FR7801687

DPh-PFC-STGI

EUR-CEA-FC-944

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February 1978

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ABSTRACT

Electron cyclotron radiation from a warm inhomogeneous plasma is investigated. A direct calculation of the emissive power of a plasma slab is performed using Rytov's method and the result is compared with the solution of the transfer equation. It is found that, for arbitrary directions of emission, the two results differ, which reflects the fact that Kirchhoff's law is not generally obeyed.

I. INTRODUCTION

The theory of the electron cyclotron emission from thermal plasmas has been considered in many papers, but often the analysis is restricted to a very tenuous medium[1] and, therefore, neglects the plasma polarization effects. The inclusion of the plasma polarization by using the cold plasma dielectric tensor[2,3] is subject to the obvious criticism that it is intrinsically inconsistent with the basic feature of the process of emission, namely, the thermal motion of the electrons. The emission of cyclotron radiation with the full warm plasma non-relativistic dielectric tensor was first briefly considered in Ref. 4. Recently[5], the problem of the plasma polarization effects in the emission near the cyclotron frequency has been reconsidered, but the analysis is subject to criticism since it predicts no emission in the ordinary mode. As it is known[6-9], in a hot plasma (T_e > 1 KeV) the ordinary mode is strongly absorbed for quasi-perpendicular propagation and, therefore, we expect that it is emitted as a black body.

The thermal emission from the plasma may be obtained from the knowledge of the absorption coefficient and the solution of the standard transfer equation, if we assume that Kirchhoff's law is valid. The validity of Kirchhoff's law, however, has not been proved for anisotropic plasmas with general dispersive properties[10,11]. In Ref. 10, the dielectric properties of the magnetized plasma are described by the cold plasma dielectric tensor with a small anti-Hermitian part due to ordinary collisions. The special case of the synchrotron radiation is discussed in Ref. 11, but for the fundamental frequency the proof of the validity of Kirchhoff's law is restricted to the case of a very tenuous plasma .

In this paper, the emissive power is obtained by Rytov's method[12], which is suitable for the calculation of the emission from a body with arbitrary dispersive properties. The procedure is to calculate from Maxwell's equations the wave field in terms of the thermal fluctuating sources and use the fluctuation -dissipation theorem to obtain the needed quadratic quantities like the Poynting vector. The calculation is specifically performed for a weakly inhomogeneous plasma slab with the dielectric properties of the medium described by the warm plasma weakly relativistic dielectric tensor. Under conditions for which the WKB solutions of the field equations are valid, it is possible to obtain a compact expression for the spectrum of the emitted radiation. This is compared with the corresponding solution of the standard transfer equation and the validity of Kirchhoff's law is then investigated.

In Sec. II, we formulate the problem. In Sec. III, the WKB solutions of the radiated wave field for a plasma slab are obtained. In Sec. IV, the spectrum of the radiated energy from the slab faces is derived and compared with the solutions of the transfer equation. The conclusions are given in Sec. V.

II. FORMULATION OF THE PROBLEM

We consider a weakly inhomogeneous plasma column located in a magnetic field $\vec{H}_o = H_o \vec{e}_z$ of constant direction (the z direction) parallel to the axis of the column. The electron density n_e, the electron temperature T_e, and H_o are assumed to vary in the xy plane, normal to the axis of the column, but constant in the z direction. For a plane wave of the form $\exp[i(\int \vec{k}.d\vec{r} - \omega t)]$, the local dispersion relation is given by[9]

$$\begin{aligned} & n_\perp^4[(\varepsilon_{xx}-n_\parallel^2)(1-\chi_{zz}) + (n_\parallel + \chi_{xz})^2] - n_\perp^2[\varepsilon_{xy}^2(1-\chi_{zz}) - \\ & 2\varepsilon_{xy}\chi_{yz}(n_\parallel + \chi_{xz}) + (\varepsilon_{xx} - n_\parallel^2)(\varepsilon_{xx} + \varepsilon_{zz} - \varepsilon_{xx}\chi_{zz} + \\ & n_\parallel^2 \chi_{zz} + 2n_\parallel \chi_{xz} + \chi_{xz}^2 - \chi_{yz}^2)] + \\ & \varepsilon_{zz}[(\varepsilon_{xx} - n_\parallel^2)^2 + \varepsilon_{xy}^2] = 0, \end{aligned} \tag{1}$$

where $n_\perp^2 = n_x^2 + n_y^2 = c^2 k_\perp^2/\omega^2$, $n_\parallel = ck_z/\omega$. In obtaining Eq. (1) we use the following dielectric tensor

$$\epsilon_{xx} = \epsilon_{yy} = \varepsilon_{xx},\ \epsilon_{xy} = -\epsilon_{yx} = \varepsilon_{xy},\ \epsilon_{zz} = \varepsilon_{zz} + n_\perp^2 \chi_{zz},$$

$$\epsilon_{xz} = n_x \chi_{xz} - n_y \chi_{yz},\ \epsilon_{zx} = n_x \chi_{xz} + n_y \chi_{yz}, \quad (2)$$

$$\epsilon_{yz} = n_y \chi_{xz} + n_x \chi_{yz},\ \epsilon_{zy} = n_y \chi_{xz} - n_x \chi_{yz},$$

where

$$\varepsilon_{xx} = 1 - \frac{X}{2(1+Y)} - X\mu W_o,\ \varepsilon_{xy} = -i\frac{X}{2(1+Y)} + iX\mu W_o,\ \varepsilon_{zz} = 1-X,$$

$$\chi_{xz} = n_\parallel\ \mu X(W_1-W_o)/Y,\ \chi_{zz} = -X[W_1+n_\parallel^2\ \mu(W_o+W_2-2W_1)]/Y^2, \quad (3)$$

$$\chi_{yz} \simeq i\chi_{xz},$$

and

$$X = \omega_p^2/\omega^2 = 4\pi n_e e^2/m_e\omega^2,\ Y = \omega_c/\omega = e H_o/m_e c\omega > o,$$

$$\mu = 2(c/v_t)^2 = m_e c^2/T_e,$$

$$W_m = -(i/2)\int_o^\infty dt(1-it)^{-m-5/2}\exp\left[i\mu t(1-Y) - \frac{\mu n_\parallel^2 t^2}{2(1-it)}\right],$$

$$= \frac{1}{2}\exp(-\mu n_\parallel^2/2)\sum_{r=o}^{\infty}(\mu n_\parallel^2/2)^r\frac{1}{r!}F_{m+r+5/2}[\mu(1-Y)-\mu n_\parallel^2/2], \quad (4)$$

$$F_q(\xi) = \sum_{P=o}^{q-3/2}(-\xi)^p\frac{\Gamma(q-1-p)}{\Gamma(q)} + \frac{\pi^{1/2}}{\Gamma(q)}(-\xi)^{q-3/2} \quad (5)$$

$$\times [i\xi^{1/2} Z(i\xi^{1/2})],$$

where Z is the Fried-Conte function[13]. The tensor (3) is obtained from the full relativistic tensor[14] by assuming that the electron Larmor radius is smaller than the field wavelength and[16] $\mu \gg 1$(the weakly relativistic approximation).

We have ignored the finite Larmor radius corrections in ϵ_{xx} and ϵ_{yy} since they have a negligible effect unless ω is close to the upper-hybrid frequency. The non-relativistic limit is obtained from Eq. (4) by letting $\mu n_{\parallel}^2/2 \gg 1$. In this case we obtain[9]

$$\varepsilon_{xx} = 1 + X\zeta_o \frac{Z_1 + Z_{-1}}{2}, \quad \varepsilon_{xy} = -iX\zeta_o \frac{Z_1 - Z_{-1}}{2},$$

$$\chi_{xz} = (X/2Y)(v_t/c)\,\zeta_o(1 + \zeta_1 Z_1), \quad \chi_{zz} = (X/2Y^2)(v_t/c)^2 \times \zeta_o\zeta_1(1 + \zeta_1 Z_1), \tag{6}$$

where

$$\zeta_m = (c/n_{\parallel} v_t)(1 - mY), \quad Z_m = Z(\zeta_m) = \Big[-2\int_o^{\zeta_m} \exp(t^2)\,dt + i\pi^{1/2}(n_{\parallel}/|n_{\parallel}|)\Big]\exp(-\zeta_m^2).$$

In Eq (1) we consider $n_{\parallel}$ as a given real constant. For problems concerning the propagation of waves from the vacuum into the plasma and the emission of radiation from the plasma into the vacuum, the boundary conditions on the wave field require that $n_z = n_{\parallel}$ be constant everywhere and equal to the direction cosine of the phase vector at the plasma-vacuum interface (Snell's law). Thus, in the plane wave representation of the wave field $\vec{k} = \vec{k}' + i\,\vec{k}''$, where $\vec{k}'$ and $\vec{k}''$ are real vectors and generally have different directions, $\vec{k}' = k_x'\,\vec{e}_x + k_y'\,\vec{e}_y + k_z\,\vec{e}_z$, $\vec{k}'' = k_x''\,\vec{e}_x + k_y''\,\vec{e}_y$. Plane waves in which $\vec{k}'$ and $\vec{k}''$ have different directions are called inhomogeneous and are generally the only ones which can satisfy simultaneously the dispersion relation and the boundary conditions. For inhomogeneous plane waves the damping is determined by $\vec{k}''$ and, therefore, the absorption coefficient in the direction of the ray path is given by the projection of $\vec{k}''$ over the group velocity of the wave-packet. The ray path and the absorption coefficient are obtained from Eq. (1) in the following way. Let $k_\perp^2 = f(k_z, \omega, \vec{r}_\perp) = f' + if''$ be a solution of Eq. (1). Assuming $|\vec{k}_\perp''| \ll |\vec{k}'|$ and $|f''| \ll |f'|$ we obtain

$$D(\vec{k}) = k_\perp^2 - f(k_z, \omega, \vec{r}) = D'(\vec{k}') + i\left[D''(\vec{k}') + \vec{k}_\perp'' \cdot \frac{\partial D'(\vec{k}')}{\partial \vec{k}'}\right] = 0,$$

and

$$D'(\vec{k}') = k_\perp'^2 - f'(k_z, \omega, \vec{r}_\perp) = 0, \qquad (7)$$

$$\vec{k}_\perp'' \cdot \frac{\partial D'(\vec{k}')}{\partial k_\perp'} = - D''(\vec{k}') = f''(k_z, \omega, \vec{r}). \qquad (8)$$

From Eq. (7) we obtain the equations for the ray path[16] ℓ

$$\frac{d\vec{k}'}{dt} = (\partial D'/\partial\omega)^{-1} (\partial D'/\partial\vec{r}), \quad \frac{d\vec{r}}{dt} = \vec{v}_g = - (\partial D'/\partial\omega)^{-1} \times (\partial D'/\partial\vec{k}'), \qquad (9)$$

and from Eq. (8) we obtain the absorption coefficient

$$\alpha = 2 \int_\ell (\vec{k}_\perp'' \cdot \vec{v}_{g\perp}/|\vec{v}_g|)d\ell = 2\,\mathrm{sign}(\partial f'/\partial\omega) \times \int_\ell (f''/|\partial D'/\partial\vec{k}'|)d\ell \qquad (10)$$

It is known (Ref. 1, p. 39) that the solution of the transfer equation is determined if ℓ and α are given [Eqs. (9) and (10)]. This is always a difficult numerical work for arbitrary inhomogeneity in the xy plane. We here consider the case of a plane stratified plasma and assume that n_e, T_e and H_o are constant in the y direction. For the slab geometry $\vec{k}'' = k_x'' \vec{e}_x$ and k_z, k_y real and constant and Eq. (1) yields k_x' and k_x''. Using Eqs. (1.100) and (1.139) of Ref. 1, we obtain :

$$\frac{dP}{d\omega\, d\Sigma\, da} = (1 - n_{\parallel}^2 - n_y^2)^{1/2} \int_{x_o}^{x_1} dx\, B_o(\omega,T_e)\ (2\,\frac{\omega}{c}\, n_x'')$$

$$\times \exp(-2\,\frac{\omega}{c} \int_{x}^{x_1} n_x''\, dx), \qquad (11)$$

where x_1 is the position of the radiating area da of the slab surface, x_o lies in the opposite face of the slab, and B_o is the source function,

$$B_o(\omega,T_e) = \omega^2\, T_e/(2\pi)^3\, c^2. \qquad (12)$$

In Eq. (11), dP is the power radiated in the spectral range $d\omega$, in the solid angle $d\Sigma = \sin\theta d\theta\, d\phi$, by the elementary area da, and $n_y = \sin\theta\, \sin\phi$, $n_{\parallel} = \cos\theta$ in polar coordinates. Equation (11) holds for each mode of propagation obtained from Eq. (1) and in the absence of reflecting boundaries.

At it is well known, the validity of the geometric optics approximation is restricted by the requirement that the real part of k_x has no singularities. Now, the real part of k_x as given by Eq. (1) is well represented[9] by the corresponding expression for a cold plasma, thus

$$n_x'^2 \approx 1 - n_{\parallel}^2 - n_y^2 - X \pm (XY/2)\,\frac{\Delta \pm Y(1 + n_{\parallel}^2)}{1 - X - Y^2} = n'^2 - n_y^2, \qquad (13)$$

where

$$\Delta = [(1 - n_{\parallel}^2)^2\, Y^2 + 4\, n_{\parallel}^2\, (1 - X)]^{1/2}.$$

From Eq. (13) we see that the extraordinary mode (the lower sign) has a resonance at $X_r = 1 - Y^2$, and the two modes interact at $X_1 = 1 + (1 - n_\parallel^2)^2\, Y^2/4n_\parallel^2$, as for the case $n_y = o$. The cut-off density X_f is now given by $n_\perp'^2 = n_y^2$ and differs from that corresponding to $n_y = o$. In the regions where $X(x) \gtrsim X_r$, X_i, X_f, the local homogeneous plasma approximation breaks down, even in a weakly inhomogeneous medium, and the conditions for the validity of the ray theory are violated. In the following we consider a plasma for which the singular regions, defined by $X(x) \gtrsim X_r$, X_i, X_f, can be avoided. This results in a restriction on the maximum value of the ratio ω_p^2/ω_c^2 and the direction of emission. For instance, in a plasma located in a monotonically decreasing magnetic field, the emission of the extraordinary mode is only possible from the high-H_o side of the slab. Moreover, for $|n_y^2| << 1$ the singularities defined by $X(x) = X_i$, X_f can be avoided[9] if $\omega_p^2/\omega_c^2 \lesssim 1$ for $-a \lesssim x \lesssim a$, where $\pm a$ are the plasma boundaries and $n_e(\pm a) = o$.

The validity of Kirchhoff's law [Eq. (12)] has been proved for a plasma with a negligible anti-Harmitian part of the dielectric tensor[10,11]. Now, for the tensor given by Eqs. (3) this is not the case, as it may easily be seen from Eqs. (6) for $\omega \gtrsim \omega_c$, and it is of interest to perform a direct calculation of dP without the assumption of the validity of Kirchhoff's law. Equations (11) and (12) are then used as a reference to infer the validity of Kirchhoff's law for the plasma slab.

III. THE WKB SOLUTION OF THE RADIATED WAVE FIELD

The electromagnetic field generated by a given source $\vec{J}(\vec{r},t)$ is obtained from the equations

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\,, \quad \nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c}\vec{J}, \tag{14}$$

where $\vec{J}$, the source of radiation, is a fluctuation quantity whose spatial and temporal correlation is known. Since we consider the medium to be homogeneous in the yz plane we

Fourier analyse the vectors $\vec{E}$, $\vec{B}$, $\vec{D}$ and $\vec{J}$ and put

$$\vec{E}(\vec{r},t) = \int_{-\infty}^{\infty} dk_y dk_z d\omega \; \vec{E}(k_y,k_z,\omega,x)\exp\left[i(k_y y+k_z z-\omega t)\right], \tag{15}$$

and a similar decomposition for $\vec{B}$, $\vec{D}$ and $\vec{J}$. The Fourier component of $\vec{D}$ is related to $\vec{E}(k_y,k_z,\omega,x)$ by the equation

$$\vec{D}(k_y,k_z,\omega,x) = \overset{\leftrightarrow}{\epsilon}\cdot \vec{E}(k_y,k_z,\omega,x), \tag{16}$$

where $\overset{\leftrightarrow}{\epsilon}$ is the dieletric tensor-operator obtained from Eq. (2) by the substitution $n_y = ck_y/\omega$, $n_\parallel = ck_z/\omega$, and $n_x = -i(c/\omega)d/dx$. Inserting Eqs. (15) and (16) in Eq. (14) we obtain

$$B_x = -i\frac{c}{\omega}(i\,k_y E_z - ik_z E_y),$$
$$B_y = -i\frac{c}{\omega}(i\,k_z E_x - \frac{dE_z}{dx}),$$
$$B_z = -i\frac{c}{\omega}(\frac{dE_y}{dx} - ik_y E_x), \tag{17}$$

$$\epsilon_{xx} E_x + E_{xy} E_y - i\frac{c}{\omega}\chi_{xz}\frac{dE_z}{dx} - \frac{c}{\omega}k_y\chi_{yz}E_z - i\frac{c}{\omega} \times (ik_y B_z - ik_z B_y) = -\frac{4\pi i}{\omega}J_x,$$

$$-\epsilon_{xy} E_x + \epsilon_{xx} E_y + \frac{c}{\omega}k_y\chi_{xz}E_z - i\frac{c}{\omega}\chi_{yz}\frac{dE_z}{dx} - i\frac{c}{\omega} \times (ik_z B_x - \frac{dB_z}{dx}) = -\frac{4\pi i}{\omega}J_y,$$

$$-i\frac{c}{\omega}\chi_{xz}\frac{dE_z}{dx} + \frac{c}{\omega}k_y\chi_{yz}E_x + \frac{c}{\omega}k_y\chi_{xz}E_y + i\frac{c}{\omega}\chi_{yz}\frac{dE_y}{dx} + \epsilon_{zz}E_z + (\frac{ck_y}{\omega})^2\chi_{zz}E_z - \frac{c^2}{\omega^2}\chi_{zz}\frac{d^2E_z}{dx^2} - i\frac{c}{\omega}(\frac{dB_y}{dx} - ik_y B_x) = -\frac{4\pi i}{\omega}J_z,$$

where

$$E_\alpha = E_\alpha(k_y,k_z,\omega,x)\,,\ B_\alpha = B_\alpha(k_y,k_z,\omega,x)\,,\ J_\alpha = J_\alpha(k_y,k_z,\omega,x)\,,$$

with $\alpha = x,y,z$. Equations (17) can be rearranged and cast in the following form

$$\frac{de_j}{dx} - i\,\frac{\omega}{c}\sum_k \tilde{T}_{jk}\,e_k = i\,\frac{\omega}{c}\,s_j\,, \tag{18}$$

where

$$e_1 = -E_z,\ e_2 = -E_y,\ e_3 = -B_z,\ e_4 = B_y,$$

and $\tilde{T}_{jk}$ is the 4 × 4 matrix

$$\tilde{T} = \begin{pmatrix} n_y\,T_{11} & T_{12} & n_y\,T_{13} & T_{14} \\ n_y^2\,T_{21} & n_y\,T_{22} & 1-(n_y^2/d_1) & n_y\,T_{24} \\ n_y\,T_{31} & T_{32} & n_y\,T_{33} & T_{34} \\ T_{41}-n_y^2 & n_y\,T_{42} & T_{43} & 0 \end{pmatrix},$$

where

$$T_{11} = n_\parallel\ \chi_{yz}/d_1,\ T_{12} = -\,n_\parallel\ \varepsilon_{xy}/d_1,\ T_{13} = -\,n_\parallel\,/d_1,$$

$$T_{14} = (\varepsilon_{xx}-n_\parallel^2\,)/d_1,$$

$$T_{21} = \chi_{yz}/d_1,\ T_{22} = -\,\varepsilon_{xy}/d_1,\ T_{24} = -\,(n_\parallel\ +\chi_{xz})/d_1,$$

$$T_{31} = n_\parallel\ +\chi_{xz} - \chi_{yz}(\varepsilon_{xy} - n_\parallel\ \chi_{yz})/d_1,\ T_{32} = \varepsilon_{xx} - n_\parallel^2 +$$

$$\varepsilon_{xy}(\varepsilon_{xy}-n_\parallel\ \chi_{yz})/d_1,$$

$$T_{33} = (\varepsilon_{xy} - n_\parallel \ \chi_{yz})/d_1, \ T_{34} = [\chi_{yz}(\varepsilon_{xx} - n_\parallel^2) + \varepsilon_{xy}(n_\parallel + \chi_{xz})]/d_1,$$

$$T_{41} = \varepsilon_{zz}(d_1/d_2), \ T_{42} = n_\parallel, \ T_{43} = - \chi_{yz}(d_1/d_2) - \varepsilon_{xy}(\chi_{xz} + n_\parallel \ \chi_{zz})/d_2,$$

and

$$s_1 = \frac{4\pi i}{\omega} \ n_\parallel \ J_x/d_1, \ s_2 = \frac{4\pi i}{\omega} n_y \ J_x/d_1, \ s_3 = \frac{4\pi i}{\omega}[- J_y + (n_\parallel \ \chi_{yz} - \varepsilon_{xy}) \ J_x/d_1],$$

$$s_4 = \frac{4\pi i}{\omega} [- J_z(d_1/d_2) - i \frac{c}{\omega} (\chi_{xz} + n_\parallel \ \chi_{zz}) \ dJ_x/d_2 \ dx + n_y \ (\chi_{xz} + n_\parallel \ \chi_{zz}) \ J_y/d_2],$$

$$d_1 = \varepsilon_{xx} + n_\parallel \ \chi_{xz}, \ d_2 = (\varepsilon_{xx} - n_\parallel^2) \ (1 - \chi_{zz}) + (n_\parallel + \chi_{xz})^2.$$

To complete Eqs. (18) we add

$$d_1 E_x = - \varepsilon_{xy} \ E_y + n_y \ \chi_{yz} \ E_z + (n_\parallel + \chi_{xz}) B_y - n_y B_z - \frac{4\pi i}{\omega} J_x, \tag{19}$$

and the first of Eqs. (17). In obtaining Eqs. (18) we neglect terms proportional to the gradients of n_e, T_e and H_o. This is consistent with the assumption of a weakly inhomogeneous plasma for which the field wave-length is much smaller than $(dn_e/n_e d_x)^{-1}$, $(dT_e/T_e dx)^{-1}$, and $(dH_o/H_o dx)^{-1}$.

The matrix $\tilde{T}_{jk}$ can be diagonalized by using the 4 × 4 matrix

$$R_{1j} \ ; \ R_{2j} \ ; \ R_{3j} \ ; \ R_{4j} = [n_j (T_{12} + T_{14} \ T_{43}) + n_y \ (\sigma_j \ T_{13} + T_{14} \ T_{13} \ T_{41} - T_{22} \ T_{43} + T_{14} \ T_{42}) \ ;$$

$$\phi_j \; ; \; n_j \, \sigma_j + n_y(\sigma_j \, T_{33} + T_{13} \, T_{34} \, T_{41}) \; ; \; T_{43} \, \sigma_j + T_{12} \, T_{41} +$$

$$T_{43} \, T_{14} \, T_{41} + n_j \, n_y(T_{33} \, T_{43} + T_{13} \, T_{41} + T_{42}) +$$

$$n_y^2(T_{33} \, T_{42} - T_{43})] \; (\phi_j F_j)^{-1/2},$$

and its inverse

$$(R^{-1})_{j1} \; ; \; (R^{-1})_{j2} \; ; \; (R^{-1})_{j3} \; ; \; (R^{-1})_{j4} = \Big\{ T_{34} \, T_{41} +$$

$$n_j \, n_y(T_{31} + T_{24} \, T_{41}) + n_y^2[T_{43} \, T_{24}(n_\parallel + \chi_{xz}) - T_{43} \, T_{21} \, \chi_{yz} +$$

$$T_{21} \, T_{42} \, T_{24} \, T_{41} - T_{34} + \sigma_j \, T_{21}] \; ;$$

$$n_j(\sigma_j - T_{34} \, T_{43}) + n_y[\sigma_j \, T_{22} + T_{22} \, T_{24} \, T_{41} \, T_{42} -$$

$$T_{43} \, T_{14} \, (n_\parallel + \chi_{xz}) - T_{43} \, T_{22} \, \chi_{yz}] \; ;$$

$$\phi_j \; ; \; n_j \, T_{34} + n_y[\sigma_j \, T_{24} + T_{14} \, T_{24} \, T_{41} + T_{22} \, \chi_{yz} +$$

$$T_{14}(n_\parallel + \chi_{xz})] \Big\} \; (\phi_j \, F_j)^{-1/2},$$

where

$$\sigma_j = n_j^2 + n_y^2 - T_{14} \, T_{41}, \; F_j = 2n_j \; (2n_j^2 - n_1^2 - n_3^2),$$

$$\phi_j = \sigma_j + n_y \, n_j(T_{24} \, T_{43} - T_{11}) + n_y^2 \; [T_{21} \, T_{43} + T_{24}(T_{42} +$$

$$T_{13} \, T_{41}) - (\sigma_j/d_1)],$$

and n_j, $j = 1, 2, 3, 4$, is given by the solutions of the equation

$$\det|\tilde{T}_{jk} - n \, \delta_{jk}| = (n^2 + n_y^2)^2 - (n^2 + n_y^2) \; (T_{32} + T_{34} \, T_{43} +$$

$$T_{14} \, T_{41}) + T_{41}(T_{14} \, T_{32} - T_{12} \, T_{34}) = o,$$

which coincides with Eq. (1) with $n_j = n_{xj}$; thus, $n_2 = - n_1$ and $n_4 = - n_3$. Letting

$$e_j = \sum_k R_{jk} f_k , \tag{20}$$

Eqs. (18) become

$$\frac{df_j}{dx} - i \frac{\omega}{c} n_j f_j = i \frac{\omega}{c} \sum_\ell (R^{-1})_{j\ell} S_\ell = \gamma_j . \tag{21}$$

In Eqs. (21) we neglect the contribution due to the terms $\sum_{\ell k} (R^{-1})_{j\ell} (dR_{\ell k}/dx) f_k$. The neglected terms become important near the singularities defined by $X(x) = X_r, X_f, X_i$. Since we consider a physical situation for which these singularities can be avoided, the neglected terms are of the order of L^{-1}, where L is the scale length of the variation of the average plasma parameters. This may be shown by considering the case $n_y = o$. Thus,

$$\Gamma_{jk} = \sum_\ell (R^{-1})_{j\ell} (dR_{\ell k}/dx) = \Big[\tfrac{1}{2}(n_j + n_k) (\rho_k\, d\rho_j/dx -$$

$$\rho_j\, d\rho_k/dx) - (n_j - n_k) d\eta/\eta dx - \tfrac{1}{2} (1 - \rho_j \rho_k)\, d(n_j - n_k)/dx -$$

$$2n_j \alpha_k \Big] (\sigma_1 \sigma_3) (\sigma_j \sigma_k F_j F_k)^{-1/2} ,$$

where

$$\eta = \varepsilon_{zz}^{-1/2}, \quad \alpha_k = \tfrac{1}{2} \Big[\frac{d_1}{d_2} \frac{d}{dx}(d_2/d_1) + \eta\, \rho_k (d_2/d_1) \frac{dT_{43}}{dx}\Big],$$

$$\rho_j = \frac{n_j^2 d_2 - \varepsilon_{zz}(\varepsilon_{xx} - n_\parallel^2)}{d_1\, \varepsilon_{zz}^{1/2}\, T_{34}} .$$

For instance, the term

$$\Gamma_{12} = - \Gamma_{21} = i(d\eta/\eta dx)/(\rho_1^2 - 1) - i\, (dn_1/2n_1\, dx) +$$

$$i\alpha_1/(\rho_1^2 - 1),$$

is singular for $n_1 = o$ or $n_1^2 = n_3^2$. The last condition implies $\rho_1 = \rho_3 = 1$ since $\rho_1\rho_3 = 1$. Away from the singular regions $|\Gamma_{jk}| \gtrsim L^{-1}$. Equations (20) and (21) describe the radiated field in the two pairs of normal modes of propagation and are valid as long as $|(c/\omega)\,\Gamma_{jk}| << 1$, which can be considered as the conditions for the validity of the WKB solution of the field equations. From Eq. (21) we obtain

$$f_j(x) = [f_j(x_o) + \int_{x_o}^{x} dx'\,\gamma_j(x')\,\exp(-i\,\frac{\omega}{c}\int_{x_o}^{x'} n_j\,dx'')]$$

$$\exp(i\,\frac{\omega}{c}\int_{x_o}^{x} n_j\,dx') = [(\tilde{\gamma}_j)_{x_o}^{x} + f_j(x_o) + \tag{22}$$

$$\int_{x_o}^{x} dx'\,\tilde{\gamma}_j(x')\,\exp(-i\,\frac{\omega}{c}\int_{x_o}^{x'} n_j\,dx'')]\,\exp(i\,\frac{\omega}{c}\int_{x_o}^{x} n_j dx'),$$

where

$$\tilde{\gamma}_j = \frac{4\pi}{c}\,(\phi_j F_j)^{-1/2}\;\vec{b}_j\cdot\vec{J},$$

$$b_{jx} = T_{41}\,T_{22} - (n_j^2 + n_y^2)\,(T_{22} + T_{24}\,T_{43}) - n_j\,n_y\,\sigma_j\,(\frac{1}{d_1} +$$

$$T_{24}\,\frac{\chi_{xz} + n_{\parallel}\,\chi_{zz}}{d_2}) - \frac{n_j n_y}{d_2}\,[(n_{\parallel} + \chi_{xz})^2\,(1 - \frac{\varepsilon_{zz}}{d_2}) +$$

$$\chi_{yz}^2],$$

$$b_{jy} = \sigma_j - n_y^2\,\sigma_j\,(\frac{1}{d_1} + T_{24}\,\frac{\chi_{xz} + n_{\parallel}\,\chi_{zz}}{d_2}) - \frac{n_y^2}{d_2} \tag{23}$$

$$\times[(n_{\parallel} + \chi_{xz})^2\,(1 - \frac{\varepsilon_{zz}}{d_2}) + \chi_{yz}^2],$$

$$b_{jz} = \Big\{ n_j T_{34} + n_y [T_{24} \sigma_j + T_{14} T_{24} T_{41} + T_{22} \chi_{yz} + T_{14} (n_\parallel + \chi_{xz})] \Big\} (d_1/d_2),$$

and

$$\tilde{\gamma}_j = \frac{4\pi i}{\omega} (\phi_j F_j)^{-1/2} [(\chi_{xz} + n_\parallel \chi_{zz})/d_2] \Big\{ n_j T_{34} + n_y \times [T_{24} \sigma_j + T_{14} T_{24} T_{41} + T_{22} \chi_{yz} + T_{14} (n_\parallel + \chi_{xz})] \Big\} \times J_x \exp(-i \frac{\omega}{c} \int_{x_0}^{x'} n_j \, dx'').$$

The wave field components are obtained from Eqs. (19) and (20)

$$E_x(x) = - \sum_{j=1}^{4} \Big\{ n_j^2 (T_{22} + T_{24} T_{43}) - T_{22} T_{41} - n_y^2 T_{33} + n_j n_y [T_{21} T_{43} + T_{24}(T_{42} + T_{13} T_{41}) - (\sigma_j/d_1)] \Big\} (\phi_j F_j)^{-1/2} f_j(x) - \frac{4\pi i}{\omega} (J_x/d_1),$$

$$E_y(x) = - \sum_{j=1}^{4} \phi_j (\phi_j F_j)^{-1/2} f_j(x), \tag{24}$$

$$E_z(x) = - \sum_{j=1}^{4} [n_j (T_{12} + T_{14} T_{43}) + n_y (\sigma_j T_{13} + T_{14} T_{13} T_{41} - T_{22} T_{43} + T_{14} T_{42})] (\phi_j F_j)^{-1/2} f_j(x).$$

To calculate the radiation flux leaving the slab faces, we need $f_j(x)$ at $x = \pm a$; the corresponding values of x_o are $x_o = \mp a$. Now, $f_1(f_2)$ and $f_3(f_4)$ represent the right (left) going waves in one mode of polarization, hence, in the absence of reflecting boundaries, we must have

$$f_1(+a) \neq o,\ f_3(+a) \neq o,\ f_2\ (+a) = o,\ f_4\ (+a) = o,$$
$$f_1\ (-a) = o,\ f_3\ (-a) = o,\ f_2\ (-a) \neq o,\ f_4\ (-a) \neq o, \tag{25}$$

and $f_j(x_o) = o$, and Eq. (22) yields

$$f_j(x_1) = \exp\left(i\,\frac{\omega}{c}\int_{x_o}^{x_1} n_j\ dx\right)\int_{x_o}^{x_1} dx\ \tilde{\gamma}(x)$$

$$\times\ \exp\left(-i\,\frac{\omega}{c}\int_{x_o}^{x} n_j\ dx'\right), \tag{26}$$

where x_1 is $\pm$ a according to conditions (25) and $(\tilde{\gamma})_{x_o}^{x_1} = o$ since $\vec{J}(\pm a) = o$.

IV. THE SPECTRUM OF THE RADIATED ENERGY

To calculate the power radiated by the slab area at x_1, we use the averaged Poynting vector

$$\vec{S}_{av}\ (c/4\pi)\ <\vec{E}(\vec{r},t)\ \times\ \vec{B}(\vec{r},t)>,$$

where the bracket means an ensemble average. Using Eq. (15) we obtain

$$S_{av}(x_1) = \frac{c}{4\pi}\iint_{-\infty}^{\infty} dk_y\ dk_z\ dk_y'\ dk_z'\ d\omega\ d\omega'\ \frac{c\vec{k}'(x_1)}{\omega'}$$

$$\times\ <\vec{E}(k_y,k_z,\omega,x_1)\cdot\vec{E}(k_y',k_z',\omega',x_1)>\ \exp\Big\{i\big[(k_y + k_y')y +$$

$$(k_z + k_z')\ z - (\omega + \omega')t\big]\Big\}, \tag{27}$$

where we assume that the predominant x dependence of $\vec{E}(k_y,k_z,\omega,x)$ is given by the factor $\exp(i\int^x k_x dx)$ with $k_x(x) = k_x(k_y,k_z,\omega,x)$ obtained from the dispersion relation (1). The field $\vec{E}(k_y,k_z,\omega,x)$ at $x = x_1$ is the sum of two modes with the same value of $\vec{k}$, namely,

$$\vec{k}(x_1) = \left\{ \pm \frac{\omega}{c} \left[1 - \left(\frac{ck_y}{\omega}\right)^2 - \left(\frac{ck_z}{\omega}\right)^2\right]^{1/2} ;\ k_y;k_z \right\},$$

and is orthogonal to $\vec{k}(x_1)$, i.e., $\vec{E}(x_1)\cdot\vec{k}(x_1) = 0$. Using the prescription $k_x(k_y,k_z,\omega,x) = -k_x^*(-k_y,-k_z,-\omega,x)$ and that the field correlation is stationary in time and homogeneous in the yz plane, Eq. (27) becomes

$$\vec{S}_{av}(x_1) = \frac{c}{2\pi} \int_0^\infty d\omega \int_{-\infty}^{\infty} dk_y dk_z \frac{c\vec{k}(x_1)}{\omega} \langle|\vec{E}(k_y,k_z,\omega,x_1)|^2\rangle, \tag{28}$$

where $\langle|\vec{E}(k_y,k_z,\omega,x_1)|^2\rangle$ is defined by the relation

$$\langle\vec{E}(k_y,k_z,\omega,x)\cdot\vec{E}^*(k_y',k_z',\omega',x_1)\rangle = \langle|\vec{E}(k_y,k_z,\omega,x_1)|^2\rangle \times \delta(k_z-k_z')\delta(k_y-k_y')\delta(\omega-\omega').$$

Equation (28) may be written as

$$\vec{S}_{av}(x_1) = \frac{c}{2\pi} \int_0^\infty d\omega \int_{-\infty}^{\infty} dk_x dk_y dk_z\ \delta\left\{k_x - \frac{\omega}{c}\left[1 - \left(\frac{ck_y}{\omega}\right)^2 - \left(\frac{ck_z}{\omega}\right)^2\right]^{1/2}\right\} \frac{c\vec{k}(x_1)}{\omega} \langle|\vec{E}(k_y,k_z,\omega,x_1)|^2\rangle = \frac{c}{2\pi}\int_0^\infty d\omega \int d\Sigma$$

$$\times(1-n_y^2-n_\parallel^2)^{1/2} \left(\frac{\omega}{c}\right)^2 \frac{c\vec{k}(x_1)}{\omega} \langle|\vec{E}(k_y,k_z,\omega,x_1)|^2\rangle, \tag{29}$$

where we use polar coordinates with $d\Sigma = \sin\theta d\theta d\phi$, the element of solid angle, and $n_y = \sin\theta\sin\phi$, $n_\parallel = \cos\theta$. Thus, the power dP radiated in the spectral range $d\omega$, in the solid angle $d\Sigma$, by the area da of the slab, is given by

$$\frac{dP}{d\omega d\Sigma da} = \frac{c}{2\pi} \left(\frac{\omega}{c}\right)^2 (1 - n_y^2 - n_{\parallel}^2) < |\vec{E}(k_y,k_z,\omega,x_1)|^2>. \qquad (30)$$

It is not difficult to show from Eqs. (24) that at X = o

$$|\vec{E}(k_y,k_z,\omega,x_1)|^2 = \frac{1}{2} (1 - n_y^2 - n_{\parallel}^2)^{-1/2} \sum_j |f_j(x_1)|^2,$$

where j = 1, 3 for x_1 = +a and j = 2, 4 for x_1 = - a, thus, using Eq. (26)

$$<|\vec{E}(k_y,k_z,\omega,x_1)|^2> = \frac{1}{2}(1-n_y^2-n_{\parallel}^2)^{-1/2} \sum_j \exp(-2 \frac{\omega}{c} \int_{x_o}^{x_1} n_j'' \, dx)$$

$$\times \int_{x_o}^{x_1} dxdy <\tilde{\gamma}_j(k_y,k_z,\omega,x) \tilde{\gamma}_j(-k_y,-k_z,-\omega, x + y) >$$

$$\times \exp(2 \frac{\omega}{c} \int_{x_o}^{x} n_j'' \, dx') \exp(i \frac{\omega}{c} \int_{x}^{x+y} n_j^* \, dx'), \qquad (31)$$

where $n_j'' = \text{Im } n_j$. Now, for a weakly inhomogeneous plasma, it is reasonable to assume that the correlation scale length of $<\tilde{\gamma}_j(x) \tilde{\gamma}_j(x+y) > \propto <J_\alpha(x) J_\beta(x+y)>$ is much shorter than the scale length of the variation of the plasma parameters n_e, T_e and H_o. The integral over y in Eq. (31) can then be evaluated in the locally homogeneous plasma approximation. By extending the limits to $\pm \infty$ and assuming that $|n_j''| << |n_j'|$ we obtain

$$\int_{-\infty}^{\infty} dy <\tilde{\gamma}_j(k_y,k_z,\omega,x) \tilde{\gamma}_j(-k_y-k_z-\omega,x+y) > \exp(i \frac{\omega}{c} \int_{x}^{x+y} n_j' dx')$$

$$\approx \int_{-\infty}^{\infty} dy < \tilde{\gamma}_j(k_y,k_z,\omega,x)\ \tilde{\gamma}_j(-k_y-k_z-\omega,x+y) > \exp[ik_x(x)y]$$

$$= 2\pi <|\tilde{\gamma}_j(\vec{k},\omega)|^2>,$$

where $<|\tilde{\gamma}_j(\vec{k},\omega)|^2>$ is defined by the relation

$$<\tilde{\gamma}_j(k_x,k_y,k_z,\omega)\ \tilde{\gamma}_j^*\ (k_x',k_y,k_z,\omega)> = <|\tilde{\gamma}(\vec{k},\omega)|^2>\delta(k_x-k_x'),$$

and Eq. (31) becomes

$$<|\vec{E}(k_y,k_z,\omega,x_1)|^2> = \pi(1-n_y^2-n_\parallel^2)^{-1/2} \sum_j \int_{x_o}^{x_1} dx <|\tilde{\gamma}_j(\vec{k},\omega)|^2>$$
$$\times \exp(-2\,\frac{\omega}{c} \int_x^{x_1} n_j''\ dx'), \tag{32}$$

where $<|\tilde{\gamma}_j(\vec{k},\omega)|^2>$ is a slowly varying function of x and $k_x(x)$ satisfies Eq. (1). Using the fluctuation-dissipation theorem (Ref. 1, p. 108)

$$<J_\alpha(\vec{k},\omega)\ J_\beta^*(\vec{k}',\omega')> = \frac{T_e}{(2\pi)^5}\ \omega\, \epsilon_{\alpha\beta}''(\vec{k},\omega)\ \delta(\vec{k}-\vec{k}')\delta(\omega-\omega'),$$

Eqs. (32) and (30) yield

$$\frac{dP}{d\omega d\Sigma da} = (1 - n_y^2 - n_\parallel^2)^{1/2} \sum_j \int_{x_o}^{x_1} dx\ B_o(\omega,T_e)\ (2\,\frac{\omega}{c}\,\beta_j)$$
$$\times \exp(-2\,\frac{\omega}{c}\int_x^{x_1} n_j''\ dx'), \tag{33}$$

where

$$\beta_j(x) = |\phi_j F_j|^{-1}\sum_{\alpha\beta} b_{j\alpha}\, b_{j\beta}\, \epsilon_{\alpha\beta}'' = |\phi_j F_j|^{-1}\ [|b_{jx} + ib_{jy}|^2$$
$$\times\epsilon_{jxx}'' + |b_{jz}|^2\ \epsilon_{jzz}'' + 2R_e(b_{jx} + ib_{jy})\ b_{jz}^*\ \epsilon_{jxz}''|, \tag{34}$$

where $\epsilon''_{\alpha\beta} = (\epsilon_{\alpha\beta} - \epsilon^*_{\beta\alpha})/2i$ is the anti-Hermitian part of the dielectric tensor (2) and we used the relations

$$\epsilon''_{xy} = -\epsilon''_{yx} = -i\epsilon''_{xx},\ \epsilon''_{zx} = \epsilon''^{*}_{xz},\ \epsilon''_{yz} = \epsilon''^{*}_{zy} = i\epsilon''_{xz},$$

$$\epsilon''_{xz} = \mathrm{Im}(n_j\ \chi_{xz}) - i\ \mathrm{Im}\ (n_y\ \chi_{xz}).$$

Equation (33) is our general result and is valid provided that the conditions for the validity of the WKB solutions of the field equations are fulfilled and that $|k''_x| << |k'_x|$. These conditions are also implicit in the definition of the quantities which enter in the transfer equation. Equation (33) is, however, more general than the corresponding result obtained from the transfer equation for it holds for arbitrary anti-Hermitian part of the dielectric tensor. Note that $|k''_x| << |k'_x|$ does not necessarily imply a negligible anti-hermitian part of $\epsilon_{\alpha\beta}$. By comparing Eqs. (11) and (33) we see that they coincide if $\beta_j(x)$ reduces to $n''_j(x)$. To extract in a unique way n''_j from β_j, it is necessary to write $\beta_j \exp(-2\frac{\omega}{c}\int_x^{x_1} n''_j\, dx')$ in terms of the power absorption of a wave in the same mode of polarization and propagating from x_1 toward x. Now, for such a wave the electric field may be written as

$$E_\alpha = E_{i\alpha}\ (\phi_i\ F_i)^{-1/2}\ f_i(x),\ \alpha = x,\ y,\ z,$$

where $E_{i\alpha}$ and $f_i(x)$ are obtained from Eqs. (21) and (24) with $\vec{J} = o$. The index i indicates a wave propagating in the direction opposite to that of the wave j and is obtained by reversing the vector $\vec{k}$, i.e., $\vec{k}_i = -\vec{k}_j$. For the i wave, the power absorption is given by

$$W_i = \frac{\omega}{8\pi}\sum_{\alpha\beta} E^*_\alpha\ E_\beta\ \epsilon''_{\alpha\beta} = |\phi_i\ F_i|^{-1}\ \frac{\omega}{8\pi}\Big[|E_{ix} - iE_{iy}|^2\ \epsilon''_{ixx} +$$

$$|E_{iz}|^2\ \epsilon''_{izz} + 2R_e(E_{ix} - iE_{iy})E^*_{iz}\ \epsilon''_{ixz}\Big]\ |f_i(x_1)|^2$$

$$\times \exp(-2\ \frac{\omega}{c}\int_{x_1}^{x} n''_i\ dx') = |\phi_j\ F_j|^{-1}\ \frac{\omega}{8\pi}\Big[|E_{jx} - iE_{jy}|^2$$

$$\epsilon''_{jxx} + |E_{jz}|^2 \epsilon''_{jzz} + 2 R_e(E_{jx} - iE_{jy})E^*_{jz} \epsilon''_{jxz}\Big]$$

$$\times |f_i(x_1)|^2 \exp(-2 \frac{\omega}{c} \int_x^{x_1} n''_j \, dx'), \tag{35}$$

since $|\phi_i \ F_i| = |\phi_j \ F_j|$, $E_{i\alpha} = E_{j\alpha}$, $\epsilon''_{i\alpha\beta} = \epsilon''_{j\alpha\beta}$ and $n''_i = -n''_j$. By comparing Eqs. (34) and (35), we deduce that $\beta_j \exp(-2 \frac{\omega}{c} \int_x^{x_1} n_j'' \, dx)$ is proportional to W_i for

$$|b_{jx} + ib_{jy}|^2 \epsilon''_{jxx} + |b_{jz}|^2 \epsilon''_{jzz} + 2 R_e(b_{jx} + ib_{jy})b^*_{jz} \epsilon''_{jxz} =$$

$$= |E_{jx} - iE_{jy}|^2 \epsilon''_{jxx} + |E_{jz}|^2 \epsilon''_{jzz} + 2\mathrm{Re}(E_{jx} - iE_{jy})E^*_{jz} \epsilon''_{jxz}. \tag{36}$$

When this occurs, we can write

$$\beta_j = \frac{W_i}{(\omega/8\pi)} \exp(2 \frac{\omega}{c} \int_x^{x_1} n_j'' \, dx') \, / \, |f_i(x_1)|^2,$$

and Eq. (33) becomes

$$\frac{dP}{d\omega \, d\Sigma \, da} = (1 - n_y^2 - n_\parallel^2)^{1/2} \sum_i \int_{x_1}^{x_0} dx \, B_o(\omega, Te) \, W_i(x)/S_{ix}(x_1), \tag{37}$$

where $S_{ix}(x_1) = -(c/16\pi) \, |f_i(x_1)|^2$ is the x component of the Poynting vector of the i wave at $x = x_1$. Equation (37) is the statement of one of Kirchhoff's laws which relates the emissive power with the absorbing power of the radiating body. For the electron cyclotron radiation emitted by an inhomogeneous plasma slab the validity of Eq. (37) is conditioned by Eq. (36). To convert Eq. (37) in Eq. (11) we use the energy conservation theorem for steady state processes

$$-\frac{d}{dx}(S_x - \frac{c}{16\pi} \sum_{\alpha\beta} \frac{\partial \epsilon'_{\alpha\beta}}{\partial n'_x} E^*_\alpha E_\beta) = W + i \frac{c}{16\pi} \sum_{\alpha\beta} \frac{\partial \epsilon''_{\alpha\beta}}{\partial n'_x}$$

$$\times (E_\beta \frac{dE^*_\alpha}{dx} - E^*_\alpha \frac{dE_\beta}{dx}), \tag{38}$$

where $\epsilon'_{\alpha\beta} = (\epsilon_{\alpha\beta} + \epsilon^*_{\beta\alpha})/2$ is the Hermitian part of $\epsilon_{\alpha\beta}$, i.e., $\epsilon_{\alpha\beta} = \epsilon'_{\alpha\beta} + i\,\epsilon''_{\alpha\beta}$. Equation (38) holds for each mode of propagation ; the x derivative takes into account the slow spatial variation of the wave packet in the inhomogeneous plasma and the wave damping represented by $\exp[-(\omega/c)\int^x n''_x\,dx]$. Assuming that the latter is the predominant x variation of E_α we obtain

$$W_i = 2(\omega/c)\, n''_i \left[S_x(x) - \frac{c}{16\pi} \sum_{\alpha\beta} \frac{\partial \epsilon'_{\alpha\beta}}{\partial n'_x} E^*_\alpha E_\beta\right]_i$$

$$\approx 2(\omega/c)n''_i \;\; S_i(x) \approx 2(\omega/c)n''_i \;\; S_{ix}(x_1) \exp(-2\,\frac{\omega}{c} \int_{x_1}^{x} n''_i dx\,). \tag{39}$$

Inserting Eq. (39) in Eq. (37) and noting that $n''_i = -\,n''_j$, we obtain Eq. (11). We now investigate the validity of Eq. (36). From Eqs. (23) and (24) we obtain

$$E_{jz} = (n_j/d_2)\left[\varepsilon_{xy}(n_\parallel + \chi_{xz}) + i\chi_{xz}\,(\varepsilon_{xx} - n_\parallel^2\right] +$$
$$n_y\left[(n_\parallel/d_1)(n_j^2 + n_y^2 + n_\parallel^2 - \varepsilon_{xx}) + i(\varepsilon_{xy}\chi_{xz}/d_2) +\right.$$
$$\left.(\varepsilon_{xy}^2/d_1 d_2)\;(\chi_{xz} + n_\parallel\;\chi_{zz})\right],$$

$$b_{jz} = (n_j/d_2)\left[\varepsilon_{xy}(n_\parallel + \chi_{xz}) + i\chi_{xz}(\varepsilon_{xx}-n_\parallel^2\;)\right] -$$
$$(n_y/d_2)\left[i\;\varepsilon_{xy}\chi_{xz} + (n_\parallel + \chi_{xz})(n_j^2 + n_y^2 + n_\parallel^2 - \varepsilon_{xx})\right],$$

$$E_{jx} - iE_{jy} = (1/d_2)\left\{(\varepsilon_{xx}-i\varepsilon_{xy}-n_\parallel^2\;)\left[(n_j^2 + n_y^2)(1-\chi_{zz}) -\right.\right.$$
$$\left.\left.\varepsilon_{zz}\right] + (n_j^2 + n_y^2)n_\parallel\;(n_\parallel + \chi_{xz})\right\} - n_y\left[(n_j-in_y)/d_1 d_2\right] \tag{40}$$
$$\times\left\{\varepsilon_{xx}\left[\varepsilon_{zz} + 2\chi_{xz}^2 - n_\parallel\;\chi_{xz}(1-2\chi_{zz}) - n_\parallel^2\;(1 - \chi_{zz}) -\right.\right.$$
$$\left.(n_j^2 + n_y^2)(1 - \chi_{zz})\right] -i\;\varepsilon_{xy}(2\;\chi_{xz}^2 + n_\parallel\;\chi_{xz} + 2n_\parallel\;\chi_{xz}\chi_{zz} +$$
$$n_\parallel^2\;\chi_{zz}) + n_\parallel\;\varepsilon_{zz}\chi_{xz} - (n_j^2 + n_y^2)(2n_\parallel + \chi_{xz})\chi_{xz} -$$

$$n_{\parallel}^2 \left[2\chi_{xz}(n_{\parallel} + 2\chi_{xz}) + n_{\parallel}\chi_{zz}(n_{\parallel} + 2\chi_{xz}) + \chi_{zz}(n_j^2 + n_y^2)\right]\Big\},$$

$$b_{jx} + ib_{jy} = (i/d_2)\Big\{(\varepsilon_{xx} - i\varepsilon_{xy} - n_{\parallel}^2)\left[(n_j^2 + n_y^2)(1-\chi_{zz}) - \varepsilon_{zz}\right] + (n_j^2 + n_y^2)\, n_{\parallel}(n_{\parallel} + \chi_{xz})\Big\} + n_y\left[(n_j + in_y)/d_2\right]$$

$$\left[\varepsilon_{zz} - n_{\parallel}(n_{\parallel} + 2\chi_{xz}) - (n_j^2 + n_y^2)(1-\chi_{zz})\right].$$

Equations (40) show that, strictly speaking, for $n_y \neq o$ Eq. (36) is not fulfilled and Eq . (37) is not valid, namely, the emission is not simply proportional to the absorption. Unfortunately, in view of the complexity of Eqs. (36) and (40) it is not possible, without a numerical evaluation, to estimate the deviation from the general Kirchhoff law. A qualitative insight of the magnitude of the deviation may be obtained by considering the emission of the ordinary mode for $n_{\parallel} \to o$. In this case

$$E_{jx} - iE_{jy} = O(n_{\parallel}^2),\ b_{jx} - ib_{jy} = O(n_{\parallel}^2),\ E_{jz} = O(n_{\parallel}),$$

$$b_{jz} = O(n_{\parallel}),$$

and the condition (36) reduces to $|E_{jz}| = |b_{jz}|$. Now, for $n_{\parallel} \to o, d_1 = \varepsilon_{xx}$, $d_2 = \varepsilon_{xx}(1 - \chi_{zz})$, $\varepsilon_{xy} = \varepsilon \approx i\chi\mu W_o$, $\varepsilon_{xx} \approx 1 + i\varepsilon$, and $|\varepsilon| >> 1$. Retaining the dominant terms in Eq. (40) we obtain

$$|E_{jz}| = |n_{\parallel}\ \left|n_y + i\left[n_j(1 + i\lambda) + n_y\lambda\right]\right|,$$
$$|b_{jz}| = |n_{\parallel}\ \left|n_y - i\left[n_j(1 + i\lambda) + n_y\lambda\right]\right|, \qquad (41)$$

where

$$\lambda = i(W_o - W_1)/W_o = i(F_{5/2} - F_{7/2})/F_{5/2}. \qquad (42)$$

The plots of $F_{5/2}$ and $F_{7/2}$ versus $\xi = \mu(1-Y)$ are presented in Ref. 6. It is seen that $|\lambda|$ can be comparable to unity, thus, for not too small $|n_y|$, $|E_{jz}|$ appreciably differs from $|b_{jz}|$. Note that for $n_\parallel \to 0$, $|\phi_j F_j| = O(n_\parallel^2)$ for the ordinary mode and from Eq. (34) we obtain β_{ord} finite. We now investigate the cases for which Eq. (36) is valid.

It is first interesting to consider the case of a plasma with a negligible anti-Hermitian part of $\epsilon_{\alpha\beta}$. This may be due to ordinary collisions, the case discussed in Ref. 10, or, for the electron cyclotron radiation, obtained from Eqs. (2) and (6) for $|\omega-\omega_c| >> k_z v_t$. In Eq. (40) we then insert

$$\varepsilon_{xx} = 1 - X/(1-Y^2),\ \varepsilon_{xy} = iYX/(1-Y^2),\ \varepsilon_{zz} = 1-X,$$

$$\chi_{xz} = \chi_{zz} = 0,$$

and we obtain

$$b_{jz} = n_y n_\parallel \frac{\varepsilon_{xx} - n_j^2 - n_y^2 - n_\parallel^2}{\varepsilon_{xx}} + i n_j n_\parallel \frac{1 - \varepsilon_{xx}}{\varepsilon_{xx}}$$

$$= - E_{jz}^*,$$

$$b_{jx} + i b_{jy} = n_j n_y \frac{\varepsilon_{zz} - n_j^2 - n_y^2 - n_\parallel^2}{\varepsilon_{xx}} +$$

$$i \frac{(n_j^2 + n_y^2)(1-n_y^2) + n_y^2(\varepsilon_{zz}-n_\parallel^2) - \varepsilon_{zz}(1-n_\parallel^2)}{\varepsilon_{xx}}$$

$$= - (E_{jx} - i\, E_{jy})^*,$$

thus, Eq. (36) is fulfilled and Eq. (37) reduces to Eq. (11). The case of a quasi-lossfree plasma is of limited interest for the electron cyclotron radiation for which the emission and the absorption take place predominantly in the region where $|\epsilon''_{\alpha\beta}| \gtrsim |\epsilon'_{\alpha\beta}|$. As it is clearly shown by Eqs. (40), for arbitrary values of $\epsilon''_{\alpha\beta}/\epsilon'_{\alpha\beta}$, Eq. (36) is fulfilled for $n_y = 0$ only. In this case

$$b_{jz} = E_{jz},\quad b_{jx} + b_{jy} = E_{jx} - iE_{jy},$$

and Eq. (36) and, consequently, Eq. (11) are fulfilled. Thus, for propagation in the xz plane, the emitted radiation can be obtained by the evaluation of n_j''. When $n_j'' \exp(-2 \frac{\omega}{c} \int_x^{x_1} n_j'' \, dx)$ has a sharp maximum near some value x_m, which may be shifted from the value for which[9] $\omega = \omega_c$, we obtain

$$\frac{dP}{d\omega \, d\Sigma \, da} = (1-n_{\parallel}^2)^{1/2} B(\omega, T_{em}) A = (1-n_{\parallel}^2)^{1/2} B(\omega, T_{em}) \times \left[1 - \exp\left(-2 \frac{\omega}{c} \int_{x_o}^{x_1} n_j'' \, dx\right)\right], \tag{43}$$

where T_{em} is the electron temperature at x_m. For arbitrary values of $n_{\parallel}$, it is not possible from Eq. (1) to obtain simple analytical expressions for n_j''. A case of particular interest is for $n_{\parallel} = o$. From Eqs. (1-5) we obtain[6]

$$2 n_1' n_1'' \approx - (1-X) \frac{X}{2Y^2} \frac{F_{7/2}''(\xi)}{\left[1 + \frac{X}{2Y^2} F_{7/2}'(\xi)\right]^2 + \left[\frac{X}{2Y^2} F_{7/2}''\right]^2}, \tag{44}$$

$$2 n_2' n_2'' \approx - \frac{\left[1 - X/(1+Y)\right]^2 G_o''}{\left[1 - X/2(1+Y) - G_o'\right]^2 + G_o''^2}, \tag{45}$$

where $G_o = G_o' + i G_o'' = (\mu X/2) F_{5/2}(\xi)$ and the indices 1 and 2 refer to the ordinary and extraordinary modes, respectively. From Eqs. (44) and (45) we deduce that $n_j'' \neq o$ for $Y > 1$ only. For the real part of n_j we may take the corresponding values given by the cold plasma approximation, namely

$$n_1^2 = 1 - X, \quad n_2^2 = 1 - \frac{X(1-X)}{1-X-Y^2}.$$

As it is seen from Eqs. (44) and (45), even for $n_{\parallel}$ = o the absorbing coefficient A must be evaluated numerically.

V. CONCLUSIONS

We have performed a direct calculation of the emissive power of a weakly inhomogeneous plasma slab for frequencies close to the electron cyclotron frequency. The plasma polarization effects are described by the warm plasma weakly relativistic tensor. Within the WKB approximation a general expression is obtained and is compared with the corresponding result given by the transfer equation. For propagation in the xz plane the two results coincide and we conclude that Kirchhoff's law is valid. The emissive power is then obtained from the absorbing power A. This has been numerically evaluated for several values of $n_{\parallel}$ for a hot tokamak plasma (Te $\approx$ 2 keV) in Ref. 9. The results obtained indicate that for quasi-perpendicular propagation the plasma emits in the ordinary mode as a black-body. Recent experiments[17] on electron cyclotron emission from tokamak plasmas have revealed back-body radiation in the ordinary mode. The extraordinary mode is also emitted like a black-body but, for the values of n_e and Te used in the numerical calculation, $n_{\parallel}$ must not be zero. An interesting feature of the emission of the extraordinary wave is the $1/n_e$ dependence of n'' for $X \gg (v_t/c)^2$. For not too dense plasma this may lead to black-body emission even for $n_{\parallel}$ = o. The direct emission is then competitive with the process of emission via mode conversion at the upper hybrid resonance considered in Ref. 18.

We have also found that for $n_y \neq$ o Eq. (33) is not identical with the result obtained from the transfer equation. For not too small values of $|n_y|$ we show that the deviation may be appreciable and we conclude that for arbitrary directions of propagation Kirchhoff's law is not strictly obeyed. This may be due to the non-validity of the usual reciprocity theorem[19] for arbitrary propagation in an anisotropic medium. Since we have found that Kirchhoff's law is valid for a plasma with a negligible $\epsilon''_{\alpha\beta}$, we infer that the presence of a large ($\epsilon''_{\alpha\beta} \gtrsim \epsilon'_{\alpha\beta}$) anti-Hermitian part of the dielectric tensor is mainly responsible for the deviation from Kirchhoff's law. This is relevant for the problem of the electron cyclotron emission from a plasma with arbitrary directions of the inhomogeneity and magnetic field.

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