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PION POLARIZABILITY IN NONLOCAL QUARK MODEL

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Поляризуемость π-мезонов в нелокальной кварковой модели

Получена амплитуда процесса $\gamma\gamma\to\pi\pi$ в четвертом порядке по теории возмущений в нелокальной кварковой модели. Найдены численные значения коэффициентов поляризуемости π-мезонов: $a_{\pi^\pm}=+0.014\frac{a}{m_\pi^3}$, $a_{\pi^0}=-0.07\frac{a}{m_\pi^3}$. Проведено сравнение с результатами расчета в других моделях.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1978

Efimov G.V., Okhlopkova V.A. **E4 - 11568**

Pion Polarizability in Nonlocal Quark Model

The $\gamma\gamma\to\pi\pi$ amplitude was calculated in nonlocal quark model in the fourth order on perturbation theory. The coefficients of electric and magnetic polarizability [redacted] determined: $a_{\pi^\pm}=+0.014\frac{a}{m_\pi^3}$, $a_{\pi^0}=-0.07\frac{a}{m_\pi^3}$. The results have been compared with calculations in other models.

The investigation has been performed at the Laboratory of Theiretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1978

Introduction

The coefficients of electric and magnetic polarizability α and β are introduced to describe the effective interaction of particles with the spatial distribution of charge and external electromagnetic field at low energies:

$$V_{int} = -\frac{\alpha}{2}E^2 - \frac{\beta}{2}H^2. \qquad (1)$$

As has been pointed our recently [1,2], there is a principal possibility to measure the electric polarizability of hadrons α from the shift of energy levels in hadronic atoms. However, to distinguish the effects associated with hadron polarizability is a rather complicated problem, since there are some other reasons for shifting energy levels (the strong interaction of hadron with nucleus, finite dimension of nucleus, nuclear polarizability and so on). Therefore, it is quite difficult to calculate the corrections to levels. The accuracy of measurement of transitions between levels of hadronic atoms presently available gives no experimental bounds on the numerical values of α and β. The only exception is nucleous for which these constraints have been found. The theoretical values of α and β calculated within different models are not in good agreement with each other (see the Table) and, thus, the experimental determination of α and β would be of great importance to distinguish between the models.

One possibility of obtaining theoretical estimations of coefficients α and β for pions comes from the consideration of the $\gamma\gamma \to \pi\pi$ amplitude.

In paper [2] this amplitude was calculated in the low-energy approximation by using current algebra and PCAC hypothesis.

The coefficient $\alpha_{\pi^\pm}$ was determined in terms of parameters of decay $\pi \to e\nu\gamma$. The effective parameter was m_π / m_ρ and relative accuracy of the result was about 30%.

In paper /3/ the $\gamma\gamma \to \pi\pi$ amplitude was calculated in the quantum theory with chiral Lagrangian in the one-loop approximation and the contribution to the amplitude from pion and baryon loops was taken into account.

Coefficients α and β were found as functions of incident energy and were shown to considerably increase with increasing energy from zero to the two-pion production threshold. The effective parameter of this model is $1/F_\pi^2$, with F_π the pion decay constant.

Paper /4/ analysed $\gamma\gamma \to \pi\pi$ in the linear σ- model. Coefficient α was found as a function of energy and has a sharp maximum at the two-pion production threshold. The calculations were made in two first perturbation orders by assuming $q_1 q_2 \ll 2m_\pi^2$.

In paper /5/ dispersion sum rules were derived for coefficients α and β by using the hypothesis of S-channel helicity conservation (SCHC). Inaccuracy in determination of $\alpha_{\pi^\pm}$, α_{π^0} is caused by uncertainty in absorptive parameters, by the introduced cut-off in integral and by the large error in the ε-meson mass.

In paper /1/ $\alpha_{\pi^\pm}$ is estimated within the nonrelativistic quark model.

In the present paper $\gamma\gamma \to \pi\pi$ amplitude is calculated within the nonlocal quark model /6/. The model is based on the hypothesis that quark does not exist as a usual particle and is a quantum-field object which can exist in a virtual state only. The free field of quarks $q(x)$ is assumed to be identicallly zero but the Green function of field $q(x)$ is nontrivial, i.e., quarks in free state do not exist but are interaction carries between mesons and baryons described by scalar and spinor fields obeying the conventional Klein-Gordon and Dirac equations. Paper /6/ gives the explicit form of the Green function of the quark field as follows

$$G(\hat{p}) = \frac{1}{M} \exp\left[l\hat{p} + \frac{l^2}{4} p^2 \right] \tag{2}$$

Table

	$\alpha_{\pi^{\pm}}\left(\frac{\alpha}{m_\pi^3}\right)$	$\alpha_{\pi^{0}}\left(\frac{\alpha}{m_\pi^3}\right)$
/2/ Terent'ev	0.16	0
/3/ Pervushin, Volkov	0.31	−0.04
/4/ Gal'perin, Kalinovsky	0.3	−0.06
/5/ L'vov, Petrun'kin	0.25 ± 0.11	0.055 ± 0.11
/1/ Degtev	0.1	
nonlocal quark model	0.014	− 0.07

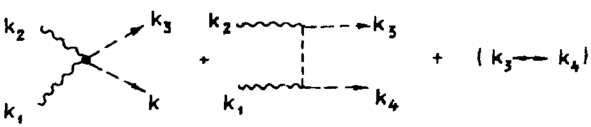

Fig.1

with ℓ, L inner parameters of the model, $M = \frac{1}{L+\ell}$, $\hat{P} = i\gamma_\mu \partial_\mu$, quantizes the field $q(x)$ and constructs the finite unitary S-matrix.

Interaction Lagrangian and Amplitude of Process $\gamma\gamma \to \pi\pi$

The basic requirements for choosing Lagrangians of the quark interaction with physical fields are symmetry with respect to certain transformation groups (gauge transformations, SU(3) group, and so on) and simplicity for Lagrangian (the absence of derivatives of higher than first order). In paper /7/ the quark interaction with an electromagnetic field is examined. Based on the results of papers /6,7/ , we take the interaction Lagrangian of quarks, pions and photons (which is gauge and SU(3)-invariant) in the following form:

$$\mathcal{L}_I(x) = -ie\left[\varphi^* \partial_\mu \varphi - \partial_\mu \varphi^* \varphi\right] A_\mu + e^2 A_\mu A_\mu \varphi^* \varphi + h^2 \Delta m \varphi^* \varphi + + ih M^{SK} (\bar{q}^K_a \gamma_5 q^S_a) + \sum_{z=1}^{3} e_z J^z_\mu A_\mu , \qquad (3)$$

where $M^{SK} = \begin{vmatrix} \pi^0/\sqrt{2} & \varphi \\ \varphi^* & -\pi^0/\sqrt{2} \end{vmatrix}$, $\varphi = \{\varphi_1, \varphi_2, \varphi_3\}$, $\varphi_3 = \varphi_3^* = \pi_0$, $\varphi_2 = -\varphi_1^* = \varphi$,

h is the coupling constant for quark and pion interaction , e_z is the charge of a r-th quark, $e_1 = \frac{1}{3}e$, $e_2 = e_3 = -\frac{2}{3}e$, a is colour index, J^z_μ is the quark current. Diagrams contributing to the process $\gamma\gamma \to \pi\pi$ in the four first perturbation orders are presented in Figs.1-4. The effective parameter of expansion in the nonlocal quark model is $\lambda_h = \frac{h^2}{(4\pi)^2 (ML)^2}$. The consideration of vector meson decays /7/ has revealed that the best agreement with experiment is achieved at L =3.12 GeV^{-1} and $\lambda_h = h^2/(4\pi)^2 (LM)^2 = 0.13$. The amplitude of $\gamma\gamma \to \pi\pi$ is expressed only through the terms of zero and second order in particle momenta. Let us write out the contribution to the amplitude from different diagrams . The part of the amplitude which corresponds to the diagrams shown in Fig.1 has nothing to do with quarks and is just the Born amplitude

$$\langle \pi^+\pi^- | S_B^{(1)} + S_B^{(2)} | \gamma\gamma \rangle = A^{\lambda_1 \lambda_2}_{\mu\nu} \cdot e^2 \left[g_{\mu\nu} - \frac{k_{4\mu} k_{3\nu}}{k_1 k_3} - \frac{k_{4\nu} k_{3\mu}}{k_1 k_4} \right] \qquad (4)$$

A set of diagrams drawn in Fig.2 corresponds to the mass renormalisation of pion by strong interactions, and the corresponding part of the amplitude is of the from

$$\langle \pi^+\pi^- | S^{(2)} + S^{(3)} + S_1^{(4)} | \gamma\gamma \rangle = A_{\mu\nu}^{\lambda_1\lambda_2} \cdot \frac{3ih^2e^2}{(2\pi)^4} \Big\{ R \Big[-\frac{g_{\mu\nu}}{2} + \frac{k_{4\mu}k_{3\nu}}{k_1k_3} + \frac{k_{4\nu}k_{3\mu}}{k_1k_4} \Big] + \frac{d}{6}\big[k_{4\mu}k_{3\nu} + k_{4\nu}k_{3\mu} \big] \Big\} . \tag{5}$$

The part of the amplitude oorresponding to the diagrams of Fig.3 is

$$\langle \pi^+\pi^- | S_2^{(4)} | \gamma\gamma \rangle = \frac{-3ih^2e^2}{(2\pi)^4} A_{\mu\nu}^{\lambda_1\lambda_2} \Big\{ \frac{d}{3}\big[k_{4\mu}k_{3\nu} + k_{4\nu}k_{3\mu} \big] - R\Big[\frac{k_{4\mu}k_{3\nu}}{k_1k_3} + \frac{k_{4\nu}k_{3\mu}}{k_1k_4} \Big] \Big\} \tag{6}$$

And finally, the tetragonal diagrams in Fig.4 give the following contribution to the total amplitude:

$$\langle \pi^+\pi^- | S_3^{(4)} | \gamma\gamma \rangle = \frac{3ih^2e^2}{(2\pi)^4} A_{\mu\nu}^{\lambda_1\lambda_2} \Big\{ \frac{R}{2} g_{\mu\nu} + \frac{N}{18}\big[g_{\mu\nu}(k_1k_2) + k_{1\mu}k_{2\nu} \big] + \frac{d}{6}\big[k_{4\nu}k_{3\mu} + k_{4\mu}k_{3\nu} \big] \Big\} \tag{7}$$

$$\langle \pi^0\pi^0 | S_3^{(4)} | \gamma\gamma \rangle = - A_{\mu\nu}^{\lambda_1\lambda_2} \cdot \frac{5e^2h^2}{18\pi^2} \big[g_{\mu\nu}(k_1k_2) - k_{1\mu}k_{2\nu} \big] \tag{8}$$

Through (4) to (8) we used the notation

$$A_{\mu\nu}^{\lambda_1\lambda_2} = \frac{i\,\delta^{(4)}(k_1+k_2-k_3-k_4)}{(2\pi)^2\, 4\sqrt{\omega_1\omega_2 k_{03}k_{04}}}\, \varepsilon_\nu^{\lambda_1}\varepsilon_\mu^{\lambda_2}$$

k_1, k_2 are the photon momenta; $\varepsilon_\nu^{\lambda_1}$, $\varepsilon_\mu^{\lambda_2}$ are the photon polarisations; k_3, k_4 are the pion momenta. Also, we took into aocount the equalities: $k_1^2 = k_2^2 = 0$, $k_3^2 = k_4^2 = m_\pi^2$, $(k_1\varepsilon_1) = (k_2\varepsilon_2) = 0$,

$$R = 4i\pi^2 \int_0^\infty du \Big\{ u^2(\theta_1')^2 + u^3(\theta_2')^2 + \frac{\mu_\pi^2}{3}\big[u^3(\theta_1'')^2 + u^4(\theta_2'')^2 \big] \Big\},$$

$$d = \frac{16i\pi^2}{M^2}\int_0^\infty du \Big\{ u^3(\theta_1'')^2 + u^4(\theta_2'')^2 \Big\}, \qquad \mu_\pi^2 = \frac{m_\pi^2 L^2}{4} ,$$

$$N = \frac{16i\pi^2}{M^2}\int_0^\infty du \Big\{ \frac{9}{2}u^2(\theta_2')^2 - \frac{3}{2}u^4(\theta_2'')^2 + \frac{10}{3}\theta_1\theta_1' + 9u(\theta_1')^2 - \frac{3}{2}u^3(\theta_1'')^2 \Big\}. \tag{9}$$

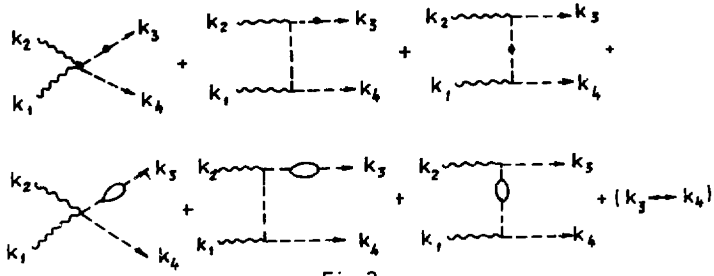

Fig.2

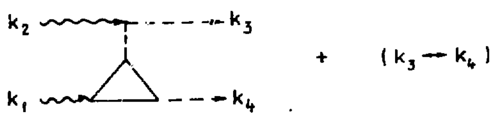

Fig.3

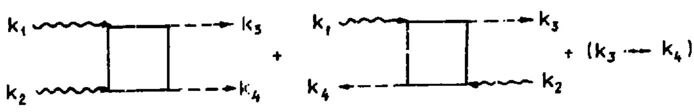

Fig.4

The quark Green function is represented in the form

$$G(\hat{q}) = G_1(-q^2) + \hat{q}\, G_2(-q^2)\,,$$

$$G_1(-q^2) = \frac{1}{M} \exp\left[\frac{l^2 q^2}{4}\right] \cos l\sqrt{-q^2}\,; \quad G_2(-q^2) = \frac{1}{M} e^{\frac{l^2 q^2}{4}} \cdot \frac{\sin l\sqrt{-q^2}}{\sqrt{-q^2}}\,.$$

In the course of calculations $G_i(a^2)$ are expanded into series and only terms of order not higher than fourth in external momenta are retained.

Summing up contributions shown in Figs.1 to 4 we find that the terms including constants R and d cancel and the amplitude of $\gamma\gamma \to \pi\pi$ takes the following form

$$\langle \pi^a(k_3)\pi^b(k_4)|S|\gamma^{\lambda_1}(k_1)\gamma^{\lambda_2}(k_2)\rangle = A^{\lambda_1\lambda_2}_{\mu\nu} \cdot T^{\lambda_1\lambda_2}_{ab}\left(k_3, k_4 \,|\, k_1, k_2\right),$$

$$T^{\lambda_1\lambda_2}_{ab} = e^2 \Big\{ (\delta_{ab} - \delta_{3a}\delta_{3b}) \Big[g_{\mu\nu} - \frac{k_{4\mu}k_{3\nu}}{k_1 k_3} - \frac{k_{4\nu}k_{3\mu}}{k_1 k_4} + \qquad (10)$$

$$+ \frac{i h^2 N \cdot 3}{(2\pi)^4 \cdot 18}\left(-g_{\mu\nu}(k_1k_2) + k_{1\mu}k_{2\nu}\right)\Big] + \delta_{3a}\delta_{3b}\, \frac{5 h^2}{18\pi^2}\left[g_{\mu\nu}(k_1k_2) + k_{1\mu}k_{2\nu}\right] \Big\}$$

with a, b 1,2,3 isotopic indices.

Coefficients of Electric and Magnetic Polarizabilities of Pions

The coefficients for the gauge-invariant combination

$$g_{\mu\nu}(k_1k_2) - k_{1\mu}k_{2\nu} \qquad (11)$$

in formula (10) define the electric and magnetic polarizabilities for pions. Since our approximation contains only one gauge-invariant structure (11) (see /3/ , App.2), the coefficients α and β are equal in magnitude and opposite in sign. With notation

$$-C_1 = 3\frac{i h^2 e^2 N}{(2\pi)^4 \cdot 18} = \alpha\, \frac{2 N i \lambda_L}{3\pi}\,, \quad C_2 = -\frac{5 h^2 e^2}{18\pi^2} = -\alpha\, \frac{40\pi\lambda_L}{9},\; \alpha = \frac{e^2}{4\pi}\,,$$

we get

$$\alpha_{\pi^\pm} = -\beta_{\pi^\pm} = \frac{C_1}{2m_\pi}\,, \quad \alpha_{\pi^0} = -\beta_{\pi^0} = \frac{C_2}{2m_\pi}\,.$$

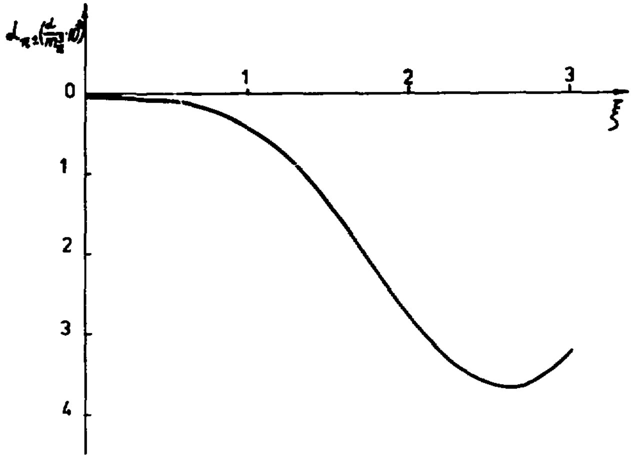

Fig. 5

The quantity N given by (9) is a function of parameter $\xi = 2\ell/\lambda$. In paper /8/ ξ was found to change in the region $0.3 \leq \xi \leq 3$ and λ_h was fixed to equal $\lambda_h = 0.13$. Basing on these results, we obtain that $\alpha_{\pi\pm}$ as a function of ξ changes from $0.89 \cdot 10^{-3}\ \alpha/m_\pi^3$ to $33.9 \cdot 10^{-3}\ \alpha/m_\pi^3$ (see Fig.5). In paper /7/ the best agreement with experimental data on vector meson decays was achieved at $\xi = 1.4$, i.e., $\alpha_{\pi\pm} = 14.5 \cdot 10^{-3}\ \alpha/m_\pi^3$. The coefficient α_{π^0} does not depend on ξ and equals $\alpha_{\pi^0} = -0.07\ \alpha/m_\pi^3$.

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