INSTITUTE OF PLASMA PHYSICS

NAGOYA UNIVERSITY

Heating of Charged Particles by Electrostatic Wave Propagating Perpendicularly to Uniform Magnetic Field

Keishiro Niu* and Takashi Shimojo**

IPPJ-323 February 1978

RESEARCH REPORT

NAGOYA, JAPAN

Heating of Charged Particles by Electrostatic Wave Propagating Perpendicularly to Uniform Magnetic Field

Keishiro Niu* and Takashi Shimojo**

IPPJ-323 February 1978

Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

Permanent address:

* Department of energy sciences, Tokyo Institute of Technology, Ohokayama, Megro-ku, Tokyo, Japan.

** Department of Physics, Tokyo Gakugei University, Nukuikita-machi, Koganei-shi, Tokyo, Japan.

Abstract

Increase in kinetic energy of a charged particle, affected by an electrostatic wave propagating perpendicularly to a uniform magnetic field, is obtained for both the initial and later stages. Detrapping time of the particle from the potential dent of the electrostatic wave and energy increase during trapping of the particle is analytically derived. Numerical simulations are carried out to support theoretical results.

1. Introduction

Charged particles increase their kinetic energies, as they are affected by an electrostatic wave which propagates in a magnetic field. This phenomenon is of interest with relation to the secondary-heating method of fusion-plasma in toroidal machines. On the other hand, high energy electrons and ions are observed in a laser-irradiated pellet. It is well known that strong electrostatic waves are induced by the parametric and/or resonance absorption of laser-light in the pellet, in which magnetic fields of several mega-gausses grow. After Smith and Kaufman,[1] Fukuyama et al.[2,3] analyzed a non-linear effect on ion motions of an electrostatic wave propagating perpendicularly to a uniform magnetic field. Their theory was based on a stochastic process which was developed from a time-dependent Hamiltonian for a test ion. The time-dependent nature of an electrostatic wave made the analysis and its compatible condition complicated. Sugihara and Midzuno[4] did calculation for an acceleration of a trapped particle in a single electrostatic wave propagating in a magnetoplasma. Their analysis could be applied to charged particles with special initial conditions.

In this papar, we simplify the analysis by observing phenomenon in a coordinate system in which an electrostatic wave as well as magnetic field is steady. Increase in the kinetic energy of a charged particle is obtained for the initial stage (for which Sugihara and Midzuno analyzed) and for the later stage (for which Fukuyama et al.[2,3] investigated).

2. Basic Equations

Let us consider a motion of a charged particle (mass is m and charge is q) in the action of an electrostatic wave ($\mathbf{E}=(E,0,0)$, $E=E_m \sin(kx+\omega t)$) propagating in a uniform magnetic field ($\mathbf{B}=(0,0,B)$). Since there is no force acting on the particle in the z-direction, the z-component of the particle velocity remains constant. Thus we limit our analysis only in the x-y space for the particle velocity ($\mathbf{v}=(v_x,v_y,0)$).

The equation of motion of the particle is

$$m d\mathbf{v}/dt=q(\mathbf{v}\times\mathbf{B})+q\mathbf{E} \tag{1}$$

that is,

$$dv_x/dt=\omega_c v_y-\omega_b^2/k.\sin(kx+\omega t), \tag{2}$$

$$dv_y/dt=-\omega_c v_x, \tag{3}$$

where t is time, $\omega_c=qB/m$ is the cyclotron frequency and $\omega_b=(kqE_m/m)^{1/2}$ is the bouncing frequency. The Hamiltonian H of eq.(1) is described by

$$H(x,y,p_x,p_y,t)=1/2m.\{p_x^2+(p_y-qBx)^2\}-qE_m/k.\cos(kx+\omega t), \tag{4}$$

where $p_x=mv_x$ and $p_y=mv_y+qBx$. Fukuyama et al. expanded the second term of the right hand side of (4) into series of Bessel functions, and then, they divided the Hamiltonian into two parts of steady and unsteady. They advanced the analysis considering that the unsteady terms were the perturbation to the steady terms.

The electrostatic wave propagates in the negative x-direction with a speed $v_c=\omega/k$. In this paper, we observe the phenomenon in a coordinate system in which the electrostatic wave is at rest. In such a coordinate system, a constant electric field $E_y=v_cB$ appears. Equations (2) and (3) reduce to

$$\partial v_x/\partial t=\omega_c v_y-\omega_b^2/k.\sin kx, \tag{5}$$

$$\partial v_y/\partial t=-\omega_c v_c \tag{6}$$

in the new coordinate system. The Hamiltonian

$$H=1/2m.\{p_x^2+(p_y-qBx)^2\}-qv_cBy-qE_m/k.\cos kx \tag{7}$$

does not depend on t. Thus,

$$H=\text{const.} \tag{8}$$

is one of the conservation equations.

3. Particle Motions

Let us suppose that the particle is near the origin ($x=x_0$, $y=0$) with the velocity $v_x=0$, $v_y=v_{y0}$ at $t=0$. Limiting ourselves into the domain of $-\pi/2k\leqq x\leqq\pi/2k$, we approximate eq.(5) by[5)]

$$\partial v_x/\partial t=\omega_c v_y-\omega_b^2 x. \tag{9}$$

From eqs.(6) and (9), we have

$$v_x=\omega_c y-\omega_b^2\int_0^t x\,dt, \tag{10}$$

$$v_y=-\omega_c x+\omega_c v_c t+v_{yo}+\omega_c x_0. \tag{11}$$

By use of (11), eq.(9) reduces to

$$d^2x/dt^2=-\Omega^2 x+\omega_c^2 v_c t+\omega_c v_{y0}+\omega_c^2 x_0, \tag{12}$$

where $\Omega^2=\omega_b^2+\omega_c^2$. The solution is given by

$$x=-\frac{\omega_c^2}{\Omega^3}v_c\sin\Omega t-\frac{\omega_c v_{y0}}{\Omega^2}\cos\Omega t+\frac{\omega_c^2 v_c}{\Omega^2}t+\frac{\omega_c v_{y0}}{\Omega^2}+x_0, \tag{13}$$

$$y=-\frac{\omega_c^3 v_c}{\Omega^4}\cos\Omega t+\frac{\omega_c^2 v_{y0}}{\Omega^3}\sin\Omega t-\frac{\omega_c^3 v_c}{2\Omega^2}t^2-\frac{\omega_c^2 v_{y0}}{\Omega^2}t+\frac{\omega_c v_c}{2}t^2+v_{y0}t+\frac{\omega_c^3 v_c}{\Omega^4}. \tag{14}$$

In order to clarify the effect of the electrostatic wave on the particle, we consider the case under the following conditions,

$$\frac{1}{2}mv_c^2 << \Phi = \frac{qE_m}{k} = \frac{m\omega_b^2}{k^2} \tag{15}$$

$$1/2.mv_0^2 = 1/2.m(v_{x0}^2 + v_{y0}^2) << \Phi. \tag{16}$$

Inequality (15) reduces to

$$\omega << \omega_b. \tag{17}$$

If we consider that the effect of the electrostatic wave is more significant than that by the magnetic field, we have another inequality

$$\omega_c << \omega_b. \tag{18}$$

In view of (17) and (18), the detrapping time t_d (since we approximate sin kx by kx, we define here that the particle is detrapped at $x=\pi/2k$, instead of at $x=\pi/k$) can be obtained from (13) as

$$\frac{\pi}{2k} - x_0 = \frac{\omega_c^2 v_c}{\omega_b^2} t_d + \frac{\omega_c v_{y0}}{\omega_b^2},$$

that is,

$$t_d = \frac{\omega_b^2}{\omega_c^2 v_c}\left(\frac{\pi}{2k} - x_0 - \frac{\omega_c v_{y0}}{\omega_b^2}\right). \tag{19}$$

We can rewrite (8) as

$$1/2.m(v_x^2 + v_y^2) - qv_c By - qE_m/k.\cos kx = 1/2.mv_0^2 - \Phi, \tag{20}$$

Therefore, the increase ΔE in the kinetic energy at the detrapping time is obtained from (20) as

$$\frac{\Delta E}{m}=\frac{\omega_b^4}{2\omega_c^2}\left(\frac{\pi}{2k}-x_0-\frac{\omega_c v_{y0}}{\omega_b^2}\right)^2+\frac{\omega_b^2 v_{y0}}{\omega_c}\left(\frac{\pi}{2k}-x_0-\frac{\omega_c v_{y0}}{\omega_b^2}\right)-\frac{\omega_b^2}{k^2}+\frac{\omega_c^4 v_c^2}{\omega_b^4}. \tag{21}$$

In case of $x_0=0$ and $v_{y0}=0$, we have

$$t_{d0}=\pi\omega_b^2/2\omega\omega_c^2, \tag{22}$$

and

$$\Delta E_0=\frac{m\pi^2\omega_b^4}{8\omega_c^2 k^2}=\frac{m\pi^2 E_m^2}{8B^2}. \tag{23}$$

Here we summarize the above results. If the initial kinetic energy of the particle is much less than the potential energy of the electrostatic wave, the particle is trapped by the wave. The trapped particle is strongly accelerated by the induced constant electric field in the y-direction. The relation (23) shows that when the particle is detrapped, the kinetic energy of the particle is elevated to a value which is much greater than that of the potential energy of the electrostatic wave.

After the detrapping, the electrostatic wave gives only a perturbing effect on the particle motion. Therefore, the particle path is basically a cycloid. Fig.1 shows a particle velocity v_x and v_y versus time t. At t=0, $v_x=v_y=0$. At first, the particle is trapped by a potential dent. For a while, v_x remains almost zero, and v_y increases nearly linearly. At $t=t_{d0}$, the particle is detrapped from the wave. Since then, v_x and v_y change sinusoidally, as the particle moves along a cycloid curve. As clearly seen from Fig.1, the electrostatic wave scarecely alters the particle path after the detrapping of the particle.

Since the gyration velocity of the particle is much larger than the drift velocity v_c after the detrapping, the cycloid curve along which the particle moves is as shown in Fig.2. At a point S or T in the Figure, $v_x=0$. If this point happens to coincide with a potential dent of the electrostatic wave, the particle is trapped again by the wave. Although the kinetic energy of the particle gyration is much larger than the potential energy of the wave, trapping or detrapping of the particle depends only on v_x regardless of the particle velocity. For such a trapped particle, we must consider that v_{y0} is very large.[6,7] At the point T, v_{y0} is negative. The constant electric field E_y accelerates the particle in the positive y-direction during the trapping, and v_{y0} can becomes zero at a later time. Since then, the behavior of the particle is similar to that of the first trapping. The order of magnitude of the kinetic energy is not changed by the trapping. On the other hand, v_{y0} is positive at the point S. In this case, the detrapping time t_d can be obtained by putting $x=\pi/2k$ in (13) as

$$\frac{\pi}{2k}=-\frac{\omega_c^2 v_c}{\Omega^3}\sin\Omega t_d-\frac{\omega_c v_{yo}}{\Omega^2}\cos\Omega t_d+\frac{\omega_c^2 v_c}{\Omega^2}t_d+\frac{\omega_c v_{y0}}{\Omega^2}+x_0. \qquad (24)$$

The detrapping time t_d which satisfies (24) must be

$$t_d=\frac{2\pi n}{\Omega}, \qquad (25)$$

where n is a large integer, and n is required to satisfy

$$\left(\frac{\pi}{2k}-x_0\right)=\frac{2\pi\omega_c^2 v_c n}{\Omega^3},$$

that is,

$$n=\left[\frac{\Omega^3}{2\pi\omega_c^2 v_c}\left(\frac{\pi}{2k}-x_0\right)\right], \tag{26}$$

where [ ] shows the integer which is equal to or larger than its argument. The increase Δv_y in the velocity component in the y-direction can be obtained by using (11) as

$$\Delta v_y=v_y-v_{y0}=\omega_c v_c t_d-\omega_c\left(\frac{\pi}{2k}-x_0\right)\simeq\frac{\Omega^2}{\omega_c}\left(\frac{\pi}{2k}-x_0\right). \tag{27}$$

From (27), the increase ΔE in the kinetic energy of the particle during trapping is given as

$$\frac{\Delta E}{m}=\frac{\Omega^4}{2\omega_c^2}\left(\frac{\pi}{2k}-x_0\right)^2+\frac{\Omega^2 v_{y0}}{\omega_c}\left(\frac{\pi}{2k}-x_0\right), \tag{28}$$

and it has maximum and minimum value when

$$x_0=\frac{\omega_c}{\Omega^2}v_{y0}-\frac{\pi}{2k} \tag{29}$$

and $x_0=\pi/2k$ respectively.

After the first detrapping, the particle gyrates with the velocity $v_{g0}=\sqrt{2\Delta E_0/m}=\pi\omega_b^2/2\omega_c k$, moving in the x-direction with the drift velocity v_c. If we put $v_{y0}=v_{g0}$ in (28) and (29), the average increase in the particle kinetic energy ΔE_1 during the second trapping is obtained from (28) as

$$\frac{\Delta E_1}{m}=\frac{3\pi^2\omega_b^4}{16k^2\omega_c^2}. \tag{30}$$

The particle velocity after the first detrapping is drawn in Fig.3. The particle has a possibility to be trapped again, if $1/2.mv_x^2<\Phi$, i.e., $v_x<\sqrt{2}\omega_b/k$, which is satisfied on QR and Q'R' in Fig.3. We omit Q'R', because v_y is negative on Q'R'. Since $v_c<<v_{g0}$, the probability P_1' that the particle is on QR is given by

$$P_1'=QR/2\pi v_{g0}=2\sqrt{2}\omega_c/\pi^2\omega_b. \tag{31}$$

In view of (29), 1/4 of the x-space corresponds to the allowable space for x_0. Accordingly, the probability P_1 that the particle is secondly trapped by the wave is

$$P_1=P_1'/4=\omega_c/\sqrt{2}\pi^2\omega_b. \tag{32}$$

Here we consider the identical N charged particles in the space as will be treated in the next section. If we neglect the interactions among particles, the sum U_k of kinetic energies of particles after t_{d0} increases with time as

$$U_k=N\Delta E_0+N\Delta E_1 P_1(t-t_{d0})=N\Delta E_0+\frac{3m\omega_b^3 N}{16\sqrt{2}k^2\omega_c}(t-t_{d0}). \tag{33}$$

Of course, the particle kinetic energies elevated to $\Delta E_0+\Delta E_1$ will be affected by the wave again. The kinetic energies increase up to $\Delta E_0+\Delta E_1+\Delta E_2$, where ΔE_2 is nearly equal to ΔE_1 in (30) except numerical factor. Thus, the particles successively affected by the wave form the high energy tail in the velocity distribution function. However, if x_0 in (28) is larger than $\pi/2k$, then the solution is meaningless. In this sense, the maximum gyration velocity of the particles which are not effected by the potential of the wave any more is $\pi\omega_b^2/\omega_c k$.

4. Numerical Simulations

In order to check the analysis in the preceding section, we carry out numerical simulations by using a particle-in-cell method. The simulation is one-dimensional in the space (x), and two-dimensional in the velocity (v_x, v_y). Initially, the distribution of electrons are Maxwellian with a low temperature. Ions are assumed to form a charge-neutralizing background and their motions are completely neglected. A constant magnetic field B in the z-direction and an electrostatic wave propagating in the negative x-direction are externally applied to the plasma. As a result of random motions of electrons, an elctric field $E_i(x)$ is automatically induced in the plasma. Thus the governing equations instead of eqs.(2) and (3) are

$$dv_x/dt=\omega_c v_y-\omega_b^2/k.\sin(kx+\omega t)+E_i/m, \tag{34}$$

$$dv_y/dt=-\omega_c v_x, \tag{35}$$

$$dE_i/dt=e(n_i-n_e), \tag{36}$$

where -e is the electron charge, n_i and n_e are the number densities of ions and electrons, respectively. A result of simulations is shown in Fig.4. The ordinate indicates the sum U_k of the kinetic energies of electrons.

References

1) G. R. Smith and A. N. Kaufman: Phys. Rev. Letters Vol.34, No.26 (1975) P1613.

2) A. Fukuyama, H. Momota and R. Itatani: Inst. of Plasma Phys. J., No.259 (1975).

3) A. Fukuyama, H. Momota, R. Itatani and T. Takizuka: Phys. Rev. Letters Vol.38, No.13 (1977) P701.

4) R. Sugihara and Y. Midzuno: Inst. of Plasma Phys. J., No.263 (1976).

5) R. Sugihara: Private communication.

6) C. S. Morawetz: Phys. Fluids Vol.4, No.8 (1961) P988.

7) K. Niu and R. J. Hung: J. Phys. Soc. Japan Vol.23, No.2 (1967) P436.

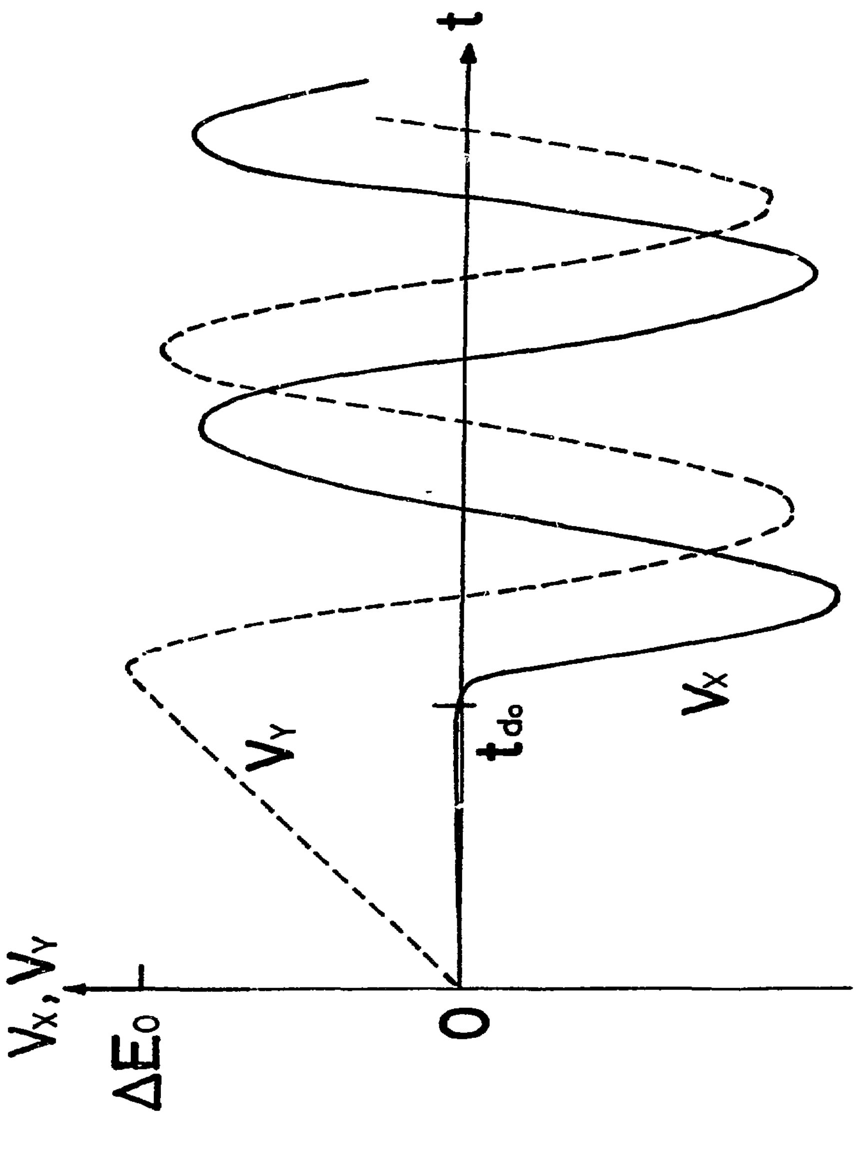
Figure Captions

Fig.1. The velocity component v_x(continuous line) and v_y (dotted line) of the particle versus time t.

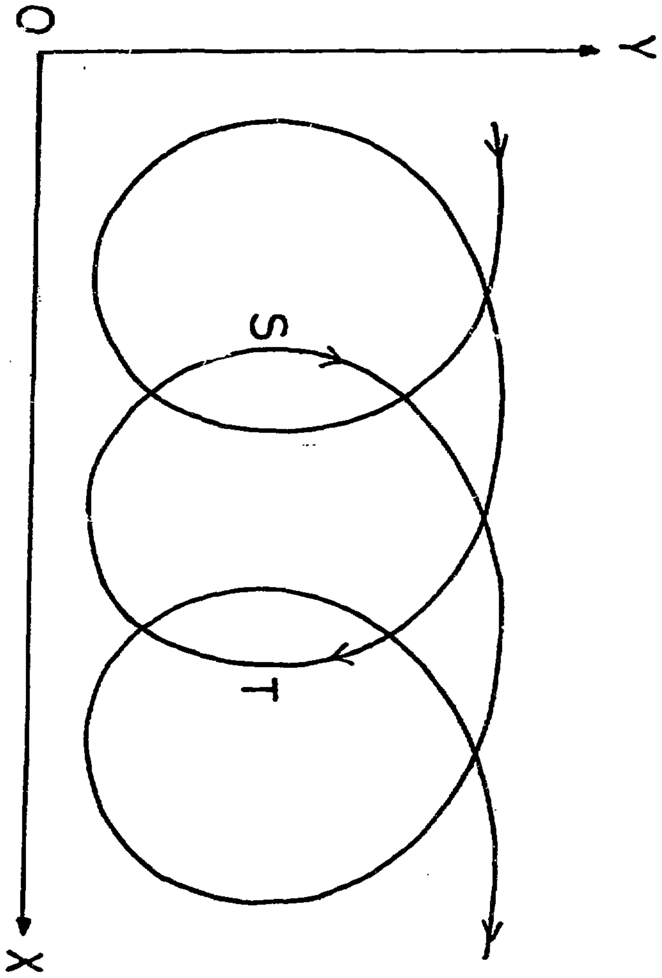
Fig.2. A cycloid curve along which the particle moves.

Fig.3. The particle velocity after the first detrapping.

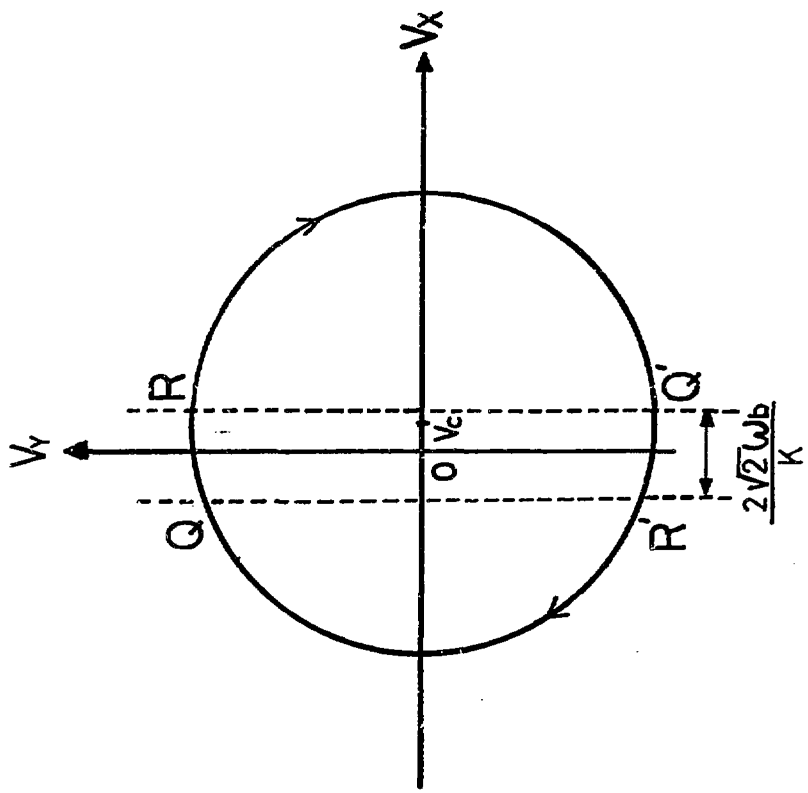
Fig.4. The sum U_k of kinetic energies of electrons versus time t. The continuous line is obtained by using eqs. (34)-(36), and the dotted line by eqs.(2) and (3). Parameters used here are; $k\lambda_D=\pi/8$, $\omega=0.6\omega_{pe}$, $\omega_c=0.5\omega_{pe}$ and $\omega_b=0.89\omega_{pe}$(where λ_D is the Debye length and ω_{pe} the electron plasma frequency).



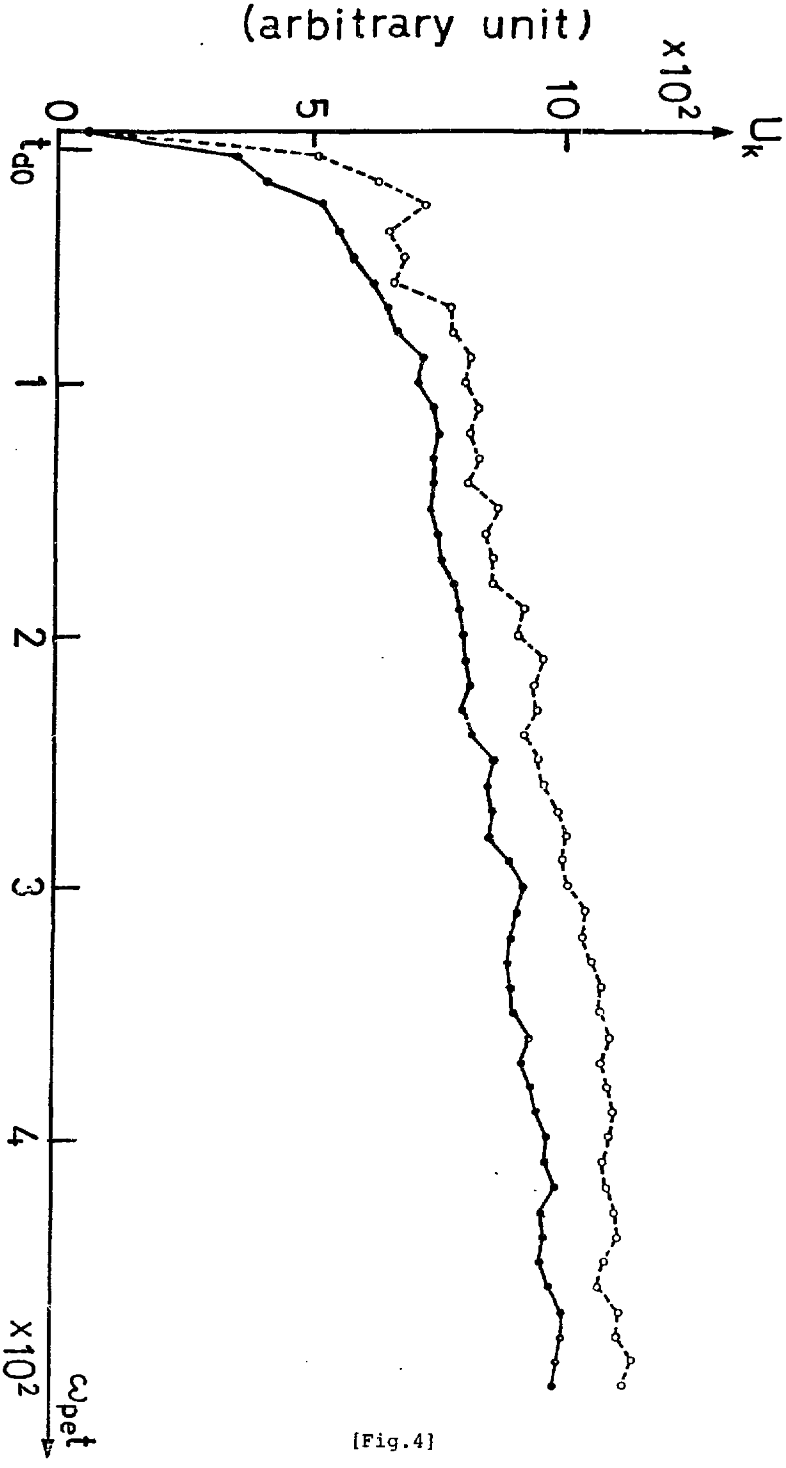
[Fig.1]



[Fig.2]



[Fig.3]



[Fig.4]