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A b s t r a c t

Relations between the transversal and longitudinal parts of elastic and quasielastic form factors are studied within the quark model. It is shown that for an even number of the constituent quarks the longitudinal part dominates while for an odd number the transversal part is the largest one. Consequences from this result are considered for deuteron form factor and for matrix elements of the electromagnetic transitions between ~~the~~ π, ρ, A_1-mesons.

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The quark model has been recently successfully applied to describe the asymptotic behaviour of the hadron electromagnetic form factors[1]. The simplest way to derive the predictions is to consider the quark graphs of the type represented in Fig.1. In particular, one finds from an analysis of these graphs that form factor falls off at large Q^2 as $Q^{-2(n-1)}$ where n is the number of the constituent quarks and $-Q^2$ is the four momentum transfer squared.

The model works well in the case of nucleon and pion. The most dramatic confirmation seems to come from the recent measurements of the deuteron form factor[2] which indicate that deuteron at short distances can be considered as a six-quark system[3]. From the theoretical point of view the use of graphs of the lowest order in the coupling constant can be justified within the quantum chromodynamics according to which interaction is relatively weak at short distances.

As is well known, the spin ½ nature of the constituents reveals itself in the dominance of the transversal cross section in the deep inelastic scattering, $\sigma_T >> \sigma_L$. It is usually thought that the same is true for the form factors as well [3,4] (with the exclusion of the pion form factor which the dominance of the longitudinal cross section is purely kinematical).

In this note we will consider in more detail the relation between the transversal and longitudinal cross sections for elastic and quasielastic scattering. Our main result is that this relation depends in fact on the number of the constituent quarks:

$$\sigma_L/\sigma_T \gg 1 \quad (Q^2\to\infty) \qquad \text{if } n \text{ is even}$$
$$\sigma_T/\sigma_L \gg 1 \quad (Q^2\to\infty) \qquad \text{if } n \text{ is odd} \tag{1}$$

and the quark counting rule refers to the dominating part of the cross section. Thus, for an even number of quarks the longitudinal part of the cross section dominates at high Q^2 despite the spinor nature of the constituents. We will give some applications of this result.

Eq.(1) follows directly from a consideration of the quark graphs of the type represented in Fig.1 under the usual assumption that the quarks in the initial and final states share the energy of hadrons, roughly speaking, on equal. It might be useful, how-ever, to indicate the line of deriving Eq.(1).

Let us start with the graphs of the type represented in Figs. 1,2a. The graphs depend on the properties of the coupling of the gluon attached to the lowest quark line and on the amplitude of the Compton-like scattering with two bosons (both gluons or a gluon and a photon) attached to the quark line. The virtuality of all the internal lines is of the order Q^2. We assume throughout the paper that gluons are vector fields and one readily finds that the interaction of the transversely polarized gluon with a quark is $(Q^2)^{1/2}$ times larger than that of the longitudinally polarized gluon. As for the amplitude of the Compton scattering it is the largest if one gluon (photon) is longitudinal and the other one is transversal. If both quanta are polarized in the similar way we get the damping factor of the order $(Q^2)^{-1/2}$. Thus, to get the dominating contribution we start with transversal gluon attached to the lowest line. The gluon coupled to the next line is longitudinal and the polarization is changed with each of the next steps. Finally, we extract the polarization of the photon which obeys Eq.(1). A similar argument can be given for the graphs of the type examplified in Fig.3. The general rule which follows from chirality conservation is that the number of transversely polarized gluons attached to each quark line must be odd. As for graphs containing there gluon vertex (Fig.2b) it can be shown that they do not contribute to the leading in $(Q^{-2})^n$ terms.

As an application of the result obtained let us consider first elastic scattering of a deuteron at large Q^2. If the deuteron can be viewed as a six-quark system at short distances, then Eq.(1) implies the vanishing of the magnetic form factor. The prediction can be checked by studying the angular distribution. The general form of the angular distribution in the lab.system is given by the Rosenbluth formula:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[A(Q^2) + B(Q^2) \tan^2 \frac{\theta_L}{2} \right] \qquad (2)$$

and we expect that $A \gg B$. A confirmation of this prediction would strengthen the argument in favour of the quark model of the deuteron at short distances.

Eq.(1) can also be tested in electromagnetic transitions between the mesons consisting of two quarks. Experimentally, these transitions can be studied in the time-like region, i.e. in the processes

$$e^+e^- \to \pi^+\pi^-,\ \pi\rho,\ \rho^+\rho^-,\ \pi A_1,\ A_1^+A_1^-, \ldots \qquad (3)$$

In all the cases we predict that σ_T is small as compared to the naive quark counting rule and we will specify the consequences from this prediction for various channels.

As was already mentioned, the dominance of σ_L in the case of the pion form factor is purely kinematical. As for the $\pi \to \rho$ transition it is described by a single form factor

$$\langle \pi | j_\alpha^{el}(0) | \rho \rangle = F_{\pi\rho}(Q^2)\, \epsilon_{\alpha\beta\gamma\delta}\, \rho_\beta k_\gamma p_\delta , \qquad (4)$$

where p and k are the momenta of the pion and ρ -meson, respectively, and ρ_β is the polarization vector of the ρ -meson. Since amplitude (4) is purely transversal the prediction is that $\pi\rho$ production at large Q^2 is small. Namely, we expect

$$\frac{\sigma(e^+e^- \to \pi\rho)}{\sigma(e^+e^- \to \pi\pi)} \propto \frac{1}{Q^2}, \qquad F_{\pi\rho}(Q^2) \propto \frac{1}{Q^4}. \qquad (5)$$

The analogous prediction is valid for transition .

Transition $\rho^\pm \to \rho^\pm$ is described by three form factors

$$\begin{aligned} \langle \rho | j_\alpha^{el}(0) | \rho \rangle = {} & F^{(1)}_{\rho\rho}(Q^2)(p_1+p_2)_\alpha (\rho^{(1)}_\beta \rho^{(2)}_\beta) + \\ & + F^{(2)}_{\rho\rho}(Q^2)\left[\rho^{(1)}_\alpha (\rho^{(2)}_\beta q_\beta) - \rho^{(2)}_\alpha (\rho^{(1)}_\beta q_\beta)\right] + \\ & + F^{(3)}_{\rho\rho}(Q^2)(p_1+p_2)_\alpha (\rho^{(1)}_\beta q_\beta)(\rho^{(2)}_\gamma q_\gamma), \end{aligned} \qquad (6)$$

where $p_{1,2}$ and $\rho^{(1)}_\beta$, $\rho^{(2)}_\beta$ are the momenta and polarization vectors of the ρ -mesons and q is the photon momentum. The terms proportional to $F^{1,3}$ contribute only to σ_L while $F^{(2)}$ contributes mostly to σ_T . Thus, Eq.(1) implies smallness of $F^{(2)}$. From a detailed analysis of the quark graphs we extract more stringent predictions:

$$F^{(0)}_{\rho\rho} \sim \frac{1}{Q^4}, \quad F^{(2)}_{\rho\rho} \sim \frac{1}{Q^4} \quad F^{(1)}_{\rho\rho} \sim \frac{1}{Q^6}. \qquad (7)$$

The smallness of $F^{(2)}$ means that in the reaction $e^+e^- \to \rho^+\rho^-$ both mesons are produced longitudinally polarized and this prediction can be checked by studying the angular distribution of the decay pions.

The predictions for the form factors of the elastic transition $A_1^{\pm} \to A_1^{\pm}$ where A_1 is a meson with quantum numbers $J^P = 1^+$ are similar to (7). This follows from the quark graphs and is most easily understood if ρ and A_1 belong to the same multiplet in the limit of exact chiral $SU(2)_L \times SU(2)_R$ symmetry. Then the asymptotic equality of the form factors of ρ and A_1-mesons follows from simple symmetry consideration.

Let us notice that in Ref. [1b, 5] it was conjectured that the form factor of A_1-meson has an extra power of Q^{-2} because of the p-wave nature of the two-quark bound state. According to our conclusions the suppression of the form factor is in fact of the order v/c where v is the relative velocity of quarks and numerically is not too strong. Moreover, the suppression factor does not depend on Q^2.

For a three quark system our conclusions are the same as of the other authors. The most extensive study of the form factors in the three-quark system is given in Ref [6].

It is worth emphasizing that all the results outlined above are obtained in the lowest order in the quark-gluon coupling constant. The graphs of the higher order can be essential at large Q^2 and, in particular, can change the power-behaviour of the suppressed form factors. It is reasonable to expect, however, that the smallness persists with the account of the higher order graphs as well.

Indeed, one can check that the proof of eq.(1) given above remains valid for the graphs with loops unless the transverse momenta of virtual gluons are much less than $\sqrt{Q^2}$. It means that for odd number of constituents the contribution of the given order in the quark-gluon constant into σ_L is one power of log Q^2 less than into σ_T and vice versa for even number of constituents. The situation is quite similar to the well known prediction for ratio σ_L/σ_T in deep inelastic scattering. In the parton model one has $\sigma_L/\sigma_T \propto 1/Q^2$. If the gluon exchanges are taken into account, then $\frac{\sigma_L}{\sigma_T} \propto \frac{1}{\log Q^2}$. Numerically, however, σ_L is predicted to be small. Thus, we believe that all the predictions given above are worth testing experimentally. Deviations from the power laws described above if detected experimentally would give additional information on the gluon exchange at short distances.

Let us conclude this note with remarks on duality between form factors and deep inelastic structure functions. Our results are dual to the predictions from the quark model for the transversal structure function. Thus, according to Ref.[7] the pion structure function $F_T(x)$ for $x \to 1$ tends to $(1-x)^2$ and is not dual to the pion form factor which is purely longitudinal. We have shown that F_T is dual to the $\gamma^* \pi \rho$ amplitude (γ^* stands for the virtual photon) which is purely transversal and is predicted to fall off faster than $\gamma^* \pi \pi$ amplitude. Thus, in the lowest order in the quark-gluon coupling constant the duality is a simple consequence of the quark graphs. Moreover, the similarity of the consideration of deep inelastic scattering given in Ref. [7] and that of elastic cross section presented above can be traced even to some details of the calculations.

For the longitudinal cross section the duality also holds although in somewhat unusual form. In particular, in the case of deuteron we get $F_L^{(d)} \propto Q^{-2}(1-x)^8$. It vanishes in the scaling limit but dual to the longitudinal form factor which falls off as Q^{-10} .

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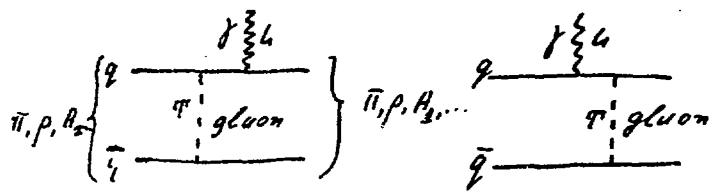

Fig. 1. Quark graphs for the electromagnetic transition form factors of mesons

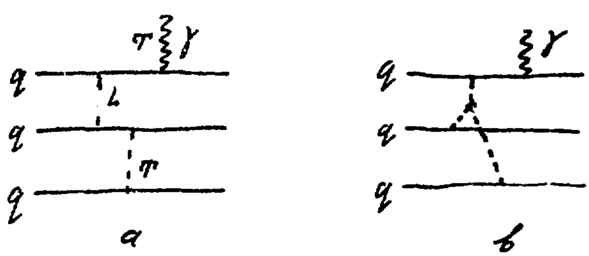

Fig. 2. Quark graphs for the form factors of barions. The dotted lines denote gluons. Symbols "L" and "T" mark the longitudinal and transversal polarizations

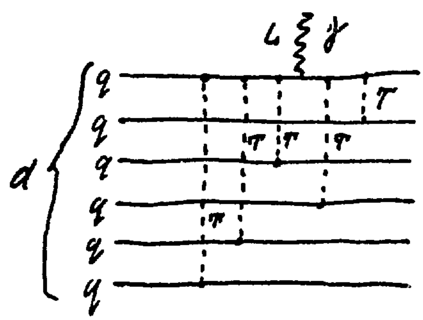

Fig. 3, An example of a quark diagram for the deuteron form factor.

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