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SCATTERING OF HIGH-ENERGY α PARTICLES ON ^{12}C

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SCATTERING OF HIGH-ENERGY α PARTICLES ON ^{12}C *

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ABSTRACT

Glauber multiple scattering theory is applied to analyse the elastic and inelastic scattering of 1.37 GeV α particles on ^{12}C. An approach which treats the N-α amplitude at the incident nucleon kinetic energy equal to the α-kinetic energy per particle as the basic interaction is adopted. Using the gaussian model for ^{4}He to obtain the N-α amplitude in terms of the NN amplitude, it is found that, in general, the experimental data are qualitatively explained. However, large discrepancies in terms of the magnitude of the cross-sections in the small angle region and the positions of the minima in the angular distribution at larger angles are generally present. Effects of the two-body correlations in the projectile as well as in the target are also investigated.

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I. INTRODUCTION

At present great interest is being shown in the use of medium- and high-energy hadronic probes for nuclear structure studies. Stimulus to this comes largely from proton scattering experiments [1,2] around 1 GeV which seem to provide valuable information on neutron density distributions and correlations in nuclei [2,3]. Perhaps attracted by its simplicity of structure, the Saclay group has very recently added the α particle to the list of high-energy hadronic probes and has obtained interesting elastic and inelastic scattering data on ^{12}C and Ca isotopes at α kinetic energy equal to 1.37 GeV. The motivation is to complement their proton scattering experiments [2] - it is not unlikely that the protons and the α particles (because of the large peripheral absorption of the latter) are sensitive to different parts of the target nucleus.

The Saclay group also presented an analysis of their α-^{12}C data, which is based on the optical potential approach of KMT theory [4]. The scattering cross-sections were calculated by constructing the optical potential in two ways. First, they applied the folding model potential as proposed by Czyz and Maximòn [5] and which takes for the input the NN amplitude (at the incident nucleon kinetic energy which is one fourth the kinetic energy of the incident α particle) and the elastic form factors of both the projectile and target nuclei. This approach predicts rather too large cross-sections in the small angle region. Second, they considered the N-α amplitude as "elementary" and expressed the potential in terms of this amplitude and the elastic form factor of the target. The former is obtained [6] from the NN amplitude as given by the phase-shift analysis at the nucleon kinetic energy of interest ($\approx$ 340 MeV) and using the oscillator model for the ground state of ^{4}He. This approach is found to give results which are in good agreement with experiment up to the momentum transfer $q \approx 2$ fm^{-1}. However, neither of the two approaches satisfactorily predict the large angle behaviour of the cross-sections. Near the second diffraction minima which appear prominently in the experimental data (except for the 3^- case) the calculations predict only a change in slope and, moreover, the calculated cross-sections generally fall rather steeply. Even the happy union between theory and experiment for $q \lesssim 2$ fm^{-1} comes under eclipse when it is noted that the same N-α amplitude which reproduces the α-^{12}C data at smaller angles fails badly in reproducing the p-α scattering [6] at 348 MeV.

In this paper we report an analysis of the elastic and inelastic scattering of 1.37 GeV α particles on ^{12}C within the framework of the Glauber multiple scattering theory [7]. Like the Saclay group [6] we also treat the projectile as a "particle" and hence consider the N-α amplitude as the elementary one. However, for calculating the N-α amplitude we use the NN amplitude at 344 MeV as given by Alberi et al. [8] and which gives quite satisfactory results for the d-d scattering at the incident deuteron lab momentum 1.75 GeV/c. Using the gaussian model for the ground state of the α particle and taking the ground state and transition densities from electron scattering experiments, we find that the α-^{12}C scattering is qualitatively reproduced. Particularly the second minima in the elastic and inelastic angular distributions, which are generally absent in the Saclay analysis, are well predicted. But, generally, large disagreement between theory and experiment is found in terms of the magnitude of the cross-sections at smaller angles and the positions of the minima in the angular distribution at larger angles. Pushing the theoretical investigation further, we find that whereas it is possible to interpret (at least partially) the large angle discrepancy by invoking the two-body correlations in the description of the two colliding nuclei, the same cannot be said for the small angle discrepancy, at least within the theoretical framework of the present paper.

II. FORMULATION

As our treatment of α-nucleus scattering resembles that of the N nucleus, it is worthwhile to discuss the latter briefly. This is motivated by some other considerations also. First, it is intended to express the hadron-nucleus scattering amplitude in terms of the intrinsic (ground state and transition) densities as measured from the centre of mass of the target without recourse to any nuclear model. In most of the previous applications of the Glauber theory to hadron-nucleus inelastic scattering, some model for the target is invoked in order to derive the expression for the hadron-nucleus scattering amplitude (e.g. Refs.3 and 9). It then looks somewhat artificial to use the electron scattering transition form factors (densities) in the calculation - as is intended here - since the models generally applied are not consistent with the electron scattering experiments for all momentum transfers. Second, the discussion would clarify our attitude towards the c.m. correlation which is generally (approximately) accounted for by multiplying the final expression for the scattering

amplitude with the oscillator model c.m. correlation factor [illegible] - an approach which sometimes gives physically unacceptable results (e.g. when it is applied together with the optical limit in nucleus-nucleus scattering [10]). And last, the N-nucleus scattering formalism to be developed below is applied later to obtain a closed expression for the N-α scattering amplitude (to be used in the α-^{12}C scattering calculation) using the realistic form factor for the ground state for ^4He and the two-body correlation function as proposed by Feshbach et al. [11].

2.1 N-nucleus scattering

According to the Glauber theory, the scattering amplitude describing the excitation of a target nucleus from the ground state $\psi_0 \equiv |0)$ to an excited state $\psi_f \equiv |f)$ is given by [7]

$$F_{f0}(q) = \frac{ik}{2\pi} \int d^2b \, e^{i\vec{q}\cdot\vec{b}} \, [\delta_{f0} - S_{f0}(\vec{b})] \, , \tag{2.1a}$$

$$S_{f0}(\vec{b}) = \left(f \left| \prod_{j=1}^{A} \{1 - \Gamma(\vec{b}-\vec{s}_j)\} \right| 0 \right) \, , \tag{2.1b}$$

where $\vec{q}$ is the momentum transfer, $\vec{b}$ the impact parameter, k the momentum of the incident particle, A the target mass number, $\vec{s}_j$ are the projections of the target nucleon co-ordinates $\vec{r}_j$ on a plane perpendicular to $\vec{k}$, and Γ is the profile function related to the elementary amplitude $f(q)$ as follows:

$$\Gamma(b) = \frac{1}{2\pi ik} \int d^2q \, e^{-i\vec{q}\cdot\vec{b}} \, f(q) \, . \tag{2.2}$$

The expression (2.1a) has been treated differently by various authors. As already mentioned, the usual approach has been to assume some model for the target nucleus to obtain the final expression for the amplitude [3),9)]. Since here we intend to use the ground-state and transition densities given by electron scattering experiments, we find it more appropriate to follow a different approach.

Defining for convenience

$$\eta_j(\vec{b}) = 1 - \Gamma(\vec{b} - \vec{s}_j) \tag{2.3}$$

and using the closure over the target states:

$$\sum_{m} |m)\,(m| = 1$$

Eq.(2.1b) may be written as

$$S_{f0}(\vec{b}) = \sum_{m_1=0} \cdots \sum_{m_{A-1}=0} (f|\eta_1|m_1)\,(m_1|\eta_2|m_2)\,\cdots\,(m_{A-1}|\eta_A|0)\ . \tag{2.4}$$

Now if all the particles are treated as identical and it is assumed that

$$(f|\eta|f) \approx (0|\eta|0)\ ,$$

then the following expansions for the elastic and inelastic matrices are easily obtained (Appx.A):

$$S_{00}(\vec{b}) = \Omega^A \left\{1 + \frac{A!}{(A-2)!2!}\,\frac{\Delta}{\Omega^2} + \frac{A!}{(A-4)!4!}\,\frac{\Delta^2}{\Omega^4} + \cdots\right\} + \cdots \tag{2.5a}$$

$$S_{f0}(\vec{b}) = -A\Gamma_{f0}\Omega^{A-1}\left\{1 + \frac{(A-1)!}{(A-3)!3!}\,\frac{\Delta}{\Omega^2} + \frac{(A-1)!}{(A-5)!5!}\,\frac{\Delta^2}{\Omega^4} + \cdots\right\} + \cdots\ , \tag{2.5b}$$

where

$$\Omega(b) = 1 - \int \rho_{00}(\vec{r})\,\Gamma(\vec{b}-\vec{s})\,d\vec{r}\ , \tag{2.6}$$

$$\Gamma_{f0}(b) = \int \rho_{f0}(\vec{r})\,\Gamma(\vec{b}-\vec{s})\,d\vec{r}\ , \tag{2.7}$$

$$\Delta(b) = \sum_{m\neq 0} (0|\Gamma(\vec{b}-\vec{s}_1)|m)\,(m|\Gamma(\vec{b}-\vec{s}_2)|0)\ , \tag{2.8a}$$

$$= \int C(\vec{r}_1,\vec{r}_2)\,\Gamma(\vec{b}-\vec{s}_1)\,\Gamma(\vec{b}-\vec{s}_2)\,d\vec{r}_1\,d\vec{r}_2\ . \tag{2.8b}$$

In the above expressions $\rho_{00}(\vec{r})$ and ρ_{f0} are the (intrinsic) ground state and transition densities of the target as measured, for example, in electron scattering experiments (assuming of course that the proton and neutron densities are identical and correcting for the proton finite size), e.g.

$$\rho_{00}(\vec{r}) = \int |\psi_0(r, r_2, r_3, \ldots, r_A)|^2\ \frac{\delta(\vec{r}+\vec{r}_2+\cdots+\vec{r}_A)}{A}\ dr_2\ldots dr_A\ .$$

The quantity $C(\vec{r}_1,\vec{r}_2)$ is the pair correlation function.

In the expansion (2.5) the curly brackets contain terms which depend upon the two-body correlations and the sum within it goes up to the power N_A in Δ, where N_A is $A/2$ for even A and is $(A-1)/2$ for odd A. The terms neglected outside the brackets depend upon the three-body and higher order correlations.

It may be pointed out that an expression for S_{00} similar to that given by Eq.(2.5a) may also be obtained [12] by using the ansatz due to Foldy and Walecka [13] for the A-body density. The two, however, differ in their higher order terms (from the point of view of calculation this is unimportant as the higher powers in Δ contribute little) and moreover the ansatz breaks down for the A-body correlation term [13].

Using Eq.(2.5), expressions for the elastic and inelastic scattering amplitudes (including also the Coulomb scattering) may be obtained in terms of the NN amplitude and the elastic and inelastic form factors of the target as in Ref.3 and need not be given here.

It may be pointed out that while using Eq.(2.5) we do not take the usual optical limit of either Ω^A or Ω^{A-1}.

2.2 α-nucleus scattering

Generalization of the expression (2.1b) to the scattering between two composite particles is straightforward. Particularizing the discussion to α-nucleus scattering the S-matrix now reads [5]

$$S_{f0}(\vec{b}) = \left(\psi_f\phi_0\left|\ \prod_{i=1}^{4}\ \prod_{j=1}^{A}\ \left\{1-\Gamma(\vec{b}-\vec{s}_j-\vec{s}_i')\right\}\right|\phi_0\psi_0\right)\ , \tag{2.9}$$

where, as before, ψ_0 and ψ_f are, respectively, the target ground and excited states; ϕ_0 is the ground state wave function of the α particle and $\vec{s}_i'$ are the projections of the projectile nucleon co-ordinates $\vec{r}_i'$ as measured from its centre of mass.

The expression (2.9) is complicated enough to be evaluated exactly, particularly for large A nuclei and therefore consideration of some suitable approximation for its evaluation seems inevitable. The problem has been discussed in some detail by Czyz and Maximon [5] who also derived a simple expression for the elastic nucleus-nucleus scattering by applying the usual independent particle for describing both the projectile and the target, and finally using the optical limit approximation. This approach leads to the so-called folding model potential mentioned earlier. We, however, do not intend to use it here, particularly because recent researches by Franco [10] cast serious doubts on the usefulness of this approach for calculating the elastic scattering. Also a recent calculation by Layly and Schaeffer [14] leads to the same conclusion.

Fortunately, the projectile we are interested in is a tightly bound system and therefore its scattering from a nucleus appears to be amenable to a physically appealing approximation. It is not unreasonable to expect that the excitations of the incident α particle between its successive scatterings with the nucleons of the target make only a small contribution to the total scattering and therefore may be neglected. In other words, the idea of treating the α particle as a "particle" for evaluating Eq.(2.9) is worth trying.

Let us assume that all the particles involved in the collision are identical and group together the factors in Eq.(2.9) which contain the same target nucleon co-ordinates. Next applying the closure over the α particle states between each group we obtain

$$S_{f0}(\vec{b}) = \sum_{m_1,\dots,m_{A-1}} \left(\psi_f\right| \left\{ (\phi_0|Z_1|\phi_{m_1}) \; (\phi_{m_1}|Z_2|\phi_{m_2}) \cdots (\phi_{m_{A-1}}|Z_A|\phi_0) \right\} \left|\psi_0\right) \qquad (2.10)$$

where

$$Z_j = \prod_{i=1}^{4} \{1 - \Gamma(\vec{b}-\vec{s}_j-\vec{s}_i{}')\} \quad . \qquad (2.11)$$

Now, if the contributions of all but the (α particle) ground state are neglected, we have

$$S_{f0}(\vec{b}) = \left(\psi_f\right| \prod_{j=1}^{A} \{1 - \Gamma_\alpha(\vec{b}-\vec{s}_j)\} \left|\psi_0\right) \quad , \qquad (2.12)$$

where

$$1 - \Gamma_\alpha(\vec{b}-\vec{s}_j) = \int |\phi_0(r_1\cdots r_4)|^2 \prod_{i=1}^{4} \{1-\Gamma(\vec{b}-\vec{s}_j-\vec{s}_i)\} \, \delta\left(\frac{\vec{r}_1+\vec{r}_2+\cdots+\vec{r}_4}{4}\right) dr_1\cdots dr_4 \, . \qquad (2.13)$$

The quantity Γ_α is nothing but the profile function for the elastic N-α scattering as follows from Eq.(2.1a) and is related to the N-α scattering amplitude through Eq.(2.2). Comparing Eq.(2.12) with (2.1b) we see that, provided our approximation works, the α nucleus scattering can be calculated in the same manner as the N nucleus scattering described in Sec.2.1, replacing of course the NN amplitude by the appropriate N-α amplitude.

III. DESCRIPTION OF THE CALCULATION

In this section we describe the calculation of the elastic and inelastic scattering of 1.37 GeV α particle on ^{12}C using the formulation of Sec.II. The main inputs of the calculation are: (i) the N-α amplitude at the incident nucleon kinetic energy 344 MeV, which is about 1/4 times the kinetic energy of the α particle and (ii) the elastic, the 2^+, 0^+ and 3^- inelastic form factors for ^{12}C.

i) As a first choice for the N-α amplitude we use the expression derived by Czyz and Lesniak [15] and Bassel and Wilkin [16] using the oscillator model for ^4He and the usual parametrization for the NN amplitude:

$$f_{NN}(q^2) = \frac{ik\sigma}{4\pi}(1-i\rho)\, e^{-\beta^2 q^2/2} \quad , \qquad (3.1)$$

where σ is the total NN cross-section, ρ the ratio of the real to imaginary parts of the imaginary amplitude and β^2 is the slope parameter.

For the NN parameters we take the values at 344 MeV as given by Alberi et al. [8] and found to give satisfactory results for the elastic d-d scattering at the incident deuteron lab momentum 1.75 GeV/c. The parameter values are:

$$\sigma_{pp} = 27 \text{ mb} \;,\quad \beta_p^2 = 0.44\ (\text{GeV/c})^{-2} \;,\quad \rho_{pp} = 0.6 \;,$$
$$\sigma_{np} = 34 \text{ mb} \;,\quad \beta_{np}^2 = 2.0\ (\text{GeV/c})^{2} \;,\quad \rho_{np} = 0 \;.$$

In the calculation we have used the average values of the neutron and proton parameters. However, it has been verified that the predictions of the average parameter values are not significantly different from the non-averaged ones.

It is well known that the oscillator model is inconsistent with the large-q behaviour of the ground state density of ^{4}He. Therefore it is tempting to study how the introduction of sophistication in the description of the ground state properties of ^{4}He affects the α-^{12}C scattering calculation. For this we use the realistic elastic charge form factor [17] for ^{4}He, the correlation function as proposed by Feshbach et al. [11] and the formalism of Sec.2.1 to obtain an expression for the N-α amplitude. Fortunately, all the integrals involved can be evaluated analytically and a closed expression is obtained. The derivation is described in some detail in Appx.B.

At this point it is tempting to compare the predictions of the N-α amplitudes so obtained with the N-α elastic differential cross-sections at 348 MeV which have also been measured by the Saclay group [6]. The comparison is made in Fig.2. The dashed curve is the prediction of the oscillator model with the charge rmsr = 1.67 fm. While the continuous and the dotted curves are, respectively, the predictions of the expression derived in Appx.B with and without the short-range correlation (we use the correlation range r_c = 0.6 fm (Ref.11)).

Two main points emerge from Fig.1. First, the NN amplitude as given by Alberi et al. [8] gives a reasonably satisfactory fit to the elastic N-α scattering data. This is a rather important point since the NN amplitude used by the Saclay group [6] in their α-^{12}C scattering analysis predicts very low p-α scattering at 348 MeV. Second, the fit to the p-α scattering in the low momentum transfer region cannot be improved simply by using the realistic charge form factor for ^{4}He. Rather, one would also have to invoke the short-range correlation to achieve the desired improvement.*) (The discrepancies present at larger angles should not be of much concern to us as this region is unlikely to contribute much to the α-^{12}C scattering calculation.)

ii) As regards the input target properties, they are taken as follows: For ^{12}C it is reasonable to assume that the neutron and proton densities are the same. The latter is obtained from the charge density as determined by Sick and McCarthy [18] after correcting for the finite size of the proton as in Ref.3.

Similarly the inelastic form factors are obtained from the inelastic charge form factors (corrections for the finite proton size applied), which are parameterized as:

$$F_{ch}^{\ell}(q^2) = B_\ell q^{n_\ell}(1 - C_\ell q^2)\, e^{-D_\ell q^2} \quad , \qquad (3.2)$$

*) Here it must be mentioned that some improvement is also achieved by using the spin-dependant NN amplitude [14].

where n_ℓ = 2 for the 2^+ and 0^+ case and 3 for the 3^- case. The parameter values for the 2^+ case are B_2 = 0.215 fm^2, C_2 = 0.137 fm^2 and D_2 = 0.549 fm^2 (Ref.19).

It is well known that the 3^- inelastic charge form factor does not give a satisfactory fit [3),9)] to the p-^{12}C scattering at 1 GeV. It highly overestimates the cross-section at the maximum and in addition predicts a well defined minimum in the angular distribution which is not found experimentally. In this case we therefore modify the electron scattering parameter values such that the height of the maximum in the 3^- p-^{12}C scattering is reproduced (the unwanted theoretical minima still present). These modified parameter values which have been used in the present work are: B_3 = 0.089 fm^3, C_3 = 0.0, D_3 = 0.76 fm^2 .

Finally it should be mentioned that we have also accounted for the Coulomb scattering in the calculation following the approach of Ref.3. However, the Coulomb phase function in the present work is obtained by folding the gaussian charge distribution of the α particle over that of the target.

IV. RESULTS AND DISCUSSION

The results of the calculation are presented and compared with experiment in Figs.2-5. The solid curves are obtained when the gaussian model N-α amplitudes are employed and also the correlation terms in Eq.(2.5) are neglected. It is seen that except for the 3^- case *) a good qualitative agreement with the data is achieved. The positions of the first minima in all the cases are well predicted. It is particularly satisfying that the second diffraction minima which are almost absent in the Saclay calculation [6] now appear prominently and that the overall situation is better.

*) The presence of oscillations in the calculated 3^- angular distribution in contrast with the rather smooth experimental behaviour is nothing but a manifestation of the existence of a similar behaviour in the 3^- p-^{13}C scattering calculations at 1 GeV using the inelastic electron scattering form factors. [3),9)]

However, appreciable discrepancies in the magnitude of the cross-sections in the small angle region where the theory is expected to work well are disturbing. That the situation does not improve by having a more realistic description of the projectile follows from the dashed curves in Figs.2-4, which have been calculated by using the realistic charge form factor for ^{4}He and incorporating the short-range correlation with the correlation range $r_c = 0.6$ fm (Ref.11). (The predictions without the short-range correlation are essentially the same as for the gaussian model.) Not unexpectedly, the situation at smaller angles becomes still worse. However, the trends at larger angles particularly for the elastic and the 2^+ inelastic scattering are in the right direction.

Next we study the effect of including the long-range correlation, i.e. the coupling between the elastic and the low-lying inelastic channels which are dominantly collective. This is achieved by evaluating $\Delta(b)$ as given by Eq.(2.8a) only for the 2^+ and 3^- states and substituting it in Eq.(2.5). For brevity, only the elastic case will be discussed here (like p-^{12}C scattering the coupling effect is relatively small in the inelastic scattering and is essentially of similar nature). The result is shown by the dotted curve *) in Fig.2. The situation seems to improve in two respects: first, the theoretical cross-sections are now closer to experimental ones at smaller angles and second that the positions of the calculated second and third minima are now very close to the positions of the corresponding experimental minima. However, the calculated large angle cross-sections are very low.

At this point it is helpful to recall the elastic p-^{12}C scattering calculation at 1 GeV (Ref.3). There the calculated angular distribution (without the coupling and correlation) agrees well with experiment up to about $\theta_{cm} \approx 18^\circ$. Beyond this the calculated cross-sections lie lower than the experimental ones and in addition the calculated second minima is shifted towards the smaller angles. When the long-range correlations (i.e. the coupling between the elastic and the 2^+ and 3^- inelastic channels) are included, the calculated curve goes down but the position of the second minimum is nearly corrected. Finally, the introduction of the correlations (Pauli, centre of mass and possible effects due to the residual interactions; all these correlations together are parametrized in terms of the correlation length $\ell_c \approx 0.72$ fm) in the intrinsic state (adiabatic model) brings the theoretical curve up leading to an agreement between theory and experiment.

*) This is calculated using the gaussian model N-α amplitude.

Ignoring for the moment the discrepancy in the magnitude of the calculated cross-sections at smaller angles, we note that the nature of the coupling effect in α-^{12}C scattering is quite the same as for the p-^{12}C scattering. It is therefore not unreasonable to expect that the effect of including the above-mentioned intrinsic-state correlations in the α-^{12}C scattering calculation *) would be essentially the same as that for the p-^{12}C scattering case. If so, then the consideration of both the coupling and the intrinsic-state correlations would leave the situation around the second maxima essentially unchanged but that at larger angles may improve in terms of both the position of the second and third minima and the magnitude of the cross-section.

Unfortunately in the α-^{12}C scattering case we cannot use the zero-range approximation for the input amplitude (here N-α amplitude) which, in the p-^{12}C scattering calculation, enables one to obtain a simple expression for the intrinsic state correlation correction [3]. Without the use of this approximation the evaluation of the correlation correction term in some reasonable way is involved and seems time consuming. We therefore leave this aspect for some future investigation **), believing on plausible grounds that the intrinsic state correlation effect on α-^{12}C scattering is qualitatively the same as found for the p-^{12}C scattering.

*) In the present formulation the effects of the coupling with the collective channels together with the intrinsic state correlations may be accounted approximately by writing Δ defined by Eq.(2.8a) as $\Delta \approx \Delta_{cp} + \Delta_{corr}$, where Δ_{cp} is calculated as stated before (i.e. considering only the 2^+ and 3^- states in the sum in Eq.(2.8a)) and

$$\Delta_{corr} = \int C_{in}(\vec{r}_1,\vec{r}_2)\, \Gamma(\vec{b}-\vec{s}_1)\, \Gamma(b-\vec{s}_2)\, d\vec{r}_1\, d\vec{r}_2 \quad ,$$

where $C_{in}(\vec{r}_1,\vec{r}_2)$ is the two-body intrinsic state correlation which may be obtained by following the arguments and approach of Ref.3. It must be admitted that there is some double counting but, as discussed in detail by Friar [20], when the overlap between the collective and the intrinsic spectra is small, the method is justified.

**) Here we also have in mind that unless the discrepancy at smaller angles is understood there is not much point in putting in labour in pushing the theory further at larger angles.

In the light of the above discussion we thus see that whereas the disagreement between theory and experiment at larger angles could be interpreted and possibly can be highly minimized, the same cannot be said for the discrepancy occurring in the small angle region. It seems hard to explain it within the present theoretical framework. Here it should be emphasised that the good agreement achieved by the Saclay group [6] in the small angle region is unrealistic because the NN amplitude used by them highly underestimates the elastic N-α cross-sections at 348 MeV - the present calculation suggests that a (spin-independent) N-α amplitude which predicts smaller N-α cross-sections gives smaller α-^{12}C scattering also (compare the continuous and dashed curves in Fig 2 and the corresponding curve in Fig.1).

Since at relatively low incident nucleon energies the NN parameters are not sharply defined, we therefore varied them (within reasonable limits) in order to see if the situation in the small angle region could be improved somewhat but did not succeed. As a matter of fact the small angle discrepancy is large enough to be explained this way in a consistent manner. One is therefore led to think that perhaps the assumption of free propagation of the incident α particles between its successive collisions with the target nucleons is unrealistic. In fact we would have felt much relaxed by blaming the theory at this point had not there appeared a recent analysis of 1.37 GeV α-Ca scattering by Lagky and Schaeffer,[14] where an approach similar to the present one works sufficiently well. The NN amplitude (spin-independent parametrization) used by these authors is about the same as in the present work and, moreover, the predictions of the spin-independent and spin-dependent NN amplitude are very similar. Thus the situation at smaller angles is much more complex than anticipated and we must find an explanation for the discrepancy which is consistent with the N-α scattering at 348 MeV, on one hand, and with the α-Ca calculation, on the other.

Finally it may be remarked that this is not the first instance of theory disagreeing greatly with experiment in the small angle region (where it is hoped to work well) at intermediate energies. We have already faced similar difficulties in 1 GeV proton scattering calculations for ^{208}Pb, ^{58}Ni, and ^{6}Li nuclei (Refs.3 and 21). It is therefore highly desirable that serious theoretical as well as experimental efforts be made to clarify the situation.

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APPENDIX A

In this appendix we outline a method of obtaining the expansion (2.5). Let

$$X_A = (0|\eta_1 \cdots \eta_A|0) \tag{A.1}$$

and assume that all the particles are identical and that

$$(m|\eta|m) \approx (0|\eta|0) \quad . \tag{A.2}$$

Further, let us denote

$$\eta_{00} = (0|\eta_i|0) \qquad (i = 1,2,\ldots,A) \tag{A.3}$$

and

$$C_2 = \sum_{m\neq 0} (0|\eta_i|m)(m|\eta_j|0) \quad ; \quad (i \neq j = 1,2,\ldots A). \tag{A.4}$$

Applying the closure once to the expression (A.1) gives

$$X_A = X_{A-1}\eta_{00} + \sum_{m_{A-1}\neq 0} (0|\eta_1\eta_2\cdots\eta_{A-1}|m_{A-1})(m_{A-1}|\eta_A|0) . \tag{A.5}$$

Similarly,

$$(0|\eta_1\eta_2\cdots\eta_{A-1}|m_{A-1}) = X_{A-2}(0|\eta_{A-1}|m_{A-1}) + \sum_{m_{A-2}\neq 0}(0|\eta_1\eta_2\cdots\eta_{A-2}|m_{A-2})(m_{A-2}|\eta_{A-1}|m_{A-1}) . \tag{A.6}$$

Substituting Eq.(A.6) in Eq.(A.5) and going on applying the closure as in Eq.(A.6), we obtain

$$\begin{aligned} X_A &= X_{A-1}\eta_{00} \\ &+ X_{A-2}\sum_{m_{A-1}\neq 0}(0|\eta_{A-1}|m_{A-1})(m_{A-1}|\eta_A|0) \\ &+ X_{A-3}\sum_{m_{A-1}\neq 0}\ \sum_{m_{A-2}\neq 0}(0|\eta_{A-2}|m_{A-2})(m_{A-2}|\eta_{A-1}|m_{A-1})(m_{A-1}|\eta_A|0) \end{aligned}$$

$$+ \cdots$$

$$+ \sum_{m_1 \neq 0} \cdots \sum_{m_{A-1} \neq 0} (0|\eta_1|m_1)(m_1|\eta_2|m_2) \cdots (m_{A-1}|\eta_A|0). \qquad (A.7)$$

Next consider the factor

$$\sum_{m_{A-r} \neq 0} \cdots \sum_{m_{A-1} \neq 0} (0|\eta_{A-r}|m_{A-r}) \cdots (m_{A-1}|\eta_A|0).$$

The term corresponding to $m_{A-r} = m_{A-r+1} = \cdots = m_{A-1}$ in the above expression is

$$\eta_{00}^{r-1} \sum_{m \neq 0} (0|\eta_{A-r}|m)\ (m|\eta_A|0) \quad ,$$

where we have used Eqs.(A.2) and (A.3). The remaining terms contain three-body and higher order correlations and will not be considered here. Thus Eq.(A.7) may be written as (using Eq.(A.4))

$$X_A = X_{A-1}\,\eta_{00} + X_{A-2}\,C_2 + X_{A-3}\,\eta_{00}\,C_2 + \cdots + \eta_{00}^{A-2}\,C_2 + \cdots \qquad (A.8)$$

Successive applications of the above relation to X_{A-1}, $X_{A-2}, \ldots$ and use of Eq.(A.2) leads to the expansion (2.5a).

Alternatively one may use the general form of the expansion containing terms up to two-body correlations,

$$X_A = \sum_{r=0} n_r(A)\, \eta_{00}^{A-2r}\, C_2^r + \cdots \quad , \qquad (A.9)$$

where $n_r(A)$ denotes the number of terms containing C_2^r. The sum in Eq.(A.9) extends up to $N_A = A/2$ for even A and $N_A = (A-1)/2$ for odd A. Obviously $n_0(A) = 1$ for all A and further $n_r(A) = 0$ for $r > N_A$.

Substituting the relation (A.9) in Eq.(A.8) we obtain the relation:

$$n_r(A) - n_r(A-1) = \sum_{t=1}^{A-1} n_{r-1}(A-t-1) \quad , \qquad (A.10)$$

which can be used to determine the coefficient of $\eta_{00}^{A-2r}\, C_2^r$ for any desired value of r.

For example, for $r = 1$ Eq.(A.10) using $n_0(A) = 1$ gives

$$n_1(A) - n_1(A-1) = A-1 \quad . \qquad (A.11)$$

From Eq. (A.11) and using $n_1(2) = 1$ one easily obtains

$$n_1(A) = A(A-1)/2 \quad . \qquad (A.12)$$

Similarly applying relation (A.10) for $r = 2$ we get

$$n_2(A) - n_2(A-1) = n_1(A-2) + n_1(A-3) + \cdots + n_1(2) \quad . \qquad (A.13)$$

Successive applications of Eq.(A.13) give

$$n_2(A) = n_1(A-2) + 2n_1(A-3) + \cdots + (A-3)\, n_1(2) \quad .$$

Using Eq.(A.12) we then obtain

$$n_2(A) = \frac{A!}{(A-4)!\,4!} \quad . \qquad (A.14)$$

The expansion for the inelastic case may be obtained similarly by applying the closure property successively and using the results for the elastic case expansion.

APPENDIX B

The appendix describes the derivation of an expression for the N-α scattering amplitude in terms of the realistic charge form factor for ^{4}He and the two-body correlation function as proposed by Feshbach et al.[11]

We follow the approach outlined in Sec.II. Considering the case for A = 4, replacing the ground-state density and the profile function with their respective Fourier transforms and substituting Eq.(2.4a) in Eq.(2.1) we obtain the following expression for the elastic scattering amplitude:

$$F_{00}(q) = F_0(q) = F_{C_1}(q) = F_{C_2}(q) \quad , \qquad (B.1)$$

where

$$F_0(q) = \frac{ik}{2\pi}\int d^2b \, e^{i\vec{q}\cdot\vec{b}} \left\{ 1 - \left[1 - \frac{1}{2\pi i k}\int d^2q \, e^{-i\vec{q}\cdot\vec{b}} f(q)\, F_\alpha(q) \right]^4 \right\}, \qquad (B.2)$$

$$F_{C_1}(q) = -\frac{3ik}{\pi}\int d^2b \, e^{i\vec{q}\cdot\vec{b}} \left[1 - \frac{1}{2\pi i k}\int d^2q \, e^{-i\vec{q}\cdot\vec{b}} f(q)\, F_\alpha(q) \right]^2 \Delta(b) \qquad (B.3)$$

and

$$F_{C_2}(q) = -\frac{ik}{2\pi}\int d^2b \, e^{i\vec{q}\cdot\vec{b}} \Delta^2(b) \, . \qquad (B.4)$$

In the above expressions $F_\alpha(q)$ is the ground state form factor for ^{4}He, $f(q)$ is the N-N amplitude and $\Delta(b)$ is the same as defined in Sec.2.1.

The quantity $F_\alpha(q)$ is obtained from the elastic charge form factor for ^{4}He as given by Frosch et al.[17] and which fits nicely the elastic electron scattering data available at present. Correcting the charge form for the finite size of the proton we have

$$F_\alpha(q) = \left[1 - (a^2q^2)^n\right] e^{-(b^2 - \langle r^2\rangle_p /6)q^2} \quad , \qquad (B.5)$$

where $\langle r^2\rangle_p$ is the charge mean square radius of the proton and the constants,

$$a = 0.316 \text{ fm} \quad , \quad b = 0.681 \text{ fm} \quad \text{and} \quad n = 6 \quad .$$

For $f(q)$ we use the parametrization given by Eq.(3.1).

Substituting Eqs.(B.5) and (3.1) in Eq.(B.2) it is found that all the integrals can be evaluated analytically. The final expression for $F_0(q)$, which depends only on the ground state density of the target, is

$$F_0(q) = \frac{ik}{2\omega^2}\sum_{r=1}^{4} (-1)^{r+1} \binom{4}{r} \frac{P^r}{r} e^{-\frac{q^2}{4r\omega^2}} (1+W) \; , \qquad (B.6)$$

$$W = \sum_{j=1}^{r} \binom{r}{j} (-C_n)^j \sum_{m_1=0}^{n} \cdots \sum_{m_j=0}^{n} \binom{n}{n-m_1} \cdots \binom{n}{n-m_j} \frac{N_j!}{m_1!\, m_2! \cdots m_j!} \left(-\frac{1}{r}\right)^{N_j} L_{N_j}\!\left(\frac{q^2}{4r\omega^2}\right),$$

where

$$\omega^2 = \frac{1}{2(\beta^2 + 2\alpha^2)} \quad , \quad \alpha^2 = b^2 - \langle r^2\rangle_p/6 \quad , \quad C_n = (2\alpha\omega)^{2n} n! \; ,$$

$$P = \frac{\omega^2\sigma}{2\pi}(1 - i\rho) \quad , \quad N_j = m_1 + m_2 + \cdots + m_j$$

and L_{N_j} is the Laguerre polynomial of order N_j .

To evaluate F_{C_1} and F_{C_2} as given by Eqs.(B.3) and (B.4) we need the two-body correlation function. Feshbach et al.[11] have discussed in detail the construction of the pair correlation function for ^{4}He within the framework of the oscillator model. Defining

$$\omega_0(\nu, \vec{u}) = \left(\frac{\nu}{\pi}\right)^{3/2} e^{-\nu u^2} \; , \qquad (B.7)$$

the pair correlation function as proposed by these authors may be written as

$$C(\vec{r}_1, \vec{r}_2) = C_{cm} + g(r)\,\omega_0(4\nu, \vec{X})\,\omega_0(\tfrac{\nu}{2}, \vec{r}) + \cdots$$

$$C_{cm} = \omega_0(4\nu, \vec{X})\,\omega(\tfrac{\nu}{2}, r) - \omega_0(\tfrac{4}{3}\nu, \vec{r}_1)\,\omega_0(\tfrac{4}{3}\nu, \vec{r}_2), \qquad (B.8)$$

where ν is the same as in Ref.11

$$\vec{X} = (\vec{r}_1 + \vec{r}_2)/2 \quad , \quad \vec{r} = \vec{r}_1 - \vec{r}_2 \quad ,$$

and $g(r)$ which describes the short-range correlation is given by

$$g(r) = -e^{-\lambda r^2} \quad . \qquad \text{(B.9)}$$

The quantity λ is related to the correlation range r_c as described in Ref.11. The correlation $C_{cm}(\vec{r}_1, \vec{r}_2)$ is due to the centre of motion constraint. Further, the quantities neglected in Eq.(B.8) are of the order of $(r_c/R)^3$ where R is the radius of the α particle [11].

Expressing $\Delta(b)$ in terms of the correlation function as stated above and substituting it in Eqs.(B.3) and (B.4) we again find that all the integrals can be evaluated analytically. The final expressions may be written as

$$F_{c1}(q) = -3i\,\Gamma^2(0) \sum_{\ell=1}^{3} g_\ell \left\{ \frac{e^{-\frac{q^2}{4\omega_\ell}}}{\omega_\ell} + \sum_{r=1}^{2} \binom{2}{r} \frac{(-\rho)^r}{(\omega_\ell + r\omega^2)} e^{-\frac{q^2}{4(\omega_\ell + r\omega^2)}}\, U \right\} \qquad \text{(B.10)}$$

$$U = 1 + \sum_{j=1}^{r} (-C_n)^j \binom{r}{j} \sum_{m_1=0}^{n} \cdots \sum_{m_j=0}^{n} \binom{n}{n-m_1} \cdots \binom{n}{n-m_j} \frac{N_j!}{m_1! \cdots m_j!} \left(\frac{-\omega^2}{\omega_\ell + r\omega^2} \right)^{N_j}$$

$$\times\, L_{N_j}\left(\frac{q^2}{4(\omega_\ell + r\omega^2)} \right)$$

$$F_{c2}(q) = -\frac{ik}{2}\, \Gamma^4(0) \sum_{\ell=1}^{3} \sum_{m=1}^{3} \frac{g_\ell\, g_m}{(\omega_\ell + \omega_m)} \exp\left\{ -\frac{q^2}{4(\omega_\ell + \omega_m)} \right\} \qquad \text{(B.11)}$$

where

$$g_1 = \frac{8\nu^2\beta^4}{(1+2\nu\beta^2)\,(1+4\nu\beta^2)} \quad , \qquad \omega_1 = \frac{4\nu}{1+4\nu\beta^2}$$

$$g_2 = -\frac{64\nu^2\beta^2}{(3+8\nu\beta^2)^2} \quad , \qquad \omega_2 = \frac{8\nu}{(3+8\nu\beta^2)}$$

$$g_3 = -\frac{8\nu^2\beta^4}{(1+4\nu\beta^2)\,(1+2\nu\beta^2+4\lambda\beta^2)} \cdot \left(\frac{\nu}{\nu+2\lambda} \right)^{\frac{1}{2}} \quad , \qquad \omega_3 = \omega_1$$

and

$$\Gamma(0) = \frac{\sigma(1 - i\rho)}{4\pi\beta^2} \quad .$$

The other quantities are the same as defined before.

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FIGURE CAPTIONS

Fig.1 Elastic scattering of 348 MeV protons on ^{4}He. Dashed curve: Oscillator model calculation corresponding to the charge rmsr 1.67 fm. Dotted curve: Calculation using the realistic charge form factor for ^{4}He and including the c.m. correlation. Continuous curve: Same as the dotted curve but with the short-range correlation (r_c = 0.6 fm).

Fig.2 Elastic scattering of 1.37 GeV α particles on ^{12}C. Solid curve: The oscillator model N-α amplitude. Dashed curve: N-α amplitude calculated using the realistic form factor for ^{4}He and including the short-range correlation. Dotted curve: Same as the solid curve but including the coupling (the 2^+ and 3^- levels).

Fig.3 The 2^+ inelastic scattering of 1.37 GeV α particles on ^{12}C. The solid and dashed curves are the same as in Fig.2.

Fig.4 Same as in Fig.3 but for the 0^+ (7.65 MeV) level.

Fig.5 The 3^- inelastic scattering of 1.37 GeV α particles on ^{12}C. Solid curve is the same as in Fig.2.

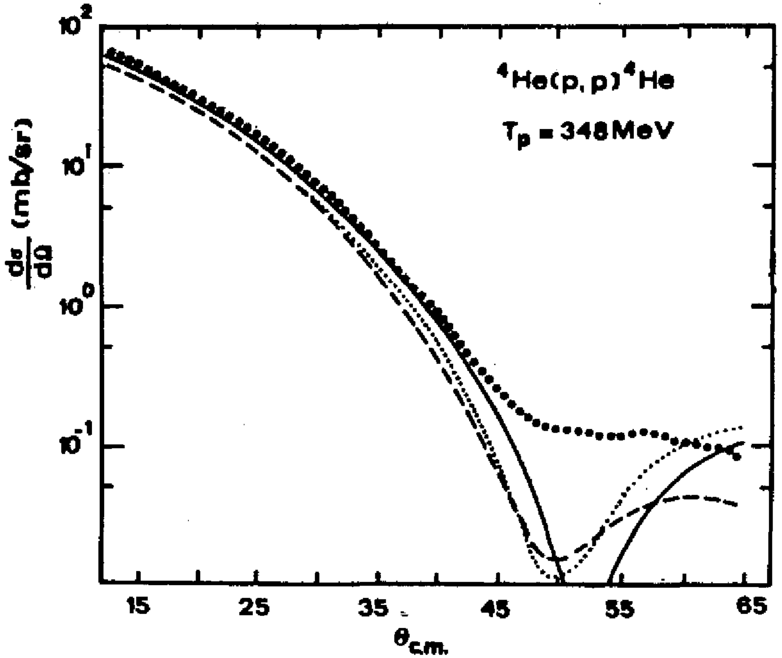

$^4He(p,p)^4He$
$T_p = 348\,MeV$
$\frac{d\sigma}{d\Omega}$ (mb/sr)
10^2
10^1
10^0
10^{-1}
15
25
35
45
55
65
$\theta_{c.m.}$

FIG. 1

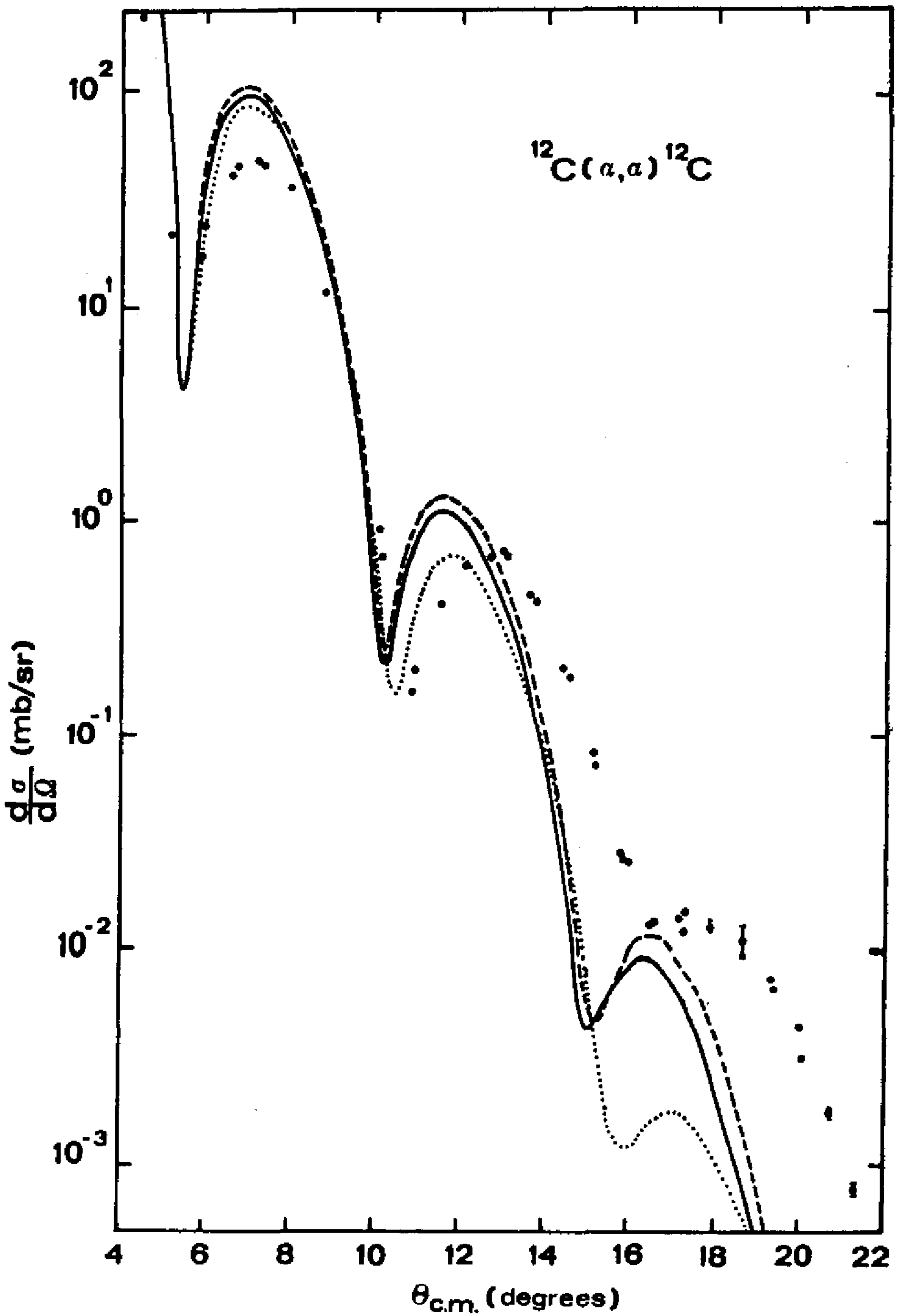

$^{12}C(\alpha,\alpha)^{12}C$
$\frac{d\sigma}{d\Omega}$ (mb/sr)
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
4
6
8
10
12
14
16
18
20
22
$\theta_{c.m.}$ (degrees)

FIG. 2

FIG. 4

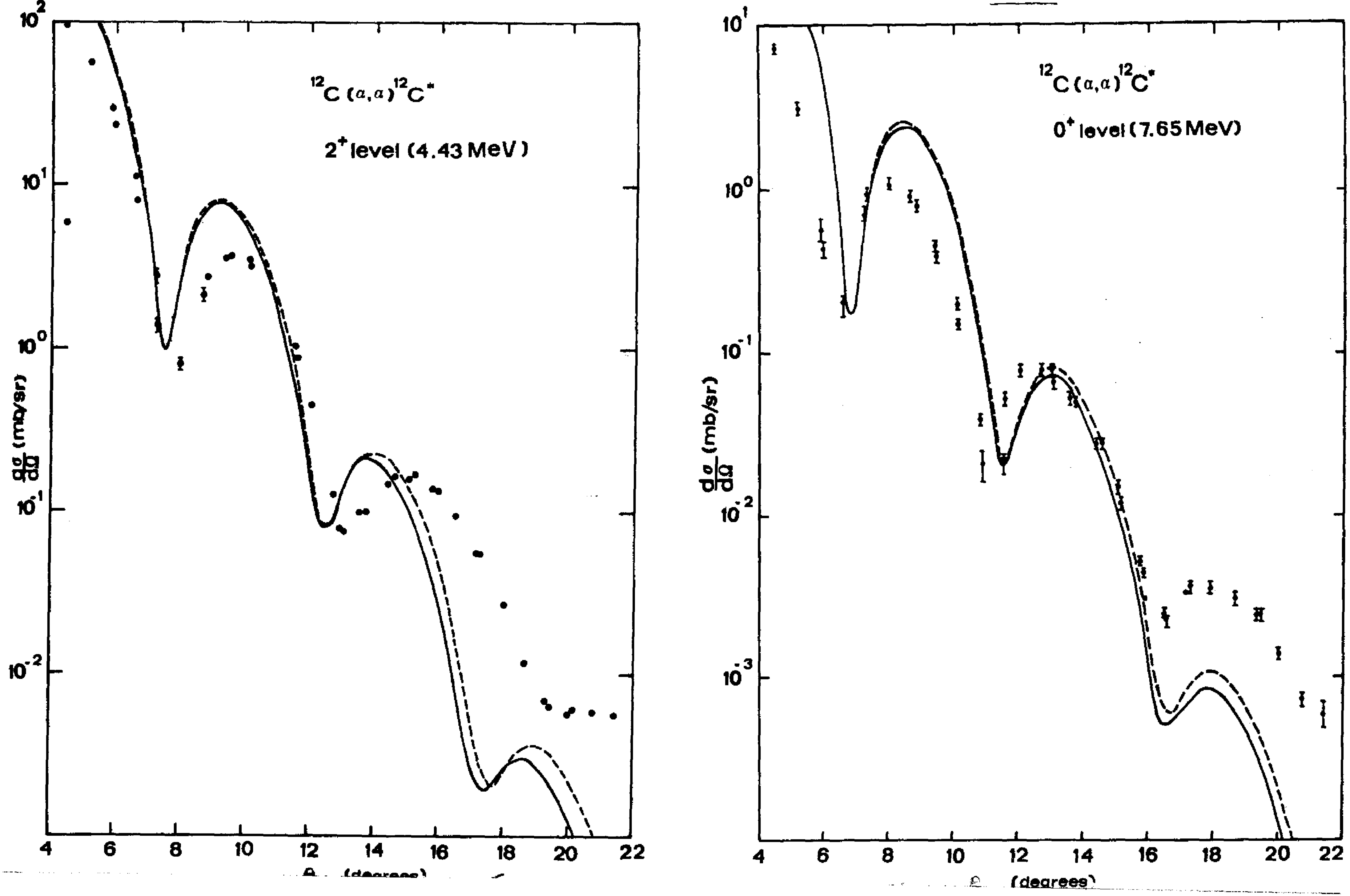
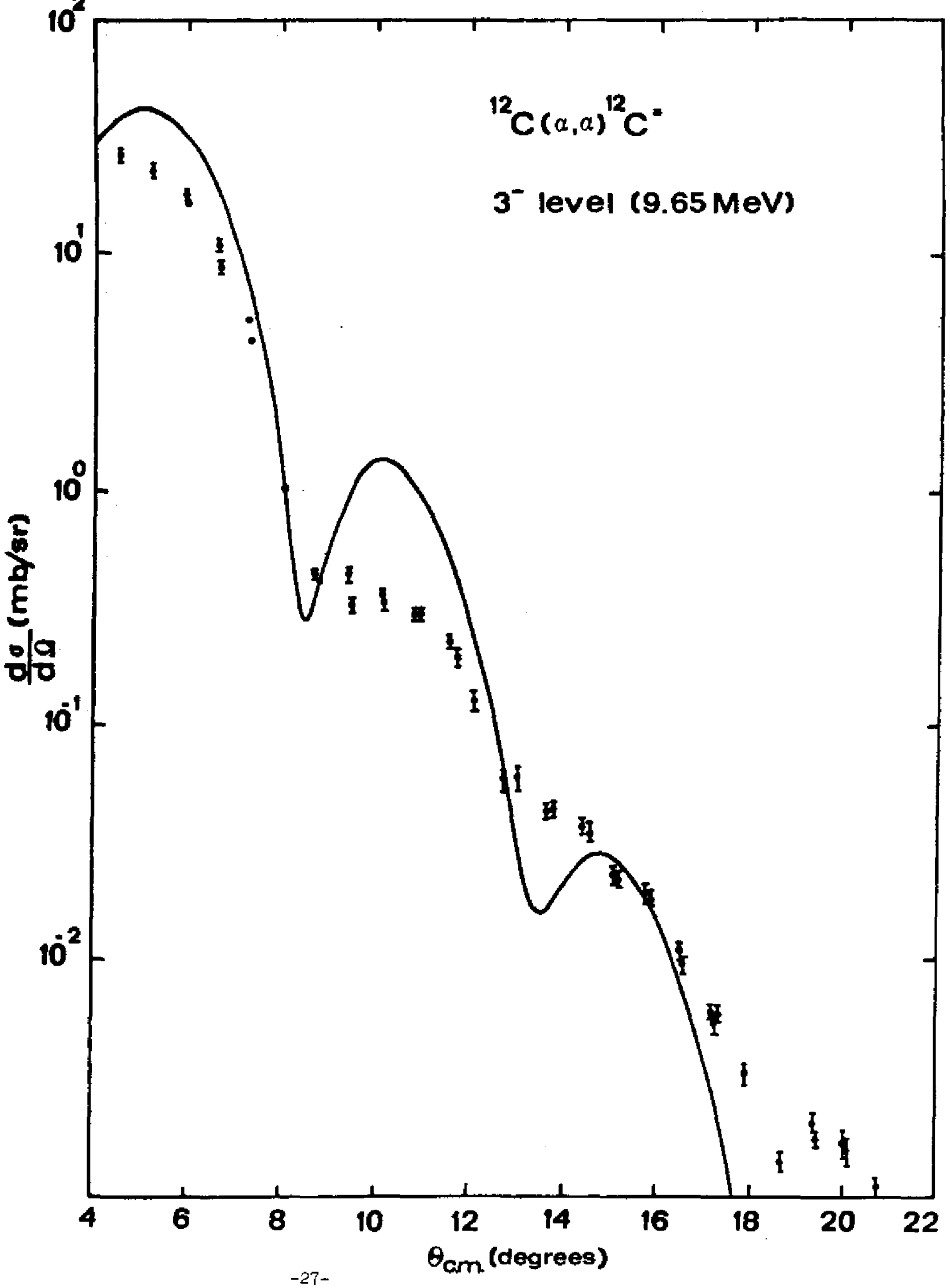

$^{12}C(\alpha,\alpha)^{12}C^*$
2^+ level (4.43 MeV)
$\frac{d\sigma}{d\Omega}$ (mb/sr)
θ (degrees)
$^{12}C(\alpha,\alpha)^{12}C^*$
0^+ level (7.65 MeV)
$\frac{d\sigma}{d\Omega}$ (mb/sr)
θ (degrees)

Fig. 5



$^{12}C(\alpha,\alpha)^{12}C^*$
3^- level (9.65 MeV)
10^2
10^1
10^0
10^{-1}
10^{-2}
$\frac{d\sigma}{d\Omega}$ (mb/sr)
4 6 8 10 12 14 16 18 20 22
$\theta_{c.m.}$ (degrees)