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MODELS OF HIGH ENERGY NUCLEAR COLLISIONS

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A. NUCLEAR COLLISIONS AT RELATIVISTIC ENERGIES

Introduction, Equation of State, Energy Thresholds

What do we know about nuclei? The literature of the last 20 or 30 years contains a wealth of fascinating detail about their structure, their energy levels and single particle aspects, their collective motion, and the way they interact with each other in collisions. Both the quantity and detail of the experimental data, and the sophistication of some of the theory is impressive. Yet what we know about nuclei concerns their properties at only one point on the graph of the equation of state of nuclear matter which is illustrated in Fig. 1. Aside from the trivial point at the origin, and the energy per nucleon at normal density, the curve drawn is a guess. The point where it crosses the axis at $\rho/\rho_0 \sim 2$ is based on nuclear matter calculations. We do not even know the curvature (compressibility) at normal density. Virtually everything we know about nuclei concerns their normal state!

Some interesting possibilities for the state of nuclear matter at high density are illustrated in Fig. 1. The Lee-Wick super dense state is illustrated, as is the effect of a phase transition, corresponding to a situation where a state of special correlation having the quantum numbers of the pion (pion condensate) becomes degenerate with ground state.

Perhaps the ultimate goal of research with relativistic energy nuclei is to study nuclear matter under abnormal conditions of high particle and energy density. This is a break from the past.

Nuclear physicists have concentrated on studying nuclei under normal conditions of low energy and temperature. High energy physicists have concentrated on putting higher and higher energy into a small volume. We do not know what surprises await us, but several possible rewards will be mentioned later.

To make it plausible why we expect to encounter new and interesting phenomena it is useful to examine Fig. 2, prepared by Swiatecki.[1] There the projectile mass for a symmetric collision is plotted on one axis, and a bombarding energy per nucleon on the other. The shaded areas indicate thresholds where qualitatively new physical features take over. The low energy region is the domain of conventional nuclear physics, and is being intensively studied at many laboratories. The region immediately adjacent to the x-axis extending to very high energies is the domain of particle physics, studied at the very large accelerators. Most of the plane is completely unknown territory. We discuss briefly the thresholds following the subsonic region of conventional nuclear physics.

Supersonic Threshold. The energy of 20 MeV per nucleon corresponds to

1) the average kinetic energy of nucleons in nuclei $\simeq \frac{3}{5} E_F$

2) minimum energy needed to compress nuclear matter to something like twice normal density (see Fig. 1).

3) estimated speed of sound in normal nuclear matter (involves the curvature at the minimum in Fig. 1).

We anticipate qualitatively different behavior as this threshold is crossed into the region of what can be labelled supersonic.

Meson Threshold. This is the threshold for particle production, starting with the pion, followed by the nucleon isobars and heavier mesons. Hot nuclear matter can be cooled by the production of these particles.

Relativistic Threshold. This corresponds to the mass of the nucleon and brings us to the full relativistic regime where not even the wave equation for s > 1/2 is known. Far into this region I have the hope that it may be possible to discover the general form of the hadronic mass spectrum, one of the most fundamental properties of matter.[2]

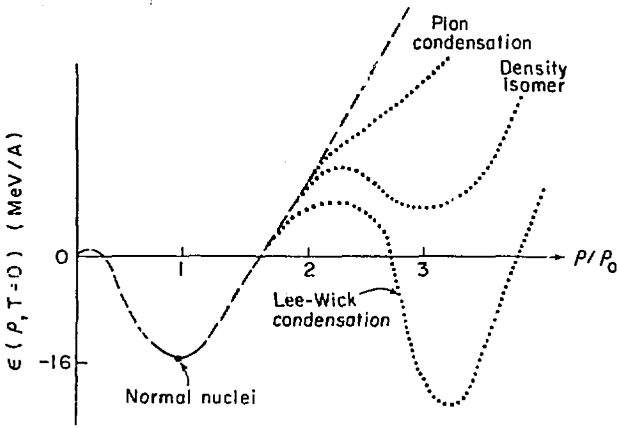

Fig. 1. Schematic of energy per nucleon versus density of nuclear matter (equation of state) showing several possible high density behaviors. (Courtesy of A. M. Poskanzer.)

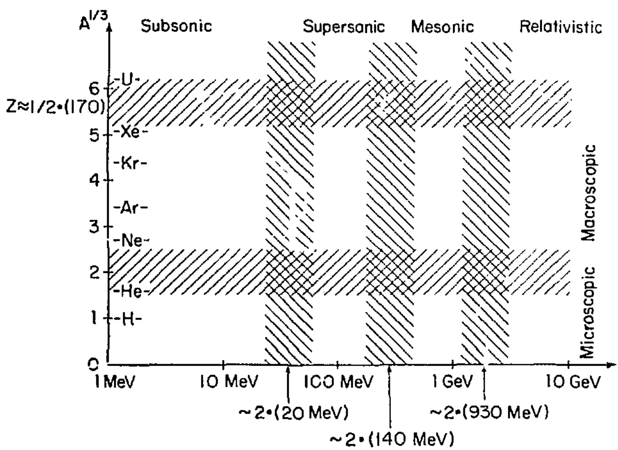

Fig. 2. Illustration by W. Swiatecki of various thresholds in the mass of projectile versus collision lab energy for symmetric collisions.

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Fig. 3. A typical peripheral collision showing forward cone of (charged) projectile fragments having virtually the projectile energy.

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Fig. 4. A central collision showing high multiplicity of charged particles.

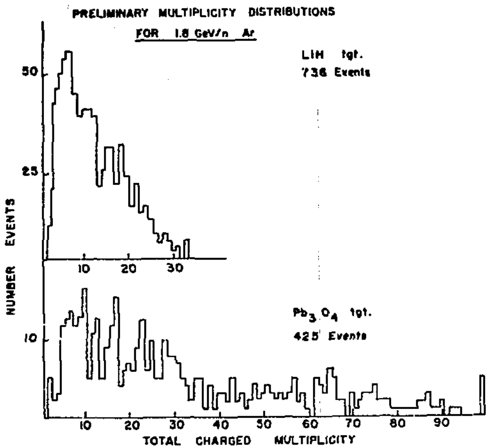

Fig. 5. Charged particle multiplicity distributions from Fung, Gorn, Kiernan, Liu, Lu, Oh, Ozawa, Poe, Van Dalen, Schroeder, and Steiner (unpublished, 1977).

Two Classes of High Energy Nucleus-Nucleus Collisions

The experimentally observed events reveal two extreme limiting types of collisions at energies in the few hundred MeV to several GeV per nucleon range.

1) Peripheral Collisions.[3] This is the most frequently observed class of collisions and is characterized by the fact that a few particles are observed in the extreme forward cone and they have almost the same speed as the projectile. They are presumably fragments of the projectile and they range in mass from one to a number of mass units, but less than the projectile. Presumably these collisions are geometrically peripheral so that a few nucleons in the overlap region are knocked out. Both the projectile and target residues are presumably excited by the sudden removal of a part of their mass, and may radiate particles after the collision (Fig. 3).

2) Central Collisions.[4] In about 10% of the collisions no fast particles having the projectile speed are observed. Instead, many particles, up to 130 charged particles are emitted even in collisions with a total initial charge of 100. Many mesons are evidently produced. No remnant of the projectile in the forward cone is observed. The projectile is presumably stopped in the target and the energy shared by many particles. Fairly fast ones come out in the forward hemisphere while high Z particles (bright tracks) come out in all directions (Fig. 4,5).

What Happens in a Central Collision?

Are nuclei opaque or transparent to an incident high energy (2 GeV/nucleon) nucleus? The de Broglie wave length is so short that we argue at first on the basis of a sequence of individual nucleon-nucleon collisions. The mean free path between collisions based on the nuclear density and N-N cross section is

$$\ell \sim \frac{1}{\rho\sigma} = [(0.17/F^3)\ (40\ \mathrm{mb})]^{-1} \sim 1.5\ F \quad .$$

The energy loss per collision is ~100 MeV so that a 2 GeV nucleon might lose a GeV energy in traversing a lead nucleus. In other words there might be a high degree of transparency. On the other hand, in roughly 10% of collisions there is an absence of high energy particles: the projectile appears to be stopped and the composite system decays or explodes. Since we deal with systems of at most several hundred particles, deviations from the mean can be large! Clearly the opaque collisions are very interesting.

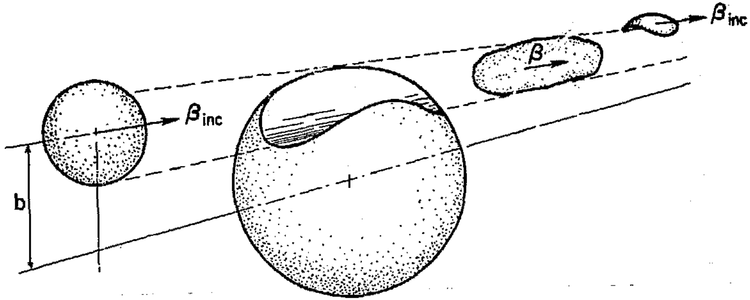

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Fig. 6. Schematic showing geometrical assumptions of fireball model. The fireball is the portion swept out by the projectile having an intermediate velocity β and a temperature corresponding to an excitation energy given by application of energy and momentum conservation.

If the projectile is stopped in the target or a part thereof the resulting system is very hot. After taking account of the escape of a certain fraction of the prompt pion production, energy and momentum conservation can be used to calculate the internal excitation or temperature (Fig. 6). The temperature can be lower than this however because 1) the prompt π's that do not immediately escape can interact with nucleons to form the nucleon isobars. This lowers the number of fermions of given type which allows cooling. 2) Collisions of hot nucleons can produce secondary π's reducing thus the kinetic energy.

The high velocity (near c) of the pions and their strong interaction with nucleons provides a fast mechanism for thermalization of the composite system in addition of course to the nucleon-nucleon collisions. Indeed computer studies suggest that thermalization can occur already after only 3 or 4 collisions. Chemical equilibrium between the various species, π, N, N*, D, t takes longer but may still be fast compared to the disassembly time of the composite. Therefore a thermodynamical description may be reasonable and indeed a free ideal gas treatment of the composite, called a nuclear fireball does qualitatively account for some of the proton and composite particle spectra observed[5,6,7] (Figs. 7,8). The temperatures so determined run as high as 100 MeV, or 10^{12} °K, perhaps the highest temperature ever produced in the laboratory (however in a very small piece of matter, i.e., ~nucleus) and the highest that have existed naturally since the beginning of time.

If a thermodynamic description does apply, it makes it very simple to investigate a very exciting prospect, the discovery of the assymptotic form of the hadronic mass spectrum. This subject will form the second part of these lectures.

High Density Phenomena. So far there is no convincing evidence for the propagation of shock waves in these collisions but the formation of regions of high density is expected in any case.[8] This is very interesting from several points of view. One is the possible creation of quark matter.[9] Quarks, if they exist, are believed to be the constituents of hadrons in which they are confined by the forces acting between them. However, Susskind finds that at least one model of confinement allows a transition to a plasma-like phase to occur at high temperature and energy density. The colored gluons form a plasma, which screens the color of quarks, allowing the quarks to become disassociated from their original hadrons, and to roam throughout the high density medium. Another interesting high density phenomenon is the Lee-Wick[10] density isomeric state of nuclear matter (see Fig. 1). Even another is the phase transition sometimes referred to as pion condensation.[11-14] At some critical density not so much greater than normal, it is believed that a collective state of special correlation having the quantum numbers of the pion will become degenerate with the ground state.

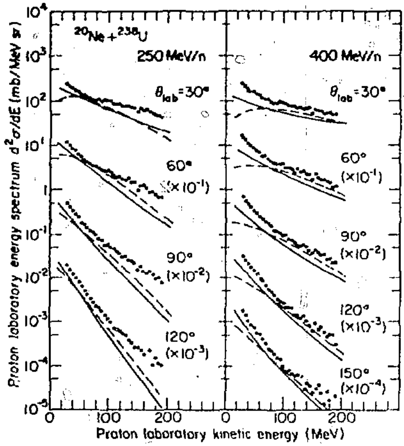


Fig. 7. Comparison of the fireball calculation[5] (dashed) and firestreak[7] (solid) with data of proton spectra.[4]

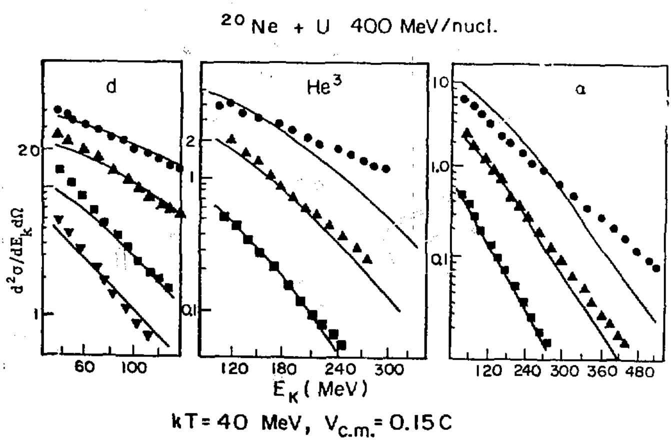


Fig. 8. Thermodynamic calculations[6] of composite particle spectra compared with data.[4]

The fact is, however, that we do not know of any way to squeeze a nucleus while keeping its temperature low. Nevertheless signals of the existence of such states, were they to exist at densities not too much greater than normal, have been sought in the spectra of ordinary nuclei.[11] The weight of recent calculations is against the existence of such pion condensates at low temperatures, and densities as low as twice normal.[12] High energy nuclear collisions may therefore give us another chance. In collisions, nuclear matter at high temperature and density is fleetingly created, offering the possibility of triggering a collective instability in the non-equilibrated matter.[13] Gyulassy has emphasized that these instabilities do not imply the participation of a large number of pions, but instead a few collective "phonons" each with the quantum numbers of a pion. The structure of the phonon is that of a spin-isospin lattice. The calculated effect of the growth of these instabilities in the nuclear medium on the scattering of nucleons passing through the medium, is to moderately increase the cross sections. As a result, if pionic instabilities occur, they will assist in rapidly thermalizing the system. However, if equilibrium is actually reached, the memory of the dynamical processes preceding it is lost! Apparently the detection of these instabilities will be very difficult.

It might be supposed that a pionic instability would lead to an enhancement in the average number of pions produced, or at the very least, change the pion-multiplicity distribution. Both these questions have been studied.[14] That the first supposition is false is a trivial consequence of the fact that at energies up to 2 GeV/nucleon most pions are produced during the non-equilibrium phase. Even at the temperature of 50 MeV, relative momenta are generally below threshold. The production rate during the pre-equilibrium phase is presumably enhanced, just as the nucleon scattering by the medium in enhanced. However the thermalization occurs sooner by a compensating factor, so that the number of pions produced is essentially unchanged by the existence of pionic instability. Even the pion multiplicity distribution is little effected.[14]

Still in a pursuit of distinguishing features of pionic instabilities, Gyulassy has made a very interesting study of correlation properties of pion produced in a medium such as colliding nuclei, which is an extension of quantum optics.[14] He studies the properties of the field equation

$$(\Box + m_\pi^2)\, \phi(\underset{\sim}{r},t) = J(\underset{\sim}{r},t)$$

where ϕ is the pion field and J is a transition current operator representing the physical processes involved in the production of pions. If J is a given C-number source, which is a reasonable assumption if the pion field is not too strong, allowing one to ignore the recoupling of the pion fields to the nucleon collision creating them, then the solution exhibits some interesting properties. In particular for a pure coherent source, the solution has the properties of a pion laser. The two pion inclusive correlation function can be used to distinguish between a coherent source, and a chaotic one. Moreover the coherent source, which stimulates the pion laser, such as the collective pion instability, can be studied mode by mode.

This will serve to give an indication of the range of phenomena under investigation, the difficulties involved in their observation and some of the theoretical approaches employed. Other theoretical approaches include hydrodynamics, Glauber theory, cascade calculations, and classical equations of motion.[15]

B. THE ASYMPTOTIC HADRON SPECTRUM

Elementary Particles

An age old question has intrigued mankind, at the latest, since the early Greek philosophers. Is matter divisible only down to fundamental particles which are not further divisible or is matter infinitely divisible into smaller pieces? The modern experience does not come down on one side of this either-or question. Instead we find that matter can be divided again and again but not into smaller and smaller particles. Instead always some of the same particles we started with reappear together with other particles. A nucleon cannot be broken up only into smaller particles! This was not known to the Greeks, and it runs counter to our experience of all matter from the macroscopic down to and including nuclei.

Einstein's law of equivalence of mass and energy tells us that in a high energy collision between nucleons mass can be created. We see mesons and baryon anti-baryon pairs produced and the nucleons may reappear as nucleon isobars. Is this an entirely trivial consequence of Einstein's law? Does a law or physical principle underlie the indeterminancy of the outcome? Presumably so. In other areas of physics we are accustomed to considering that the state of system comprises, virtually, all possible configurations of the same symmetry. What these "configurations" are is the goal of high energy physics. Whatever the mathematical description of the modern answer to the age old question, whether it be bootstraps or quarks or whatever, it seems quite certain that we know only a few of the particles and resonances that can be produced in high energy collisions; that their number and variety is staggering.

The Known Hadrons. There are 56 named hadrons representing about 1000 hadronic states with spin, isospin, baryonic charge and strangeness quantum numbers measured (Table 1).[16] The lightest of these are the three pions $m_\pi \sim 140$ MeV. They become quite densely spaced as their mass increases to about $10\ m_\pi$. Thereafter the spectrum becomes sparse. Presumably however the cutoff is an experimental one. Figure 9 plots the number of hadronic states per pion mass interval. Already at $m = 10\ m_\pi$ there are 34 non-strange states per pion mass interval and the average width at this mass is $\Gamma \sim 100$ MeV. Production rates are expected to decrease with m. The experimental problem becomes one of intensity and resolution.

The Hadrons that Will Never be Known by Name. Theories of hadronic structure, in contrast to the known spectrum, imply that it continues indefinitely. The bootstrap hypothesis predicts a spectrum that increases exponentially.[17] The hypothesis can be stated simply as follows: From among the known particles or resonances select two (or more) and combine their quantum numbers. The multiplet so obtained are also particles or resonances (at something like the sum of the masses). Add these to the pool of known particles and continue. The spectrum thereby generated by Hamer and Frautschi[18] is also shown in Fig. 9. The implication is astonishing. The number of particles and resonances grows so fast that at only 2.5 GeV the number expected in a pion mass interval, on the basis of the bootstrap hypothesis, is $\sim 10^4$. The number of known particles is $\lesssim 10^2$ at that mass. If new particles were discovered at the rate of one a day it would require about a hundred years to verify the bootstrap prediction by a direct count, and that at only one mass!

Most high energy physicists currently favor the quark hypothesis, and there is even a theory, quantum chromodynamics (QCD), that is a candidate for the dynamical description of hadrons. The theory has not been solved in any general sense so far. The mesons are thought to consist of a quark anti-quark pair and baryons of three quarks, which in both cases are referred to as the valence quarks, and in addition there is believed to exist in each hadron, a sea of quark anti-quark pairs. Whether all the quark flavors have been identified is an open question. The modes of excitation of the quarks within a hadron, like the radial and angular momentum states in a nucleus, are also unknown.

Table I. The families of light mass multiplets, their average masses in MeV, and their baryon and strangeness quantum numbers (B, S). Total multiplicity including the unlisted multiplets is indicated in the bottom row for each family.

Family (B, S)	Π (0,0)	K (0,1)	N (1,0)	Λ (1,-1)	Σ (1,-1)	Ξ (1,-2)	Ω (1,-3)
Multiplets	π(138)	K(495)	N(940)	1116	1193	1318	1672
	η(549)	K*(892)	N*(1430)	1405	1385	1533	
	ρ(773)	K*(1421)	N*(1520)	1519	1670		
	ω(783)		N*(1515)	1670	1745		
	η'(958)		Δ(1232)	1690	1773		
	⋮		⋮	⋮	⋮		
Total Multiplicity	103	18	248	38	108	12	4 = 531

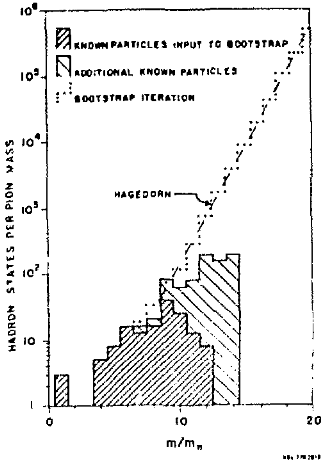


Fig. 9. The density of different hadrons, ρ, in unit interval is plotted as a function of the mass in units of the pion mass. The experimentally known particles and resonances with their multiplicities are shown in the shaded areas. The dotted histogram is a bootstrap iteration[18] on the known spectrum and the solid curve is a Hagedorn type spectrum, fitted to the bootstrap.

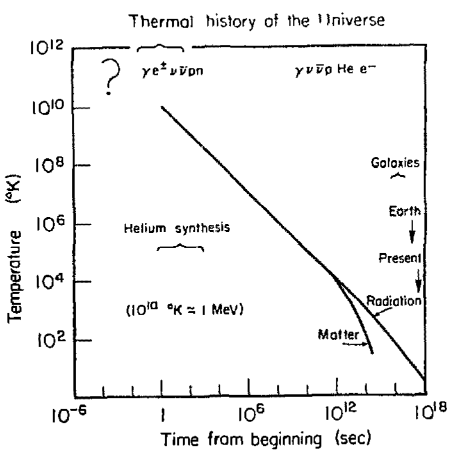


Fig. 10. Thermal history of the universe. The question mark indicating ignorance as to temperature and composition is due in part to the unknown hadronic spectrum.

QCD is not an easy theory. It is possible that the number of excitations (i.e., hadrons) are unbounded in number and mass, that the density of states grows with mass, perhaps exponentially as the bootstrap. It should be borne in mind throughout the rest of this paper, that the exponential spectrum, here associated explicitly with the bootstrap hypothesis, may also be the form of the spectrum, based on the quark hypothesis.

In any case, we have seen that it is out of the question to determine even the general form of the hadronic mass spectrum at even relatively low masses like 2 to 3 GeV, much less in the high mass region, by a direct count of individual particles and resonances. The sheer density of states is not only very large, but the widths are at least a pion mass, so of the order 10^4 or more states fall with the width of any one.

Is it Interesting? Is it important or even interesting to know the density of hadronic states in the region were they cannot be individually discovered and given a name? I think so. It is both interesting and important. Interesting because it is a fundamental property of matter on the smallest scale, and important for two reasons that I can think of. It is important in particle physics because the density of hadron states at high mass provides an asymptotic constraint on theories of hadronic structure. Let me elaborate. The properties of the low mass particles that can be individually identified provide important clues as to the group structure of the theory. Their quantum numbers (spin, isospin, strangeness) which are determined by the decay modes and so on, and their masses suggest particular classifications which any theory of hadronic structure must account for. But the symmetries are not perfect. This fact leaves a lot of room for competing theories. Certainly the quark theory is favored by many particle physicists today, but there are a number of quark models. If quarks are the fundamental building blocks, there is still no agreement as to the nature of the glue that holds them together. And because the symmetries are broken, the light particle spectroscopy cannot provide a unique way of discriminating between the theories. Yet any theory of hadronic structure, when sufficiently developed, can be made to yield a prediction of the asymptotic region (i.e., high mass). It is in this sense that the asymptotic behavior of the hadronic spectrum may become decisive in particle theory.

The asymptotic region is important also in cosmology. The thermal history of the universe can be guessed with considerable confidence back to the time of helium synthesis at temperatures of about 1 MeV. Figure 10 traces this history, backward in time from the present, through the formation of planets, the galaxies, and heavy element synthesis to the high temperature of 1 MeV (10^{10} °K). For much earlier times when the energy density was extremely high, the composition of the universe must have been very different in kind, not merely in density and temperature, from what we see today. That there were no nuclei is clear, but that there were no nucleons is likely. What there was was in fact determined by the spectrum of hadrons and leptons that could energetically exist at the energy densities prevalent. At even earlier times, at extreme particle and energy density, the hadrons may have been dissolved into a quark soup which only later condensed into hadrons. Whether there are any residual signals in the universe left over from these early instants I do not know, but doubt. Probably the temperature and composition in the earliest instants will remain forever a subject of speculation, uninformed at the present, but informed, if the form of the hadronic spectrum can be determined.

Are there contemporary astronomical events tha bear on the high mass region of the hadron spectrum? I was fascinated to learn a short time ago of Hawking's work on the quantum theory of black holes.[19] I had believed that black holes are really black. That no matter or radiation can escape from them. No so in quantum theory. Black holes evaporate, at first slowly but eventually catastrophically. A process by which this evaporation can occur is illustrated in Fig. 11. The loops represent the spontaneous fluctuations of the vacuum, when a particle and anti-particle momentarily appear and then mutually annihilate. However in the vicinity of a black hole, if one member of such a pair is captured by the black hole, the other has lost the partner with which to annihilate. It appears therefore as radiation from the black hole. It is not easy to follow the reasoning that leads to the conclusion that the temperature of the black hole increases as a result. It is easy to understand, as will develop later, that the energy released in the ultimate explosion of the black hole, depends crucially on the hadronic spectrum, being many orders of magnitude greater, if it increases exponentially than otherwise. The reason is simply that in the former case, energy can be stored in the benign form of mass rather than kinetic energy. In the latter case, where this possibility is limited, the explosion occurs, so to speak, prematurely.

I am sure that I have convinced you by now of two things. The general form of the hadronic spectrum is a most interesting thing to know, and it cannot be discovered by looking for the individual particles of which it is composed.

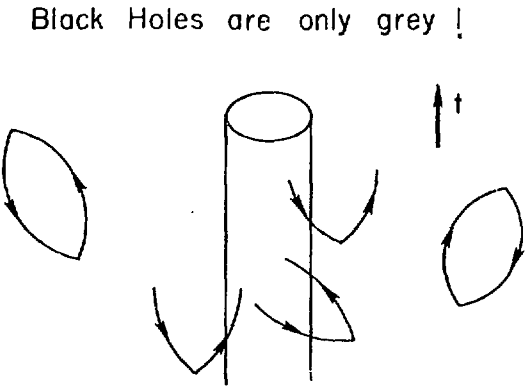
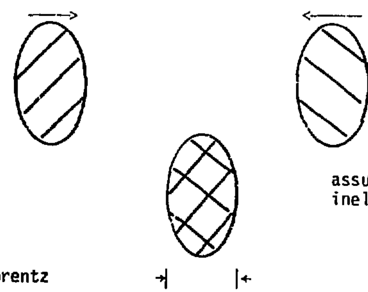


Fig. 11. A mechanism for the evaporation of a black hole showing the capture by the hole of one partner of pair fluctuations of the vacuum.

INITIAL FIREBALL

Symmetric collision of Z = N nuclei in C.M.



assume perfect inelasticity

Lorentz

$$\frac{1}{\gamma} = \frac{m}{E}$$

$$\rho_{\text{initial}} = 2\gamma\rho_0 = 2\,\frac{E}{m}\,\rho_0$$

Fig. 12. Illustrating the collision.

How Then Can it be Discovered?

Perhaps by creating as large a piece of matter as possible at high energy density and studying its properties. This is the only way I can think of. I am sure that you appreciate for example, that the specific heat of material objects depends upon their compositions. The composition of matter at high energy density depends in turn upon the number, type, and masses of hadrons that can energetically exist at that density--both those that are known, and those that are unknown, and never will be known by name!

The only means we have of producing matter at very high energy density is in collisions, by no means an ideal situation for performing calorimetric measurements. Yet it is our only hope.

So we have in mind collisions between large nuclei at high energy. Two questions come immediately to mind. 1) What is the dynamical description of the reaction? and 2) At what energy can different assumptions about the hadronic spectrum be expected to yield observable difference in the outcome of the collision?

For the dynamics I can envision two extremes. Either the collision of two nuclei at high energy 1) develops as a sequence of independent collisions, or 2) it attains thermal equilibrium and then decays.

If the first is true then for the purpose at hand, at least, there is no point in studying nuclear collisions rather than nucleon-nucleon collisions.

But I think it highly unlikely that the first is true. More likely the truth lies between the extremes.

A moment's reflection makes clear that a complete dynamical description of a collision between nuclei at very high energy involves something like the full complexity of a relativistic quantum field theory. Of course if all the ingredients of such a theory were at hand, the question raised by this paper would be moot. Since however the ultimate theory of particle structure or its solution is unlikely to emerge in the near future, it seems reasonable to attempt a model description of the dynamics of a nuclear collision.

Before attempting to explore too elaborate a model it seems prudent to me to assess whether it is worth doing so. For example, it might turn out that the energy at which sensitivity to the hadronic spectrum is achieved is so high as to be out of sight; that by no stretch of the imagination would it ever be possible to produce the required energy in the laboratory.

Therefore, Y. Karant[20] and I have assumed thermal equilibrium as a model of high energy collisions for the purpose of accessing the prospects of learning from nuclear collisions the form of the hadronic spectrum and the possibility of distinguishing between various theories of hadronic structure. If the results of such a study give an optimistic prognosis, we will feel encouraged to try harder in our treatment of the dynamics.

The attainment of a state of thermal equilibrium in a nuclear collision may seem strange at first. But at the energies in question a very large phase space is opened up by particle production. The high velocity (near c) of the pions and their strong interaction with nucleons provides a fast mechanism for thermalization in addition of course to the hadron-hadron collisions. Indeed computer studies suggest that thermalization can occur already after 3 or 4 collisions.[21] Chemical equilibrium among the various species π, N N^* Δ... takes longer but may still be fast compared to the disassembly time of the composite. The extended size of the initial nuclear composite for geometrical reasons alone, slows the disassembly of the interior.[6]

There is a very extensive and beautiful literature on the thermodynamic theory of hadronic structure.[17] Also for nuclear collisions, at lower energy than we have in mind, a thermodynamic model has been introduced.[4,5,6,7,22] Inspired by the analogy of hadron thermodynamics, the hot composite system was referred to as a nuclear fireball. The model has been refined[7] and applied[23] to data on pion, proton, and composite particle spectra at various energies between 200 MeV per nucleon to 2 GeV per nucleon laboratory kinetic energy. The overall agreement with such a wide range of data is quite impressive.

Thermodynamics of Hadronic Matter

In this section we discuss the thermodynamics of the nuclear fireball in terms of an ideal relativistic gas. It may seem strange that a strongly interacting hadron system is described in such a way. However, Hagedorn[17] has argued convincingly, on the basis of statistical mechanical techniques introduced by Beth and Uhlenbeck[24] and Belenkij[25] that the hadronic spectrum <u>is</u> the manifestation of the interactions; that by introducing the complete spectrum one has accounted for their interaction completely.

The partition function and momentum distribution for an ideal relativistic gas of Fermions or Bosons of mass m and statistical weight g = (2J+1)(2I+1) occupying a volume V at temperature T are.[17] (Units are $\hbar = c = k_{Boltzman} = 1$).

$$Z(V,T) = \frac{gV}{2\pi^2} m^2 T \sum_1^\infty \frac{(\mp)^{n+1}}{n} K_2\left(\frac{nm}{T}\right) \quad , \begin{pmatrix} F \\ B \end{pmatrix} \tag{1}$$

$$f(p,T)d^3p = \frac{gV}{2\pi^2} \frac{p^2 dp}{\exp\left(\frac{1}{T}\sqrt{p^2+m^2}\right) \pm 1} \quad , \begin{pmatrix} F \\ B \end{pmatrix} \tag{2}$$

from which the various thermodynamic quantities can be calculated. We want to describe a gas of Baryons and Mesons distributed in mass according to some unknown functions $\rho_\alpha(m)$. (α labels the families of particles, ordinary and strange baryons and mesons, some of which are shown in Table I). There are two important quantum numbers that have to be conserved, the net baryon number and strangeness. This is achieved as usual in thermodynamics, by introducing chemical potentials. If we specialize to symmetric collisions between Z = N nuclei then the conservation of baryon number and strangeness implies conservation of electric charge, since on the average $\langle Q \rangle = \frac{B+S}{2}$. We therefore make this specialization.

The average number and energy for the family of particles labelled α are

$$N_\alpha = \frac{VT}{2\pi^2} \int_{m_\alpha}^\infty dm \rho_\alpha(m) m^2 \sum_1^\infty \frac{(\mp)^{n+1}}{n} K_2\left(\frac{nm}{T}\right) \exp \frac{n\mu_\alpha}{T} \tag{3}$$

$$E_\alpha = \frac{VT}{2\pi^2} \int_{m_\alpha}^\infty dm \rho_\alpha(m) m^3 \sum_1^\infty \frac{(\mp)^{n+1}}{n} \left[K_1\left(\frac{nm}{T}\right) + \frac{3T}{nm} K_2\left(\frac{nm}{T}\right)\right] \exp\left(\frac{n\mu_\alpha}{T}\right) \tag{4}$$

Here μ_α is the chemical potential, m_α is the threshold, i.e., lowest mass particle in the family α, and K is a Kelvin function.

The baryonic charge of the system is clearly

$$B = \sum_\alpha N_\alpha B_\alpha \tag{5}$$

the sum being extended over the seven families of particles indicated in Table I(α = Π, K, N, ...) (As a family label, Π does not designate only the pions, but all the ordinary mesons, π, ρ, η, etc., and similarly for K, N, etc.) The number of antiparticles of type α and their energy are given by the above two equations (3) and (4) with $\mu \rightarrow \bar{\mu}$. We indicate by a bar, the antiparticle quantity, e.g., $\bar{N}$. Then the net baryon charge is

$$A = B - \bar{B}. \tag{6}$$

This quantity is conserved, and equal to the initial number of nucleons in the collision. The net strangeness is also conserved and is zero

$$0 = S - \bar{S} \tag{7}$$

where

$$S = \sum_\alpha N_\alpha S_\alpha \tag{8}$$

(The sign of the strangeness is opposite for particle and antiparticle.)

The two conditions (6) and (7) expressing baryon and strangeness conservation clearly couple all thermodynamic quantities T, μ_α. The reactions possible between the various particles dictate certain relations among the chemical potentials with the result that there are only two independent potentials, that for the nucleon and that for the kaon. The scheme we use to solve for the energy and particle populations as a function of temperature is basically the following. Choose a temperature T and find the values of the two chemical potentials that satisfy equations (6) and (7). When these are found then the populations and energies can be found from (3) and (4) and the total energy is of course

$$E = \sum_\alpha E_\alpha \tag{9}$$

The initial condition of the fireball is a little more complicated to solve. We consider symmetric collisions in the center of mass frame between nuclei of atomic number A/2. Each nucleus is Lorentz contracted by the factor $1/\gamma = m/E$. If the volume per nucleon in the rest frame of each nucleus is $v_0 = \frac{4}{3}\pi(1.2)^3$, then in the C.M. frame it is $v_0\gamma$. We assume that the collision is perfectly inelastic; that each nucleus is stopped by the other. Then the largest possible volume, in which all nucleons are contained, just after the nuclei have stopped each other is the contracted volume occupied originally by one.[18] So the initial baryon density of the fireball is

$$\rho_{initial} = \frac{2\gamma}{v_0} = \frac{2E}{mv_0} = v^{-1} \qquad (10)$$

and the volume per baryon is the reciprocal (see Fig. 12). Hence the volume V multiplying all quantities (3), (4), is a function of the as yet to be determined energy. How this problem is solved can be found in Appendix B.

In case the assumption of inelasticity is questioned, Fig. 13 shows the stopping of a very high energy proton by a nucleus which is clearly highly inelastic.

Three Examples of Hadronic Spectra

The object of the rest of the paper is to show how and at what energies the thermodynamic nuclear fireball would differ under the three different assumptions for the hadronic spectrum discussed below. For brevity we shall sometimes refer to the results for different spectra as being different worlds. The ultimate object, toward which this paper is a modest start is to discover which is most like our world.

a) The Known Hadrons: As one extreme case we might suppose that all of the hadrons have already been discovered. They are listed with their properties in the Particle Data Tables[16] and their density is plotted in Fig. 9 with the exception of recent discoveries. There are 56 different multiplets known with a total particle multiplicity of 531. Together with the antiparticles these comprise the 1000 or so known hadronic states mentioned earlier. We include them all by using the average mass and width for each multiplet. For our purpose, they fall into the seven families shown in Table I.

b) Bootstrap (exponential) Spectrum: There are several mathematical formulations of the bootstrap hypothesis but the thermodynamic theory of Hagedorn[17] is most useful to us because it yields an asymptotic form for the bootstrap hadron spectrum. The bootstrap spectrum lies at the opposite extreme from the "known" spectrum since it rises exponentially and without bound. We shall test the consequences of a bootstrap theory by using the Hagedorn form of the spectrum for the non-strange mesons and baryons in the region $m > 12\ m_\pi$. Below this mass we use the discrete known particles for these two families and all known strange particles.

We normalize the Hagedorn spectrum to agree with the average density of states in five pion mass intervals around 10 m_π. Thus

$$\rho_{Bootstrap}(m) = \begin{cases} \dfrac{1.12\ e^{m/T_0}}{(m/T_0)^3}/\text{pion mass} & m > 12\ m_\pi \\ \\ \text{discrete non-strange particles} & m \leq 12\ m_\pi \end{cases} \quad (13)$$

$$+ \text{ all strange particles}$$

$$T_0 = 0.958\ m_\pi \qquad m_\pi = 140\ \text{MeV}$$

We assume that there is an equal number of ordinary mesons and baryons in the continuous region. We might, but do not yet include continua for the families of strange particles because there is generally an insufficient number to estimate the normalization of the continua.

Recall that the quark hypothesis might also lead to an exponential spectrum, but possibly with different constants than those determined by a fit (Fig. 9) to the bootstrap iteration on the known particles.

c) Rigid Quark Bag: As an intermediate case, and so as to bring out where the sensitivity is achieved under less extreme alternatives than the first two, we consider a naive rigid quark bag. A meson is considered to be composed of 2 quarks, and a baryon of three. The walls are considered rigid and no new quark pairs are created within a hadron. Frautschi[26] finds that the density of such objects rises as m^2 and m^5 respectively. Normalizing at $m = 10\ m_\pi$ to the same value as the Hagedorn spectrum at that mass, we have for the continuous spectra for ordinary mesons (π) and baryons (N)

$$\rho_{Bag} = \begin{cases} \left.\begin{array}{l} \rho_\pi(m) = 0.154\ (m/m_\pi)^2/\text{pion mass} \\ \rho_N(m) = 1.36 \times 10^{-4}\ (m/m_\pi)^5/\text{pion mass} \end{array}\right\} & m > 12\ m_\pi \\ \text{discrete non-strange particles} & m \leq 12\ m_\pi \end{cases} \quad (14)$$

$$+ \text{ all strange particles}$$

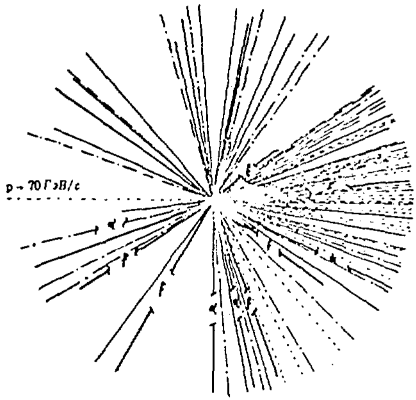


Fig. 13. Complete destruction of Ag, Br, Pb nuclei by 70 GeV/c protons and 17 GeV/c alphas (150 events analyzed). From O. Akhrorov et al., JINR (Dubna, 1976).

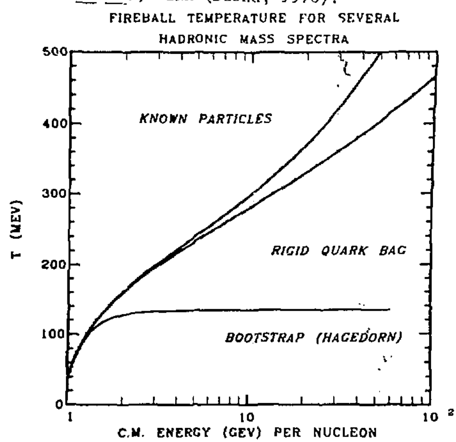


Fig. 14. For the three "worlds" considered, the temperature of hot hadronic matter assumed to be produced as a symmetric nuclear collision is plotted as a function of the C.M. total energy per nucleon of the colliding nuclei for a volume corresponding to the initial Lorentz contracted fireball.

The Temperature

The first crude indication of differences between hadronic matter constructed from the three assumed spectra is registered in the initial temperature they would be heated to for the same energy content. Since we assume a perfectly elastic collision, the C.M. collision energy per nucleon including rest energy is the total fireball energy per nucleon. These temperatures are shown in Fig. 14. For matter composed of a hadronic spectrum limited to the known particles, the temperature is by far highest at energies greater than several GeV. Because energy goes into making additional particles in the quark bag spectrum that were not present in the known spectrum, the temperature is lower at any corresponding energy. For the exponentially rising spectrum, as first discovered and emphasized by Hagedron, the temperature is limited to a maximum value corresponding to the constant T_0 in the spectrum eq. (13).

While T_0 appears to be nearly the pion mass, its value is not determined within the theory of Hagedorn. Instead it is deduced from a comparison with data. While the data often used are p_t measurements, we chose to fit the Frautschi[18] bootstrap iteration on the known particles.

The limiting temperature of matter, if composed of hadrons obeying the bootstrap condition (more precisely the exponential rise) is a truly remarkable property which has no analogies in other physical systems that I know of. (The boiling point of water is sometimes mentioned. This is a false analogue. The temperature of matter is limited even though the energy input is increased indefinitely! The limit to water temperature is reached because the energy is carried off by the steam. It is by comparison a trivial limit and totally different in origin.)

The mathematical nature of the limit can be seen by referring to eq. (4). For large masses, $m \gg T$, the Kelvin functions decay exponentially like

$$K(x) \propto \frac{1}{\sqrt{x}} \; e^{-x}$$

Inserting the Hagedorn spectrum we find

$$E \propto \int_M \frac{dm}{m^{1/2}} \; \exp - \left(\frac{T_0 - T}{T_0 T}\right) m \propto \left(\frac{T\,T_0}{T_0 - T}\right)^{1/2}$$

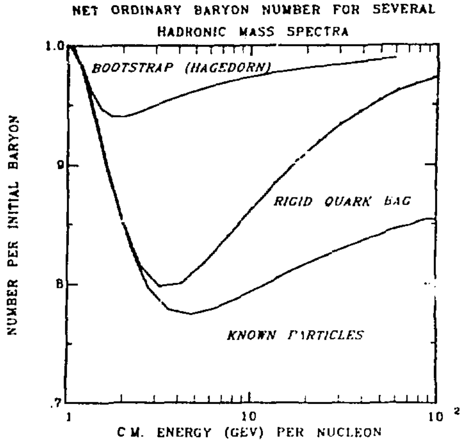


Fig. 15. The number of ordinary baryons is depleted owing to creation of strange baryons. The difference between unity and the curve corresponds to the baryon change resident in strange particles, and is also equal to the strange meson (K) population, because the net strangeness is zero.

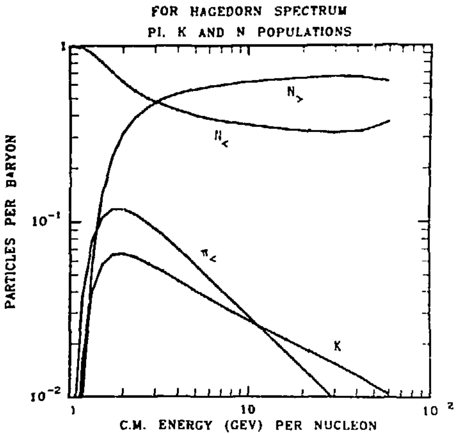
Thus as long as $T < T_0$ the integral converges; the energy is finite. But for $T \geq T_0$ the integral diverges. It would require infinite energy to raise the temperature to T_0 or beyond!

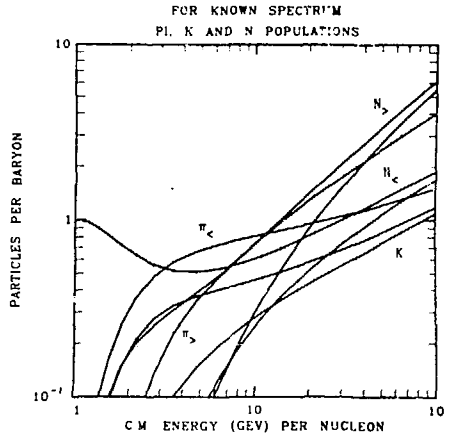
Composition of the Initial Fireball

Neither the temperature nor composition of the initial fireball are observables because any conceivable experiment must look at the products of the collision after the fireball has disassembled. Nonetheless it is interesting to look at the calculated populations because they are the starting point of the subsequent expansion or decay of the fireball. They also give us a glimpse of what the composition of the universe might have looked like at the beginning of time for very high energy and particle density. Because of the time scales involved we do not have to consider photons and leptons in equilibrium with the hadrons, so this is an important difference from the cosmological problem.

A very immediate impression of how the three worlds differ is given by Fig. 15 which shows the degree to which the ordinary (non-strange) baryon number is depleted. Initially all of the baryonic charge resides in non-strange baryons (the original neutrons and protons). As the energy is increased, the strange particles begin to be populated. The difference between unity and the plotted curves is this strange baryon population. This is a loose statement since the strangeness quantum numbers are not limited to the value unity. The succeeding statement for kaons is exact. Since however there was initially no net strangeness, this is exactly counter balanced by kaon populations (the strange mesons). We see that in all cases, there is a sudden rise in the strange particle populations which however is quenched quickly in the bootstrap world but rises to almost 25% in the case of the "known" world. At about 5 GeV almost 25% of the baryonic charge is converted to strange particles and to corresponding kaons!

Because there are so many discrete particles, not to mention the continua, we make the following arbitrary groupings to display more detailed information. Each family of particles is broken up into light particles comprising the lightest five (when there are that many) and heavy particles comprising all the rest, including continuum particles in the case of the quark bag and bootstrap worlds. We sum the populations in each group and plot only the summed populations. Thus the ordinary (non-strange) mesons are represented by two curves, for light and heavy mesons. There are no heavy kaons, but there are anti-kaons so there are curves for both. The ordinary baryons are represented by four curves, light and heavy baryons and anti-baryons. And so on.





Figs. 16-17. Corresponding to the three "worlds" investigated, the populations of the light and heavy members of the family of ordinary mesons (Π), strange-mesons (K) and ordinary baryons (N), are plotted as a function of energy. Light, refers to the first five multiplets (if that many) of each family, and are denoted by < Heavy, refers to all others, including the continuum where applicable, and are denoted by >. Anti-particles, except for the bootstrap, approach the particle populations at high energy. Refer to Table I for some of the members of the families.

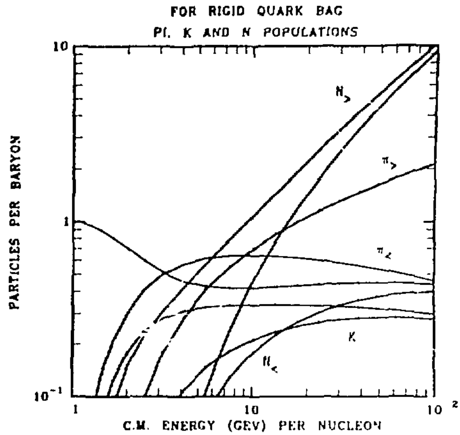
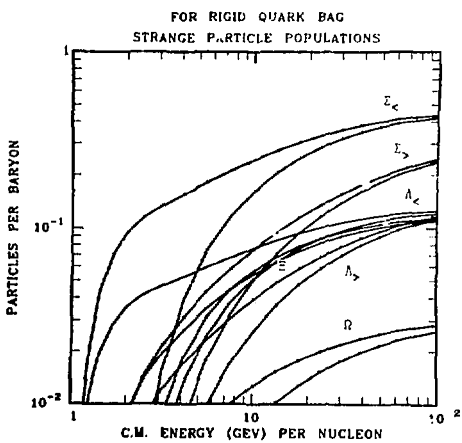


Fig. 17. See caption for Fig. 16.



Figs. 18-19. The population of the strange baryons as a function of energy for the three "worlds". Notation similar to above.

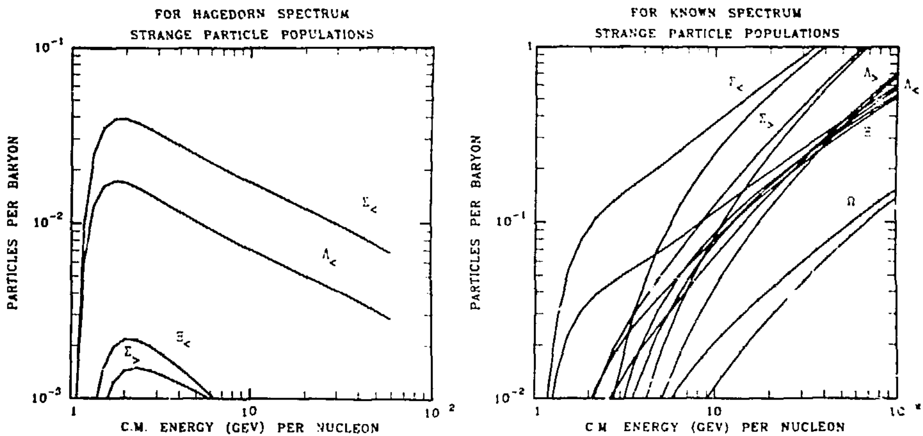


Fig. 19. See caption for Fig. 18.

Figures 16-19 show truly remarkable differences of the three fireballs depending on which is the underlying hadronic spectrum. For both the known spectrum and the quark bag, the heavy baryon and anti-baryon populations eventually dominate with heavy mesons the next most populous group. In the case of the quark bag this happens at rather low energy (on a particle creation scale). The heavy mesons follow. The composition at one GeV is of course all nucleon, but the light baryon and anti-baryons become less populated than the heavy ones in the bag model at energy above 10 GeV. The Hagedorn or bootstrap world is remarkably different. The light meson population rises to 10% and then falls. The heavy baryon population rises sharply and above 3 GeV the fireball is composed of more heavy baryons than light ones. By 10 GeV about 60% of the baryons are heavy and only 40% are light.

There is another remarkable difference. In the "known" and "bag" worlds, all particle-anti-particle populations approach each other at high energy (with anti-particles slightly less numerous). In the bootstrap (exponential) world, the anti-particles and mesons have microscopic populations. It is a world dominated at high energy and density by heavy baryons. This is an inevitable consequence of the exponential rise in the bootstrap density. At high temperature the system wants to produce heavy particles. Since however baryon conservation is forced, the energy is committed to making heavy baryons to the exclusion of mesons.

Expansion of the Fireball

If the S-matrix of the strong interactions (for each of our model worlds) were known, then we could calculate the observables that reach the counters. It is not, and that is what led us to the statistical mechanical treatment and the reasonable assumption of an equilibrated initial state. Now we must model the subsequent expansion of the system that carries the particles to the counting apparatus. It is possibly reasonable to assume, as in cosmology, that the expansion occurs through a series of equilibrated states. At some point during the expansion, when the density falls below a critical value, thermal contact between the particles is broken. This is called the freezeout.[27] Relative populations do not change thereafter, except by the decay of isolated particles. Since however, there is no membrane surrounding the expanding fireball, fast outward moving particles can escape from the equilibrated region prior to freezeout. Therefore there are two (indistinguishable) components to the particles that reach the counters, those that escape from the fireball during its expansion and prior to freezeout, and those that remain in thermal equilibrium until the freezeout. This is the scenario that we wish to model in order to calculate the spectra of the observed stable particles.

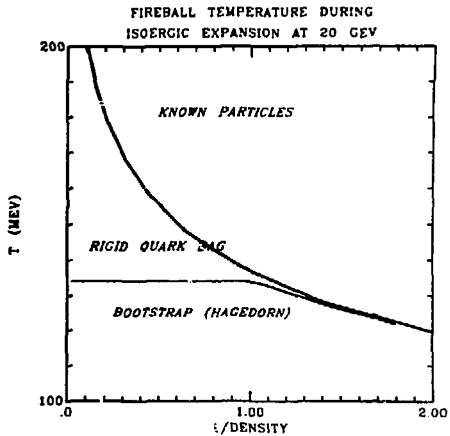


Fig. 20. The temperature of the fireball as it expands with constant energy equal to 20 GeV. The ordinate is the reciprocal of the total hadron density in units of the density of normal nuclei (0.17 Fm^{-3}). On this scale, 2 corresponds to a density such that each hadron has a share of the volume corresponding to a sphere with radius equal to the pion Compton wavelength 1.4 Fm.

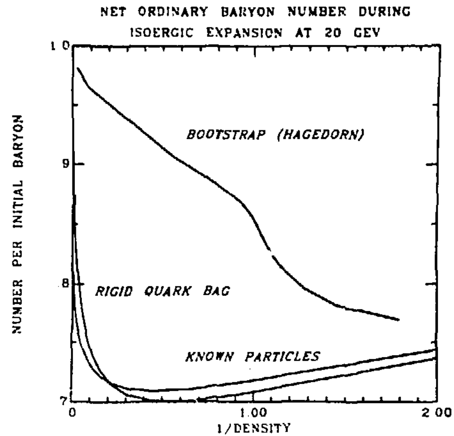


Fig. 21. Similar to Fig. 15 for the expansion at 20 GeV.

The importance of the pre-freezeout radiation should be commented on. An equilibrated system retains no memory of earlier states of the system. As the equilibrated region expands, its temperature falls, and the distinction between the three worlds will fade. It is the pre-freezeout radiation that carries the vital information on the early high-temperature condition of the fireball. Those particles that remain in equilibrium until the end are an unwanted background.

Isoergic Expansion with no Pre-freezeout Radiation

As a first orientation to gain insight into the evolution of the composition and temperature of the fireball in the three worlds, during an expansion, we consider an isoergic expansion in which no particles leave the system prior to the freezeout density, which is the density below which thermal contact is lost.

Presumably, the freezeout density, ρ_F, is less than normal nuclear density, $\rho_N \simeq 0.17\ \mathrm{fm}^{-3}$, but is not less than the density corresponding to each particle having a sphere of radius equal to a pion wavelength, $\rho_\lambda \simeq 0.085\ \mathrm{fm}^{-3}$.

$$\rho_\lambda < \rho_F < \rho_N$$

We shall plot our results for the thermal expansion stage as a function of $1/\rho$ where the density is measured in units of the nuclear density. On such a scale, the freezeout presumably occurs between 1 and 2. (ρ is the hadron density.)

Of course the temperature falls monotomically during the expansion to the freezeout point. Therefore the temperature characterizing particles that remain in thermal contact until the freezeout is the lowest temperature the fireball possessed. In this connection it is worth remarking that the claim in the literature[28] that the ultimate temperature has been measured in hadron-hadron collisions is possibly unwarranted. Unfortunately no one possesses a thermometer that he can insert into the initially formed fireball, but instead he must wait until the fireball expands and its constituents arrive at the counters.

The fall in temperature during an isoergic expansion at 20 GeV is shown in Fig. 20. For the "known" and rigid bag worlds it drops precipitously while for the bootstrap world it remains nearly constant until a late stage. At somewhat less than normal nuclear density, it begins to fall at the more rapid rate of the other worlds. This merely corresponds to the fact that at low enough temperature all worlds look the same. This figure suggests that temperature measurements near freezeout cannot distinguish between various hadronic spectra, as remarked earlier. The possible measurement of a temperature of T = 119 MeV in hadron-hadron collisions at 28 GeV reported in the literature[28] would correspond, on our figure, to a freezeout a little to the right of the frame where all worlds have fallen to virtually the same temperature.

The way in which the ordinary baryon charge is depleted during the expansion at 20 GeV is shown in Fig. 21. The bootstrap world is again remarkably different from the others, but although there was an initial large difference between the bag and "known" world (Fig. 15) it rapidly diminishes.

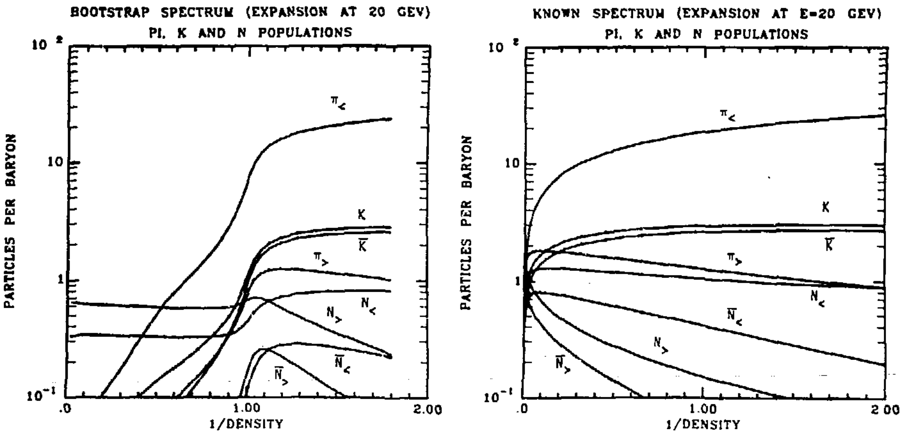
The populations of the various groups during the expansion at 20 GeV are shown in Figs. 22-23. The composition of the bootstrap fireball is remarkably different from the others during the early stage. However if thermal contact were sustained for <u>all</u> time it is clear that all worlds must appear the same at low enough temperature and density. Indeed they would just return to the original neutron proton composition. The breaking of thermal contact interrupts this return however. As stated earlier, we expect freezeout to occur between $1 \lesssim \frac{1}{\rho} \lesssim 2$. It is in precisely this range however that all <u>light</u> particle populations have already become quite similar in the three worlds.

It has now become clear that <u>if</u> thermal contact between all constituents is sustained during an expansion to a freezeout density equal to the nuclear density or less, it is impossible to distinguish between the three worlds. On the other hand it seems most likely that some of the constituents of the fireball will escape prior to the freezeout.

Isentropic Expansion of the Fireball

In an isentropic expansion, energy is lost to that part of the fireball that remains in thermal contact. I interpret the loss to be balanced by radiation of particles from the surface during the course of the expansion. This radiation produces the pressure against which the particles remaining in the fireball work during the expansion.

Figure 24 shows the energy remaining in the fireball for an isentropic expansion starting from an energy of 20 GeV. What this picture immediately suggests is that the pre-freezeout radiation is much more copious for the "known" and bag worlds than for the



Figs. 22-23. For the expansion at 20 GeV the populations for the three "worlds".

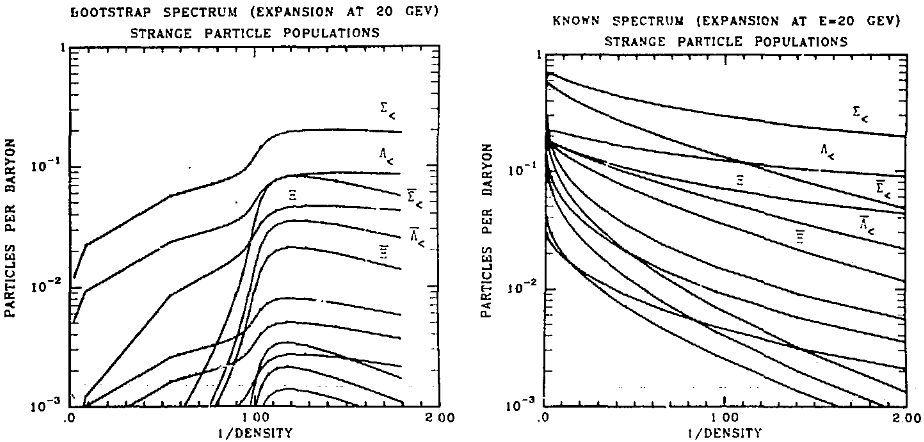


Fig. 23. See caption for Fig. 22.

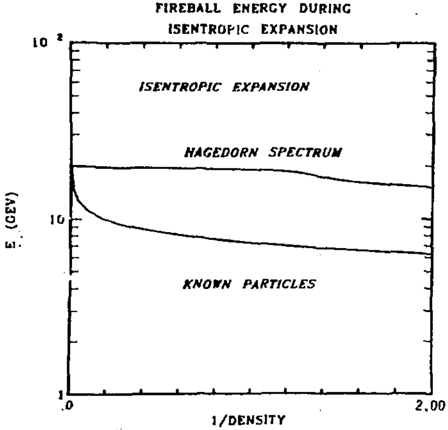
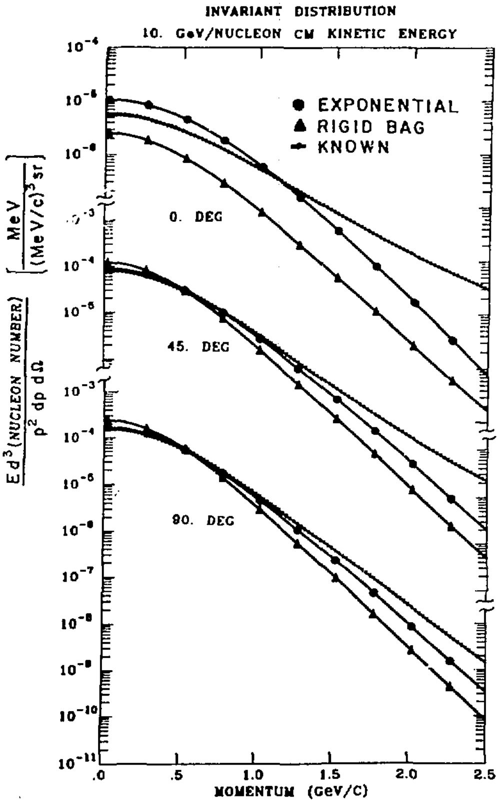


Fig. 24. The energy of the fireball decreases during an inentropic expansion. The energy lost to the thermal region is assumed to be carried off by particle radiation. No strange particles were included in this particular calculation.



Figs. 25-28. For a symmetrical head-on collision of two A = 50 nuclei with C.M. kinetic energy of 10 GeV/nucleon (colliding beams) the number, N, of particles, nucleons, anti-nucleons, pions, and kaons) in invariant phase space, $Ed^3N/(p^2dpd\Omega)$ is plotted as a function of C.M. momentum at three angles.

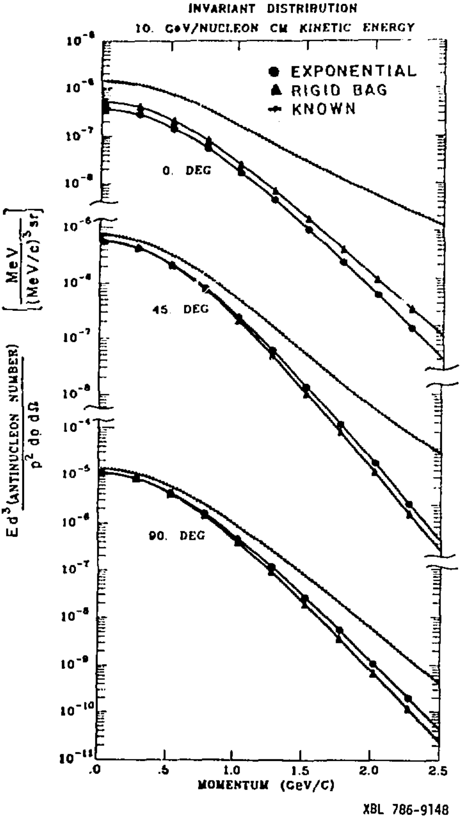


Fig. 26. See caption for Fig. 25.

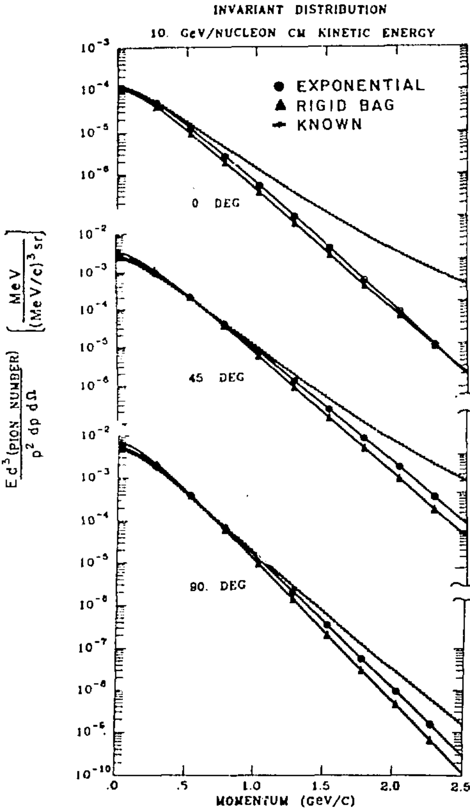


Fig. 27. See caption for Fig. 25.

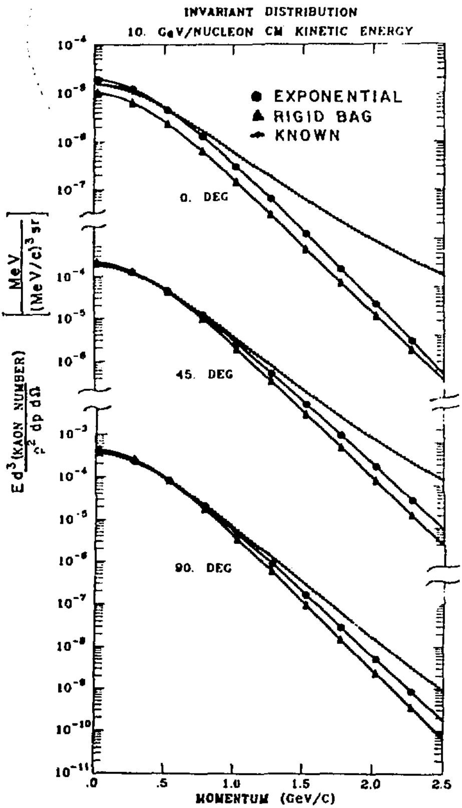


Fig. 28. See caption for Fig. 25.

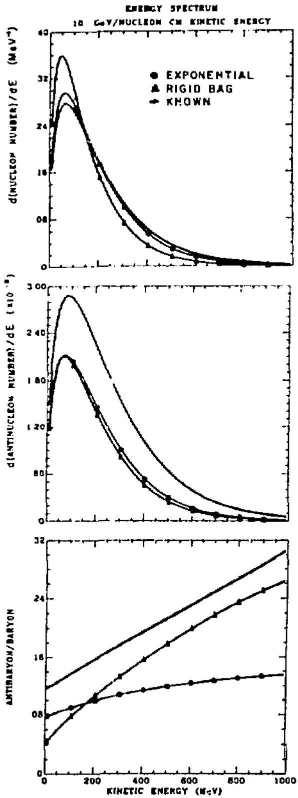


Fig. 29. For the collision described in Fig. 25 the spectrum (integrated over all angles) of nucleons, anti-nucleons and the ratio anti-baryon/baryon is plotted as a function of C.M. kinetic energy.

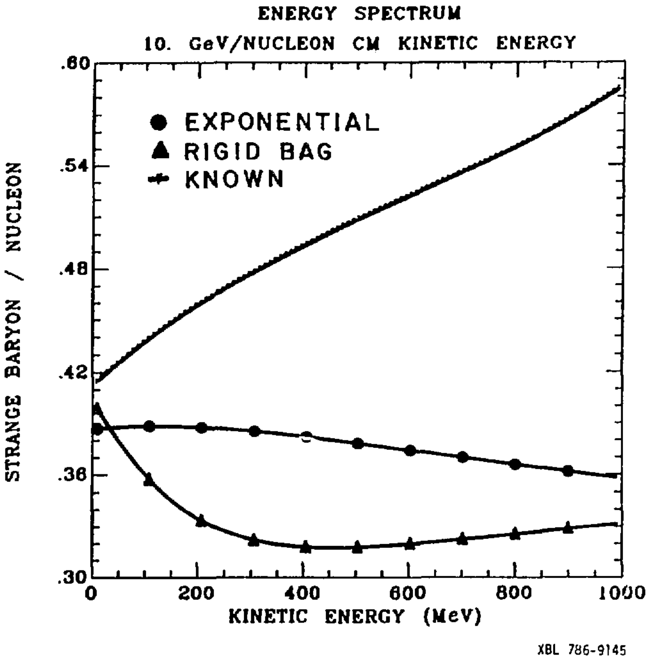


Fig. 30. Ratio of strange baryons to nucleons.

bootstrap world. The energy is trapped in massive baryons till a later stage in the expansion. Otherwise the qualitative difference between the worlds discussed in connection with the isoergic expansion appear in this expansion also.

Quasi-Dynamical Expansion

The hint gained in the preceeding study, that the pre-freezeout radiation from the three worlds will be different, because of the differing populations in the early, high-temperature phase of the expansion, provides the motive for attempting to follow the time development of the expansion.

We shall assume that at any instant the particles that lie within a mean free path of the surface of the fireball, and are directed outward will move into vacuum. Those of them that are unstable will decay, within a resonance mean life, into lighter stable and instable particles, so that in the immediate vicinity of the surface, the density remains high. Therefore we take this to define an instantaneous new surface and we assume that a new quasi-equilibrium state is established in this new volume. Meanwhile, those of the original outward moving particles that are stable, and moving faster than the unstable ones that established the position of the new surface, escape to vacuum. Their quantum numbers and energy are subtracted from those defining the state of the new quasi-equilibrated fireball. These steps are iterated until the density has dropped to the critical density or the fireball contains negligible energy and conserved quantum numbers in resonance states, whichever comes sooner. At that point the remaining particles move freely to the vacuum.

Of course the expansion does not occur isotropically in the C.M. because of the initial Lorentz contracted shape of the fireball. It is clear from the geometry that the shape of the fireball will evolve from the oblate spheroidal shape to a prolate spheroid.

We have calculated the distributions of the various stable particles and anti-particles [π, K, η, n, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$, $\Omega(1672)$]. As anticipated, the differences between the three worlds does not register so dramatically in the final products as it did in the initial hot fireballs, since the products are emitted over the lifetime of the fireball, from its initial hot state to its cooler final state. Note that the expansion to freezeout is enormous, beginning with a Lorentz contracted nuclear volume, and ending at freezeout some 20-30 nuclear volumes.

A sample of the preliminary results, corresponding to two mass 50 nuclei impinging on each other with 10 GeV kinetic energy per nucleon (colliding beams) are shown in Fig. 25-29. At three different angles, the invariant quantity, $E\, dN/d^3p$, is plotted as a function of the momentum, for nucleons, anti-nucleons, pions and kaons. While the differences are not large, they become of the order of 10 for the high momentum particles, which are emitted predominantly during the early stage of the fireball expansion. A particularly strong signal, and easy to measure, is the ratio of anti-baryons to baryons shown in Fig. 29.

Anti-Nuclei, Hyper-Nuclei, and Quark Phase

We come now to a most remarkable difference between nuclear collisions and hadron-hadron collisions. To the extent that thermodynamics applies to each, then all I have said until now applies to nuclear fireballs as to hadron fireballs.

A reexamination of the populations reveals that the anti-baryons and also strange baryons have significant populations. As an example, for the bootstrap world during the expansion phase at densities below nuclear density, (1 on the ordinate) the population of the light Σ family is ~ 0.2 per baryon. That means that for a collision involving a hundred nucleons, 20 Σ's appear at the freezeout! There are even more light anti-baryons present, about 27. Thus although we have not yet calculated composite particle populations, we can anticipate significant production of light anti-nuclei and hyper-nuclei and possibly even strange nuclei, i.e., nuclei composed entirely of strange baryons. This appears to be a fascinating possibility. I presume little is known of the binding properties of such objects, except of course anti-nuclei, which would have to be the same as ordinary nuclei.

Moreover, we note that pre-freezeout radiation of these as with single particles would be quite different in the worlds examined.

A quark phase can also be discussed. If the whole system remains in thermal contact until the density has fallen to a freezeout density below which interactions cease, then a quark phase would be hidden. (Unless of course quarks can exist as free particles in which case some of them may not find a partner to recombine with before freezeout.) The total energy, since it is still shared by the whole system, insures that when the quarks recondense into the hadron phase, the composition will evolve with density to the freezeout in exactly the same way as if the quark phase had never existed. However, if some particles do escape from the equilibrated region before freezeout, which does seem plausible, then the numbers and types of escaping particles would depend upon whether, during part of the expansion stage, the matter was in a quark phase. During the quark phase there would be no radiation (assuming no asymptotically free quarks), or if there were a mixture of the two phases, say quark matter in the interior surrounded by a hadron halo, the radiation would likely be different than if only the hadron phase existed throughout. Assuming that quarks cannot exist as asymptotically free particles, I conclude that detection of a quark phase may be possible, but its detection would depend upon an accurate description of the disassembly stage of the fireball.

CONCLUSIONS

The calculations reported here confirm our hope that it will be possible to make a statement concerning the assymptotic form of the hadronic spectrum through a study of the products of nuclear collisions at energies from about 5 GeV per nucleon in the C.M. In particular, it should be easy to distinguish between a finite spectrum and an unbounded one. The energies required are high but they can be attained in conceivable accelerators.

We concede that the dynamics of high energy collisions is probably more complicated than thermodynamics supplemented by a quasi-equiliorium expansion. But I can see no reason to expect that a complete dynamical description, were it possible, would exhibit much less sensitivity to the outcome, than our model.

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APPENDIX A: CHEMICAL POTENTIALS

The chemical potentials, as is usual in thermodynamics, must obey certain relations that are dictated by the possible reactions. For example

$$2p \rightarrow 2p + \pi^0 \text{ implies } \mu_\pi = 0$$

and $pn \rightarrow p\Delta^0$

$$pn \rightarrow n\Delta^+$$

$$pp \rightarrow p\Delta^+$$

$$pp \rightarrow p\Delta^{++}$$

$$nn \rightarrow p\Delta^-$$

imply that

$$\mu_{\Delta^0} = \mu_n,\ \mu_{\Delta^+} = \mu_p,\ \mu_{\Delta^{++}} = 2\mu_p - \mu_n,\ \mu_{\Delta^-} = 2\mu_n - \mu_p$$

We will ignore the proton, neutron mass difference so that $\mu_n = \mu_p - \mu_B$. Then

$$\mu_\Delta = \mu_B$$

and in general all multiplets belonging to the same family have the same chemical potential.

There are also relationships between the chemical potentials of different families. The reactions

$$NN \rightarrow NK\Lambda$$

$$NN \rightarrow NK\Sigma$$

$$N\pi \rightarrow \Lambda K$$

$$NN \rightarrow N\Xi KK$$

$$NN \rightarrow N\Omega KKK$$

imply

$$\mu_B = \mu_K + \mu_\Lambda = \mu_K + \mu_\Sigma = \mu_\Lambda + \mu_\Sigma = \mu_\Xi + 2\mu_k = \mu_\Omega + 3\mu_K$$

Call $\mu_K = \mu_S$; then a solution to the equation is:

$$\mu_\Lambda = \mu_\Sigma = \mu_B - \mu_S$$

$$\mu_\Xi = \mu_B - 2\mu_S$$

$$\mu_\Omega = \mu_B - 3\mu_S$$

Now all the chemical potentials are expressed in terms of the two, μ_B and μ_S.

APPENDIX B: CONTRACTED INITIAL FIREBALL

As indicated in eq.(10) the initial volume, which multiplies all quantities, depends on the as yet undetermined energy because of the Lorentz contraction. To solve the initial fireball equations, write eqs. (3), (4), (10) as

$$N_\alpha = V\, n_\alpha(\mu,T) \qquad E_\alpha = V\, \mathcal{E}_\alpha(\mu,T) \tag{B1}$$

$$E = V\,\mathcal{E}(\mu,T) \qquad \mathcal{E} = \Sigma\, \mathcal{E}_\alpha$$

where μ stands for μ_S, μ_B, the two independent chemical potentials. Since eq. (10)

$$V = Av = A\frac{mv_0}{2E} \tag{B2}$$

we find from (B1) and (B2)

$$E = \left(\frac{v_0\, m\, \mathcal{E}}{2}\right)^{1/2}$$

$$V = A\left(\frac{mv_0}{2\mathcal{E}}\right)^{1/2} \tag{B3}$$

So the equation for baryon conservation, eq. (6), becomes

$$1 = \left(\frac{m\, v_0}{2\,\mathcal{E}(\mu,T)}\right)^{1/2} (b(\mu,T) - \bar{b}(\bar{\mu},T)) \tag{B4}$$

The equations that define the initial contracted fireball are (7) and (B4) which are to be solved for μ_B and μ_S for chosen T.

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