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INVESTIGATION OF THE CHARGE EXCHANGE PROCESS $\pi^- d \to \pi^0 nn$ *

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ABSTRACT

The transition rate has been calculated for the charge exchange of stopped π^- in the deuteron. Our present result is $\hbar\omega(\pi^- d \to \pi^0 nn) = 0.695 \times 10^{-4}$ eV. Making use of the value $\hbar\omega(\pi^- d \to nn) = 0.682$ eV, which was previously obtained, we have estimated the branching ratio $\omega(\pi^- d \to \pi^0 nn)/\omega(\pi^- d \to nn) = 1.02 \times 10^{-4}$.

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The branching ratio $\omega(\pi^- d \to \pi^0 nn)/\omega(\pi^- d \to \text{all})$ was recently measured for the first time by stopping the π^- in a liquid-deuterium target [1]. The result was $(1.45 \pm 0.19) \times 10^{-4}$, which was obtained by making use of the average value, $\omega(\pi^- d \to nn)/\omega(\pi^- d \to \gamma nn) = 2.97 \pm 0.07$, of those produced by two groups [2),3)].

The negative pion is generally absorbed by a deuteron from the K orbit. Since π^- and π^0 have, of course, the same parity and the two-neutron system has the ^3P state in the final state, π^0 is obviously in the p-state relative to the centre-of-mass of the two neutrons. From the fact that the overlap of p- and s-states is small, it might be speculated that the reaction $\pi^- d \to \pi^0 nn$ is highly forbidden. In the impulse approximation, MacDonald et al. [1] derived the predicted branching ratio $\omega(\pi^- d \to \pi^0 nn)/\omega(\pi^- d \to \text{all})$, which was in the range 1.39×10^{-4} to 1.59×10^{-4} . Their result was obtained by limiting the cross-section of π^- in flight to zero energy. It is also interesting to test the microscopic description for processes of the stopped π^- absorption by the deuteron.

Five years ago, on the basis of the dispersion theory [2], an effective interaction was proposed to analyse the pion absorption and production at low energy [4]. With this interaction, the absorption rate for the process $\pi^- d \to nn$ at rest is given as [5]

$$\hbar\omega(\pi^- d \to nn) = \frac{16 g^2 \hbar^3 k}{3 a_0^3 M_n \tilde{\mu}_\pi c} [F(k)]^2 , \qquad (1)$$

where the coupling constant $g^2 = 0.08$, a_0 and c are the Bohr radius and the speed of light, M_n and $\tilde{\mu}_\pi$ are the neutron mass and the pion reduced mass and $k = 1.854$ fm^{-1} is the momentum of relative motion of the two outgoing neutrons. The matrix element $F(k) = 0.3642$ fm$^{1/2}$ is given in Ref.4. After some numerical calculations, we obtain the transition rate for the process $\pi^- d \to nn$ as [5]

$$\hbar\omega(\pi^- d \to nn) = 0.682 \text{ eV} . \qquad (2)$$

By the same method as used in the calculation of the two-nucleon process $\pi^- d \to nn$, we can evaluate the transition rate of the charge exchange, $\pi^- d \to \pi^0 nn$. Fig.1 shows the Feynman diagram for this process. Since the pion is absorbed from the K orbit and the outgoing neutral pion is in low energy, it is a sufficiently good approximation to take in each vertex the following interaction [6]:

$$H'(1) = [D(p_0)\,\tau_i^{(1)}\tau_j^{(1)} + E(p_0)\,\tau_j^{(1)}\tau_i^{(1)}]\;\phi_j^+(q)\;\phi_i(p)\;, \qquad (3)$$

where τ is the pion isospin, ϕ is the pion field and D and E are given by the pion-nucleon scattering length as

$$D(p_0) = E(-p_0) = \frac{2\pi}{3}\;(a_1 + 2a_3) - \frac{2\pi p_0}{3\mu_\pi c}\,(a_1 - a_3)\quad . \qquad (4)$$

After the action of the isospin operators on the wave functions, one can easily find the effective interaction

$$\begin{aligned} H_I &= \langle \text{isospin}|H'(1) + H'(2)|\text{isospin}\rangle \\ &= [D(\mu_\pi) - E(\mu_\pi)]\;\phi_\pi^+(q)\;\phi_\pi(p) \\ &= -\frac{4\pi}{3}\;(a_1 - a_3)\;\phi_\pi^+(q)\;\phi_\pi(p)\quad . \end{aligned} \qquad (5)$$

In the present case, the pion wave functions are expressed as

$$\langle 1|\phi_1^+(q)|0\rangle = \frac{\hbar c}{\sqrt{2E_q L^3}}\;f_\ell^\pi(qr_2)\;Y_{\ell m}(\hat{q})\qquad , \qquad (6)$$

$$\langle 0|\phi_1(k)|1\rangle = \frac{\hbar c}{\sqrt{2\tilde{\mu}_\pi c^2}}\;2\left(\frac{Z}{a_0}\right)^{3/2}\exp\left(-\frac{r_1}{2a_0}\right)\;Y_{00}(\hat{r}_1)\quad , \qquad (7)$$

where $E_q = (\mu_\pi^2 c^4 + \hbar^2 q^2 c^2)^{1/2}$, $\tilde{\mu}_\pi$ is the reduced mass of the bound pion, Z is the atomic number and L is the length of the normalized box. It is convenient to separate the co-ordinates into the centre-of-mass and relative co-ordinates of the two nucleons. Then, we have

$$\langle 1|\phi^+(q)|0\rangle = \frac{\hbar c}{\sqrt{2E_q L^3(2\pi)^3}}\;2\sqrt{4\pi}\;i\left[\sqrt{3}\;f_1^\pi\left(\frac{q}{2}\,r\right)\;Y_{10}(\hat{r})\right]\exp(iq\cdot R) \qquad (8)$$

for the p-wave of π^0 . Since the wave length of the bound pion is much longer than the deuteron size, we can write the wave function of the bound pion as

$$\langle 0|\phi_\pi|1\rangle = \frac{\hbar c}{\sqrt{2\tilde{\mu}_\pi c}}\;2\left(\frac{Z}{a_0}\right)^{3/2}\exp\left(-\frac{r}{2a_0}\right)\;Y_{00}(\hat{r})\;. \qquad (9)$$

Some work in the algebra leads to the transition rate for the single scattering diagram (Fig.1a)

$$\omega_1(\pi^- d \to \pi^0 nn) = \frac{8M\hbar}{(2\pi)^3\,\tilde{\mu}_\pi^2\,a_0^3}\left[\frac{4\pi}{3}\,(a_1 - a_3)\right]^2 G(F)\;, \qquad (10)$$

where the matrix element $G(F)$ is given by

$$G(F) = \int_0^\xi \left[\frac{8}{25}\,F_d^2(k,q) + \left\{F_s(k,q) + \frac{\sqrt{2}}{5}\,F_d(k,q)\right\}^2\right] kq^2\,dq\;, \qquad (11)$$

with the momentum k of relative motion of the two outgoing neutrons. By energy conservation, we find

$$k = \left[\frac{M_n}{\hbar}\left\{1.0878\text{ MeV} - \frac{\hbar^2 q^2}{2\tilde{\mu}_\pi}\right\}\right]^{1/2}\quad . \qquad (12)$$

The upper limit of the integral ξ is determined by Eq.(12), i.e. $\xi = \left[\frac{2\tilde{\mu}_\pi}{\hbar^2}\times 1.0878\text{ MeV}\right]^{1/2}$. The functions $F_s(k,q)$ and $F_d(k,q)$ associated with s- and d-states of the deuteron are given as

$$F_{s,d}(k,q) \equiv \int_0^\infty g_1^{NN}(kr)\;f_1^\pi\left(\frac{q}{2}\,r\right)\;\exp\left(-\frac{r}{2a_0}\right)\;\psi_{s,d}(r)\;r^2\,dr\quad , \qquad (13)$$

where $\psi_{s,d}(r)$ are the radial parts of the deuteron wave function and $g_1^{NN}(kr)$ and $f_1^\pi\left(\frac{q}{2}\,r\right)$ are the radial wave functions of the p-state for relative motion of the two outgoing neutrons and the neutral pion relative to the centre-of-mass of the two neutrons, respectively. It is generally expressed by the phase shifts as

$$f_\ell^\pi\left(\frac{q}{2}\,r\right) = e^{i\delta_\ell}\left[\cos\,\delta_\ell j_\ell\left(\frac{q}{2}\,r\right) - \sin\,\delta_\ell n_\ell\left(\frac{q}{2}\,r\right)\right]\quad , \qquad (14)$$

where j_ℓ and n_ℓ are the spherical Bessel and Neuman functions. $g_\ell^{NN}(kr)$ is also expressed in the same form as $f_\ell^\pi\left(\frac{q}{2}\,r\right)$. Since energies of the outgoing particles are very low, the wave functions $g_1^{NN}(kr)$ and $f_1^\pi\left(\frac{q}{2}\,r\right)$ are approximated by the spherical Bessel functions with sufficient guarantee. In the same fashion as given in Ref.4, the rescattering process (Fig.1b) can be calculated and the resultant transition rate is

$$\omega_2(\pi^- d \to \pi^0 nn) = \frac{8M_n\hbar}{(2\pi)^3\,\tilde{\mu}_\pi^2\,a_0^3}\left[\frac{\lambda_0}{2(2\pi)^2}\right]^2 G(\tilde{F})\;, \qquad (15)$$

where $\lambda_0 = \frac{8\pi^2}{9}\,(2a_1^2 + 8a_1a_3 - a_3^2)$ and $G(\tilde{F})$ is given by the same function as Eq.(11), provided $F(k,q)$ is replaced by

$$\tilde{F}_{s,d}(k,q) = \int_0^\infty g_1^{NN}(kr)\; f_1^\pi\!\left(\frac{q}{2}r\right)\, \exp\!\left(-\frac{r}{2a_0}\right)\, \psi_{s,d}(r)\; r\, dr \quad . \tag{16}$$

The difference between F and $\tilde{F}$ appears only in the power of r, because the rescattering term has the matrix element such as $\langle|\frac{1}{r}|\rangle$.

In the numerical calculation we have used the factorized deuteron wave function with $L = N = 4$ (which is the number of factorization parameters).[7] This wave function contains the d-state of 6.964% and the hard core range is 0.485 fm. With the values $a_1 - a_3 = 0.291\ \lambda_\pi$ [8] and $a_1 + 2a_3 = -0.055\ \lambda_\pi$,[4] we obtain

$$\hbar\omega(\pi^-d \to \pi^0 nn) = 0.695 \times 10^{-4}\ \mathrm{eV} \quad , \tag{17}$$

where contribution from the d-state of the deuteron wave function is only 4.2%. The rescattering effects are negligibly small, i.e. 1.25×10^{-10} eV. Fig.2 shows the momentum distribution of the transition rate. There is a peak at $q = 0.069\ \mathrm{fm}^{-1}$. Because of low intensity, it might be difficult to measure the π^0 momentum distribution. However, the measurement is, in principle, possible since π^0 decays into two gamma rays. With the results (2) and (17), we find the branching ratio

$$\frac{\omega(\pi^-d \to \pi^0 nn)}{\omega(\pi^-d \to nn)} = 1.02 \times 10^{-4} \quad . \tag{18}$$

The experimental value of this branching ratio [1)-3)] is $(1.93 \pm 0.25) \times 10^{-4}$.

Recently, Gibbs et al.[9] calculated the radiative pion capture rate, $\pi^-d \to \gamma nn$, although the rescattering effects were neglected because they were less than 6%. Their result is

$$\hbar\omega(\pi^-d \to \gamma nn) = 0.283\ \mathrm{eV}. \tag{19}$$

With this value we obtain the branching ratios

$$\frac{\omega(\pi^-d \to nn)}{\omega(\pi^-d \to \gamma nn)} = 2.41 \quad ,$$

$$\frac{\omega(\pi^-d \to \pi^0 nn)}{\omega(\pi^-d \to \gamma nn)} = 2.46 \times 10^{-4} \quad , \tag{20}$$

$$\frac{\omega(\pi^-d \to \pi^0 nn)}{\omega(\pi^-d \to \mathrm{all})} = 0.720 \times 10^{-4} \quad ,$$

for which the experimental results [1] are (2.97 ± 0.07), $(5.76 \pm 0.87) \times 10^{-4}$ and $(1.45 \pm 0.19) \times 10^{-4}$, respectively. Our results are reasonably good. However, our result for the transition rate $\omega(\pi^-d \to \pi^0 nn)$ is smaller than the experimental value [1] by a factor of 2.

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REFERENCES

1) R. MacDonald, D.S. Beder, D. Berghofer, M.D. Hasinoff, D.F. Measday, M. Salomon, J. Spuller, T. Suzuki, J.M. Poutissou, R. Poutissou, P. Depommier and J.K.P. Lee, Phys. Rev. Letters 38, 746 (1977).

2) P.K. Kloeppel, Nuovo Cimento 34, 11 (1964).

3) J.W. Ryan, Phys. Rev. 130, 1554 (1963).

4) Il-T. Cheon and J. Cugnon, Nucl. Phys. B66, 412 (1973).

5) Here the misprint in Ref.4 is corrected.

6) H. Miyazawa, Phys. Rev. 104, 1741 (1956).

7) J.W. Humberston and J.B.G. Wallace, Nucl. Phys. A141, 362 (1970).

8) V.K. Samaranayake and W.S. Woolcock, Phys. Rev. Letters 15, 936 (1965).

9) W.R. Gibbs, B.F. Gibson and G.J. Stephenson, Jr., Phys. Rev. C16, 327 (1977).

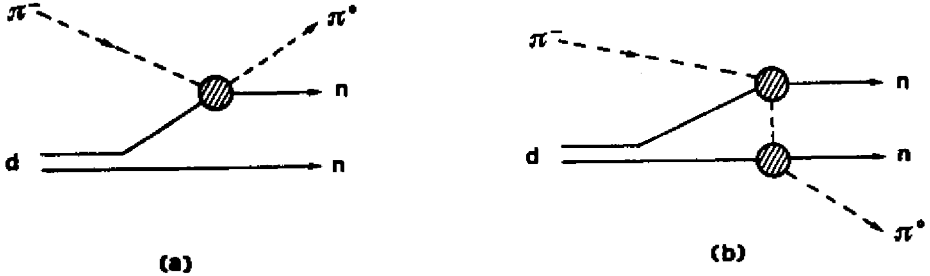

Fig. 1

Feynman diagrams for the charge exchange process $\pi^- d \to \pi^0 nn$.

The momentum distribution of the transition rate $\log[d\Gamma\omega(\pi^- d \to \pi^0 nn)/dq]$. q is the momentum of π^0 and k is the momentum of relative motion of the two outgoing neutrons (see Eq.(12)in the text).

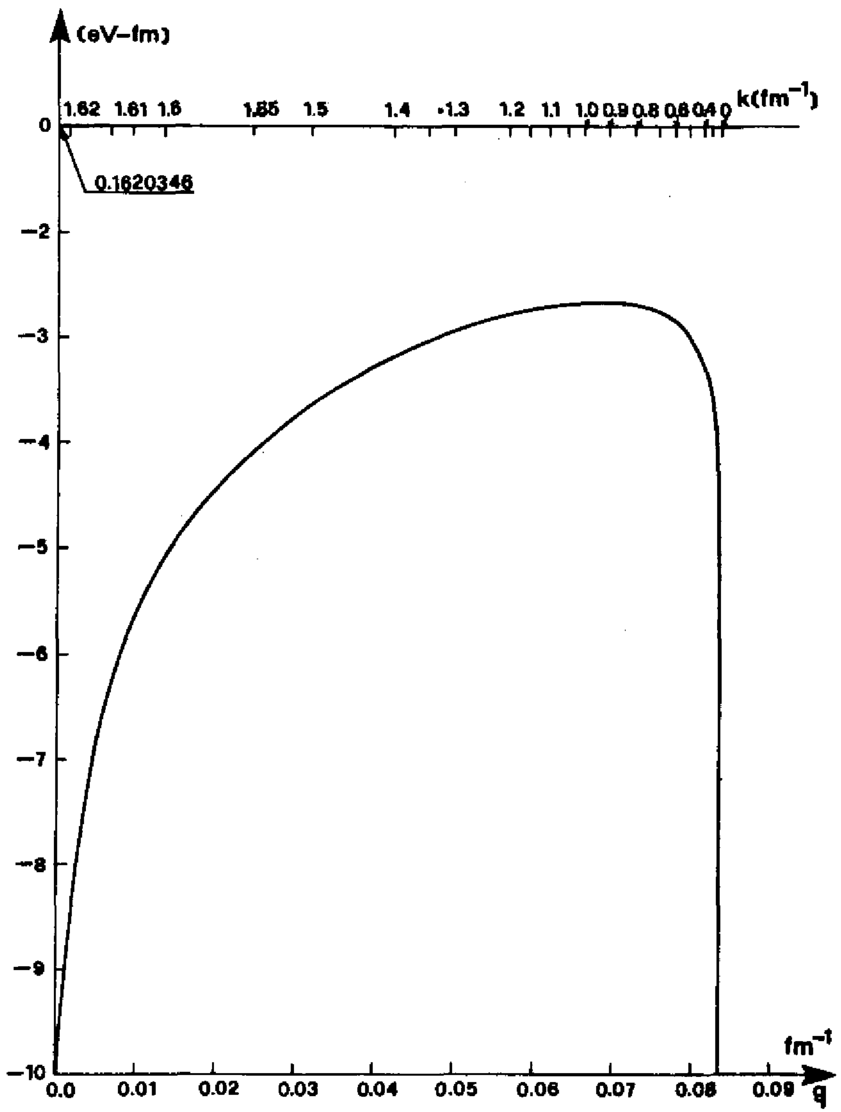

Fig. 2