ABSTRACT: The propagation of a pressure wave in fluid filled tubes is significantly affected by the pipe wall motion and vice versa. A computer code TMOC (Transients by the Method of Characteristics) is being developed for the analysis of the coupled fluid and pipe wall transients. Because of the structural feedback the pressure can be calculated more accurately than in the programs commonly used.

1. INTRODUCTION

Several authors [1-6] have investigated the effect of structural flexibility on the propagation of pressure pulses in fluid filled pipes. In the classical water-hammer theory [1] the flexibility of the pipe is taken into account by modifying the wave speed as

\[ c_m = \frac{c}{\left(\frac{2\rho \varepsilon}{3E}\right)^{0.65}} \]

where \( \rho \) and \( c \) are the density and the sonic velocity of water, \( E \) is the tube material's Young's modulus and \( R \) and \( e \) are the tube's radius and thickness. This theory has been further improved by Youngdahl and Kot [2] who assumed that the pipe deformation could also occur in the plastic region.

In the axial direction the wave propagation phenomena occur in the fluid and in the tube wall. The latter phenomenon is called a precursor wave because its wave speed is generally greater than the
sonic velocity of the fluid. The precursor wave produces a radial contraction or expansion through the Poisson effect and thus changes the fluid pressure. Furthermore, if the pipe can move, the moving pipe ends have a strong effect on the fluid pressure [3]. Williams [5] has found experimentally that the pipe motion causes mechanical damping of the pressure wave that cannot be ignored.

2. DESCRIPTION OF THE TMOC MODEL

2.1. One-Dimensional Flow Model

To describe the tube motion phenomena in the TMOC program the following equations for axisymmetric elastic deformations are used:

\[ \frac{\partial \sigma}{\partial t} - \frac{\partial \sigma}{\partial z} = 0 \]  \hspace{1cm} (2)

\[ \frac{\partial \sigma}{\partial t} - E^* \frac{\partial v}{\partial z} - \frac{v E^* w}{R} = 0 \]  \hspace{1cm} (3)

\[ \frac{\partial \tau}{\partial t} - E^* v \frac{\partial v}{\partial z} - \frac{E^* w}{R} = 0 \]  \hspace{1cm} (4)

\[ \frac{\partial \omega}{\partial t} - \frac{p}{\rho_p} + \frac{\sigma_\theta}{\rho_p R} = 0 \]  \hspace{1cm} (5)

Here \( v \) and \( w \) are the tube's axial and radial velocities, \( \sigma \) and \( \sigma_\theta \) are the tube's axial and circumferential stresses, \( v \) is Poisson's ratio, \( E^* = E/(1-v^2) \) where \( E \) is Young's modulus of elasticity and \( e, R \) and \( \rho_p \) are the thickness, mean radius and density of the tube material.

Similar equations were used also by Walker and Phillips [4] who investigated short (duration < 1 ms) pressure pulses.
For the hydraulic model of TMOC it is assumed that
- liquid and steam flow at equal velocities \( u \),
- steam is always saturated, whereas water can be either subcooled or superheated, and
- the flow area \( A \) is a function of time.

With these assumptions the conservation equations for the mass, momentum and energy of the mixture and the equation for the mass of the steam are

\[
\frac{\partial \rho A}{\partial t} + \frac{\partial \rho A u}{\partial z} = 0 \tag{6}
\]

\[
\frac{\partial \rho A u}{\partial t} + \frac{\partial \rho A u^2}{\partial z} + A \frac{\partial p}{\partial z} + AF + A p g \sin \theta = 0 \tag{7}
\]

\[
\frac{\partial}{\partial t} \left[ \left( \rho h - \rho A u^2 \right) A \right] + \rho A \frac{\partial A}{\partial t} + \frac{\partial}{\partial z} \left[ \left( \rho h + \frac{1}{2} \rho u^2 \right) A u \right] - QA - A p g u \sin \theta = 0 \tag{8}
\]

\[
\frac{\partial \rho p A}{\partial t} + \frac{\partial \rho A u}{\partial z} - AM = 0 \tag{9}
\]

Above \( \rho \) is the density of the liquid, \( p \) is the pressure, \( F \) is the friction, \( M \) is the evaporation rate, \( \theta \) is the angle between the pipe and the horizontal plane, \( \alpha \) is the steam volume fraction and

\[
\rho = \rho_\alpha + \rho_1 (1 - \alpha) \tag{10}
\]

\[
\rho h = \rho_\alpha h_\alpha + \rho_1 (1 - \alpha) h_1 \tag{11}
\]

\[
\Delta h_1 = h_1 - h_{1\text{sat}} \tag{12}
\]

Here \( h \) is enthalpy and \( h_{\text{sat}} \) the enthalpy of saturated liquid.

After some tedious manipulation Eqs. (2)-(6) can be written in the form

\[
\frac{\partial u}{\partial t} + B \frac{\partial u}{\partial z} + C(u) = 0 , \tag{13}
\]

where \( U = [u, p, \alpha, \Delta h_1, v, \sigma, w, \theta]^T \).
The derivatives of the flow area $A$ can be written in terms of the pipe deformation. It can be shown that the axial derivatives of $A$ are small and so the only interaction with the wall occurs via the radial tube motion and the boundary conditions.

In the computer program the set of equations is solved by the method of characteristics. The matrix $B$ has eight eigenvalues: $\pm c_p, u \pm c, u$ (twice), and 0 (twice).

$c_p$ is the velocity of the precursor wave which is the same as the velocity of sound in the tube material. The corresponding equations along each characteristic are given in the Appendix. In the computer program the user can specify the characteristics that are used. Hence, transients can be calculated by 2 to 8 equations depending on the physical situation to be analyzed.

2.2. Boundary Conditions and Flashing

The computational system of TMOC is a piping network where pipes are connected together by suitable boundary conditions. The boundaries are treated explicitly except an open pipe end where critical flow may occur. At a closed pipe end fluid and tube transients are coupled together by the condition \( \text{fluid velocity} = \text{velocity of the pipe end} \).

Critical Flow

The critical flow is calculated from

\[
\frac{\partial u}{\partial t} = (1 - \frac{u^2}{c^2}) \frac{P_n - P_{\text{ext}}}{\Delta z}
\]  

(14)

Here $\Delta z$ is the axial space differential, $P_{\text{ext}}$ is the outside pressure and $P_n$ is the pressure at the exit. Eq. (14) is taken from the NORA-program [7]. This equation automatically limits the criti-
cal velocity to the sonic velocity. Eq. (14) is solved iteratively together with three characteristic equations.

\[ M = \frac{c_2}{\rho \sigma^2} \left[ \frac{1}{\rho_1^2} \left( h_g - h_1 \right) - \frac{1}{\rho_2} \frac{1}{\rho_g} \right] \Delta h_1 \]  

(15)

Here \( c_2 \) is a constant (see Appendix) and \( \tau \) is relaxation time to the saturated condition. The constant \( \tau \) has to be determined by experimental data.

3. APPLICATIONS

The code is still under development, but it is already advanced enough to allow certain types of applications. A few examples follow.

Standard Problem No. 1 of the NEA/CSNI [6]

The well known Edward's pipe experiment was conducted with a straight pipe (length 4-1 m, internal diameter 7.3 cm) initially filled with compressed water at 6.89 MPa and 230°C. The sudden rupture of the glass disc at one end of the tube produced a decompression which dropped the pressure to the saturation value or slightly below it. At the closed end of the pipe the pressure dropped to a much lower value due to the reflection from the closed end of the pipe. This experiment was simulated with 41 axial seg...
ments by the TMOC program. The short term pressure histories as calculated by using three different flashing constants are shown in Figs. 1 and 2.

**Standard Problem No. 6 of the NEA/CSNI [8]**

For the further testing of the flashing constant a much slower transient, Standard Problem 6, was calculated. This experiment consists of a BWR-pressure vessel simulator (height about 11 m, internal diameter 0.77 m) initially filled with approximately saturated water and steam at 7 MPa. To calculate the correct pressure history the water must be slightly subcooled. Pressure curves for this experiment are plotted in Fig. 3. For this transient the best flashing constant has a much greater value than in the Edward's pipe experiment.

**The effect of radial expansion [2]**

Youngdahl and Kot studied the effect of a plastic deformation of a tube wall on the pressure. The uppermost pressure curve in Fig. 4 is produced by a pulse gun. The curve is somewhat damped because of numerical diffusion. In the other two curves the transient is calculated assuming either an elastic or a "plastic" response of the tube. After the radial strain has reached a certain value corresponding to the pressure of about 5 MPa, the tube deforms much more flexibly in the plastic case thus limiting effectively the pressure rise.
The effect of precursor waves [5]

Williams has investigated precursor waves in flexible (A.B.S.) and rigid (steel) pipes. The experimental setup consists of a reservoir and a straight pipe section (9.5 m), which is free to move. The test case where the reservoir end of the pipe was also free, is calculated in Fig. 5. The precursor wave reduces the pressure significantly. After 30 ms the rarefaction wave from the reservoir drops the pressure below the saturation conditions.

4. CONCLUSION

The calculations performed show that the TMOC program is capable of analyzing several phenomena concerning pressure pulse propagation in fluid filled tubes.

The important features of the code are
- thermal nonequilibrium in the water phase, and
- a simple model for the pipe movement.

According to the calculated experiments these both features have an essential effect on pressure pulse propagation. For better simulation of these phenomena the TMOC-code is under further development.
REFERENCES


APPENDIX

The hydraulic characteristic equations of TMQC are

\[
\frac{du}{dt} = \frac{1}{pc} \frac{dp}{dt} + \frac{fu}{2d} - \frac{c}{\rho_1 \rho_1^2} \frac{\partial p}{\partial h} Q \frac{1}{\rho_1 \rho_1^2} \left( h - h_1 \right)
\]

\[-\left( \frac{1}{\rho_1} - \frac{1}{\rho_g} \right) \frac{2cw}{R} = 0 \quad (A1, A2)
\]

along the characteristic \( \frac{dz}{dt} = u \pm c \) \hspace{1cm} (A3, A4)

\[
\frac{da}{dt} + \frac{1}{c_1} \frac{dp}{dt} + M \left[ \frac{\alpha}{\rho_1^2} \frac{\partial p}{\partial h} (h - h_1) - \frac{\alpha}{\rho_1^2} \left( \frac{1}{\rho_1} - \frac{1}{\rho_g} \right) \right] - \frac{\alpha}{\rho_1^2} \frac{\partial p}{\partial h} Q = 0 \quad (A5)
\]

\[
\frac{d\Delta h}{dt} + \frac{1}{c_2} \frac{dp}{dt} - \frac{M(h - h_1)}{c_2^2 c_3} + \frac{Q}{c_2 c_3} = 0 \quad (A6)
\]

Eqs. (5) and (6) hold along the characteristic

\[
\frac{dz}{dt} = u \quad (A7)
\]

\[
c_1 = \frac{-pc^2}{\alpha(1-pc^2 \frac{\partial p}{\partial h})} \quad (A8)
\]

\[
c_2 = \frac{-p_1(1-\alpha)}{c_3} \quad (A9)
\]

\[
c_3 = 1 - \alpha p_1 \frac{dh_{g \text{ sat}}}{dp} - (1-\alpha) p_1 \frac{dh_{1 \text{ sat}}}{dp} \quad (A10)
\]

The velocity of sound is

\[
c = \sqrt{\left[ \rho_g p_1^2 / \rho_1 \left( \frac{\partial p}{\partial h} \frac{c_3}{c_1} + \rho_1 \alpha \left( 1 - \frac{\alpha}{\rho_1^2} \frac{\partial p}{\partial h} \right) \right) \right]^{1/2}} \quad (A11)
\]

For the flashing term it is assumed using equation A6 that the pressure difference from the saturated value is \( \Delta h_1 \). Hence from Eqs. A1 and A2

\[
M = \frac{c_2}{pc^2 \left( \frac{1}{\rho_1^2} \frac{\partial p}{\partial h} (h - h_1) - \frac{1}{\rho_1} \frac{1}{\rho_g} \right)} \times \Delta h_1 \quad (A12)
\]
Fig. 1. Short term pressure history at the closed end of Edwards' pipe (Standard Problem 1).

Fig. 2. Short term pressure history at the middle section of Edwards' pipe (Standard Problem 1).
Fig. 3. Standard Problem 6. Pressure at level B (1.7 m from the bottom of the vessel).

Fig. 4. SRI pipe experiment [2]. Pressure predictions at point 3.
Fig. 5. The effect of precursor waves. Pressure history at the valve.