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A.Sandulescu, I.Silisteanu, R.Wünsch

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A.Sandulescu,[1] I.Silisteanu,[1] R.Wünsch[2]

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[1] On leave from Institute for Atomic Physics, Bucharest, Romania.

[2] On leave from Zentralinstitut für Kernforschung Rossendorf, GDR.

Сандулеску А., Силиштяну И., Вьюнш Р. E4 - 10771

Альфа-распад в рамках теории Фешбаха

В рамках теории ядерных реакций Фешбаха ширины альфа-распада определяются альфа-дочерним ядерным потенциалом и факторами формирования. Показано, что рассчитанные абсолютные значения альфа-ширин для легких изотопов Po находятся в хорошем согласии с экспериментальными данными, если используется реальная часть оптического потенциала с параметрами, подобранными по нижней энергии α-рассеяния.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Alpha -Decay within Feshbach Theory of Nuclear Reactions

In the frame of Feshbach theory of nuclear reactions the alpha-decay widths are determined by the alpha-daughter nucleus optical potential and by the preformation factors. It is shown that the calculated absolute values of the alpha widths for the light Po isotopes are in good agreement with ~~the~~ experimental data, if the real part of the optical potential with the parameters fitted by the low energy α-scattering is used.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

It is well known, that the relative alpha decay widths calculated in the framework of R-matrix theory of nuclear reactions [1,2] are in reasonable agreement with the experimental data, however, the predicted absolute values deviate drastically from the experimental ones [3]. Applying the R-matrix theory to alpha decay, one finds that the alpha widths are exclusively determined by the surface terms like penetrabilities and reduced widths and that the absolute values are strongly dependent on the channel radius parameter.

In order to avoid these difficulties new methods were proposed for determining the alpha widths without using the channel radius [4-7].

The present paper is aimed to describe the alpha decay within the Feshbach theory of nuclear reactions [8] in complete analogy to the continuum shell model for nucleons [9]. This theory unifies the advantages of the shell model description of the parent and daughter nuclei with the optical model for the emitted alpha particle. This model is explained in details in section 2, where a new expression for the alpha decay widths

is obtained. In the last section this expression is applied to the alpha-decay of the Po isotopes. It is shown that the absolute alpha-decay widths are in reasonable agreement with the experimental data for these isotopes.

2. THE DECAY CONSTANT

The starting point of our considerations is the fact that for the description of such a phenomenon like alpha-decay we need two sets of basic states. The first set consists of the bound states $|\Phi_k\rangle$ of the parent nucleus obtained by diagonalizing the shell model Hamiltonian H^{SM} within the functional subspace of bound states, denoted as the Q-space, which reproduce the energy spectrum of the parent nucleus. The second set involves the scattering states which consist of products of the intrinsic states of the daughter nucleus $|\Phi^D\rangle$ and of the alpha particle $|\Phi^\alpha\rangle$ of the angular $|\ell m\rangle$ and radial $|\Phi_{\epsilon\ell}\rangle$ parts of the relative motion characterized by the angular momentum ℓ, its projection m and the energy ϵ of the relative motion.

$$|\chi_E^c\rangle = [\,|\Phi^D\rangle|\Phi^\alpha\rangle|\ell\rangle|\Phi_{\epsilon\ell}\rangle\,]_c \qquad (1)$$

In the above expression, we have denoted by the channel index c all discrete quantum numbers necessary for the description of the alpha-particle, daughter nucleus and the relative motion and by E the continuous quantum number which describes the total energy of the system, i.e., the sum of the ground and excitation energies of the daughter nucleus, of the alpha-particle and the energy ϵ of the relative motion.

The space formed by the scattering states $|\chi^c_E>$ with $\epsilon > 0$ will be denoted as the P-space.

Both basic states are normalized

$$<\Phi_k|\Phi_{k'}> = \delta_{kk'} \tag{2}$$

$$<\chi^c_E|\chi^{c'}_{E'}> = \delta_{cc'}\delta(E-E') \tag{3}$$

so that we can define the projection operators

$$P = \sum_c \int dE\, |\chi^c_E><\chi^c_E| \tag{4}$$

and

$$Q = \sum_k |\Phi_k><\Phi_k|. \tag{5}$$

The projection operators P and Q determine two orthogonal subspaces of the complete functional space of the α-decaying nucleus which describes the behaviour of the $A+4$ nucleons-system. If there are no other open channels, the sum of these two spaces is a complete functional space of the $A+4$ nucleons-system, and we get

$$P \cdot Q = Q \cdot P = 0 \qquad P + Q = 1. \tag{6}$$

If there are other open channels than the α-channel, for example fission, the completeness relation (6) is fulfilled only approximately. In that case we can try to describe the influence of the additional subspace P' by an effective Hamiltonian containing an imaginary part in the nuclear potential.

Let us denote the exact Hamiltonian by H. Thus, the basic states $|\Phi_k>$ and $|\chi^c_E>$ are the

eigenstates of the projected Hamiltonian in the corresponding subspaces ($H_{QQ}=QHQ$, $H_{PP}=PHP$)

$$(E_k - H_{QQ})|\Phi_k\rangle = 0 \tag{7}$$

$$(E - H_{PP})|\chi_E^c\rangle = 0. \tag{8}$$

While the operators H_{QQ} and H_{PP} describe the behaviour of our process in the corresponding subspaces, the exact Hamiltonian $H=H_{PP}+H_{QQ}+H_{QP}+H_{PQ}$ ($H_{QP}=PHQ$) describes both the bound and the scattering states and the connection between them.

Due to the orthogonality of the subspaces (6), the scattering states $|\chi_E^c\rangle$ must be orthogonal to the bound states

$$\langle\Phi_k|\chi_E^c\rangle = 0. \tag{9}$$

Thus, the eigenstates $|\Psi\rangle$ of the exact Hamiltonian H

$$(E-H)|\Psi\rangle = 0 \tag{10}$$

can be written as a sum of two components corresponding to the two subspaces

$$|\Psi\rangle = P|\Psi\rangle + Q|\Psi\rangle. \tag{11}$$

Using the expansion (11) and the orthogonality (6) we get from the Schrödinger eq. (10) the following system of coupled equations for the two components $P|\Psi\rangle$ and $Q|\Psi\rangle$:

$$(E-H_{QQ})Q|\Psi\rangle = H_{QP}P|\Psi\rangle \tag{12}$$

$$(E-H_{PP})P|\Psi> = H_{PQ}Q|\Psi> . \qquad (13)$$

With the help of the Green operator of the P-subspace

$$G_P^+ = P\frac{1}{E^+-H}P \qquad (E^+ = E + i\,) , \qquad (14)$$

which produces only outgoing waves for the α-particles, we can rewrite eq. (13) in the form

$$P|\Psi> = G_P^+(E)H_{PQ}|\Psi> . \qquad (15)$$

Contrary to the description of a scattering process eq. (15) does not contain a solution of the homogeneous equation, because there is no incoming flux in the decay process. Inserting expression (15) into eq. (12) we get a homogeneous equation for the bound component $Q|\Psi>$:

$$(E-H_{QQ}-H_{QP}G_P^+(E)H_{PQ})Q|\Psi> = 0. \qquad (16)$$

Due to the imaginary part of the Green operator $G_P^+(E)$ the component $Q|\Psi>$ is an eigenstate of a nonhermitian operator, i.e., it describes a set of resonance states whose energies and widths are given by the real and imaginary parts of the complex eigenvalues of eq. (16).

Using the relations (4-6) and the Schrödinger eq. (7), multiplying eq. (16) from the left by $<\Phi_k|$ and integrating we get

$$\sum_{k'}[(E-E_{k'})\delta_{kk'} - \sum_{c'}\int dE' \frac{<\Phi_k|H|\chi_{E'}^{c'}><\chi_{E'}^{c'}|H|\Phi_{k'}>}{E^+-E'}]<\Phi_{k'}|\Psi> \qquad (17)$$

or

$$\sum_{k'} [(E-E_{k'})\delta_{kk'} - \Delta E_{kk'} + \frac{i}{2}\Gamma_{kk'}] <\Phi_{k'}|\Psi>, \qquad (18)$$

where the real part is denoted by

$$\Delta E_{kk'} = \sum_{c'} \mathcal{P} \int dE' \frac{<\Phi_k|H|\chi^{c'}_{E'}><\chi^{c'}_{E'}|H|\Phi_{k'}>}{E-E'} \qquad (19)$$

in which $\mathcal{P}$ means the principal value of the integral and the imaginary part by

$$\Gamma_{kk'} = 2\pi \sum_{c} <\Phi_k|H|\chi^{c}_{E}><\chi^{c}_{E}|H|\Phi_{k'}>. \qquad (20)$$

In order to obtain the energy and widths of the resonances, we have to diagonalize the matrix eqs. (18). If the widths are much smaller than the decay energy, the matrix is practically diagonal and the width Γ_k of a state k is given by the diagonal element

$$\Gamma_k = \Gamma_{kk} = 2\pi \sum_{c} |<\Phi_k|H|\chi^{c}_{E}>|^2. \qquad (21)$$

In such a case we can also neglect the small energy shift ΔE_{kk}, which is of the same order of magnitude as Γ_{kk}.

While the bound basic states $|\Phi_k>$ can be obtained directly by diagonalizing a shell model Hamiltonian in a truncated space, we have still to calculate the scattering states $|\chi^{c}_{E}>$. From eq. (8), using eq. (9) we have

$$(E-H)|\chi^{c}_{E}> = -QH|\chi^{c}_{E}> = -\sum_{k} <\Phi_k|H|\chi^{c}_{E}>|\Phi_k>, \qquad (22)$$

i.e., an integral equation.

Denoting by $|\chi^{c}_{E;0}>$ the solution of the homogeneous equation

$$(E-H)|\chi^{c}_{E;0}> = 0 \qquad (23)$$

which fulfils the usual boundary conditions for the scattering states (incoming waves in the channel c and outgoing waves in all open channels) and by $|\chi^{c}_{E;k}>$ the solution of the unhomogeneous equation

$$(E-H)|\chi^{c}_{E;k}> = |\Phi_{k}> \qquad (24)$$

which vanishes at large separations, the solution of the integral equation (22) can be written as

$$|\chi^{c}_{E}> = |\chi^{c}_{E;0}> - \sum_{k} D^{k}_{c} |\chi^{c}_{E;k}>, \qquad (25)$$

where D^{k}_{c} are the unknown matrix elements $<\Phi_{k}|H|\chi^{c}_{E}>$.

On the other hand the coefficients D^{k}_{c} can be determined from the condition (9) as solutions of the following system of algebraic equations

$$\sum_{k'} D^{k'}_{c} <\Phi_{k}|\chi^{c}_{E;k'}> = <\Phi_{k}|\chi^{c}_{E;0}>. \qquad (26)$$

Inserting (25) into eq. (20), we get

$$\Gamma_{kk'} = 2\pi \sum_{c} D^{k}_{c} D^{k'*}_{c}. \qquad (27)$$

If we consider only one level k of the parent nucleus, we get the same expression like in ref. [7].

In the R-matrix theory of alpha decay, due to the fact that we divide the total space into two completely separated subspaces with two different Hamiltonians with no connections between them ($H_{PQ} = H_{QP} = 0$), our

coupled system of equations (22) reduces to the uncoupled eq. (23). It is evident that the orthogonality relation (9) is already satisfied and the alpha widths (21) can be reduced to a surface integral with the help of the usual approximations.

An equation which depends only on one coordinate can be easily obtained. Indeed, denoting by H^D and H^α the intrinsic Hamiltonians of the separated fragments (daughter nucleus and alpha particle)

$$(E_D - H^D)|\Phi^D\rangle = 0. \tag{28}$$

$$(E_\alpha - H^\alpha)|\Phi^\alpha\rangle = 0. \tag{29}$$

where E_D, E_α and $|\Phi^D\rangle, |\Phi^\alpha\rangle$ are the eigenvalues and the eigenfunctions of the considered fragments, we can write the exact Hamiltonian H as a sum of Hamiltonians for separated fragments and their relative motion

$$H = H^D + H^\alpha + T^{\alpha D} + V^{\alpha D}. \tag{30}$$

where $T^{\alpha D}$ is the relative kinetic operator and

$$V^{\alpha D} = \sum_{i=1}^{4} \sum_{j=5}^{A+4} V_{ij}(r_{ij}) \tag{31}$$

is the sum of the two-body interactions between the nucleons of different fragments.

Assuming that $V^{\alpha D}$ can be approximated by a nuclear potential which depends only on the relative distance between the centers of mass of the fragments, we get by multiplying to the left the eqs. (24-25) with the channel wave function

$$|c> = [|\Phi^{a}(\xi)>|\Phi^{D}(\eta)>|Y_{\ell}(\Omega)>]_{c} \qquad (32)$$

and integrating over the intrinsic coordinates of the fragments ξ and η and the angular variables of the relative motion Ω, the following differential equations for the radial parts of the homogeneous solution $\phi_{c}^{0}(r) \equiv <rc|\chi_{E;0}^{c}>$ and of the unhomogeneous solution $\phi_{c}^{k}(r) \equiv <rc|\chi_{E;k}^{c}>$

$$\left[\frac{h^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right) - V^{aD}(r) + \epsilon\right] \begin{Bmatrix} \phi_{c}^{0}(r) \\ \phi_{c}^{k}(r) \end{Bmatrix} = \begin{Bmatrix} 0 \\ I_{c}^{k}(r) \end{Bmatrix}. \qquad (33)$$

where μ is the reduced mass of the fragments, ϵ is the energy of the relative motion

$$\epsilon = E - E_{D} - E_{a} \qquad (34)$$

and $I_{c}^{k}(r)$ the overlap integral

$$I_{c}^{k}(r) = r\int d\xi\, d\eta\, d\Omega\, [\Phi^{D}(\xi)\Phi^{a}(\eta)Y_{\ell}(\Omega)]_{c}^{*}\Phi_{k}(\vec{r}_{i}). \qquad (35)$$

In order to compute the above integral we have to pass the wave function $\Phi_{k}(\vec{r}_{i})$ of the parent nucleus, which is a function of all individual coordinates $\vec{r}_{i}$ of the nucleons, to the same coordinates as the channel wave function (32). The procedure of calculating the overlap is given in ref. [10-14]. After calculating the overlap integral (35), we can solve numerically the differential equations (33) with the boundary conditions mentioned above.

Because the normalization of a Schrödinger equation of the type (22) is determined by its asymptotic behaviour, we have only to normalize the homogeneous solution, in order to satisfy the condition (3). Then, we can solve the system of algebraic equations (26) whose matrix elements can be expressed by a simple integral over the overlap integral (35) and the radial solutions $\phi_c^0(r)$ and $\phi_c^k(r)$ of the homogeneous and unhomogeneous equations (23) and (24)

$$\sum_{k'} D_c^{k'} \int dr\, I_c^{k*}(r)\phi_c^{k'}(r) = \int dr\, I_c^{k*}(r)\phi_c^0(r). \qquad (36)$$

The corresponding widths follow from eq. (27).

3. APPLICATION TO THE Po ISOTOPES

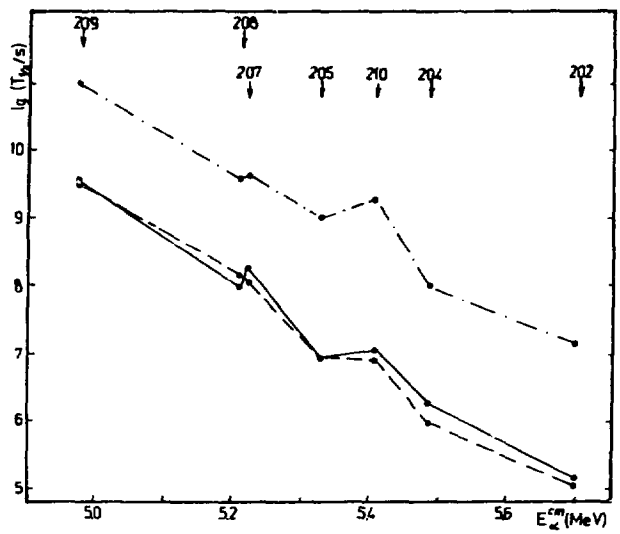
In the following we shall assume that the bound states can be described by the simplest shell model with no residual interactions and harmonic oscillator radial single particle wave functions, and only one bound state $|\Phi_k\rangle$ participates in the decay.

Due to the fact that for open shell model configurations we have to consider the mixture of configurations, we restrict ourselves to the alpha decay of the light polonium isotopes ($N \leq 126$). For the parent nuclei (Po isotopes) the shell model consists of a core (the double magic nucleus ^{208}Pb) with two additional protons in the $1h_{9/2}$ - shell and a certain number of neutron holes, and for daughter nuclei (Pb isotopes) the same core and two additional holes in 3p-2f- -shell relative to the parent nucleus.

For the intrinsic wave functions of the alpha particle we have taken a Gauss func-

tion in the relative coordinates with the parameters from ref.[3] and for the alpha-nucleus optical potential a Wood-Saxon shape with the parameters obtained by fitting the elastic and reaction cross section data in the lead region at energies close to the Coulomb barrier[12]. For completeness also the imaginary part has been considered.

The results are shown in the figure. It is evident that the calculated absolute alpha widths with the real part of the optical potential are in good agreement with the experimental data for all considered isotopes, in spite of the simplicity of the assumptions.



Absolute alpha decay widths of the light Po isotopes with (—·—·—) and without (— — — —) the imaginary part of the optical potential. The experimental values (———) are taken from ref.[20].

The inclusion of the imaginary part of the optical potential gives absolute values for the alpha decay half-lifes much larger than the values obtained if only the real part of the optical potential is considered. In principle, our model may contain also imaginary potentials but we may exclude these potentials since in a decay process with a given initial state there are no other open channels which can participate in the process like in the elastic scattering. Thus, it is reasonable in our calculations to consider only the real part of the optical potential.

We have concluded that the proposed model is able to describe the absolute values of the alpha widths using the real part of the optical potential whose parameters were obtained by fitting the elastic and reaction cross section data at energies close to the Coulomb barrier. Of course, in order to have final conclusions additional studies are needed like: the inclusion of the Pauli principle in the relative motion [13-15], the improvement of the radial single particle shell model wave functions (Woods-Saxon type) [10,16,17], the introduction of a more complex intrinsic alpha particle wave function [4,18], or of the residual interaction of a pairing type [19].

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