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# INTRODUCTION TO HYDRODYNAMICS


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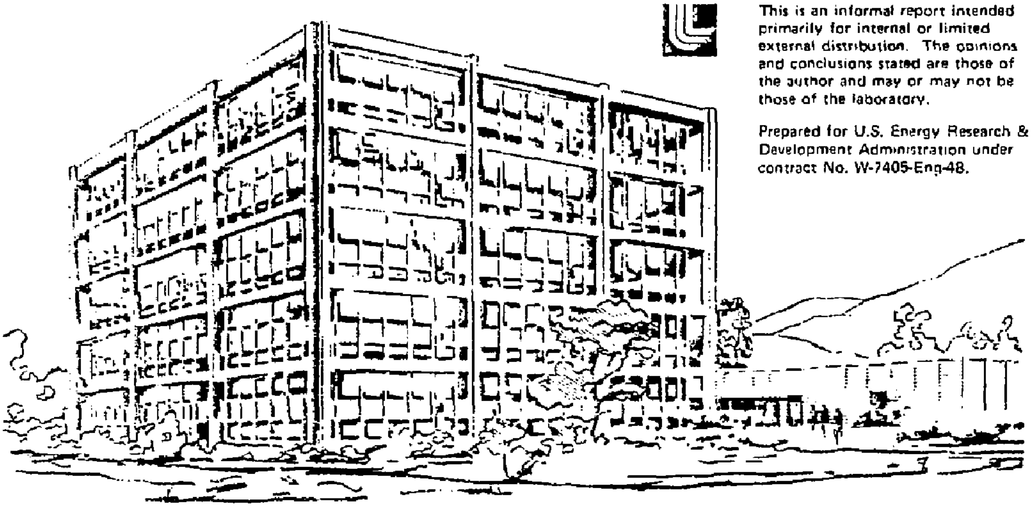




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# INTRODUCTION TO HYDRODYNAMICS

## ABSTRACT

This report is an introduction to various aspects of hydrodynamics and elastic-plastic flow. Its chief purpose is to define hydrodynamic terms and explain what some of the important hydrodynamic concepts are. The first part of the report covers hydrodynamic theory; it discusses fundamental hydrodynamic equations, discontinuities, and shock, detonation, and elastic-plastic waves. The second part of the report deals with applications of hydrodynamic theory to material equations of state, spall, Taylor instabilities, and detonation pressure measurements.

# I. HYDRODYNAMIC THEORY

## A. The Medium

The term fluid is used here in the ordinary sense, i.e., a material medium that is continuously deformable, has very little cohesion between its different particles, and may be compressible or incompressible. The material may take totally or partially the shape of its container. The concept of a continuum is also implied. This means that pressure, temperature, and density vary continuously from point to point. Hydrodynamics is the study of such a system.

The hydrodynamic approach to a given problem therefore assumes that the thermodynamic variables have a definite value no matter where one attempts to measure them. The whole system doesn't have to be in equilibrium, but any arbitrarily small region within the system must be in equilibrium. The space considered can't be too small, such as of the order of a molecular mean free path, because pressure and temperature, for example, lose their macroscopic meaning. Therefore hydrodynamics does not apply if local equilibrium does not exist, i.e., the thermodynamic variables are changing over dimensions of the order of a molecular mean free path. Examples are strong shock fronts and the reaction zones of detonations. For such cases a kinetic theoretical approach to the problem must be taken and the fluid variables replaced by molecular distribution functions.

However, the fact that hydrodynamics does not apply in the interior of detonation and shock wave fronts is not a limitation on the studies under consideration here. Because these zones are very small they can be replaced mathematically by a discontinuous surface on either side of which the macroscopic model of hydrodynamics is again valid. For example, the reaction zones of explosives are typically $\sim 0.1$ cm thick. Shock widths are of the order of a molecular mean free path which, for example, in the case of air is about $10^{-4}$ cm. The dimensions of the physical problems to be considered here are many times larger.

Except when the motion is discontinuous, heat conduction and viscosity in the medium will be neglected.

Hydrodynamics as described above applies to a liquid or a gas. Many of the results also apply to solid media, metals for example.

B. Fundamental Equations

1. Equation of Motion: As was stated above, viscosity and heat conduction are neglected as are all exterior forces.

The fundamental equation of mechanics (i.e. Newton's second law) applied to a fluid element leads to Euler's equation:

$$\rho \frac{d\vec{U}}{dt} = -\nabla P \tag{1}$$

$\rho$ = density

$t$ = time

$\vec{U}$ = velocity vector

$P$ = pressure

This equation expresses the fact that the momentum of a fluid element can only be changed by the pressures this element receives from its neighbors.

2. Co tinuity Equation: For flow that does not have any sources or sinks, the principle of conservation of mass applied to the fluid during the motion is expressed by:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{U}) = 0 \qquad (2)$$

3. Energy Equation: The neglect of viscosity and heat conduction is equivalent to assuming that the internal energy of a fluid element can only be changed by the work done by the pressure of neighboring elements.

$$dE/dt + P\, dV/dt = 0 \qquad (3)$$

E = internal energy per unit mass

V = specific volume = $1/\rho$

From the first law of thermodynamics $dE + PdV = Tds$ where

T = temperature

s = entropy of unit mass

Thus equation (3) expresses the fact that the entropy of a fluid element does not change. The assumption therefore is that the change in state of each fluid element is adiabatic and reversible.

4. Equation of State: At each instant and each point of the fluid there is a state of thermodynamic equilibrium defined by:

P - pressure

E - internal energy per unit mass

$\rho$ - density

s - entropy per unit mass

T - temperature

From thermodynamics it is known that only two of these parameters are independent.

It is advantageous to choose E and $\rho$ as the two independent variables. The equation of state then is the equation that relates P to E and .

$$P = P(\rho,E) \tag{4}$$

As an example we will consider the equation of state for an ideal gas.

(i) $P = R\rho T$

where the gas constant R is related to the specific heat at constant volume $C_v$ and the specific heat at constant pressure $C_p$ by:

(ii) $R = C_p - C_v$

For an ideal gas the internal energy E is a function of the temperature T above.

If in particular this function is:

(iii) $E = C_v T$

the gas is called polytropic.

It is customary to designate the ratio of the specific heats by $\gamma$.

(iv) $$\gamma = C_p/C_V$$

The equation of state (i) can now be rewritten as

$$P = (\gamma-1)\rho E$$

or

(v) $$P = (\gamma-1)E/V \text{ where } V = 1/\rho$$

It turns out that the equation of state given by (v) applies to a large number of real situations. For example with $\gamma = 3$ equation (v) describes to a good approximation the detonation product gases of a high explosive. When equation (v) is substituted into equation (3) and the resultant expression is integrated the result is the familiar formula:

(vi) $$PV^{\gamma} = \text{constant}$$

C. Solutions to the Fundamental Equations

Equations 1-4 describe the behavior of a hydrodynamic system, (except at discontinuities which will be discussed later). They are nonlinear partial differential equations and can be solved in closed form for only a limited number of special cases. The fundamental work on these equations was done over a hundred years ago by G. Monge, B. deSaint-Venant, Lord Rayleigh, G. Stokes and H. Hugoniot and others.

Except for the work of a few ingenious people in the field of mechanics, this area of hydrodynamics has been dormant. This is probably due to a reluctance of researchers to work in a field where the fundamental equations cannot be solved. Technological developments in recent years have established a strong requirement for understanding nonlinear wave motion. The field of hydrodynamics is receiving much greater interest especially now that large high speed computers can solve the fundamental equations.

D. Propagations of Discontinuities

1. Sound Speed: If a perturbation is introduced at a point in a fluid it will propagate and modify the physical and kinetic characteristics of the fluid. This perturbation will describe a wave front or wave surface that at any time t will separate the disturbed fluid, 1, from the undisturbed fluid, 0. As an example, consider a fluid initially at rest in a tube. At one end a piston is moved into the fluid. The fluid molecules adjacent to the piston will be put into motion while those further away will still be at rest. Thus two distinct states have been created separated by a wave front which moves in the fluid. If the piston has been put into motion without a discontinuity in its velocity the wave produced will travel at the speed of sound with respect to the fluid. The velocity U of the fluid particles as well as the pressure P, density $\rho$ and specific internal energy E will be continuous functions of the position coordinate X. Only the space derivatives of these parameters will have a discontinuity at the wave front.

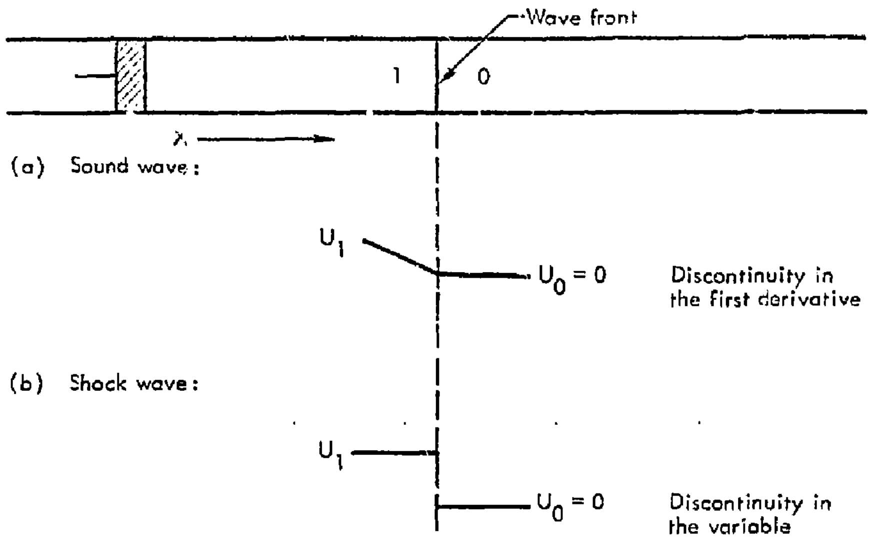


Figure 1

Piston Moving into a Fluid at Rest

In the next section it is shown that if there is a discontinuity in the space derivative of the velocity (case (a) $\frac{\partial U}{\partial X} \neq 0$) then there will be concurrent discontinuities in the derivatives of the other parameters also. The discontinuity will travel in the fluid at a velocity equal to: $U \pm \sqrt{dP/d\rho}$. The quantity $\sqrt{dP/d\rho}$ is called the sound speed.

2. <u>Speed of Discontinuity Propagation</u>. For motion in only one space direction X the hydrodynamic parameters are a function of X and t.

In this case since $\frac{d(\ )}{dt} = \frac{\partial(\ )}{\partial t}\cdot\frac{dt}{dt} + \frac{\partial(\ )}{\partial X}\cdot\frac{dX}{dt}$ and $\frac{dX}{dt} = U$ the hydrodynamic equations become:

Conservation of momentum

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + \frac{1}{\rho}\frac{\partial P}{\partial X} = 0 \qquad (5)$$

Conservation of mass

$$\frac{\partial \rho}{\partial t} + U\frac{\partial \rho}{\partial X} + \rho\frac{\partial U}{\partial X} = 0 \qquad (6)$$

Equation of state and the isentropic assumption.

$$P = F(\rho) \qquad (7)$$

If the fluid is separated into two regions at the wave front, (see Fig. 1), each of the states corresponding to $(U_0, P_0, \rho_0)$ and $(U_1, P_1, \rho_1)$ must satisfy equations 5, 6 and 7.

At the wave front the following difference parameters are defined:

$$L = U_1 - U_0 = 0$$

$$M = \rho_1 - \rho_0 = 0$$

$$N = P_1 - P_0 = 0$$

it is assumed that at least one space derivative of L, M or N is not equal to zero.

Calling S the speed of the wave, the condition that L, M and N are always zero on S is described by:

$$\frac{\partial Z}{\partial t} + S\,\frac{\partial Z}{\partial X} = 0 \tag{8}$$

where Z is L, M or N.

If the equations (5) and (6) are written for the system where U, P and ρ are $U_1$, $P_1$ and $\rho_1$, and again where U, P and ρ are $U_0$, $P_0$ and $\rho_0$ and then a term by term subtraction is made the result is:

$$\begin{aligned} &\frac{\partial L}{\partial t} + U\,\frac{\partial L}{\partial X} + \frac{1}{\rho}\,\frac{\partial N}{\partial X} = 0 \\ &\frac{\partial M}{\partial t} + U\,\frac{\partial M}{\partial X} + \rho\,\frac{\partial L}{\partial X} = 0 \end{aligned} \tag{9}$$

Equations (9) apply at the wave front where $U = U_1 = U_0$; $P = P_1 = P_0$ and $\rho = \rho_1 = \rho_0$.

Starting with equation (7), $P = F(\rho)$, it follows that:

$$\frac{dP}{dt} = \frac{dF}{d\rho}\,\frac{d\rho}{dt}$$

$$\frac{\partial P}{\partial t} + U\,\frac{\partial P}{\partial X} = \frac{dF}{d\rho}\left(\frac{\partial \rho}{\partial t} + U\,\frac{\partial \rho}{\partial X}\right)$$

A term by term subtraction similar to the above gives:

$$\frac{\partial N}{\partial t} + U\frac{\partial N}{\partial X} = \frac{dF}{d\rho}\left(\frac{\partial M}{\partial t} + U\frac{\partial M}{\partial X}\right) \tag{10}$$

If the time derivatives of equations (9) and (10) are eliminated by use of equations (8) the following system of equations results:

$$\begin{aligned} \frac{1}{\rho}\frac{\partial N}{\partial X} + (U-S)\frac{\partial L}{\partial X} &= 0 \\ \rho\frac{\partial L}{\partial X} + (U-S)\frac{\partial M}{\partial X} &= 0 \\ (U-S)\frac{\partial N}{\partial X} - (U-S)\frac{\partial M}{\partial X}\frac{dF}{d\rho} &= 0 \end{aligned} \tag{11}$$

Equations (11) are three linear equations in $\partial L/\partial X$, $\partial M/\partial X$ and $\partial N/\partial X$ that can't all be zero since the original hypothesis was that at least one discontinuity existed.

Equations (11) can be satisfied in two ways:

(a) $S-U = 0$ This is not really a wave since the disturbance is traveling at the fluid velocity.

(b) None of the partial derivatives is zero. This is possible only if:

$$(S-U)^2 = \frac{dF}{d\rho}$$

$$S = U \pm\sqrt{\left.\frac{dP}{d\rho}\right|_S}$$

Thus it is seen that (1) if there is a discontinuity in the derivative of one parameter there is a discontinuity in the derivatives of all the parameters; and (2) the discontinuity travels in the fluid at the velocity of sound. The sign $\pm$ indicates that the propagation is in either direction.

3. <u>Characteristics.</u> Once again equations (1) and (2) are applied to motion in one space dimension X.

Conservation of momentum

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + \frac{1}{\rho}\frac{\partial P}{\partial X} = 0 \tag{12}$$

Conservation of mass

$$\frac{\partial \rho}{\partial t} + U\frac{\partial \rho}{\partial X} + \rho\frac{\partial U}{\partial X} = 0$$

Equations (3) and (4) will be specified through the sound speed C.

$$C^2 = \left.\frac{dP}{d\rho}\right|_S \tag{13}$$

At this point the Riemann parameter $\sigma$ is defined

$$\sigma = \int C\,\frac{d\rho}{\rho}$$

The derivatives of ρ and P can be restated in terms of

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial t} = \frac{\rho}{C} \cdot \frac{\partial \sigma}{\partial t}$$

$$\frac{\partial \rho}{\partial X} = \frac{\rho}{C} \frac{\partial \sigma}{\partial X} \qquad (14)$$

$$\frac{\partial P}{\partial X} = \rho C \frac{\partial \sigma}{\partial X}$$

Substitution of equations (14) into equations (12) yields:

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + C \frac{\partial \sigma}{\partial X} = 0$$

$$(15)$$

$$\frac{\partial \sigma}{\partial t} + U \frac{\partial \sigma}{\partial X} + C \frac{\partial U}{\partial X} = 0$$

Equations (15) can be rewritten as:

$$\frac{\partial (U-\sigma)}{\partial t} + (U-C) \frac{\partial (U-\sigma)}{\partial X} = 0$$

$$(16)$$

$$\frac{\partial (U+\sigma)}{\partial t} + (U+C) \frac{\partial (U+\sigma)}{\partial X} = 0$$

Now for a variable Z that is a function of two parameters C and t the total derivative is:

$$\frac{dZ}{dt} = \frac{\partial Z}{\partial t}\frac{dt}{dt} + \frac{\partial Z}{\partial X}\frac{dX}{dt} \tag{17}$$

If $(U-\sigma)$ and $(U+\sigma)$ are used in place of the parameter Z equation (17) becomes:

$$\begin{aligned} \frac{d(U-\sigma)}{dt} &= \frac{\partial(U-\sigma)}{\partial t} + \frac{\partial(U-\sigma)}{\partial X}\frac{dX}{dt} \\ \frac{d(U+\sigma)}{dt} &= \frac{\partial(U+\sigma)}{\partial t} + \frac{\partial(U+\sigma)}{\partial X}\frac{dX}{dt} \end{aligned} \tag{18}$$

Comparison of equations (18) and equations (16) shows us that along curves where: $dX/dt = U \pm C$ the quantity $U \pm \sigma$ = constant. Curves with this property are called <u>characteristics</u>. From the results of the preceding section it is seen that discontinuities propagate along characteristic curves. Until now no distinction has been made in the choice of motion of the piston, i.e., whether it moves into the fluid and transmits a compression wave or moves away from the fluid and transmits a decompression wave. The discontinuities induced follow characteristic curves as discussed for either motion. However, the subsequant results are very much different. In general, the sound speed is an increasing function of P (or of $\rho$). Thus in a simple compression pulse, (see Fig. 2), the high pressure portion of the pulse will travel faster than the low pressure portion. The compression wave becomes progressively steeper until it eventually approaches a discontinuity. The wave front is now called a <u>shock wave</u> and the parameters U, P, $\rho$, and E become discontinuous in space across the wave front.

If, on the other hand, the fluid is already under pressure and a pressure decreasing action is made by moving the piston away from the fluid a rarefaction is introduced into the fluid. In this case, since the high pressure portions of the signal can travel faster than the low pressure portion the wave will tend to flatten with time.

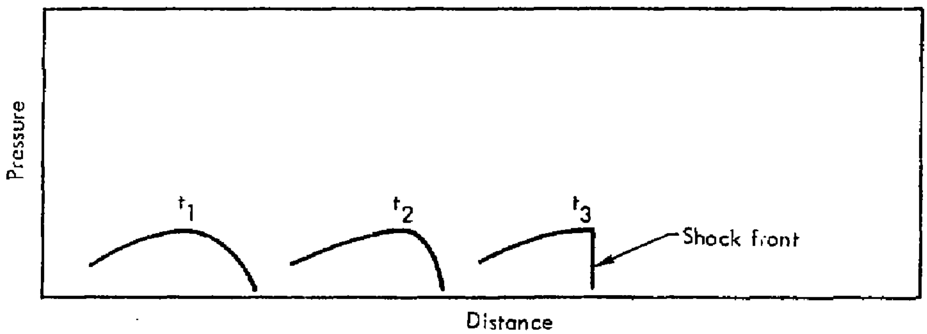


Figure 2

Shock forming at time $t_3$ from a compression wave at time $t_1$. The compression portion steepens while the expansion portion flattens.

E. Shock Waves

In the previous section it was seen that differential equations of hydrodynamics allowed discontinuities in the derivatives of the variables to propagate. It was shown that these discontinuities propagated along characteristic curves. However, the non-linear hydrodynamic equations break down completely when there is a discontinuity in the parameters themselves (shock wave). This situation occurs from the gradual steepening of a compression wave front. A shock wave may also be initiated by suddenly giving the piston of Fig. 1 a velocity. The fluid in contact with it will jump discontinuously from a velocity zero to a velocity not zero, Fig. 1b.

In the physical shock phenomenon there are irreversible thermodynamic processes caused by friction and heat conduction taking place in the shock region. The neglect of viscosity and heat conduction in the mathematical formulation of the problem is the cause of the difficulties encountered when a shock forms. However, the mathematics becomes overwhelmingly complicated when these effects are included.

Fortunately, the real shock phenomenon usually takes place over a very narrow region, as was discussed earlier. Outside of this region the fluid flow obeys the isentropic formulation given here. The smallness of this region suggests its replacement by a surface across which pressure, density, and velocity change in a discontinuous manner. The values of $P$, $\rho$ and $U$ on the two sides of the shock must of course obey the laws of conservation of mass, momentum and energy. In this way the effect of viscosity and heat conduction, necessary to describe the real irreversible process, can be incorporated without actually specifying them. The conservation equations connecting the shocked and unshocked fluids were first given by Hugoniot. The three equations may be written as: (see UCRL 6797 for details)

$$\left.\begin{aligned} &\text{(i)} \qquad S = \frac{(U_1-U_0)V_0}{V_0-V_1} \\ &\text{(ii)} \qquad P-P_0 = \rho_0 S(U_1-U_0) \\ &\text{(iii)} \qquad E_1-E_0 = \frac{1}{2}(P_0+P_1)(V_0-V_1) \end{aligned}\right\} \text{Hugoniot equations} \qquad (19)$$

Equations (i) and (ii) can be combined to give the very useful relations:

$$\text{(iv)} \qquad S^2 = V_0^2\left(\frac{P_1-P_0}{V_0-V_1}\right) \qquad \text{where } V = \frac{1}{\rho}$$

$$\text{(v)} \qquad (U_1-U_0)^2 = (P_1-P_0)\ (V_0-V_1)$$

Here S is the shock velocity and $V = 1/\rho$; the other parameters are defined as before. The subscript 0 refers to the state ahead of the shock and the subscript 1 refers to the state behind the shock.

The first two of equations (19) give the relation between the dynamic and thermodynamic variables. The third equation is a relation between the thermodynamic quantities alone. For a given equation of state of the form $P = P(V,E)$ the energy E can in principle be eliminated with the third Hugoniot equation above. The result will be a curve in the P-V plane, which is the locus of all P, V states that can be attained by a shock from a given initial state $P_0$, $V_0$. This is called a Hugoniot curve. It can be shown (UCRL 6797) that at $P_0$, $V_0$ the Hugoniot curve makes a second order contact with the adiabat through $P_0$, $V_0$. The change in entropy across a shock increases with increasing shock strength, but the entropy increase is only of 3rd order compared to the shock strength. Shock strength can be measured by a change in the pressure, $(P_1-P_0)$, or particle velocity, $(U_1-U_0)$, of a shocked medium. Given the shock strength and the initial conditions of a medium all of the other quantities describing the medium in its shocked condition can be readily calculated from the medium equation of state.

For strong shocks in solids initially at atmospheric pressure it is usual to set $P_0 = 0$ since $P_1$ is so much larger. Figure 3 shows the Hugoniot and the isentrope for a typical solid material with initial conditions $P_0$, $V_0$.

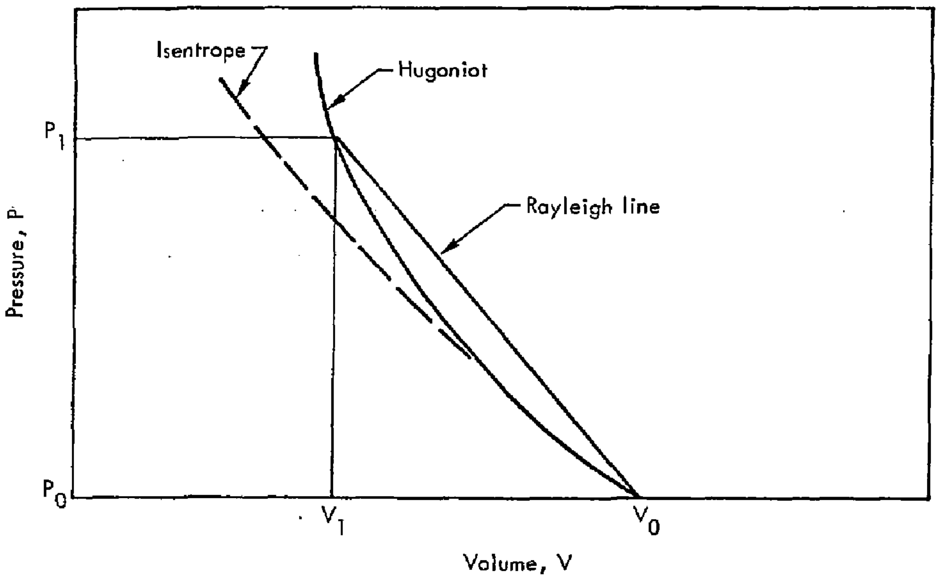


Figure 3

Typical Hugoniot and Adiabat for a Solid Material

The fourth of equations (19) describes a straight line of slope $(\rho_0 S)^2$ in the P-V plane. This line is called the Rayleigh line and represents the locus of all permissible P, V, states consistent with a particular shock velocity S. The intersection of this line with the Hugoniot curve gives the P, V point consistent with the Hugoniot curve. This is an example of the fact that only one shock parameter is necessary to determine the other parameters when the equation of state and initial conditions are known.

The isentrope sound speed as defined earlier is $C^2 = dP/d\rho$. This can be written as:

(i) $$C^2 = -V^2 \frac{dP}{dV}$$

or

(ii) $$\rho^0 C^2 = -\frac{dP}{dV} \qquad (20)$$

where

$$V = 1/\rho$$

The Rayleigh line has the same form as (ii) above except that the differential becomes a finite difference

$$(\rho^0 S)^2 = \frac{P_1 - P_0}{V_0 - V_1} \qquad (21)$$

It follows that

$$U + C > S > C_0 \qquad (22)$$

This inequality expresses the fact that shocks are supersonic with respect to the material ahead and subsonic with respect to the material behind.

From the third equation (19) it is seen that the change in internal energy across a shock is the area under the Rayleigh line, Fig. 3. From the fifth equation (19) it is seen that the change in kinetic energy per unit mass, $1/2\ U^2$, also equals this area. Here we have considered the initial state to be $P_0 = 0$ and $U_0 = 0$.

F. Detonation Waves

In the preceding section it was seen that a shock is a dissipative process and that furthermore a rarefaction from behind the shock wave will always overtake it since signals travel at U+C and U+C > S. This means that shocks will ultimately die out unless energy is continuously supplied from behind the shock wave. If, however, the passage of the wave involves a release of chemical energy in the medium the wave propagation can be self-sustaining. Such a wave does exist in a high explosive and is called a detonation wave.

The calculation of a detonation wave differs in two principle ways from a shock wave.

1. The Hugoniot conditions across the wave front still apply but it is necessary to supply the chemical energy released at the front.
2. The wave propagation is not controlled by conditions behind the front as was true in the example of the piston considered earlier.

The three Hugoniot equations (19) are not sufficient to determine the four unknowns P, ρ, U and D. (Here D is the detonation velocity and replaces S in equations (19)). A supplementary condition is given by the assumption that at the detonation front a small disturbance travels at the same speed D as the front itself. This is called the Chapman-Jouguet (C-J) hypothesis and is stated mathematically as:

$$D = U+C \tag{23}$$

It has not been possible to supply a rigorous demonstration of this hypothesis, but it does give results verifiable by experiments and has been the basis of much fruitful work on high explosives.

In reality there is a reaction zone at the detonation wave front where an irreversible decomposition of the explosive takes place. Thermodynamic equilibrium is assumed to exist immediately behind the reaction zone and if there are any further chemical reactions occurring in this region they will not affect the detonation velocity. It is in the region immediately behind the reaction zone that the C-J hypothesis is applied.

G. Elastic-Plastic Waves

In contrast to fluids, solids resist shear distortion and as a result the equations of motion and the thermodynamic description applied to solids are much more complicated. It can be argued that at stress levels greatly in excess of the shear strength the stress system is effectively isotropic and equivalent to a hydrostatic pressure. With this assumption the hydrodynamic analysis discussed here can be applied to solids. However, in recent years it has been found that the presence of a small shear stress component has a large effect on the manner by which a pressure wave attenuates. (see UCPL 7322)

Furthermore, it has been observed experimentally that the shear strength of some solids increases with increasing pressure. These facts have led to the development of an elastic-plastic model instead of a fluid model to describe the behavior of solids even at high pressures.

Some of the outstanding features of an elastic-plastic material can be demonstrated by considering a one-dimensional compression wave. The stress, $\sigma$, is considered to be composed of a hydrostatic pressure, P, and a distortion stress, s. The hydrostatic pressure can be thought of in the same sense as a fluid pressure and described by an equation of state. The distortion stress, following elasticity theory, is considered to be a linear function of strain. There is an upper limit to the magnitude of the distortion stress and this limit is stated by a yield condition. After the yield point has been attained the material deforms plastically for any additional loading. A material is said to be elastic when the stress is proportional to strain and plastic when the stress is no longer proportional to strain.

For the one-dimensional strain considered here the stress $\sigma$ is given by:

$$\sigma = P+s$$

where $$P = k\left(\frac{V^0-V}{V}\right)$$

$$s = \frac{4}{3}\ \mu\left(\frac{V^0-V}{V}\right)$$

$k$ = bulk modulus

$\mu$ = shear modulus

$V$ = specific volume

These equations apply until $s$ reaches a maximum value, $s = (2/3)Y$, where $Y$ is the yield strength in simple tension. For all subsequent compression the material is taken to deform plastically with $s$ remaining equal to its maximum value and $P$ increasing. The parameter $k$ is initially constant but then increases with increasing pressure. In the P-V plane the pressure curve will be concave upward similar to fluids. Thus the sound speed increases with pressure and shocks can form.

The Hugoniot equations (19) still apply where now the pressure $P$ is replaced by the total stress $\sigma$.

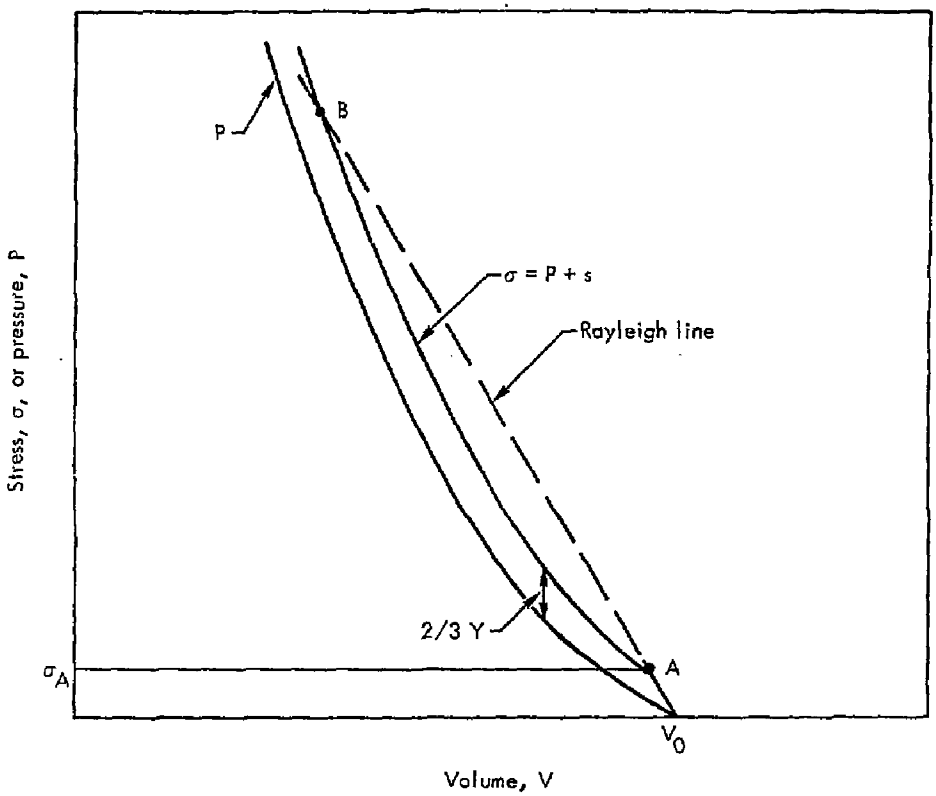


Figure 4

One-dimensional stress-strain for an elastic-plastic material

In Figure 4, point A is where the distortion stress component, s, has reached its maximum value and is referred to as the Hugoniot elastic limit. The discontinuous decrease in slope at point A will cause the stress wave to break into two steps for stress levels that are between points A and B. An elastic precursor of stress level $\sigma_A$ will travel at the elastic velocity $C_E$.

$$C_E = \left[\frac{k + \frac{4}{3}\mu}{\rho^0}\right]^{1/2}$$

This stress will be followed by a plastic wave traveling at the shock velocity, $S_p$, given by applying equation *IV) of equations (19).

$$S_p^2 = V_A^2 \left(\frac{\sigma-\sigma_A}{V_A-V}\right)$$

where $\sigma_A < \sigma < \sigma_B$

For $\sigma > \sigma_B$ the plastic wave velocity, $S_p$, will be greater than the elastic precursor velocity, $C_E$, and the stress will propagate as a single shock. This follows from noting that the slope of the Rayleigh line is greater than the slope OA for stress points above B.

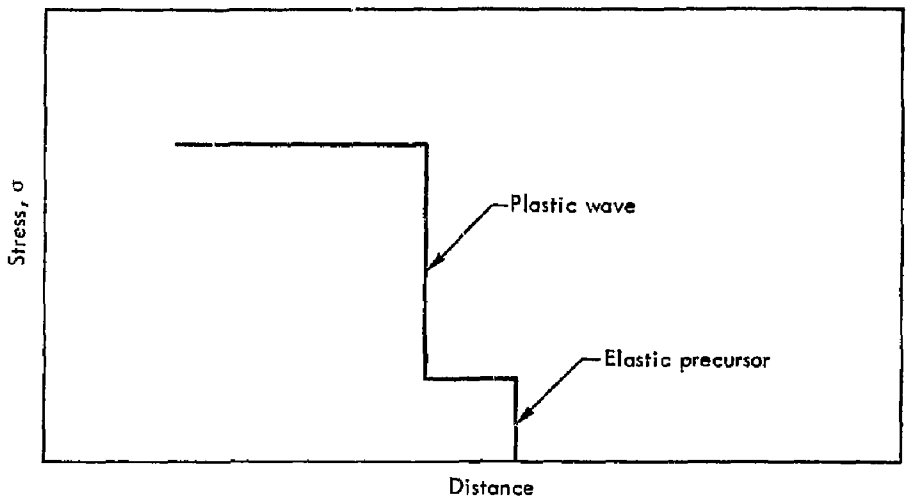


Figure 5

Space profile of an elastic-plastic stress wave

## H. Units and Orders of Magnitude

Pressure is a force per unit area. It is convenient to use a system of units that fits in with the c.g.s System. The pressure unit is the kilobar (kb) = $10^3$ bars or the megabar (Mb) = $10^6$ bars.

Here 1 bar = $10^6$ dynes/cm$^2$

1 atmosphere = 76 cm of mercury = $1.012 \times 10^6$ dynes/cm$^2$

Hence one bar is approximately equal to one atmosphere.

A consistent set of units is:

| | |
|---|---|
| Pressure | - Mb |
| Distance | - cm |
| Time | - μs = $10^{-6}$ seconds |
| Velocity | - cm/μs |
| Density | - gms/$cm^3$ |
| Energy | - $10^{12}$ ergs/gm |

A common source of high pressures in experimental hydrodynamics work is high explosives. Typical explosive detonation velocities are 0.8 cm/μs and detonation pressures are ∿ 0.3 Mb. A 0.3 Mb detonation pressure can induce pressures between 0.1 and 0.6 Mb in a material placed in contact with the high explosive, depending on the equation of state of the material. By accelerating a metal plate with a high explosive and allowing the plate to strike a target plate, pressures up to 2 Mb may be attained.

# II. APPLICATIONS OF HYDRODYNAMIC THEORY

## A. Equation of State

1. Experimental Methods. The greatest uncertainty in applying hydrodynamic theory to physical situations lies in the equation of state of the materials. Shock wave theory, however, provides an experimental method to obtain information on the equations of state.

As discussed earlier a shock propagating into a material having a known state is completely specified by any two of the variables P, ρ, U, E and S and the three Hugoniot relations (19). The variables S and U are the easiest to obtain experimentally and one of three methods may be employed.

a) When a free-flying plate is allowed to strike a target plate the interface will acquire a new velocity and the pressure on either side will be the same from the principle of action and reaction. If the two materials are the same, the interface velocity will be one-half the free-flying plate velocity.

Electric probes at fixed positions can measure the times of arrival of the flying plate, and the shock wave in the target plate. From the known space separation of the probes the velocity of the flying plate and the shock velocity in the target can be determined. The probes or "pins" are simple switches that close when a disturbance reaches them, usually by metallic contact. By repeating these measurements for different flying plate velocities the P-V curve of the material can be determined.

b) When a shock wave reaches a free surface the subsequent velocity of the free surface will be the result of the contributions of (1) the shock particle velocity U associated with the change of state from $P_0$ to P, $\rho_0$ to $\rho$ etc., and (2) the isentropic velocity when a rarefaction proceeds back into the material decompressing the material and reducing the pressure from P to the boundary pressure at the free surface P=0. It can be shown that these two velocities are nearly equal for shocks where $\rho/\rho_0$ is less than about 1.4. The front surface velocity of the target material can be measured by pins; one-half this velocity will then be the required particle velocity. The shock velocity can be measured as before.

c) Once a complete P-U curve for one material is known it can be used with a material whose P-U relation is not known and only the shock speed S need be measured for the new material to determine its complete state. Consider a high explosive in contact with material A whose P-U relation is known; next to this material is placed material B whose P-U relation is to be determined. A shock from the high explosive will traverse material A and enter material B. A shock will always enter material B, but the wave reflected into material A at the interface may be:

(1) A reflected shock if material B has the greater shock impedance.

(2) A reflected rarefaction if material B has the smaller shock impedance.

(3) Neither a reflected shock nor a reflected rarefaction. In this case materials A and B have the same shock impedance.

Figure 6 shows the known P-U curve for material A. One-half the measured front surface velocity of a free surface of material A will give the particle velocity $U_1$ and pressure $P_1$ that is present just before the shock reaches the interface of materials A and B. It is assumed that the reflected wave into material A still follows the second Hugoniot quation (19) i.e. $\Delta P = \rho^0 S \Delta U$. For case (1) the pressure increases with a decrease in particle velocity, while for the case (2) the pressure decreases with an increase in the particle velocity. Graphically these states are given by reflecting a mirror image of the P-U curve of material A about the point $P_1$, $U_1$.

Now, if the reference density $\rho_0$ of material B is known and the transmitted shock speed $S_T$ has been measured then $P/U = \rho_0 S_T$ is the locus of all points that satisfy the second Hugoniot equation (19). The intersection of this line with the reflected P-U curve A' is the desired $P_T$, $U_T$ state in material B. This is called the impedance match method.

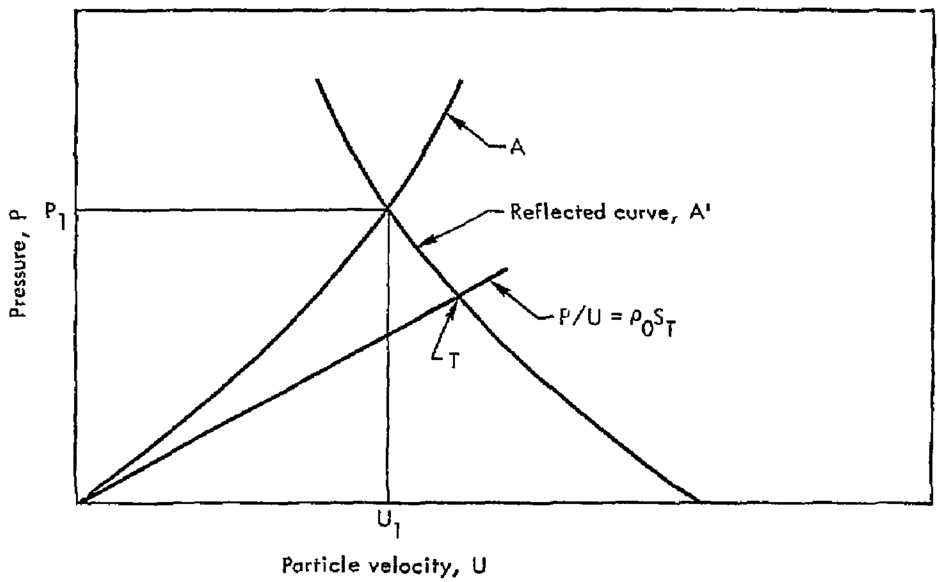


Figure 6
Pressure-particle velocity curves to illustrate the impedance match method to obtain equation of state data.

2. Form of the Equation of State for Solids. Many equations of state are expressed in terms of pressure, volume and temperature. The use of temperature as a variable requires data on the specific heat so that an expression for energy may be obtained for the shock wave equations (19). Since hydrodynamic applications do not require temperature explicitly, an equation of state relating P, V and E is much more preferable. A form that has been very successful for describing metals at high pressures is the Mie-Gruneisen equation of state.

$$P = P_L + P_T \tag{24}$$

where $$P_L = -\frac{dE_L}{dV} \quad \text{and} \quad P_T = \frac{\gamma_g}{V}(E-E_L)$$

The interpretation is that the total pressure P is the sum of the lattice pressure $P_L$ due to the lattice potential energy $E_L$ at absolute zero and the thermal pressure $P_T$ due to the lattice vibrational energy $E-E_L$. Here $\gamma_g$ is called the Gruneisen ratio and is assumed to be a function of volume only.

The thermal energy $E_T$ due to a shock is shown schematically in Fig. 7 as the striped area. The corresponding energy $E_L$ due to the compressibility at absolute zero is also shown. The total energy change due to a shock is the sum of these two energies.

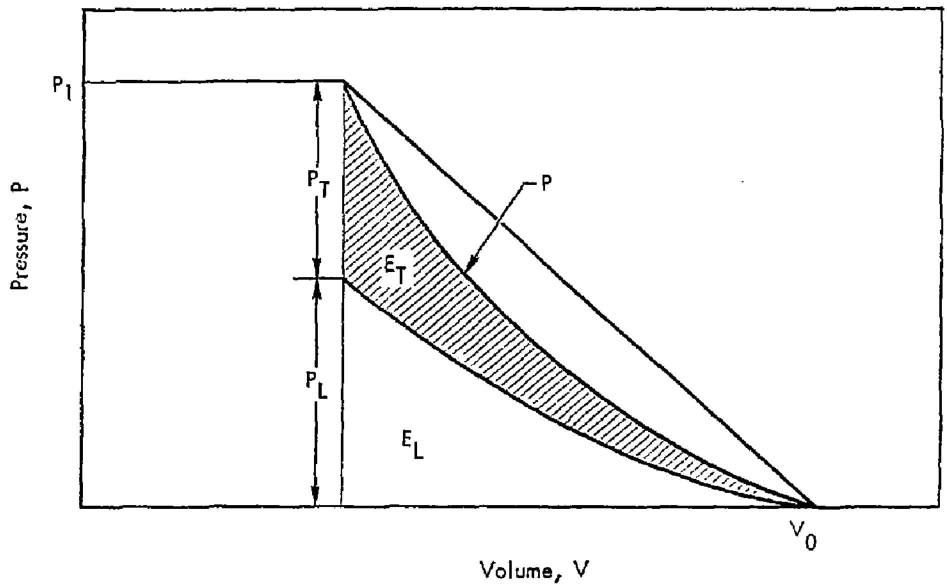


Figure 7

Shock Compressibility

The behavior of a shocked material can be seen by substituting the third Hugoniot equation (19) into equation of state (24) where $P_0 = 0$, $E_0 = 0$.

$$P = \frac{P_L - \frac{\gamma_0}{V} E_L}{1 - \frac{\gamma_0}{2}\left[\frac{V_0}{V} - 1\right]} \tag{25}$$

Equation (25) gives the locus of all P-V states reached by a single shock where the initial state is $P_0$, $V_0$. It is seen that the pressure P becomes infinite when the relative compression $\eta = \frac{V_0}{V} = 1 + \frac{2}{\gamma_0}$. For metals the limiting compression is $\sim 2$, implying $\gamma_0 \cong 2$.

Experimental P, V data along the Hugoniot curve may be used with equation (25) and $P_L = -\frac{dE_L}{dV}$ of equation (24) to derive consistent values of the functions $E_L(V)$, $P(V)$ and $\gamma_g(V)$. (No distinction is made here between the absolute zero isotherm and the room temperature isotherm.) When this is done equation (19) will describe all P, V, E states of the material. It must be realized that the shock wave data gives P and V and from this a P, V, E relation is developed, i.e. a line has been used to generate a surface. For this reason it can be expected that the experimental equation of state will be valid only in regions near the Hugoniot curve.

At pressures above 100 mb the electronic shells of atoms are crushed and lose their individual structure. The Thomas-Fermi-Dirac (TFD) statistical model of the atom can be used to describe the compressibility in this region. The experimentally derived, $E_L$, $P_L$ and $\gamma_g$ relations can therefore be extrapolated from the experimentally determined portion, to the TFD zero temperature isotherms. This is a rather long extrapolation since experimental data ends at 2 Mb and the TFD data begins at 100 Mb.

B. Spall

One of the difficulties encountered in attempting to accelerate solid materials by high explosives is the dynamic fracture phenomenon, spall. When a shock pressure wave reaches a free surface a rarefaction is generated that reduces the pressure to zero. Since rarefactions always overtake shocks, the shock wave is being attenuated by a rarefaction from behind the shock wave before the shock wave reaches the free surface. The crossing of the two rarefaction waves causes a drop in pressure from two sources and the material is sent into tension (see Fig. 8). If the tension falls below the dynamic strength of the material, the material will spall. The strain rates associated with stresses induced by high explosives are of the order of $10^6$/sec. It has been found experimentally that materials under this type of loading can sustain tensions of the order of 10 times their static strength before fracturing.

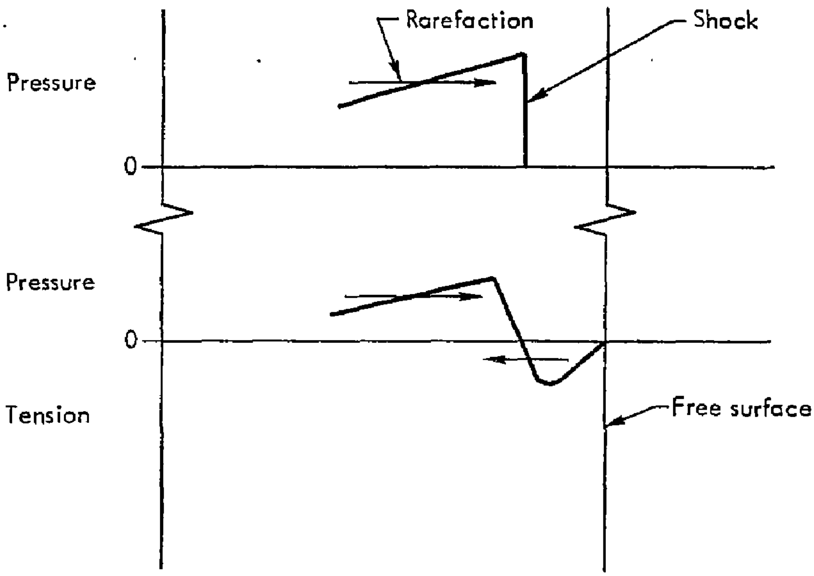


Figure 8

Reflection from a free surface of a shock wave followed by a rarefaction.

C. Taylor Instability

For two fluids that have a common boundary, any perturbation on the interface will tend to grow when an acceleration is directed from the less dense to the more dense medium. This effect is known as Taylor instability. The rate of growth of a given perturbation increases with increasing acceleration. Calculations show that the rate of growth is decreased and can be completely eliminated by the shear strength of the material being accelerated.

A situation where Taylor instability can show up is when a high explosive gas accelerates a metal plate. Experiments with explosively driven plates show that for low strength materials, perturbations grow in a manner consistent with theoretical predictions and for high strength materials they do not grow.

Figure 9 is a calculation that shows the growth of a sinusoidal perturbation on the upper surface of a compressible copper plate where a uniform pressure of 200 Kb is applied. The original amplitude of the perturbation is $10^{-3}$ cm. In this problem the copper is considered to behave like a fluid as is the usual assumption for explosively driven plate problems.

Figure 10 is a calculation with the same initial conditions, but here the copper is assumed to have a material strength Y = 10 Kb. Here Y is the yield strength as would be determined by a simple tension test and where the Von Mises condition for plastic flow is satisfied.

Figure 11 shows a calculation similar to Figure 10 except that the yield strength Y = 20 Kb. Analysis of the results showed that the perturbation grew only to a small extent and that further growth was prevented by the 20 Kb material strength. If the driving pressure were made greater the material resistance would be overcome and a growth similar to Figure 10 would occur.

These calculations illustrate the important role that material strength plays in Taylor instability. It is known that material strength increases with loading pressure. It can therefore be assumed that strength greater than the static values are operating when plates are accelerated by high explosives. In general the actual material strength as a function of pressure is not reliably known. Hence, at the present time, it is not known what pressure levels may be used to accelerate a given material and still avoid Taylor instability. It is known, however, that to minimize the effect, smooth surfaces are required.

D. Detonation Pressure Measurement

A charge of high explosives detonated in contact with a metal witness plate will transmit a shock wave into the plate. If the Hugoniot curve for the metal plate is already known, a measurement of the plate free surface velocity will determine the pressure in the metal. Repeating the experiment for metals with different known Hugoniot curves will determine additional pressures. These pressures lie on the reflected Hugoniot of the detonation products of the high explosive; i.e.; they represent states reached by the detonation products where the initial state was the detonation pressure $P_d$, detonation density $\rho_d$, and detonation particle velocity $U_d$. A measurement of the detonation velocity and the original high explosive density and the second Hugoniot equation (19), $P/U = \rho_0 D$, determine a line in the P-U plane. The intersection of the line with the curve traced out by the experiments with different metals gives the detonation parameters $P_d$ and $U_d$ (see Fig. 12).

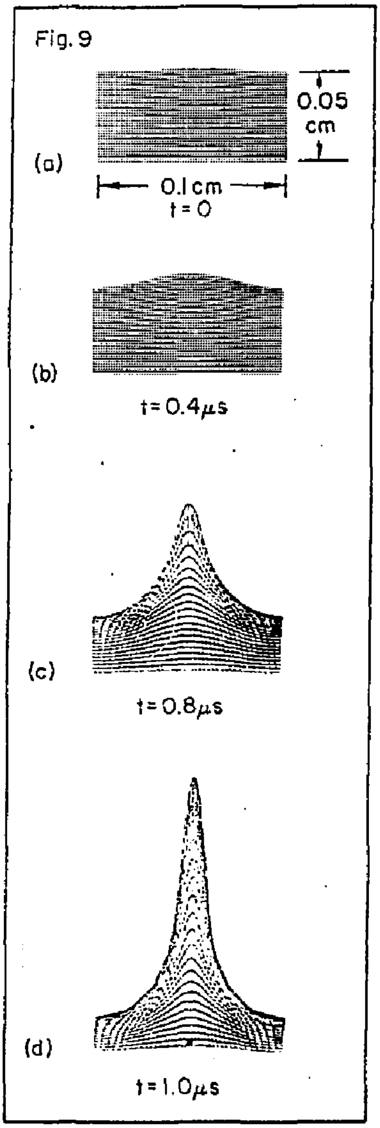


FIG. 9. Time Sequence of the growth of Taylor instability of a copper plate. Initial conditions: sine wave of amplitude $10^{-3}$ cm on the top surface and a uniform 200-kb pressure applied. No yield strength.

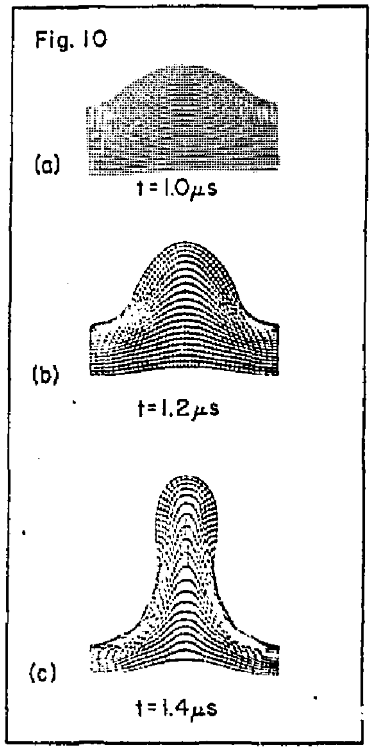


FIG. 10. Initial conditions same as Fig. 9, but yield strength Y = 10 kb.

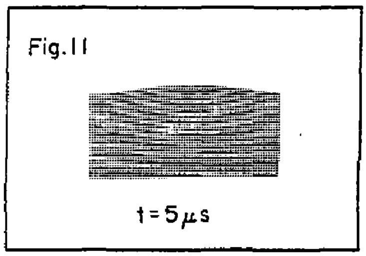


FIG. 11. Conditions same as Fig. 10, but yield strength Y = 20 kb. Here the instability started to develop and was stopped by the material strength.

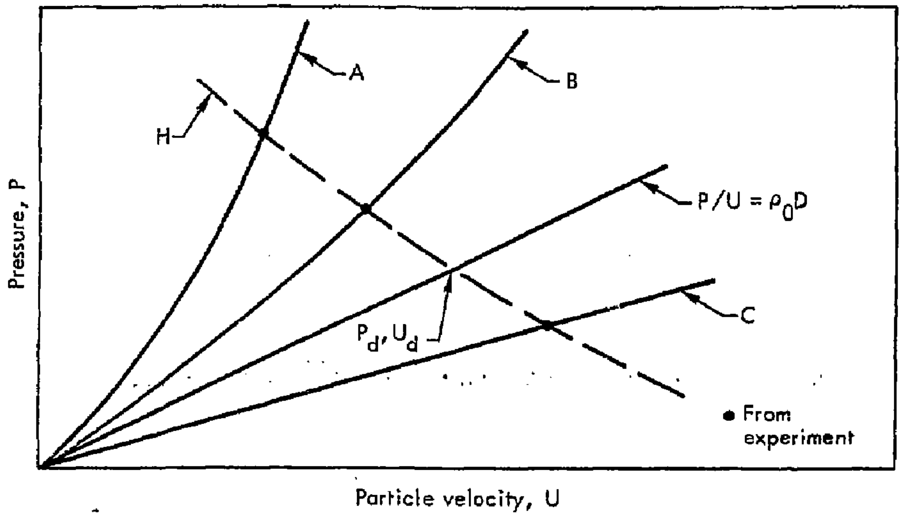


Figure 12

A, B and C are known curves. Curve H is the locus of P, U states where the initial state is $P_d$, $U_d$.

In measuring the detonation pressure care must be taken that the experiment is done in one-dimensional geometry. Also, it is the pressure at essentially zero plate thickness that is required experimentally so a suitable extrapolation must be made from the finite plate thicknesses used (see UCRL 14531).

This series of experiments provides a means of checking the Chapman-Jouguet hypothesis. The second Hugoniot equation (19) can be used to describe the shocked states of the detonation product gases where the initial state is the detonation point, $P_d$, $U_d$ and $\rho_d$.

$$P-P_d = \rho_d S(U-U_d) \tag{26}$$

or

$$\frac{P-P_d}{U-U_d} = \rho_d S$$

In the limit of shock states very close to the detonation state, $P_d$, $U_d$, equation (26) leads to:

$$\frac{dP}{dU} = -\rho_d C_d \tag{27}$$

Here the shock speed S has been replaced by the sound speed at the detonation state $C_d$. Equation (27) gives the slope of curve H at point $P_d$, $U_d$ (Fig. 12).

The C-J hypothesis states:

$$D = U_d + C_d \tag{28}$$

The first Hugoniot equation (19) is used to express conservation of mass across the detonation front.

$$\frac{D-U_d}{D} = \frac{\rho_0}{\rho_d} \tag{29}$$

Combining equations (28) and (29):

$$C_d \rho_d = \rho_0 D \tag{30}$$

Therefore, according to C-J theory equation (27) becomes:

$$\frac{dP}{dU} = -\rho_0 D \qquad (31)$$

When the slope of curve H is measured (Fig. 12), for solid explosives it is in fact equal to $-\rho_0 D$, thus supporting the C-J hypothesis.

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