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RELATIVISTIC AMPLITUDES IN TERMS OF WAVE FUNCTIONS

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Abstract

In the framework of the invariant diagram technique which arises at the formulation of the field theory on the light front the question about conditions at which the relativistic amplitudes can be expressed through the wave functions is investigated. The amplitudes obtained depend on four-vector ω , determining the light front surface. The recipe is formulated of finding such ~~the~~ values of the four-vector ω , at which the contribution of diagrams not expressed through wave functions is minimal. The investigation carried out is equivalent to study the dependence of amplitudes of the old-fashioned perturbation theory in the infinite momentum frame on direction of the infinite momentum.

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1. Introduction

Investigation of nuclei in the region of the relativistic nucleons momenta ($q \sim m$) and nuclear reactions at high momentum transfer required developing the formalism of the relativistic wave functions (WF), which are relativistically covariant, admit a probabilistic interpretation, depend on three- vectors like their nonrelativistic counterparts and go over in nonrelativistic limit into the usual nonrelativistic WFs. In paper by Shapiro [1] the WFs in relativistic coordinate space were considered. The problem of relativistic WFs was discussed in paper [2] . In paper [3] the formalism of WFs which have the properties pointed out above has been developed. In this approach the WFs are non-equal-time Fock components of the state vector in the invariant Schrödinger representation (ISchR) on the surface of a light wave front $\omega x = 0$ (ω is four- vector on light cone: $\omega = (\omega_0 , \vec{\omega})$, $\omega^2 = 0$, $\omega_0 > 0$).

In order to extract the information of WF from experimental data it is necessary that the processes amplitudes were expressed through WF of bound system. For expressing the amplitudes through the non-equal-time WFs the non-equal-time calculative formalism is required. Such the formalism in ISchR has been developed by Kadyshevsky [4] and formulated for the case of light front in paper [3] . It combines in itself the positive features of Feynman diagram technique (explicit relativistic inveriance) and of the old fashioned perturbation theory (possibility to work with the probabilistic interpretated WFs). The unitarity and casuality in ~~the~~ field theory formulated on the light front was studied in Ref. [5] .

However the problem of expressing the relativistic amplitudes through WFs is complicated by that in the scattering amplitude on the bound system not only the diagrams expressed through the bound system WF contribute but also the diagrams which is not expressed via WF. The reason of this phenomenon which is absent in the nonrelativistic case we shall explain in more detail in Sect. 2. Now we note only that it is connected with follows. Because of the virtual particles criation the state vectors $|in\rangle$ and $\langle out|$ which give complete description of the interacting particle system (e.g., electron and deuteron) contain such Fock components which can not be expressed via Fock components of the deuteron state vector. Besides this the amplitudes depend on the four- vector ω defining the light front surface, the relative contribution of the diagrams expressed and not expressed through WF depending also on ω.

Note that the same problems take place in the old fashioned perturbation theory in the infinite momentum frame (IMF). E.g., the analogue of the dependence on ω of the ISchR diagrams is dependence of the diagrams of old fashioned perturbation theory in IMF on the direction of infinite momentum.

In order to obtain unambiguous expression for the amplitudes through WFs it is necessary, firstly, to find the conditions in which the relative contribution of the diagrams not expressed via WFs is minimal and, secondly, to fix the ω position on the light cone relative to the momenta of the particles taking part in a reaction.

The aim of the present paper is to find the method allowing to obtain unambiguous expressions of the processes amplitudes through WFs. We shall obtain the recipe of finding such values of the four- vector ω (depending on the type of diagram), that the contribution of the diagrams not expressed via WFs will be minimal. Consequently, the processes amplitudes will be unambiguosly expressed through the bound system WF.

In Sect. 2 we reminde the principal properties of non-equal-time WFs, studied in Ref. [3], and diagram technique of ISchR. We explain the reasons of arising the diagrams which are not expressed through WF, and the reasons of appearence the dependence of amplitudes on four- vector ω. We also illustrate the diagrams properties by examples. In Sect. 3 we show that the diagram technique in ISchR is invariant formulation of the old fashioned perturbation theory in IMF (after replacing the variables the expressions for amplitudes coincide). By this way, all the results obtained in the present paper apply equally to old fashioned perturbation theory in IMF. Dependence of the amplitudes on ω means that these amplitudes depend on additional invariant variables. These variables are the scalar products of ω with the four- momenta of the particles paticipating in a reaction. We introduce these invariant variables and find their physical region. In Sect. 4 we investigate the singularities of the amplitudes in ISchR. The positions of some these singularities turned out to depend on the values of invariant variables connected with ω. In Sect. 5, having investigated the character of the amplitudes dependence on the invariant variables connected with ω and using the information on the position of the singularities depending on the scalar products of ω with four- momenta we obtain the recipe of finding such values of these invariants (it is equivalent to finding the four- vector ω), at which the contribution of the diagrams not expressed through WFs is minimal. Section 6 contains concluding remarks. In appendix we study the parametrization of the ISchR amplitudes off the energy shell.

2. Relativistic WFs and Diagram Technique

Let us remind the main results of Ref. [3], related to relativistic WFs. Here we consider the WF's properties qualitatively*).

Let $\Psi(\vec{x}_1, \vec{x}_2, t)$ be the two particle Fock component having sense of the probability amplitude to find the particles in the system A at the points $\vec{x}_1$ and $\vec{x}_2$ at the time moment t. In the system A' this WF is not equal-time one. It describes the particles at the points $\vec{x}_1'$, $\vec{x}_2'$ at the time moments t_1', t_2'. WF $\Psi'(\vec{x}_1', \vec{x}_2', t')$ which is equal-time in system A' does not coincide with $\Psi(\vec{x}_1, \vec{x}_2, t)$ since during the time $\Delta t = t_2' - t_1'$ the particles position is changed. Therefore equal-time WF is not covariant quantity, Ψ and Ψ' are not connected kinematically. Connection between them is dynamical one and contains the system Hamiltonian.

The covariant quantity is non-equal-time WF $\Psi(x_1, x_2, \lambda)$, defined on the arbitrary space-like surface. For the sake of simplicity we shall consider flat surface $\lambda x = 0$; where $\lambda = (\lambda_0, \vec{\lambda})$, $\lambda^2 = 1$, $\lambda_0 > 0$. Such WF is transformed kinematically since in the system A' it describes the particles at the same space-time points as in the system A. The coordinates of these points and

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*) My attention to the qualitative consideration stated below was drawn by I.S.Shapiro.

the surface position in the systems A and A' are different, they are connected by a Lorentz transformation.

In the case of nonrelativistic bound system the particles position during the time $\Delta t = t_2' - t_1'$, does not change considerably. Therefore non-equal-time WFs coincide in nonrelativistic limit with usual equal-time WFs.

In the momentum space the qualitive aspect of the problem is reduced to follows. In nonrelativistic theory the deuteron momentum $\vec{p}$ and nucleons momenta $\vec{k}_1$ and $\vec{k}_2$ are connected by the equality:

$$\vec{k}_1 + \vec{k}_2 = \vec{p}. \tag{1}$$

In relativistic theory the equality (1) can take place only in the reference frame, where WF is equal-time one, since in any other reference frame we add the particle momenta taking in the different time moments. Since the deuteron momentum is conserved it is clear that for two time moments t_1 and t_2 we have:

$$\vec{k}_1(t_1) + \vec{k}_2(t_2) \neq \vec{p}(t_1) = \vec{p}(t_2).$$

In the reference frame, where $\vec{k}_1 = -\vec{k}_2 \equiv \vec{q}$ the deuteron momentum is not equal to zero ($\vec{p} \neq 0$), and besides the vector variable $\vec{q}$ (relative relativistic momentum) one more vector variable $\vec{p}$ remains. In other words, in relativistic deuteron one never can separate the center-of-mass variables, while it can be done in nonrelativistic theory. The exception is a system of non-interacting particles. In this case equality (1) is satisfied always, because the momenta $\vec{k}_i$ are not changed in a time.

Most simple parametrization may be obtained for WF on the light front $\omega x = 0$. This WF is equal-time in IMF. In reference system where $\vec{p} \to \infty$ this WF does not depend on $|\vec{p}|$, but depend on variable $\vec{n} = \vec{p}/|\vec{p}|$:

$$\Psi = \Psi(\vec{q}, \vec{n}). \qquad (2)$$

Due to invariance of non-equal-time WFs the parametrization (2) takes place in any reference system. In addition in WF defined on the light front the vacuum fluctuations do not contribute. Namely in this case the concept of WF acquires the clear sence.

Going over to the four-dimensional notation we note that not equaled to zero difference of the four- momenta $k_1 + k_2$ and p (all the four momenta are on the mass shells) should be proportional to the four- vector λ :

$$k_1 + k_2 - p = \lambda\tau \qquad (3)$$

where τ is scalar parameter. Indeed, λ is the only four- vector characterizing the surface where WF is defined. In reference frame, where WF is equal-time one, we have $\vec{\lambda} = 0$, $\lambda_0 = 1$, and we return to equality (1).

In case of WF defined on the light front the equation (3) acquires the form:

$$k_1 + k_2 = p + \omega\tau \qquad (4)$$

WF depends on 4 four- vectors, connected by the conservation law (4): $\Psi = \Psi(k_1, k_2, p, \omega\tau)$. Therefore it is convenient to represent WF graphically in the form of the four- point diagram (Fig. 1) and to consider the four-momentum $\omega\tau$ as belonging to the fictive particle (spurion). The WF, as any amplitude of the reaction $1 + 2 \to 3 + 4$, depends on the relative momentum $\vec{q}$ of the "final" particles (see Fig.1) and on the variable $\vec{n}\vec{q}$ connected with the "scattering angle" ($\vec{n}$ is the momentum direction of the "initial" spurion in c.m.-system of the "reaction"). This parametrization coincides with the Eq.(2). In the nonrelativistic limit the dependence on $\vec{n}$ disappears.

Let us emphasize that introduction of the spurion does not mean the presence in the state vector of unphysical particle.

It is not difficult to express the variables $\vec{q}$, $\vec{n}\vec{q}$ via the invariants:

$$\vec{q}^{\,2} = S/4 - m^2, \quad \frac{\vec{n}\vec{q}}{\varepsilon(\vec{q})} = \frac{(u-t)\sqrt{s}}{2(s-M^2)} \tag{5}$$

where $s=(p+\omega\tau)^2=(k_1+k_2)^2$, $t=(p-k_1)^2$, $u=(p-k_2)^2$

One can introduce the variables analogeous to variables $\vec{k}_\perp$ and x of the parton model in IMF ($\vec{k}_\perp$ is the transverse momentum projection, x is the momentum portion of one of the particles from the total system momentum):

$\psi = \psi(\vec{k}_\perp^{\,2}, x)$. The connection between $\vec{k}_\perp$, x and $\vec{q}$, $\vec{n}$ is follows [3]:

$$\vec{k}_\perp^{\,2} = \vec{q}^{\,2} - (\vec{n}\vec{q})^2, \quad x = \frac{1}{2}\left(1 - \frac{\vec{n}\vec{q}}{\varepsilon(\vec{q})}\right).$$

In terms of these variables a nonrelativistic WF depends on the following combination of $\vec{k}_\perp$ and x:

$$\vec{q}^{\,2} = \frac{\vec{k}_\perp^{\,2} + m^2}{4x(1-x)} - m^2 .$$

Thus, the relativistic WF depends on additional argument having the form of the unit vector $\vec{n}$. The WF dependence on $\vec{n}$ is determined by dynamics and describes the properties of the bound system just as the dependence on the relative momentum $\vec{q}$. The consideration of the simplest models shows that typical parameter determining the dependence of the nuclear WFs on the variable $\vec{n}\vec{q}$ is the nucleon mass. The investigation of this dependence being the quate new aspect of the problem of the nuclear WFs is of considerable interest.

The diagram technique in ISchR is obtained at solving the Schrödinger equation for the state vector in terms of the "oblique" time (along the direction of ω). The ISchR diagrams can be obtained from the Feynman ones by ordering in a "time" of the vertices by all possible ways. Assuming the "time" axis is directed in a graph from left to right and numbering the vertices from right to left one should connect them by directed spurion line in the order of increasing numbers (the larger number corresponds to the smaller time moment). The particles lines should be oriented from left to right. The four- momenta in the vertices including the spurion momentum, just as in WFs, are connected by the conservation law. To each spurion line with four- momentum $\omega\tau_i$ one should set in correspondence the factor $1/2\pi(\tau_i - i0)$, to each internal particle line with four- momentum k_i one should associate the propagator $\theta(\omega k_i)\delta(k_i^2-m_i^2)$ (for the spinless particles). The diagrams with the particles criation from the vacuum do not contribute, just as in the old fashioned perturbation theory in IMF. The rules of the diagrem technique in ISchR is states in Refs. [4, 3].

The vertex part Γ having the same diagram representation as WF (Fig.1) is connected with WF by the formula [3]:

$$\psi(k_1,k_2,p,\omega\tau)=\frac{1}{2\pi(s-M^2)}\Gamma(k_1,k_2,p,\omega\tau). \quad (6)$$

Let us illustrate the ISchR diagram technique by the example of the elastic ed- scattering. The part of diagrams contributing in the ed- scattering amplitude is shown in Fig. 2. The expression for the amplitude of the diagram 2a has the form:

$$M = \int \Gamma_1 \Gamma_4\, \theta(\omega(p-q))\, \delta((p+\omega\tau_3-q)^2 - m^2) \times$$
$$\times\, \theta(\omega(p'-q))\, \delta((p'+\omega\tau_1-q)^2-m^2)\, \theta(\omega q)\, \delta(q^2-m^2) \times$$
$$\times\, \theta(\omega(k-k'))\, \delta((k-k'+\omega\tau_2-\omega\tau_3)^2) \times$$
$$\times\, d^4q \frac{d\tau_1}{2\pi(\tau_1-i0)} \times \frac{d\tau_2}{2\pi(\tau_2-i0)} \frac{d\tau_3}{2\pi(\tau_3-i0)}. \tag{7}$$

Integrating over $d\tau_i$ ($i = 1, 2, 3$), over dq_0 and connecting Γ with WF by means of eq.(6), we come to the expression:

$$M = \frac{1}{2\pi} \int \Psi_1\, \theta\left(1 - \frac{\omega q}{\omega p}\right) \Psi_4\, \theta\left(1 - \frac{\omega q}{\omega p'}\right) \times$$
$$\times \frac{\theta(\omega(p'-p))}{\frac{\omega(p'-p)}{\omega(p-q)}[m^2-(p-q)^2] - (p-p')^2 - i0} \cdot \frac{d^3q}{2\varepsilon_q} \tag{8}$$

where WFs Ψ_1 and Ψ_4 depend on the four- momenta in the vertices 1 and 4.

From the expression (8) one can see that the amplitude M depends not only on the invariant $t = (p-p')^2$, but on the scalar products ωp and $\omega p'$, M depending only on the ratio of there scalar products $y = \omega p'/\omega p$. The last assertion becomes to be evident if one note that expression (8) is not changed at multiplying ω by the number.

The diagrams 2b and 2c are not expressed through deuteron WF. The reasons of appearing of such diagram one can explaine as follows. As it was mentioned the complete description of the interacting deuteron and electron is given by the state vectors of the continuous spectrum $|\,in\rangle$ and $\langle out\,|$, and S- matrix is dertermined by their scalar product: $S = \langle out\,|\,in\rangle$. Diagrams just show what virtual particles are contained in the state vectors $|\,in\rangle$ and $\langle out\,|$. In nonrelativistic theory in intermediate states besides the projectile particle only the same particles are present which are contained in the deuteron state vector. They appeare due to the virtual deuteron decays. Therefore in nonrelativistic theory the amplitudes always are expressed through WFs. In relativistic theory, due to possibility of the additional particles creation, the situation is changed. Thus, in the diagram Fig. 2b in the intermediate state between the "time moments" 3 and 2 the $N\bar{N}$ pair is present, and there are no the vertex of the deuteron decay in nucleons in Fig. 2b. In the diagram Fig. 2c also the intermediate state containing only nucleons from the initial deuteron and the electron is absent (in state 3 - 2 the γ- quantum is present), and the vertex 3 is not also expressed through WF). It is the presence in the state vectors $|\,in\rangle$ and $\langle out\,|$ of such Fock components that comes to appearence of the diagrams, which are not expressed via WF.

The method of suppression of the diagrams not expressed through WFs will consist in the following. As it has been explained, the WFs depend on the hyper surface, on which they are defined. The magnitudes of "undesirable" components of the state vectors $|\,in\rangle$ (and $\langle out\,|$) and their contribution in the processes amplitudes also depend on this hypersurface. Below we investigate the dependence of these amplitudes on the hypersurface (i.e., on the four- vector ω) and show how to find such values of ω , at which the contribution of the diagrams non-expressed through WFs is minimal. In the example of ed- scattering considered above it turned out to be possible to find such value of ω , at which the contribution of the diagrams, non-expressed through WF, turns into zero. After fixing ω by this condition the ed- scattering amplitude (Eq.(8)) becomes to be quite unambiguous.

3. Connection with the Old Fashioned Perturbation Theory

As it has been noted, the non-equal-time dynamical scheme in ISchR retains the positive features of Feynman diagram technique (the possibility to carry out the explicitly relativisticly invariant calculations in any reference system) and the old fashioned perturbation theory (separation of particles from antiparticles and, consequently, possibility to attach sense to conception of the composite system and its description by means of WF). In order to explain the non-equal-time diagram technique in more detail we shall show in explicite form that calculation formalism of ISchR is relativisticly invariant form of the old fashioned perturbation theory in IMF.

Let us consider arbitrary intermediate state shown in Fig. 3. The expression for the amplitude of diagram Fig.3 has the form:

$$M = \int M_1 M_2 \delta^{(4)}\Big(\sum_1^m p_j - \sum_1^n k_j + \omega\tau\Big)\delta^{(4)}\Big(\sum_1^n k_j + \omega\tau - \sum p_j'\Big) \times \frac{d\tau}{2\pi(\tau - i0)} \prod_{j=1}^n \theta(\omega k_j)\delta(k_j^2 - m_j^2) d^4k_j . \qquad (9)$$

Taking into account that $\tau = [(\Sigma k_j)^2 - (\Sigma p_j)^2] / 2\omega \Sigma p_j$ (it follows from equality $\Sigma p_j + \omega\tau = \Sigma k_j$) and integrating over dk_{oj} we obtain:

$$M = \delta^{(4)}(\Sigma p_j - \Sigma p_j^{\prime}) \times$$

$$\times \int \frac{M_1 M_2 (-2\omega \Sigma p_j) \delta^{(4)}(\Sigma p_j - \Sigma k_j + \omega\tau) d\tau}{2\pi [(\Sigma p_j)^2 - (\Sigma k_j)^2 + i0]} \prod_j \frac{d^3 k_j}{2\varepsilon_j} \quad (10)$$

For transformation Eq.(10) into tne form of the old fashioned perturbation theory in IMF we introduce the following variables:

$$R_j^{ext} = p_j - y_j \sum_i p_i , \quad y_j = \omega p_j / (\omega \sum_i p_i)$$

$$R_j = k_j - x_j \sum_i k_i , \quad x_j = \omega k_j / (\omega \sum_i k_i). \quad (11)$$

Note that $\Sigma y_j = \Sigma x_j = 1$ and four- vectors R satisfy to the condition $R_j \cdot \omega = R_j^{ext} \cdot \omega = 0$. Introducing the projections: $R = (R_o , \vec{R}_\perp , \vec{R}_\parallel)$, where $\vec{R}_\perp \vec{\omega} = 0$, $\vec{R}_\parallel \| \vec{\omega}$, and taking into account the equality: $R^2 = - \vec{R}_\perp^2$, we obtain that in terms of variables R_j^{ext} , R_j the denominator in Eq.(10) aquares the form:

$$\Big[\sum_1^m \frac{(\vec{R}_{j\perp}^{ext})^2 + m_j^2}{y_j} - \sum_1^n \frac{\vec{R}_{j\perp}^2 + m_j^2}{x_j} + i0 \Big] \quad (12)$$

which coincides with the form which is given by Weinberg's rules [6] . The expression $d^3 k_j / 2\varepsilon_j$ in terms of these variables turns into $d^2 R_j dx_j / 2x_j$, the integrals over dx_j having the limits from 0 to 1. The expression $(\omega \Sigma p_j) \times$ $\times \; \delta^{(4)}(\Sigma p_j - \Sigma k_j + \omega\tau) d\tau$ turns into $\delta^{(2)}(\Sigma \vec{R}_{j\perp}^{ext} - \Sigma \vec{R}_{j\perp}) \times$

$\times\ \delta(\Sigma X_j - 1)$. By exactly the same way one can transform the expression for any intermediate state. In this way we obtain the same expression for amplitude as in the old fashioned perturbation theory in IMF. The vectors $\vec{K}_\perp$ play the role of the momenta transverse to the motion direction of IMF. The variables x , y play the role of the portions of the particles momenta from the infinite momentum (divided by the sum of the initial particles momenta portions).

From Eq. (12) one can see that amplitudes in ISchR depend on four- vector ω by means of the ratios of the scalar products of ω with the particles four- momenta (variables y_j). In the end it is connected with the invariance of theory under the replacement: $\omega \to \omega' = \lambda\omega$. Note that the number of independent variables is equal to 2 (there are three independent scalar products of ω with four- momenta and, consequently, two independent ratios of these scalar products). In the case of two-body reaction $1 + 2 \to 3 + 4$ it is convenient to introduce the following variables (besides s and t):

$$y_1 = \frac{\omega p}{\omega(p+k)}\ , \quad y_2 = \frac{\omega k}{\omega(p+k)}$$

$$y_3 = \frac{\omega p'}{\omega(p+k)}\ , \quad y_4 = \frac{\omega k'}{\omega(p+k)} \qquad (13)$$

where p, k and p', k' are four- momenta of initial and final particles ($p^2 = p'^2 = M^2$, $k^2 = k'^2 = \mu^2$). Due to equations $y_1 + y_2 = 1$, $y_3 + y_4 = 1$, there are two independent variables. We choose y_1 and y_3 . So, for two-body amplitude corresponding to ISchR diagram we obtain: $M = M(s, t, y_1, y_3)$. In the framework of the perturbation theory the sum of the amplitudes for all ISchR diagrams obtained from given Feynman diagram by all possible time ordering coincides with the Feynman amplitude and does not depend on y_1, y_3. The problem of finding of the hypersurface on which the Fock components of the state vectors $|\,in\rangle$, $\langle out\,|$ not expressed through the deuteron WF give the minimal contribution in amplitude is reduced now to finding such values of variables y_1, y_3 at which the relative contribution of the diagrams not expressed through WF is minimal.

In conclusion of the present Section let us find the physical regions in which the variables connected with ω change at fixed s and t and at arbitrary change of ω. At first we shall find kinematical region of variable $y = \omega p'/\omega p$. In particular, the ed-scattering amplitude (see Eq.(8)) depend on this variable. In reference frame, where $\vec{p} = 0$, we have $y = (\varepsilon(\vec{p}\,') - p'\cos\theta)/M$ (θ is the angle between $\vec{\omega}$ and $\vec{p}\,'$), and y is changed in the limits $y_- \leqslant y \leqslant y_+$, where $y_\pm = (\varepsilon(\vec{p}\,') \pm p')/M$. In arbitrary frame we obtain:

$$y_\pm = \frac{(pp') \pm \sqrt{(pp')^2 - M^4}}{M^2}. \tag{14}$$

The region of changing of variables y_1, y_3 (see Eq.(13)) is found analogously. In the case of elastic scattering it is the ellipsis in the plane (y_1, y_3). The coordinates of the elipsis center are $(y_1, y_3) = (\varepsilon^*/\sqrt{s}, \varepsilon^*/\sqrt{s})$, half-axis is situated at the line $y_1 = y_3$ and equals to $a = \sqrt{2}\,\frac{p^*}{\sqrt{s}}\cos\frac{\theta^*}{2}$. The second half-axis equals to: $b = \sqrt{2}\,\frac{p^*}{\sqrt{s}}\sin\frac{\theta^*}{2}$ (ε^*, p^* are the energy and momentum of the particle of mass M in c.m.-system). This region is shown in Fig. 4.

4. The Singularities of Amplitudes

In order to find the values of the invariant variables y_i at which the contribution of diagrams not expressed through WF is minimal it will be necessary for us to know the situation of singularities of the ISchR amplitudes. The method of finding the singularities we explain in example of the simplest diagrams shown in Fig. 5. By the internal wavy line the exchange by the meson of the mass μ is shown.

The expression for the amplitude of the diagram Fig.5a integrated over $d\tau_i$ and over dq_0 has the form:

$$M_a = F(t, y)/(t-m^2), \quad y = \omega p'/\omega p,$$

$$F(t, y) = c\int \frac{\theta(\omega(p-q))}{m^2-(p-q)^2-i0} \cdot \frac{\theta(\omega(p-p'-q))}{\mu^2-(p-p'-q)^2-i0} \frac{d^3q}{2\varepsilon_q}. \tag{15}$$

The problem of finding the singularities of the function $F(t, y)$ differs from the case of Feynman diagram by that the integration momentum q is on the mass shell and the integration region is limited by θ - function $\theta(\omega(p-p'-q))$ (at $\omega(p-p'-q)>0$ we have $\theta(\omega(p-q))=1$ and do not obtain additional restrictions). The latter restriction results in appearing the singularities corresponding to approach of the integrand singularity to the bound of the region of variable q .

If we did not take into account the restrictions mentioned, then according to usual method of finding of the Feynman amplitude singularities (see paper by Landau [7]) these singularities would be determined from condition of the extremum of the following function: $\varphi_1 = \alpha_1(m^2-(p-q)^2)+ + \alpha_2(\mu^2-(p-p'-q)^2)$. Taking into account of these constraints results in the problem of the conditional extremum. This problem can be solved by introducing in φ_1 the corresponding terms with the Lagrange multipliers. Thus, we come to the problem of finding the extremum of the following function:

$$\varphi = \alpha_1 (m^2 - (p-q)^2) + \alpha_2 (\mu^2 - (p-p'-q)^2) + \\ + \gamma_1 (m^2 - q^2) + \gamma_2 \omega (p-p'-q) \tag{16}$$

us function of the variables α_1, α_2, γ_1, γ_2 and q, where γ_1 and γ_2 are the Lagrange multipliers. It is necessary also to find the extremums of the functions obtained from φ by equaling to zero in terms all the coefficients except γ_1.

The case $\gamma_2 = 0$ returns us to expression for φ considered at finding the singularities of the Feynman triangle diagram. This singularities is situated at the points:

$$t = (m+\mu)^2 \tag{17}$$

$$t = m^2 + 2\mu^2 . \tag{18}$$

The situation of the singularity (18) was found in approximation $|\varepsilon| = |M - 2m| \ll m$ (M is mass of the particle with four- momentum p).

In the case $\gamma_2 \neq 0$, $\alpha_2 = 0$ by differentiating φ over q, α_1, γ_1, and γ_2 we obtain the equation:

$$2\alpha_1 (p-q) - 2\gamma_1 q - \gamma_2 \omega = 0 \tag{19}$$

at the conditions

$$(p-q)^2 = m^2, \quad q^2 = m^2, \quad \omega(p-p'-q) = 0.$$

Multiplying Eq.(19) in turn by q , p and ω and equaling to zero the determinant of the equations system obtained we come to the quadratic equation concerning $y = \omega p'/\omega p$. This equation gives the situation of the singularities at the points:

$$y = \frac{1}{2} \pm \frac{i}{2}\sqrt{\frac{|\varepsilon|}{m}} . \tag{20}$$

Thus, besides the singularities of Feynman diagram the amplitude of diagram Fig. 5a has the singularities in variable y.

Analogously one can find the singularities of the diagrams Fig. 5b and 5c. The amplitude of diagram 5b has the singularity in y , determined by Eq.(20). It also has the singularities in t . These singularities depend on the value of y and is situated at the points:

$$t = m^2 + \frac{1-y}{y}\mu(\mu + 2m) \tag{21}$$

$$t = m^2 + \frac{1-y}{y}2\mu^2 . \tag{22}$$

The amplitude of diagram 5c has the singularities determined by Eqs.(18), (20), (21) and (22).

Note that sum of diagrams Fig. 5a,b and c results in the Feynman amplitude. Therefore the y - singularities and t - singularities depending on y can not be present in the only ISchR amplitude, since in sum of the ISchR diagrams resulting in Feynman diagram these singularities must be cancel.

5. Finding the Values of Additional Variables in Amplitudes

The problem of finding the values of additional variables in the scattering amplitudes we shall consider in example of the double-scattering diagrams Fig.6.

The amplitude of the diagram Fig.6a has the form:

$$M = \frac{1}{2\pi}\int \Psi_4^* \frac{\theta(\omega(p-q_1))}{1-\frac{\omega q_1}{\omega p}} \frac{\theta(\omega(p+k-q_1-q_2))}{\mu^2-(p+k-q_1-q_2)^2-i0} \times$$

$$\times \frac{\theta(\omega(p'-q_2))}{1-\frac{\omega q_2}{\omega p'}} \Psi_1 \frac{d^3q_1}{2\varepsilon_1}\frac{d^3q_2}{2\varepsilon_2} \qquad (23)$$

where Ψ_1 and Ψ_4 depend on the momenta in the vertices 1 and 4.

Going over to the variables $R_1 = q_1 - \frac{\omega q_1}{\omega p}p$, $R_2 = q_2 - \frac{\omega q_2}{\omega p'}p'$ we obtain:

$$M = \frac{1}{2\pi}\int \frac{\Psi^*(\vec{R}_{1\perp}+x_1\vec{Q}_{1\perp}, x_1)\Psi(\vec{R}_{2\perp}+x_2\vec{Q}_{2\perp}, x_2)}{\mu^2+(\vec{R}_{1\perp}+\vec{R}_{2\perp})^2+(1-x_1y_1-x_2y_3)\left(\frac{m^2+\vec{R}_{1\perp}^2}{x_1y_1}+\frac{m^2+\vec{R}_{2\perp}^2}{x_2y_3}-S\right)-i0} \times$$

$$\times\, \theta(1-x_1)\theta(1-x_2)\theta(1-x_1y_1-x_2y_3)\frac{d^2R_{1\perp}dx_1}{2x_1(1-x_1)}\cdot\frac{d^2R_{2\perp}dx_2}{2x_2(1-x_2)} \qquad (24)$$

where $\vec{Q}_{1\perp}$, $\vec{Q}_{2\perp}$ are the orthogonal to $\vec{\omega}$ projectios of the space parts of the four- momenta $Q_1 = p - y_1(p+k)$, $Q_2 = p' - y_3(p+k)$; y_1 and y_3 are defined by Eqs.(13).

The amplitude of the diagram Fig.6b (and the amplitudes of other analogous diagrams) are not expressed through WF, since the vertex 3 in Fig.6b is not connected with WF.

Let us choose the variables y_1 and y_3 so as to suppress the contribution of the diagrams of the Fig.6b type. For this purpose we note that the amplitude of the diagram Fig.6b contains the product of the following θ - functions: $\theta(\omega(q_1-p))\theta(\omega(k+p-q_1-q_2))\theta(\omega q_1)\theta(\omega q_2)$ corresponding to the lines 43, 42, 32 and 41. Introdicing the variables $z_{1,2}=\omega q_{1,2}/\omega(p+k)$ and y_1 we obtain $\theta(z_1-y_1)\theta(1-z_1-z_2)\times\theta(z_1)\theta(z_2)$. Therefore the amplitude of the diagram Fig.6b can be represented in the form:

$$M_b=\int_{y_1}^{1}F_b(s,t,z_1,y_1)\,dz_1\,. \tag{25}$$

Taking into account the diagram resulted from Fig.6a by changing the order of the vertices 1 and 2 results in the integral with the limits from y_3 to 1.

If the function F_b in the formula (25) does not tend to infinity at z_1, $y_1 \to 1$, then at $y_1 \to 1$ the amplitude M_b decreases. However, as the examples show, the function F_b as a function of z_1 can have the singularities, which tend to 1 at $y_1 \to 1$. But, as the further investigation shows, these singularities are found to be weak and do not prevent the amplitude from decreasing at $y_1 \to 1$. To find the character of the function F_b singularities we use the formula by Landau [7]:

$$F_b\sim\varphi^{\frac{m}{2}-n}$$

where φ is value near the singularity of denominator of the integrand determining F_b , m is the number of integrations, n is degree in which φ is contained in denominator. There are three intermediate states in diagram Fig.6b for M_b , therefore after using the Feynman parametrization we obtain $n=3$. There are two integration contours over $d^3q_{1,2}$ and integration over the parameters α_i ($i=1, 2, 3$) in the amplitude. Taking into account the condition $\Sigma \alpha_i = 1$ and that F_6 contains by one integration lesser than M_6 we obtain $m = 7$. It gives the square root singularity. Taking into account the singularities connected with θ - functions and consideration of examples of more complicated diagrams do not result in the pole or more strong singularities.

In this way, to suppress the diagram of type 6b it is necessary to choose the variables y_1 and y_3 to be more close to 1. The point which is closest to 1 in the physical region of variables y_1, y_3 (see Fig.4) is the point $y_1 = y_3 = y_{max}$, where

$$y_{max} = (\mathcal{E}(p^*) + p^* \cos\frac{\theta^*}{2})/\sqrt{s} \tag{26}$$

$\mathcal{E}(p^*)$, p^* are the deuteron energy and momentum in c.m.--system, θ^* is the scattering angle. The condition $|y_{1,3} - 1| \ll 1$ at which the amplitude 6b is suppressed is valid at high energy and at small scattering angle.

The physical reason of suppression of diagram Fig.6b is decrease the phase volume in which the intermediate particles can be. In any vertex of diagram 6b the "reaction" admissible by the four- momentum conservation law (with the spurion four- momentum) takes place. Since the direction of the spurion momentum is not changed, this puts the restrictions on the particles momenta, at which such reactions are admissible kinematically. The condition $|y_{1,3} - 1| \ll 1$ means the choice of such direction of ω, at which the kinematically admissible region is minimal.

Keeping in mind that the denominators, corresponding to the intermediate states, contain in terms of variables $\vec{k}_\perp$, x the expressions $\Sigma(\vec{k}_{\perp i}^2 + m^2)/x_i$, one can say that at the conditions $|y_{1,3} - 1| \ll 1$ the intermediate states of diagrames not expressed through WFs are most far from the physical region. And on the contrary, the intermediate states of the diagrams expressed through WF are most close to the physical region.

It seems on the first glance that the ambiquities connected with the diagrams of the type 6b disappear completely if one chooses the variables y_1 , y_3 equaled to 1. Indeed, the amplitude M_6 (see (25)) at $y_1 = y_3 = 1$ turns into zero. But at approaching y_1 and y_3 to value 1, which is beyond the bound of the physical region Fig.4, the singularities of the integrand in (25) move, cross the real axis, hook the integration contours and transfer them in the complex region. The extrapolation in the complex region of the underintegral functions in Eq.(24) is rather ambiguous.

The recipe of findind the y_i variables values it is easy formulates for the general diagram. Let us notice that the reason of the diagram 6b suppression at y_1 closed to 1 ($y_2 = 1 - y_1 << 1$) lies in the fact that this diagram contains in the vertex 4 only outgoing internal lines. Since the sum of corresponding to these lines variables z_i ($z_i > 0$) equals to y_2 , then at $y_2 \to 0$ the integration region over the variables z_i tends to zero. Let us consider the arbitrary diagram which contains the external line incoming in a vertex and the internal lines only outgoing from this vertex. Similary one can consider the diagrams with the external line out going from a vertex which contains only incoming internal lines. Namely the diagrames with such vertices are expressed through WF. To enhamce such diagrams one should choose the variable y_i , corresponding to these external lines to be maximal in the physical region. On the contrary, to suppress the diagrams with the vertices of such type (e.g., vertex 1 in diagram Fig.5c) one should choose the corresponding variables y_i to be minimal.

Note that these results apparently one can also obtain by direct calculation of the amplitudes asymptotics in region where minimal and maximal values of variables y_i tend to 0 or 1.

For illustration we shall return to the expression (8) for ed- scattering amplitude. The external line relative to the triangle diagram Fig.2 is the photon line. To suppress the diagrams of type 2b one should choose the variable $y_{in} = \omega q/\omega(p+k)$ (where $q = p - p' + \omega\tau_1 - \omega\tau_3$ is the photon four- momentum) equaled to zero. It results in the condition $\omega(p-p') = 0$, or $y = \omega p'/\omega p = 1$, Since $t = (p-p')^2 < 0$, the condition $y = 1$ is valid in the physical region of variable y, and the diagram 2b does not contribute. One can show that at $y = 1$ the diagram 2c also turns in zero. The expression (8) for the only diagram not equaled to zero at $y = 1$ obtains the form:

$$M = -\frac{1}{t}F(t)$$

where the form-factor of the scalar deuteron $F(t)$ in terms of variables $\vec{R}_\perp$, X has the form:

$$F(t) = \frac{1}{2\pi}\int \Psi(\vec{R}_\perp^{\,2}, x)\,\Psi((\vec{R}_\perp + x\vec{Q}_\perp)^2, x)\frac{d^2R_\perp\, dx}{2x(1-x)^2}. \quad (27)$$

The expression (27) coincides with the formula for formfactor obtained by means of Weinberg's rules in IMF, moving in the direction which is orthogonal to the space part of the momentum transfer (see., e.g., Ref. [8]). Note that if the non-nucleon components of the Fock column are essential (for instance, $\Delta\Delta$ in deuteron), they give the additional contribution in form-factor, which is not eliminated by choice of ω .

6. Conclusion

The recipes forulated above allow to express unambiguously the processes amplitudes through WFs. This was achieved due to that after transition on the light front the diagrams describing the particles production forom vacuum turned into zero, and after suitable choice of the light front surface position the diagrams not expressed through WFs becomed minimal. In the framework of the old fashioned perturbation theory in IMF such diagrams are minimal at the infinite momentum direction along vector $\vec{\omega}$. The $\vec{\omega}$ direction relative to the particles momenta is easy determined by found values of variables y_i .

By that way, the formalism developed allows, after including spin, to approach consistently the theoretical description of the relativistic nuclear reactions at high momentum transfer and study of the high momentum components of the nuclear WFs.

One of the primary tasks at present is the solution of the mechanism identification problem of the reactions at high momentum transfer. It requires the indication the distinctive "marks" of one or another mechanism, which allow to identify this mechanism surely. This in it's turn would allow to obtain from experimental data the information about relativistic WFs, and, especially, about the character of their dependence on the variable $\vec{n}$. Modern status of the mechanism determination problem of a number of reactions at high momentum transfer has been descoussed in the review [9] . Also it seems important to investigate the general properties of the WFs dependence on their argumets in relativistic region, the asymptotical behaviour of WFs.

The author is sincerely graatful to I.S.Shapiro for useful discussions and valuable remarks.

Appendix

Parametrization of Amplitudes off Energy Shell

Let us consider the parametrization of two-body amplitude off energy shell, shown in Fig.7. Let at first $\tau_2 = 0$. Then the amplitude, as any five-point function, depends on five invariant variables. Adding one external spurion line with four- momentum $\omega\tau_2$ results in arising only one additional variable, since the spurion four- momenta are "parallel". By that way, two-body reaction amplitude off energy shell besides $s=(k+p)^2$ and $t=(p-p')^2$ depends on four additional variables, which we choose as follows:

$$s_1=(p+\omega\tau_1)^2,\ s_2=(p'+\omega\tau_2)^2,\ s_3=(p'+\omega\tau_1)^2$$
$$s_4=(k+p-\omega\tau_1)^2=(k'+p'-\omega\tau_2)^2. \tag{28}$$

Arbitrary n - point amplitude also depends on four additional veriables. Note, that, in principle, there are amplitudes with the arbitrary number of external spurion lines in this formalism, adding each spurion line, begining from the second, resulting in the appiarence of one additional variable.

Let us consider now the parametrization of the particle form-factor off the energy shell. Such form-factor, for instance, comes into the ed- scattering amplitude (diagram Fig.2). The off shell amplitude of the electron scattering on a particle is shown in Fig. 8a. The expression for amplitude (integrated over $d\tau_3$) has the form:

$$M = \frac{e^2}{(2\pi)^{5/2}} \int \frac{d\tau_4}{(2\omega q \tau_4 - q^2 - i\varepsilon)(\tau_4 - i0)} F_1(\tau_4) \quad (29)$$

where $q = k - k'$.

Since the four- momenta k and k' come into one photon exchange amplitude (29) in the form of difference $k - k'$, this amplitude depends on variables $t = (k-k')^2$ and s_1, s_2, s_3, but it does not depend on s and s_4. Let us consider this amplitude under condition $\omega q = 0$. This condition is used in calculation of the deuteron form-factor (see Sect.5). At $\omega q = 0$ the amplitude M obtains the form:

$$M = -\frac{e^2}{(2\pi)^{1/2}} \frac{1}{t} F, \quad (30)$$

where $F = \int \frac{d\tau_4}{(2\pi)^2 (\tau_4 - i0)} F_1(\tau_4)$.

The condition $\omega q = 0$ is equivalent to one $s_1 = s_3$, and, consequantly, the form-factor F depends besides t on two variables s_1 and s_2 (see Eqs.(28)): $F = F(t, s_1, s_2)$ and can be represented in Fig. 8b with the virtual proton.

As the last example we consider the structure function of the deep inelastic eN - scattering off energy shell. This function is connected with off energy shell $eN \to eX$ amplitude squared and summarized over all final states of X. It appeares in calculations of deep inelastic ed- scattering in impules approximation (see Fig. 9a). Keeping in mind that at calculation of diagram Fig. 9a the condition $\omega Q = 0$ is used, we represented the amplitude in Fig. 9b, analogously to the case of form-factor, with the virtual photon with four- momentum Q, not "enmeshed" by the spurion line. One can easy see that this structure function besides the arguments Q^2 and pQ (coming in the structure function on the energy shell in the scaling region in the form of ratio $x = -\frac{Q^2}{2pQ}$) depends on the variable $s = (p + \omega\tau)^2$ (since there are no any other independent scalar products). Therefore, in general, it does not coincide with the structure function, measured in experiments on nucleons. These conclusions remains valid and for calculations in the framework of the old fashioned perturbation theory in IMF. The character of the off energy shell form-factor and amplitude dependence on the additional variables is determined by dynamics.

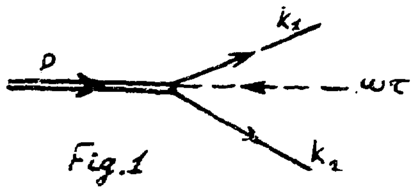
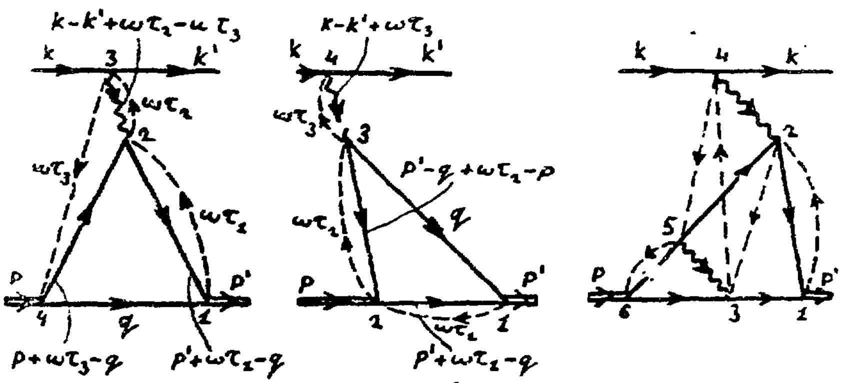


Fig.1



a b c

Fig.2

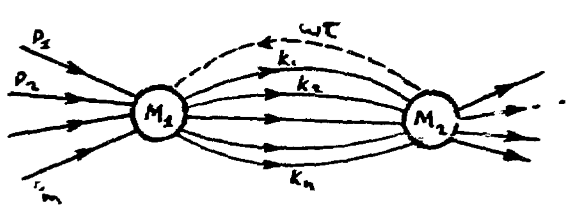


Fig.3

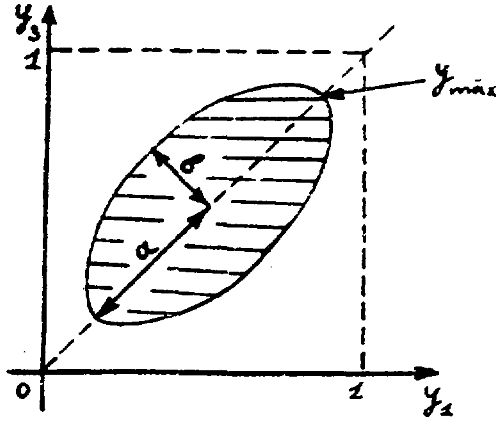
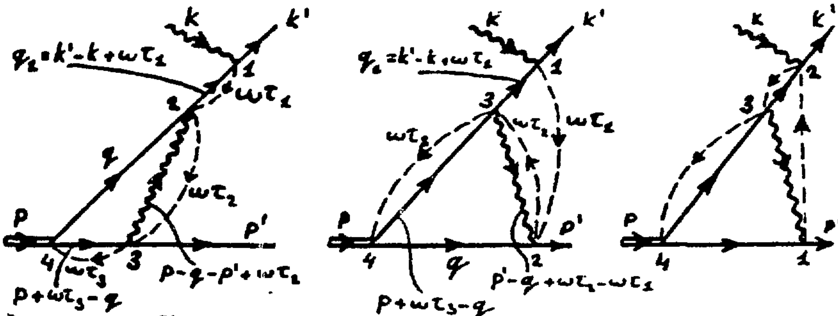
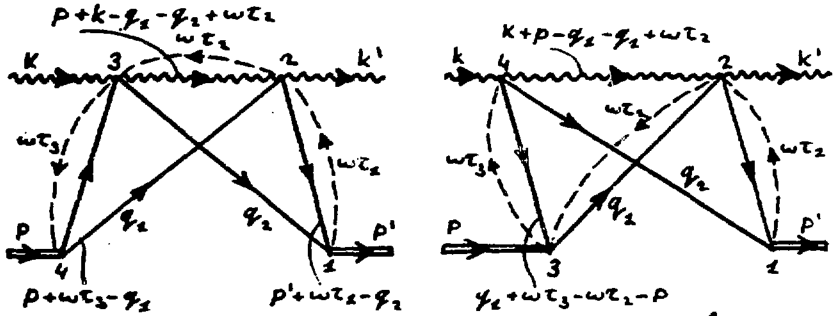


Fig. 4



a Fig. 5 b c



a b

Fig. 6

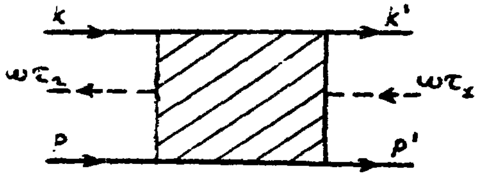


Fig. 7

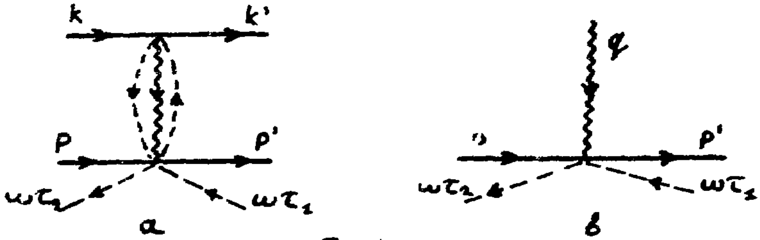


Fig. 8

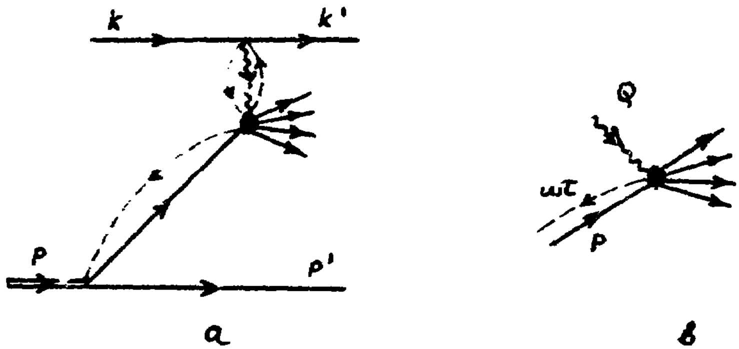


Fig. 9

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