u> have the rotational properties of a harmonic solid tensor $T^{(3)}_{\mu\nu}(\vec{b})$ of the variable $\vec{b}$ since, even if the spin is absent, $\Gamma(|\vec{b}-\vec{s}|)$ is not a three-dimensional scalar, but only a scalar in the plane perpendicular to $\vec{k}$.

The mistake of considering (2.7) as proportional to $T^{(3)}_{\mu\nu}(\vec{b})$ is in fact quite common in the past literature.

To proceed correctly, it is convenient, from the property of $\Gamma(|\vec{b}-\vec{s}|)$ just observed, to separate the longitudinal and transverse parts of $T^{(3)}_{\mu\nu}(\hat{x})$, writing

$$T^{f0}_{33}(\vec{q}) = \frac{ikA}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}}\bar{f}(b)^{A-1}\int d_3x\, \Gamma(|\vec{b}-\vec{s}|)\,\rho_{f0}(x^2)\sqrt{\frac{3}{2}}\left(\frac{z^2}{r^2}-\frac{1}{3}\right)\delta_{33}\,,$$

$$T^{f0}_{ij}(\vec{q}) = \frac{ikA}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}}\bar{f}(b)^{A-1}\int d_3x\, \Gamma(|\vec{b}-\vec{s}|)\,\rho_{f0}(x^2)\, T^{(3)}_{ij}(\hat{x})\,, \qquad (2.8)$$

where

$$\mu,\nu = 1,2,3\;; \quad i,j = 1,2\;;$$

the index 3 denotes the longitudinal direction $(\vec{k})$. Due to the symmetry of the d^3x integration, the components $T^{f0}_{i3}(\vec{q})$ vanish.

We observe now that

$$\int d_3x\, \Gamma(|\vec{b}-\vec{s}|)\,\rho_{f0}(z^2+s^2)\left(\frac{z^2}{r^2}-\frac{1}{3}\right) \qquad (2.9)$$

is a scalar in the plane orthogonal to $\vec{k}$, while $T^{(3)}_{ij}(\hat{x})$ can be rewritten as

$$T^{(3)}_{ij}(\hat{x}) = \frac{1}{r^2}T^{(3)}_{ij}(\vec{x}) = \frac{1}{r^2}\left\{\frac{\sqrt{3}}{2}s^2\,T^{(2)}_{ij}(\hat{s}) - \sqrt{\frac{3}{2}}\,\frac{1}{2}\left(\frac{2}{3}z^2-\frac{4}{3}s^2\right)\delta_{ij}\right\}, \qquad (2.10)$$

where $T^{(2)}_{ij}(\hat{s})$ is now a solid harmonic tensor of rank 2 in 2 dimensions, a function of the unit vector $\hat{s}$. In this way we have separated terms which have different properties under rotations in the orthogonal plane.

From the observation that the coefficients of both δ_{33} and δ_{ij} are proportional to

$$\int d_3x\, \Gamma(|\vec{b}-\vec{s}|)\,\rho_{f0}(x^2)\left(z^2-\frac{1}{3}r^2\right)/r^2 = \int d_2s\, \Gamma(|\vec{b}-\vec{s}|)\int_{-\infty}^{+\infty} dz\, \rho_{f0}\cdot\frac{z^2-\frac{1}{3}r^2}{r^2}\,, \qquad (2.11)$$

which is a scalar function of b^2, while

$$\int d_3x\, \Gamma(|\vec{b}-\vec{s}|)\,\rho_{f0}(x^2)\frac{s^2}{r^2}T^{(2)}_{ij}(\hat{s}) = \int d_2s\, \Gamma(|\vec{b}-\vec{s}|)\,s^2\,T^{(2)}_{ij}(\hat{s})\int_{-\infty}^{+\infty} dz\,\frac{\rho_{f0}}{r^2} \qquad (2.12)$$

has the rotational properties of $T^{(2)}_{ij}(\hat{b})$, we can write

$$\sqrt{\frac{3}{2}}\int d_3x\, \Gamma(|\vec{b}-\vec{s}|)\,\rho_{f0}(x^2)\left(\frac{z^2}{r^2}-\frac{1}{3}\right) = E_0(b^2)\,,$$

$$\frac{\sqrt{3}}{2}\int d_3x\, \Gamma(|\vec{b}-\vec{s}|)\,\rho_{f0}(x^2)\frac{s^2}{r^2}T^{(2)}_{ij}(\hat{s}) = E_2(b^2)\,T^{(2)}_{ij}(\hat{b})\,, \qquad (2.13)$$

with

$$E_2(b^2) = \int d_3x\, \frac{\sqrt{3}}{2}\,\Gamma(|\vec{b}-\vec{s}|)\,\rho_{f0}(x^2)\frac{s^2}{r^2}\,G_2(\cos\theta)\,. \qquad (2.14)$$

where

$$G_2(\cos\theta) = T^{(2)}_{ij}(\hat{b})\cdot T^{(2)}_{ij}(\hat{s})$$

is the Gegenbauer polynomial of the second order ($\theta = \widehat{bs}$).

With a similar argument, we obtain

$$T^{f0}_{33}(\vec{q}) = \frac{ikA}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}}\bar{f}(b)^{A-1}E_0(b^2) = \bar{E}_0(q^2)\,\delta_{33}\,,$$

$$T^{f0}_{ij}(\vec{q}) = \bar{E}_2(q^2)\,T^{(2)}_{ij}(\hat{q}) - \frac{1}{2}\bar{E}_0(q^2)\,\delta_{ij}\,. \qquad (2.15)$$

where

$$\bar{E}_2(q^2) = \frac{ikA}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}}\bar{f}(b)^{A-1}G_2(\cos\varphi)\,E_2(b^2)\,, \qquad \varphi = \hat{b}\cdot\hat{q}\,. \qquad (2.16)$$

These expressions can be rewritten in a convenient way going over to momentum space. Using the inversions (2.2) and

$$\rho_{f0}(x^2)\,T^{(3)}_{\mu\nu}(\hat{x}) = \frac{1}{(2\pi)^3}\int d_3q'\, e^{-i\vec{q}'\cdot\vec{x}}F_{f0}(q')\,T^{(3)}_{\mu\nu}(\hat{q}')\,,$$

one gets for the scattering amplitude $T^{f0}_{\mu\nu}(\vec{q})$:

$$T^{f0}_{\mu\nu}(\vec{q}) = A\,\frac{i}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}}\bar{f}(b)^{A-1}\frac{1}{2\pi}\int d_2q'\, e^{-i\vec{q}'\cdot\vec{b}}f(q')\,F_{f0}(q')\,T^{(3)}_{\mu\nu}(\hat{q}') \qquad (2.17)$$

and, from the relation

$$\frac{1}{2\pi}\int_0^{2\pi} d\varphi\, e^{iqb\cos\varphi}\,G_n(\cos\varphi) = (-1)^{n/2}J_n(qb)\,,$$

separating as before the longitudinal and transverse parts, one gets

$$E_0(q^2) = A\,\frac{i}{\sqrt{6}}\int_0^\infty b\,db\, J_0(qb)\,\bar{f}(b)^{A-1}\int_0^\infty q'dq'\, J_0(q'b)\,f(q')\,F_{f0}(q')\,,$$

$$E_2(q^2) = A\,\frac{\sqrt{3}}{2}\int_0^\infty b\,db\, J_2(qb)\,\bar{f}(b)^{A-1}\int_0^\infty q'dq'\, J_2(q'b)\,f(q')\,F_{f0}(q')\,. \qquad (2.18)$$

The differential cross-section is given by (summing over the final nucleus polarizations)

$$\frac{d\sigma}{dq^2} = \frac{\pi}{k^2}\frac{d\sigma}{d\Omega} = \frac{\pi}{k^2} T^{fo}_{\mu\nu}(\vec{q})\, T^{fo}_{\mu\nu}(q)^* \tag{2.19}$$

and, since δ_{ij} and $T^{(2)}_{ij}$ are orthogonal

$$\frac{d\sigma}{dq^2} = \frac{\pi}{k^2}\left[\left| E_2(q^2)\right|^2 + \frac{3}{2}\left|E_0(q^2)\right|^2 \right] . \tag{2.20}$$

The same expression (2.20) has been obtained by Ahmad [8], using a different formalism.

Before introducing the spin-flip amplitude for proton scattering, we pause for some considerations.

The amplitude for the excitations from incident electrons can be obtained from (2.4) neglecting the absorption, and therefore putting $\bar{f}(b) = 1$ (besides a proper redefinition of $\Gamma(|\vec{b} - \vec{s}|)$ as the impact-parameter transform of the Rutherford amplitude). In this case, the integration over the impact parameter can be performed, and we obtain

$$T^{fo}_{\mu\nu}(\vec{q}) = A\, f(q^2)\, F_{fo}(q)\, T^{(3)}_{\mu\nu}(\hat{q}) . \tag{2.21}$$

We see, therefore, that the amplitude $T^{fo}_{\mu\nu}(\vec{q})$ is proportional to $T^{(3)}_{\mu\nu}(\hat{q})$ only when the absorption is neglected.

It is also interesting to consider the effect of using the zero range approximation for the elementary hadron-nucleon scattering amplitude, which amounts to writing

$$\Gamma(|\vec{b}-\vec{s}|) = \frac{2\pi f(0)}{ik}\, \delta^{(2)}(\vec{b}-\vec{s}) . \tag{2.22}$$

In this way, always starting from (2.4), one gets

$$\int d_3x\, \Gamma(|\vec{b}-\vec{s}|)\, \rho_{fo}(x)\, T^{(3)}_{\mu\nu}(\hat{x}) = \frac{2\pi f(0)}{ik}\int_{-\infty}^{+\infty} dz\, \rho_{fo}(b^2+z^2)\, T^{(3)}_{\mu\nu}(\hat{b}+\hat{z}) , \tag{2.23}$$

which, however, does not now have the property of a solid harmonic tensor of second rank in 3 dimensions.

We must remark, however, that this zero range approximation, even if the expression (2.23) is treated correctly, leads to a serious overestimation of the differential cross-section at the first maximum, when it is used for the profile function of the excitation amplitude. This can again easily be understood when one takes as an example the non-absorption limit ($\bar{f}(b) = 1$), which, in the zero range approximation, would lead to

$$\frac{d\sigma}{d\Omega} = A^2 |f(0)|^2 F^2_{fo}(q) , \tag{2.24}$$

to be compared with the correct result (obtained from (2.21))

$$\frac{d\sigma}{d\Omega} = A^2 |f(q)|^2 F^2_{fo}(q) . \tag{2.25}$$

The overestimate of using the zero range approximation corresponds, therefore, to a factor $\exp(bq^2)$, $\left[|f(q^2)|^2 = |f(0)|^2 . \exp(-bq^2)\right]$, which, on the maximum ($q^2 \sim 0.055\ (\mathrm{GeV}/c)^2$) corresponds roughly to $\exp(0.5)$.

We now turn to the introduction of the spin variables.

As both the ground and excited levels have a total spin S = 0, the target nucleon which is excited on the 2^+ level cannot flip its spin if we make the reasonable simplifying hypothesis of neglecting the spin-flip in the absorption factor $\bar{f}(b)$.

As a consequence, in the pion nucleon amplitude only the non-spin flip amplitude can contribute to the reaction, while in the nucleon-nucleon amplitude not all the tensor terms can contribute (since they correspond to a spin flip of both the projectile and target nucleon), while the single spin flip amplitude (γ) comes in only with a factor $\frac{1}{2}$ (corresponding to the spin-flip of the projectile nucleon).

If we impact-parameter transform the spin-flip amplitude, the contribution to the excitation amplitude will be

$$T^{fo\,(\gamma)}_{\mu\nu}(\vec{q}) = i\, T^{fo}_{\mu\nu\ell}(\vec{q})\cdot\left[\vec{\sigma}\times\hat{k}\right]_\ell = i\left[\vec{\sigma}\times\hat{k}\right]_\ell \cdot \frac{ik}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}}\, \bar{f}(b)^{A-1} \cdot \int d_3x\, \gamma(|\vec{b}-\vec{s}|)\, T^{(1)}_\ell(\vec{b}-\vec{s})\, \rho_{fo}(x)\, T^{(3)}_{\mu\nu}(\hat{x}) , \tag{2.26}$$

which contains the product of a vector in the two-dimensional space ($T^{(2)}_l(\vec{b}-\vec{s}) = (\vec{b}-\vec{s})_l$) and a tensor of rank 2 in the three-dimensional space. Again, we shall separate the longitudinal and transverse coordinates in $T^{(3)}_{\mu\nu}(\hat{x})$ and compute the tensors $T^{fo}_{ijl}(\vec{q})$ and $T^{fo}_{33l}(\vec{q})$.

For $T^{fo}_{33l}(\vec{q})$ one immediately obtains

$$T^{fo}_{33\ell}(\vec{q}) = \hat{q}_\ell\, \bar{M}(q^2)\, \delta_{33} , \tag{2.27}$$

where

$$\bar{M}(q^2) = \frac{ikA}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}} f(b)^{A-1}\, \hat{q}\cdot\hat{b}\, M(b^2) .$$

$$M(b^2) = \sqrt{\frac{3}{2}}\int d_3x\, \gamma(|\vec{b}-\vec{s}|)\, \rho_{f_0}(x^2)\left(\frac{z^2}{x^2}-\frac{1}{3}\right)(\vec{b}-\vec{s})\cdot\hat{b} \quad . \tag{2.28}$$

Now expressing $T^{(3)}_{ij}(\hat{x})$ in terms of $T^{(2)}_{ij}(\hat{s})$ and δ_{ij}, as done earlier, we easily find that the coefficient of δ_{ij} is equal to $-\frac{1}{2}\hat{q}_\ell M(q^2)$.

We still have to compute the term which contains the tensor

$$N_{ij\ell}(\vec{b}) = \frac{\sqrt{3}}{2}\int d_3x\, \rho_{f_0}(x^2)\, \gamma(|\vec{b}-\vec{s}|)\, (\vec{b}-\vec{s})_\ell\, T^{(2)}_{ij}(\hat{s}) . \tag{2.29}$$

The tensor $N_{ijl}(\vec{b})$ is a third-rank tensor in the two-dimensional space. It has the following properties

$$N_{ij\ell}\,\delta_{ij} = 0 \quad ; \quad N_{ij\ell} = N_{ji\ell} \quad . \tag{2.30}$$

It is not symmetric, however, under the exchange of the pair of indices i,ℓ (or j,ℓ). Therefore, it can be expanded on the basis of two independent traceless orthogonal tensors. As the basis for the expansion we shall use the following choice:

$$\hat{b}_\ell\, T^{(2)}_{ij}(\hat{b}) = \sqrt{2}\,\hat{b}_\ell\left(\hat{b}_i\hat{b}_j - \frac{1}{2}\delta_{ij}\right) \quad ; \quad P_{ij\ell}(\hat{b}) = \frac{1}{\sqrt{2}}\left(\delta_{i\ell}\hat{b}_j + \delta_{j\ell}\hat{b}_i - 2\hat{b}_i\hat{b}_j\hat{b}_\ell\right). \tag{2.31}$$

One can easily verify that the following properties hold:

$$\hat{b}_\ell T^{(2)}_{ij}(\hat{b})\cdot\hat{b}_\ell T^{(2)}_{ij}(\hat{b}) = 1 \quad ; \quad P_{ij\ell}(\hat{b})\cdot P_{ij\ell}(\hat{b}) = 1 \; ; \quad \hat{b}_\ell T^{(2)}_{ij}(\hat{b})\cdot P_{ij\ell}(\hat{b}) = 0 ,$$

$$P_{ij\ell}(\hat{n})\cdot\hat{b}_\ell T_{ij}(\hat{b}) = 2\left[\hat{n}\cdot\hat{b} - (\hat{n}\cdot\hat{b})^3\right] = \frac{1}{2}\left[G_1(\cos\varphi) - G_3(\cos\varphi)\right] .$$

$$\hat{b}_\ell T^{(2)}_{ij}(\hat{b})\,\hat{n}_\ell T^{(2)}_{ij}(\hat{n}) = (\hat{n}\cdot\hat{b})\left[2(\hat{n}\cdot\hat{b})^2-1\right] = \frac{1}{2}\left[G_3(\cos\varphi)+G_1(\cos\varphi)\right] . \tag{2.32}$$

$$P_{ij\ell}(\hat{b})\cdot P_{ij\ell}(\hat{n}) = 2(\hat{n}\cdot\hat{b})^3 - \hat{n}\cdot\hat{b} = \frac{1}{2}\left[G_3(\cos\varphi) + G_1(\cos\varphi)\right]$$

$$\varphi = \widehat{n\cdot b} \quad .$$

We can therefore write

$$N_{ij\ell}(\vec{b}) = A_1(b^2)\,\hat{b}_\ell T^{(2)}_{ij}(\hat{b}) + B_1(b^2)\,P_{ij\ell}(\hat{b}) , \tag{2.33}$$

where

$$A_1(b^2) = \frac{\sqrt{3}}{2}\int d_3x\, \rho_{f_0}(x^2)\,\gamma(|\vec{b}-\vec{s}|)\,(\vec{b}-\vec{s})\cdot\hat{b}\; T^{(2)}_{ij}(\hat{s})\cdot T^{(2)}_{ij}(\hat{b}) ,$$

$$B_1(b^2) = \frac{\sqrt{3}}{2}\int d_3x\, \rho_{f_0}(x^2)\,\gamma(|\vec{b}-\vec{s}|)\,(\vec{b}-\vec{s})_\ell\; T^{(2)}_{ij}(\hat{s})\, P_{ij\ell}(\hat{b}) . \tag{2.34}$$

The contribution of this term to the scattering amplitude will therefore be of the form

$$\bar{N}_{ij\ell}(\vec{q}) = \bar{C}_1(q^2)\,\hat{q}_\ell\, T^{(2)}_{ij}(\hat{q}) + \bar{D}_1(q^2)\, P_{ij\ell}(\hat{q}) , \tag{2.35}$$

where

$$\bar{C}_1(q^2) = \frac{ikA}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}}\bar{f}(b)^{A-1}\left[A_1(b^2)\,\hat{b}\cdot\hat{q}\, T^{(2)}_{ij}(\hat{b})T^{(2)}_{ij}(\hat{q}) + B_1(b^2)\,P_{ij\ell}(\hat{b})\,\hat{q}_\ell T^{(2)}_{ij}(\hat{q})\right] ,$$

$$\bar{D}_1(q^2) = \frac{ikA}{2\pi}\int d_2b\, e^{i\vec{q}\cdot\vec{b}}\bar{f}(b)^{A-1}\left[A_1(b^2)\,\hat{b}_\ell T^{(2)}_{ij}(\hat{b})\, P_{ij\ell}(\hat{q}) + B_1(b^2)\,P_{ij\ell}(\hat{b})\,P_{ij\ell}(\hat{q})\right] \tag{2.36}$$

The contribution of the spin-flip amplitude to the cross-section, summed over the nuclear polarization as well as over the nucleon polarization, will therefore be

$$\frac{d\sigma^{(2)}}{dq^2} = \frac{\pi}{k^2}\frac{d\sigma^{(2)}}{d\Omega} = \frac{\pi}{k^2}\left[\frac{3}{2}|\bar{M}(q)|^2 + |\bar{C}_1(q)|^2 + |\bar{D}_1(q)|^2\right] , \tag{2.37}$$

and therefore the cross-section for the proton-initiated reaction will be given by the sum of (2.20) and (2.37).

Given the non-spin-flip and the spin-flip amplitude, we can compute the nucleon polarization, which turns out to be

$$P = \left(2\,\mathrm{Re}\left\{\bar{C}_1(q^2)\cdot\bar{E}^*_2(q^2) + \frac{3}{2}\bar{M}(q^2)\,\bar{E}^*_0(q^2)\right\}\right)\Big/\frac{d\sigma}{d\Omega} \quad . \tag{2.38}$$

It is, however, more interesting to compute the nuclear polarization density matrix (summed over the nucleon polarization). In the cartesian basis we find that the density matrix is diagonal, with the expressions

$$\frac{d\sigma}{d\Omega}\rho_{\hat{q}\hat{q}} = \frac{1}{2}\left\{|\bar{E}_2(q^2)|^2 + |\bar{C}_1(q^2)|^2 + \frac{1}{2}|\bar{E}_0(q^2)|^2 + \frac{1}{2}|\bar{M}(q^2)|^2 - 2\,\mathrm{Re}\left(\bar{E}_0(q^2)\cdot\bar{E}^*_2(q^2) + \bar{C}_1(q^2)\cdot\bar{M}^*(q^2)\right)\right\} ,$$

$$\frac{d\sigma}{d\Omega}\rho_{\hat{k}\hat{k}} = |\bar{E}_0(q^2)|^2 + |\bar{M}(q^2)|^2 \tag{2.39}$$

(where we have denoted the cartesian axes by the unit vectors $\hat{k}$ and $\hat{q}$), the third diagonal non-zero element being determined by $\mathrm{Tr}(\rho_{\mu\nu})=1$.

On the other hand, it is more instructive to express the density matrix elements in an angular momentum basis, with the quantization axis along the direction of the momentum transfer vector $\vec{q}$. The following expressions are found for the amplitudes with definite values of the component M of the angular momentum along $\vec{q}$:

$$T^{fo}_{M=0}(\vec{q}) = \frac{\sqrt{3}}{2}\left[\left(\bar{E}_2(q^2) - \frac{\bar{E}_0(q^2)}{\sqrt{2}}\right) + i\,(\vec{\sigma}\times\hat{k})\cdot\hat{q}\left(\bar{C}_1(q^2) - \frac{\bar{M}(q^2)}{\sqrt{2}}\right)\right],$$

$$T^{fo}_{M=\pm1}(\vec{q}) = \frac{1}{\sqrt{2}}(\vec{\sigma}\times\hat{k})\cdot\hat{q}\,\bar{D}_1(q^2), \qquad (2.40)$$

$$T^{fo}_{M=\pm2}(\vec{q}) = \frac{1}{2}\left[\left(\frac{\bar{E}_2(q^2)}{\sqrt{2}} + \frac{3}{2}\bar{E}_0(q^2)\right) + i\,(\vec{\sigma}\times\hat{k})\cdot\hat{q}\left(\frac{\bar{C}_1(q^2)}{\sqrt{2}} + \frac{3}{2}\bar{M}(q^2)\right)\right],$$

from which one gets the density matrix elements $\rho_{MM'}$

$$\frac{d\sigma}{d\Omega}\cdot\rho_{00} = \frac{3}{4}\left[\left|\left(\bar{E}_2(q^2) - \frac{\bar{E}_0}{\sqrt{2}}\right)\right|^2 + \left|\left(\bar{C}_1(q^2) - \frac{\bar{M}(q^2)}{\sqrt{2}}\right)\right|^2\right] ; \quad \rho_{1,0} = \rho_{-1,0} = 0 ,$$

$$\frac{d\sigma}{d\Omega}\cdot\rho_{11} = \frac{d\sigma}{d\Omega}\rho_{1,-1} = \frac{1}{2}\left|\bar{D}_1(q^2)\right|^2 , \qquad (2.41)$$

$$\frac{d\sigma}{d\Omega}\cdot\rho_{22} = \frac{d\sigma}{d\Omega}\rho_{2,-2} = \frac{1}{4}\left[\left|\left(\frac{\bar{E}_2(q^2)}{\sqrt{2}} + \frac{3}{2}\bar{E}_0(q^2)\right)\right|^2 + \left|\left(\frac{\bar{C}_1(q^2)}{\sqrt{2}} + \frac{3}{2}\bar{M}(q^2)\right)\right|^2\right] ,$$

$$\frac{d\sigma}{d\Omega}\cdot\rho_{20} = \frac{d\sigma}{d\Omega}\rho_{-2,0} = \frac{\sqrt{3}}{4}\left[\left(\frac{\bar{E}_2(q^2)}{\sqrt{2}} + \frac{3}{2}\bar{E}_0(q^2)\right)^*\cdot\left(\bar{E}_2(q^2) - \frac{\bar{E}_0(q^2)}{\sqrt{2}}\right) + \left(\frac{\bar{C}_1(q^2)}{\sqrt{2}} + \frac{3}{2}\bar{M}(q^2)\right)^*\cdot\left(\bar{C}_1(q^2) - \frac{\bar{M}(q^2)}{\sqrt{2}}\right)\right] .$$

The following remarks are relevant:

- In the spinless case, in the absence of absorption, only the state with M=0 is populated, so that $\rho_{00}=1$ and all the other density matrix elements vanish. The role of the absorption is to produce a non-vanishing amplitude with M=2 , so that also ρ_{22} and ρ_{20} acquire a non-zero value;
- When the beam has spin $\frac{1}{2}$, also the state with M=1 is populated, and the amplitude is proportional to the coefficient $D_1(q^2)$ of the tensor $P_{ij1}(\hat{q})$. Therefore, at least in principle, the measure of the density matrix elements ρ_{11} or $\rho_{1,-1}$ is a direct measurement of the single spin-flip nucleon-nucleon amplitude, and not as in the polarization measurements of its interference with other nucleon-nucleon amplitudes.

3. Comparison with the experimental data

The theory developed in Sec. 2 has been compared with the following set of experimental data:

a) The differential cross-section for the semicoherent scattering of protons at 1.04 GeV [1] and of pions at 4.5 GeV/c [2-4];

b) The integrated cross-sections for the same reaction for incident protons at 1.04 GeV [1], at 0.68, 0.8, 1.0 , 1.2, 2.0, 4.0 GeV/c from Ref. 2 , and 6.0 GeV/c from Ref. 10 ; for incident pions at 0.62, 1.0, 3.0, 4.0 GeV/c from Ref. 2 , at 4.5 GeV/c from Refs. 2-4 , at 6.0 GeV/c from Ref. 10 and at 25 and 40 GeV/c from Ref. 11 ; and finally for incident K^- at 6.0 GeV/c from Ref. 6;

c) The nuclear density matrix elements for the same reactions, for incident pions at 4.5 GeV/c [4] and protons at 6 GeV/c [10].

To actually perform the calculation we need:

- The pion-nucleon non-spin-flip amplitude, and the nucleon-nucleon non-spin-flip and spin-flip amplitudes, as a function of the momentum transfer, in the isospin symmetric state;
- The ^{12}C ground state form factor, which enters in the rescattering factor (2.3), or equivalently the ground state density;
- The transition form factor $F_{fo}(q)$.

For the elastic form factor, we have used the ground-state density given by Lesniak [6]

$$\rho(x^2) = \frac{1}{3}\frac{1}{(\sqrt{\pi}R)^3}\left(1 + \frac{4}{3}\frac{x^2}{R^2}\right)e^{-\frac{x^2}{R^2}} , \qquad (3.1)$$

with R=1.6 fm. We have, however, checked that all the numerical results, in the region of the first maximum of the angular distribution, differ by at most 2 % if we use the density given by Sick and McCarthy [12].

For the transition form factor we have used the form given by Ahmad [8]

$$F_{fo}(q^2) = B q^2 (1 - C q^2)\, e^{-Dq^2} , \qquad (3.2)$$

with $B=0.215\ fm^2$, $C=0.137\ fm^2$ and $D=.549\ fm^2$, which produces a good fit to the electron excitation of the 2^+ level.

As regards the elementary pion-nucleon and K-nucleon scattering amplitudes, they have been parametrized in the standard form,

$$f(q) = ik\sigma_T^{(\pm)}(1-i\alpha^{(\pm)})/(4\pi)\exp(-bq^2/2)\ ,$$

and the values of the parameters, which are reported in Table I, have been extracted from standard data tables.

When looking at the proton-initiated reactions, we have used the same procedure as for pions for the non-spin-flip amplitude; the spin-flip amplitude, however, is quite poorly known, except at 1.04 GeV, where we have used the parametrization given by Viollier[13],

$$\alpha(q^2) = a_\alpha e^{-g_\alpha q^2}\ ,$$

$$\gamma(q^2) = a_\gamma e^{-g_\gamma q^2}\ ,$$

with

$$a_\alpha = -1.+3.09i\ (fm)\ ; \qquad g_\alpha = .102\ (fm^2)\ ,$$

$$a_\gamma = .818-.637i\ (fm)\ ; \qquad g_\gamma = .153\ (fm^2)\ .$$

Moreover, since at the other incident momenta we are only interested in the integrated cross-section and, as we shall see, the contribution of the spin-flip amplitude to the cross-section is of the order of 10%, we are not making a large error in assuming for the other energies the same ratio between the amplitude α and γ as at 1.04 GeV. Anyway, the results for the proton-initiated reactions suffer from this ambiguity (except at 1.04 GeV).

The results of the calculation of the differential cross-section are shown in Fig.1 (pions at 4.5 GeV/c) and in Fig. 2 (protons at 1.04 GeV.).

The following remarks are relevant:

- For the proton angular distribution, the theory (which is parameter free) predicts both the correct magnitude and shape; this would not be so if the contribution of the spin-flip amplitude (dashed curve in Fig. 2) had been neglected. In this case one would have underestimated the cross-section by about 10 %.
- For the pion angular distribution, the cross-section is overestimated at the maximum, but only by about 10 - 15 %

This result must be compared with an overestimate by a factor 1.9, found in Refs.5 and 6, in which, however, either a zero-range approximation has been used for the elementary amplitude (as usually done in DWBA) or the rotational properties have been mistreated. This result must also be compared with the result of Ahmad[8], who finds for the cross-section at the peak a value of ~ 34 mb/(GeV/c)^2, while we find 26.7. We have no explanation for this difference, since (for pions) we have used the same theoretical formulation and the same parameters for the amplitude and form factors. The only possible explanation is a numerical mistake in Ahmad's calculation. We have checked the reliability of our numerical procedure, performing the numerical calculation in two different ways. In the first, we have performed numerically a double integration in coordinate space with a Gaussian method; in the second, in momentum space, we have performed one integral analytically and only the second numerically. The results always coincided for all four digits required, confirming the accuracy of the numerical procedure.
- Both for pions and protons, the experimental results are higher than the theory for $q^2 > 0.1$ (GeV/c)^2. In our opinion, this is related to the behaviour of the inelastic form factor at large q^2, which is probably not well represented by a simple form like (3.2). We have, however, not tried to use a more sophisticated form, looking for a simultaneous fit of electron, pion and proton data at large q^2, since this was not the spirit of our calculation.

As far as the results of the calculation of the integrated cross-sections are concerned, they are displayed in Table I. In general, we see that the agreement between theory and experiment is satisfactory, except maybe for protons at 2.0 GeV/c, where the experimental result is anomalously small, and for pions at 1.0 GeV/c, where the large value of the theoretical cross-section is due to the maximum in the symmetric total pion-nucleon cross-section. A singular case is also the preliminary result of the Milan group[11] at 40 GeV/c, where the experimental cross-section seems to show an appreciable drop from 25 to 40 GeV/c, while the elementary parameters used in the theoretical description show only a very slow variation.

If we turn now to the density matrix elements, we show in Figs. 3 and 4 the results for the dominant diagonal element ρ_{00} and the real part of ρ_{20} (the other diagonal element, ρ_{22}, is very small, of the order of 1 or 2 %, in the region of the maximum of the angular distribution). These curves must be compared with the values [4] extracted from the angular distribution of the gamma ray emitted in the de-excitation of the 2^+ level, which is equal to [4] 0.13 ± 0.05 for the ratio $\mathrm{Re}\,\rho_{20}/\rho_{00}$ in the q^2 interval between 0.0247 and 0.0987 $(\mathrm{GeV}/c)^2$. We see that, within this large error, our theoretical prediction agrees with the experiment.

It would also be interesting to compute the density matrix elements for proton scattering at 6 GeV/c, which have also been measured [10]. Unfortunately, the calculation requires the knowledge of the spin-flip amplitude for nucleon-nucleon scattering, which is not available at that energy.

We have, however, computed the density matrix elements at 1.04 GeV for incident protons, mainly to estimate the size of the element ρ_{11}. The results are practically the same as for incident pions for ρ_{00}, ρ_{22} and $\mathrm{Re}\,\rho_{20}$, while ρ_{11} has been found to be very small, of the order of 0.5 % in the region of the first maximum of the angular distribution. It therefore looks hopeless to be able to measure ρ_{11} to extract information on the nucleon-nucleon spin-flip amplitude.

For reference, the experimental values for the density matrix elements for incident protons at 6.0 GeV are the following [10]:

$$\rho_{00} = 0.843;\ \rho_{11} = 0.041\ ;\ \rho_{22} = 0.038\ ;\ \rho_{2,-2} = 0.02\ ;\ \mathrm{Re}\,\rho_{02} = 0.178$$

We have also computed, for incident protons at 1.04 GeV, the polarization of the incident protons. Here we shall not give details, which can be found elsewhere [14]; the general result is, however, that, for q^2 up to $0.1 (\mathrm{GeV}/c)^2$, the polarization is quite large, of the order of 70-80 % of the nucleon-nucleon polarization. Therefore it is conceivable to use the semi-coherent excitation of the 2^+ level of ^{12}C as an analyser of the nucleon-nucleon non-spin-flip and single spin-flip amplitudes.

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TABLE I

Integrated cross-section for the reaction hadron+$^{12}C \rightarrow$ hadron+$^{12}C(4.4)^*$ for incident momenta k > 0.5 GeV/c

Incident particle	Momentum k or Kin. Energy T	PARAMETERS $\sigma_T^{(+)}$ (mb)	$\alpha^{(+)}$	b $(GeV/c)^2$	$\sigma^{int.}_{exp}$ (mb)	$\sigma^{int.}_{th}$ (mb)	Ref.
p	k=0.68 GeV/c	34	0.1	1.0	2.9 ± 0.5	3.1	2
p	k=0.8 "	32	0.05	1.0	2.9 ± 0.5	2.77	2
p	k=1.0 "	31	-0.4	1.5	2.8 ± 0.5	3.44	2
p	k=1.2 "	39	-0.4	4.6	3.2 ± 0.5	3.85	2
p	T=1.04 GeV	44.15	-0.356	5.2	3.95±0.3 (*)	3.87	1
p	k=2.0 GeV/c	42	-0.3	6.4	2.6 ± 0.5	3.5	2
p	k=4.0 "	43	-0.15	6.6	3.3 ± 0.5	3.15	2
p	k=6.0 "	41.5	-0.12	8.0	3.3 ± 0.3	2.8	10
π	k=0.62 GeV/c	25.2	-0.57	5.0	1.6 ± 0.2	2.15	2
π	k=1.0 "	42.6	-0.08	6.5	1.8 ± 0.3	2.8	2
π	k=3.0 "	30.8	-0.33	7.5	1.9 ± 0.3	2.1	2
π	k=4.0 "	29.2	-0.3	8.0	1.7 ± 0.3	1.9	2
π	k=4.5 "	28.4	-0.3	8.2	2.0 ± 0.3	1.8	2,3
π	k=6.0 "	27.1	-0.2	8.5	1.9 ± 0.2	1.55	10 (++)
π	k=25 "	24.1	-0.12	9.5	1.05±0.25	1.12	11
π	k=40 "	23.7	-0.1	10.0	.8 ± 0.3	1.20	11
K^-	k=6.0 GeV/c	22.0	0.0	7.0	1.4 ± 0.4	1.30	6

(*) Deduced from the integration of $d\sigma/dq^2$.
(++) Both the exp. result and the theory for the interval $q^2 \lesssim 0.09\ (GeV/c)^2$.

The parameters are those of the non-spin-flip scattering amplitude, which has the form $f^{(+)}(q) = ik\sigma_T^{(+)}/(4\pi)(1-i\alpha^{(+)})\exp(-bq^2/2)$.

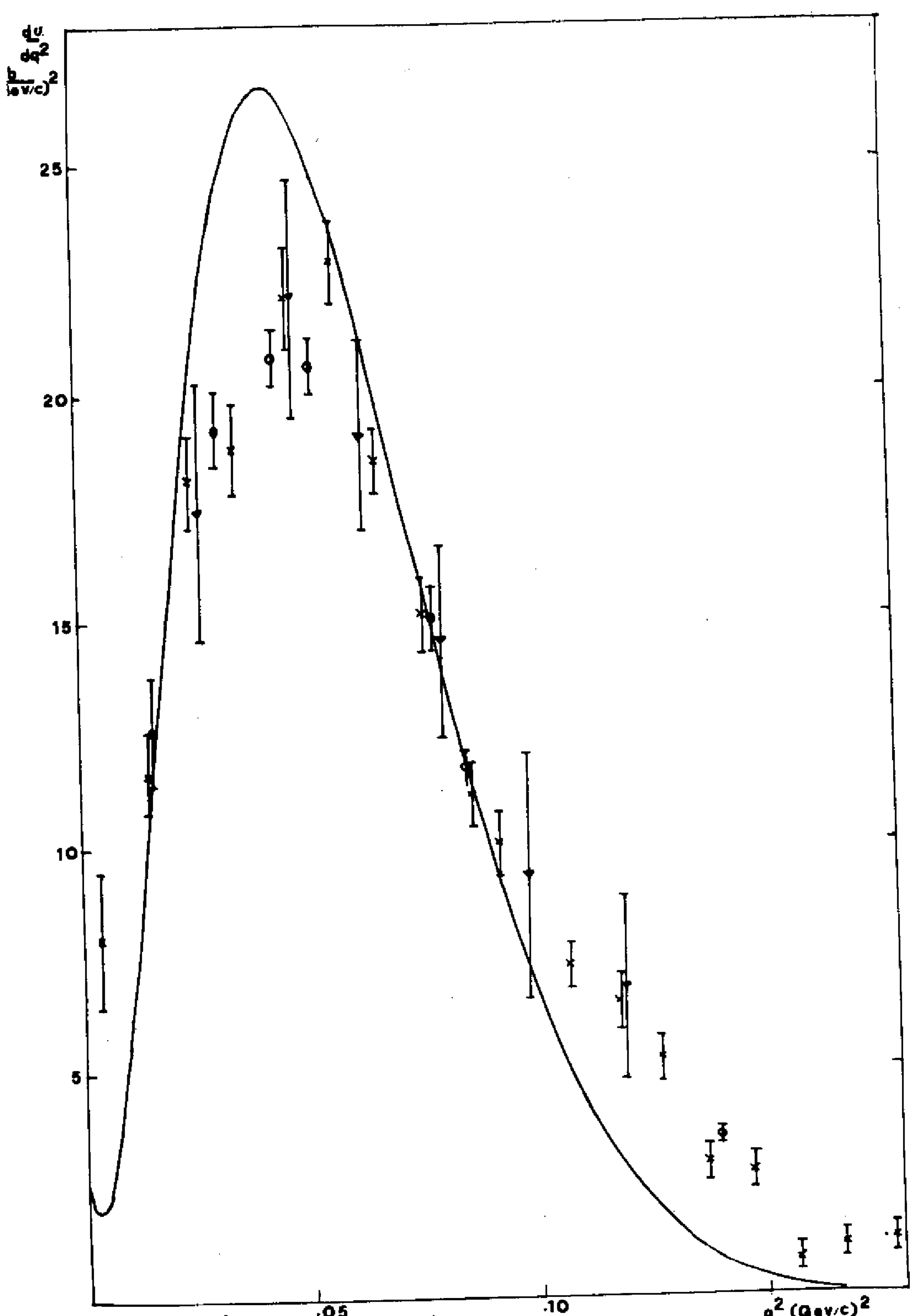
FIGURE CAPTIONS

Fig. 1 - The differential cross-section for the pion-initiated reaction at 4.5 GeV/c. The experimental points are from Refs. 2-4, the continuos curve is the theoretical prediction.

Fig. 2 - The differential cross-section for the proton-initiated reaction at T=1.04 GeV. The experimental points are from Ref. 1 , the continuous curve is the theoretical prediction for the sum of the non-spin-flip and the spin-flip contributions, the dotted curve the spin-flip contribution.

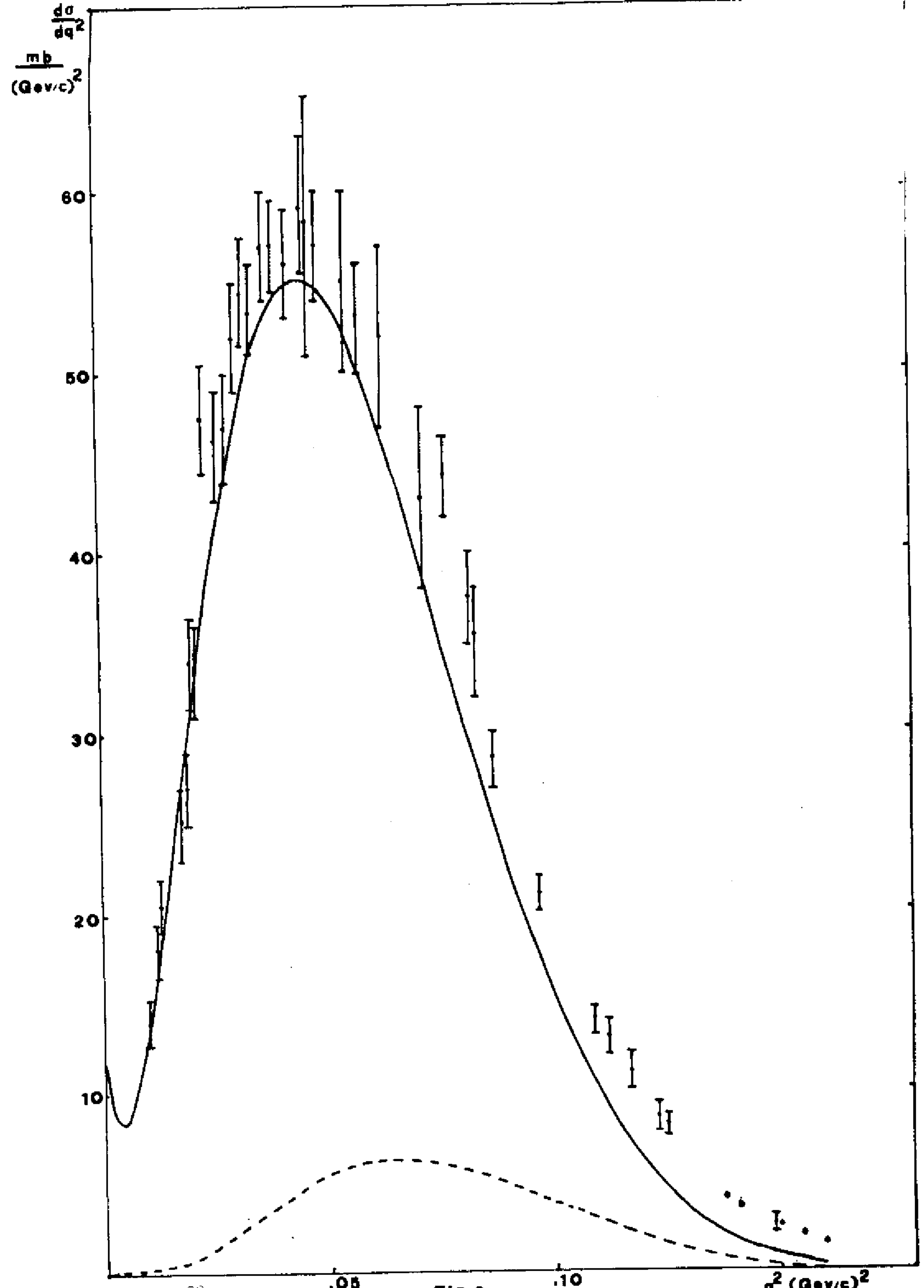
Fig. 3 - The density matrix element ρ_{00} for the pion-initiated reaction at 4.5 GeV/c as a function of the momentum transfer q^2.

Fig. 4 - The same as Fig. 3 for the real part of ρ_{20}.



dσ/dq²
25
20
15
10
5
.05
.10
q² (Gev/c)²

Fig.1



dσ/dq²
mb/(Gev/c)²
60
50
40
30
20
10
.05
.10
q² (Gev/c)²

Fig.2

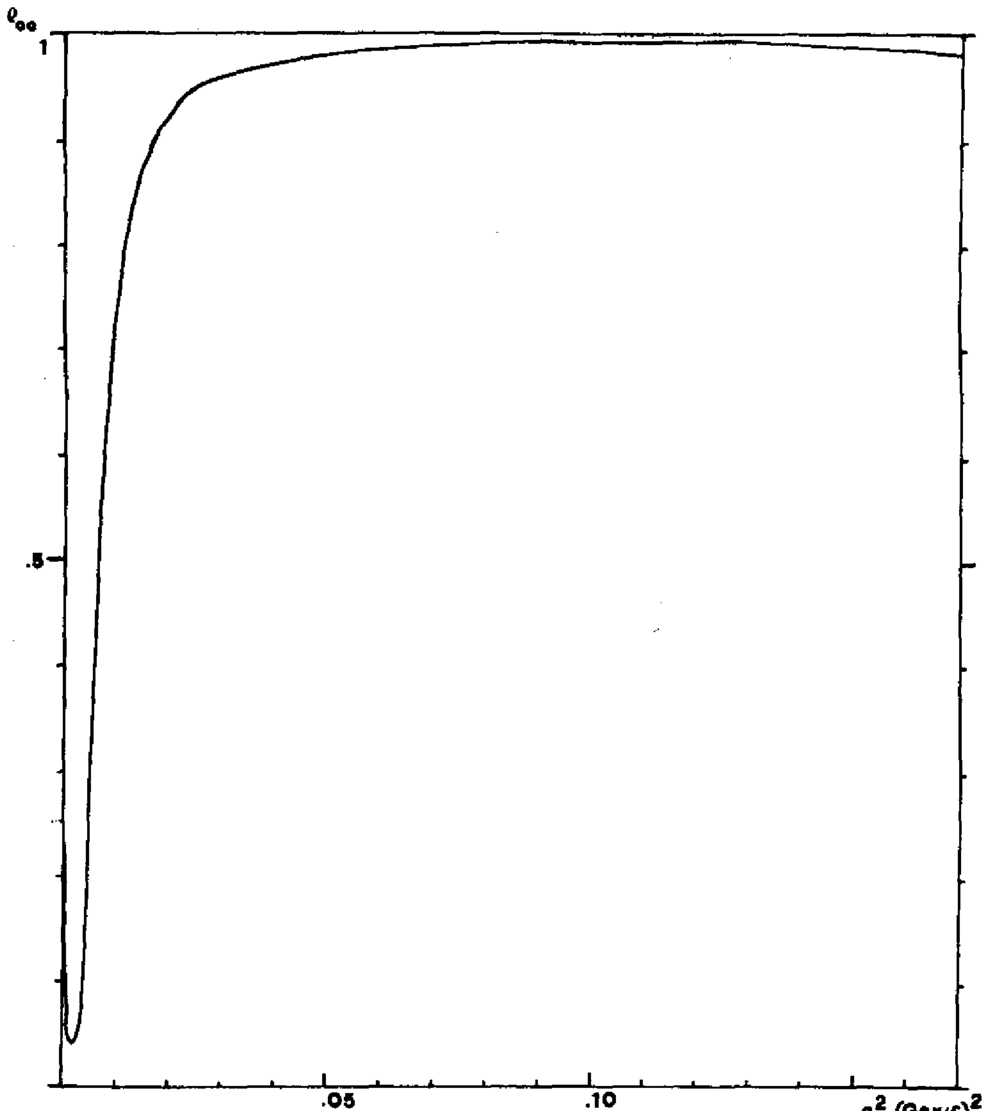


Fig.3

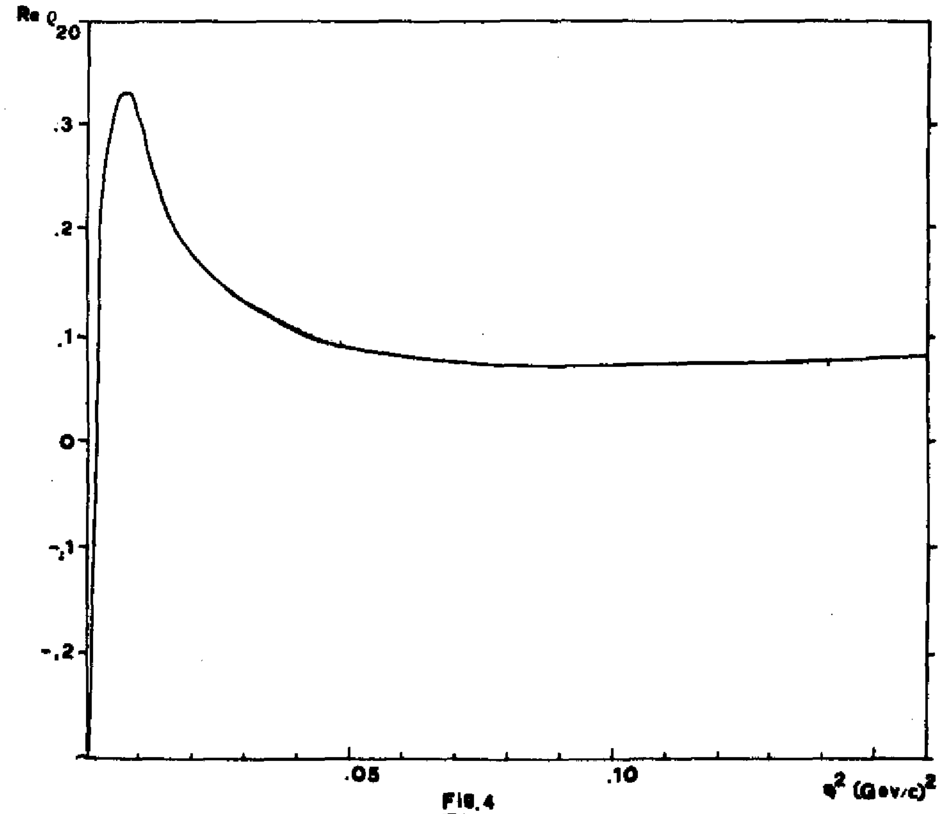


Fig.4