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NON-ADIABATIC STABILITY ANALYSIS OF CURRENT AND MAGNETIC CURVATURE DRIVEN MODES IN COLD PLASMAS PENETRATED BY NEUTRAL GAS

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Abstract

Previous stability theories concerning electrostatic current and magnetic curvature driven modes in cold plasma mantle boundary layers are generalized. In particular the commonly used adiabatic approximation is relaxed. In the general theory presented important new effects associated with heat conduction, ionization and ohmic heating are found. In combination with viscosity and resistivity these effects introduce additional stabilizing as well as destabilizing effects. Furthermore the present theory typically predicts similar stability properties as the adiabatic theory in the limit $|d(\ln T)/d(\ln n)| \ll 1$. However in the limit $|d(\ln T)/d(\ln n)| \gg 1$ the general theory predicts less favourable stability properties. One may speculate that these conclusions also apply to more general types of electrostatic modes associated with density and temperature gradients in cold plasma mantel boundary layers.

1. Introduction

In several previous papers we discussed the stability properties of the boundary regions of cold mantle systems [1-10]. These investigations could be of particular importance when considering the possible existence of cold blanket systems [11]. At first sight one would not expect unstable boundary regions to affect the global plasma properties to any larger extent. This is however not necessarily true. In high density cold mantle systems a considerable fraction of the input power is spent on ionization and excitation work on the incoming neutrals balancing the outflux of charged particles [11]. If the boundary regions are unstable, anomalous particle losses might lead to excessive neutral influx, resulting in its turn in large power losses not being compatible or posing severe constraints on the possible existence of a steady state. Thus, excessive power losses in the boundary region in high density Tokamak discharges for example, have been shown to lead to a loss of equilibrium. In this case, however, the modified temperature profiles lead to MHD unstable current profiles, in their turn leading to global disruptive instabilities [12-13].

One of the main reasons for studying cold mantle systems is connected with the suggestion that the surrounding neutral gas could act as a fuel reservoir in a future cold mantle thermonuclear reactor [7,12]. In order for this to be a viable fueling scheme the fuel has to diffuse to central parts at a rate determined by the thermonuclear burnup time. In this connection investigation of the stability properties of the boundary regions and the anomalous transport associated with possible unstable modes becomes important. The transport properties being anomalous or classical, in different parts of the plasma including the boundary regions, provides crucial pieces of information towards ultimate answer whether a gas mantle can be used for fueling in a future thermonuclear reactor.

The present paper forms essentially an extension of previous stability analysis [1,3,6]. In this paper we relax assumptions made in previous investigations, specifically the adiabatic approximation, i.e. the assumption that the pressure p is related to the density n by the expression $pn^{-\gamma} = \text{const}$. The word adiabatic is here used in a somewhat restricted sense since we have assumed $pn^{-\gamma}$ to be independent of any spatial coordinate i.e. $\gamma = 1 + d(\ln T)/d(\ln n)$.

In this paper we present a general theory which involves solving the particle and heat balance equations separately. First we consider current driven modes which arise due to the joint effect of a resistivity gradient perpendicular to the lines of force and an applied electric field parallel to the magnetic field. Second we consider flute or resistive flute modes driven by magnetic curvature and a pressure gradient. For arbitrary values of the ratio between the density and temperature scale lengths we find that the dispersion relation derived become of higher order than the corresponding dispersion relation derived in the adiabatic approximation. Consequently additional branches are introduced in a more general theory. We will present a detailed comparison Between the stability criteria derived in the two approaches including discussion of new physical effects introduced in the more complete theory. Furthermore the results are illustrated in stability diagrams for data typical of cold mantle boundary layers.

2. Magnetically Confined Plasmas in Cylindrical Geometry

We will now consider a magnetized quasi-neutral plasma penetrated by neutral gas. We will restrict us to cases when the macroscopic fluid equations are valid i.e. we consider length and time scales much larger than the characteristic mean free paths and collisional times. Further we consider an axisymmetric cylindrical plasma neutral gas column (r,θ,z) with the main confining magnetic field in the axial direction.

2.1. Model Equations

The conservation laws for the neutral gas can be expressed as [15].

$$\frac{\partial n_n}{\partial t} + \mathrm{div}(n_n\vec{v}_n) = -nn_n\xi \tag{1}$$

$$n_n m_n \frac{d\vec{v}_n}{dt} = -\mathrm{grad}\ p_n - nm_i\nu(\vec{v}_n - \vec{v}) + \mu_n(\mathrm{grad})^2\vec{v}_n \tag{2}$$

$$\frac{3}{2}\frac{\partial p_n}{\partial t} + \frac{3}{2}\mathrm{div}(p_n\vec{v}_n) + p_n\mathrm{div}(\vec{v}_n) = -\frac{3}{2}nn_n\xi kT_n +$$

$$+\frac{3}{2}nk\left[\bar{f}_{in}\nu_{in}(T_i - T_n) + f_{en}\nu_{en}(T_e - T_n)\right] - \mathrm{div}\ \vec{q}_n \tag{3}$$

Here n_n denotes the neutral particle density, n ion density, v_n the macroscopic neutral fluid velocity, $\vec{v}$ charged particle fluid velocity, ξ the ionization rate [16]

$$\xi = 1.64 \times 10^2\ k\left(\frac{2e}{\pi m_e}\right)^{1/2}(\phi\chi)^{3/2}\left(1 + \frac{1}{2\chi}\right)e^{-1/\chi} \tag{4}$$

Here k Boltzmann's constant, e electric charge, m_e electron mass, ϕ ionization potential, $W = e\phi$, $\chi = kT_e/W$, T_e electron temperature. Further m_n denotes neutral mass, d/dt convective derivative, p_n neutral scalar pressure, $p_n = n_n kT_n$, T_n neutral temperature, m_i ion mass, ν an effective collision frequency between ions and neutrals also including ionization

$$\nu = n_n \sigma_{in} w_{in} + n_n \tag{5}$$

Here σ_{in} denotes the cross section for elastic ion-neutral collisions, w_{in} relative thermal velocity between ions and neutrals. Further μ_n denotes the viscosity coefficient characteristic for the neutral gas [1]

$$\mu_n = \frac{n_n kT_n}{3\nu_n} \tag{6}$$

$$\nu_n = n\sigma_{in} w_{in} + n_n \sigma_{nn} w_n \tag{7}$$

Here σ_{nn} denotes cross section for elastic neutral-neutral collisions and w_n neutral thermal velocity. Further f_{in} denotes the fraction of energy lost by ions in collisions with neutrals, ν_{in} elastic collision frequency between ions and neutrals, $\nu_{in} = n_n \sigma_{in} w_{in}$, T_i ion temperature, f_{en} fraction of energy lost by electrons in collisions with neutrals, ν_{en} an effective collision frequency for elastic electron-neutral collisions, $\vec{q}_n$ neutral gas heat flow, $\vec{q}_n = -\lambda_n \operatorname{grad} T_n$, λ_n heat conduction coefficient characteristic for the neutral gas [17]

$$\lambda_n = \frac{75 n_n k^2 T_n}{16\sqrt{2} m_n \nu_n} \tag{8}$$

The conservation laws for the charged particles can be expressed as [15]

$$\frac{\partial n}{\partial t} + \mathrm{div}(n\vec{v}) = nn_n\xi \tag{9}$$

$$\mathrm{div}\,\vec{j} = 0 \tag{10}$$

$$nm\,\frac{d\vec{v}}{dt} = \vec{j} \times \vec{B} - \mathrm{grad}\ p - nm_i\nu(\vec{v}-\vec{v}_n) + \mu(\mathrm{grad})^2\vec{v} \tag{11}$$

$$\overleftrightarrow{\eta}\vec{j} = \vec{E} + \vec{v} \times \vec{B} - \frac{\vec{j} \times \vec{B}}{ne} + \frac{\mathrm{grad}\ p_e}{ne} + \frac{3\eta_\perp nk}{2B^2}\vec{B} \times \mathrm{grad}\ T_e \tag{12}$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \frac{3}{2}\,\mathrm{div}(p\vec{v}) + p\ \mathrm{div}\,\vec{v} = \frac{E_\parallel^2}{\eta_\parallel} +$$

$$+ \frac{3}{2}\,nn_n\xi kT_n - nn_nW\xi - \frac{3}{2}nk\left[f_{in}\nu_{in}(T_i - T_n) +\right.$$

$$\left. + f_{en}\nu_{en}(T_e - T_n) + f_{exc}\nu_{en}T_e\right] - \mathrm{div}\,\vec{q} - \pi \tag{13}$$

Here $\vec{j}$ denotes the current density, $m = m_e + m_i$, $\vec{B}$ magnetic field strength, p scalar pressure of charged particles, $p = nk(T_i + T_e)$, μ viscosity coefficient due to ion-ion collisions [18]

$$\mu = 2.7 \times 10^{-47} \times \frac{g\, n^2 A^{3/2} (\ell n \Lambda)}{B^2 T_i^{1/2}} \quad (14) \qquad g = \frac{\omega_{ci}^2}{\omega_{ci}^2 + \nu_{ii}^2} \quad (15)$$

Here A denotes mass number, Λ = ratio between impact parameter and Debye length, ω_{ci} ion gyro frequency and ν_{ii} ion-ion collision frequency, $\nu_{ii} = 6 \times 10^{-8} n(\ell n\Lambda)/A^{1/2} T_i^{3/2}$ [18].
Further $\vec{\eta}$ denotes resistivity tensor, $\eta_\perp$ resistivity perpendicular to the lines of force, $\eta_\parallel$ resistivity parallel to the lines of force [18]

$$\eta_\perp = 1.3 \times 10^2 \frac{\ell n\Lambda}{T_e^{3/2}} \quad (16) \qquad \eta_\parallel = 6.5 \times 10^1 \frac{\ell n\Lambda}{T_e^{3/2}} \quad (17)$$

$\vec{E}$ denotes electric field strength, p_e electron scalar pressure, $p_e = nkT_e$, $\vec{E}_\parallel$ electric field strength parallel to the lines of force, f_{exc} fraction of energy lost by excitation radiation in electron-neutral collisions, $\vec{q}$ charged particle heat flow, $\vec{q} = -\lambda_\perp \mathrm{grad}_\perp T$ [18]

$$\lambda_\perp = 1.5\times10^{-42} \frac{g\, n^2 A^{1/2} (\ell n\Lambda)}{B^2 T_i^{1/2}} \quad (18)$$

Furthermore π denotes the total radiation losses except losses due to excitation radiation.

We will now assume that $T \cong T_i \cong T_e \cong T_n$ and that the plasma and neutral gas are strongly coupled i.e. $\vec{v} = \vec{v}_n$ (it can be shown that this is a good approximation in the parameter range of interest [3-6]). Combining the momentum and heat balance equation for the neutral and charged fluids thus yields the following set of equations

$$nm_c \frac{d\vec{v}}{dt} = \vec{j} \times \vec{B} - \text{grad}(p + p_n) - \mu_c(\text{grad})^2 \vec{v} \tag{19}$$

$$\frac{\partial T}{\partial t} + \text{div}(T\vec{v}) + \frac{2}{3} T \,\text{div}\, \vec{v} = c(-\frac{1}{2} f_{exc}\nu_{en}T -$$

$$- \frac{1}{2} n_n \xi T - \frac{n_n W\xi}{3k} + \frac{E^2}{n_n 3nk} - \frac{1}{3nk} \text{div}\, \vec{q}_c) \tag{20}$$

Here $m_c = m(1 + n_n m_n/nm)$, $\mu_c = \mu + \mu_n$, $c = 1/(1 + n_n/2n)$, $\vec{q}_c = \vec{q}_n + \vec{q}$.

2.2. Stationary State

In this investigation we will not explicitly analyze the equilibrium properties of cold mantle boundary layers.Numerous investigations of the steady state properties of the boundary regions have been reported previously [11,15, 19-22]. We will merely assume that an equilibrium exists in the parameter range of interest i.e. $10^{20}m^{-3} < n < 10^{22}m^{-3}$, $10^{20}m^{-3} < n_n < 10^{22}m^{-3}$, $10^4K < T < 10^6K$, $0.1T < B < 10T$.

2.3. Linearized Stability Theory

We will now consider localized electrostatic perturbation of the form $\exp\left[i(k_r r + m\theta + 2\pi nz/L - \omega t)\right]$. Here L denotes an effective length of the plasma column. A curved bar (~) will be used to denote perturbed quantities.

2.3.1. Current Driven Modes

Using the momentum balance Eq. (19) taking account of plasma neutral gas interaction effects by introducing an enlarged mass and viscosity coefficients we can express the quasi-neutrality condition Eq. (10) as

$$\frac{\mu_c k_\perp^2 - i\omega n m_c}{B^2}\,\mathrm{grad}(\vec{B}\times\tilde{\vec{v}}) + \vec{B}\cdot\mathrm{grad}\left(\frac{\tilde{j}_\parallel}{B}\right) = 0 \tag{21}$$

Here $k_\perp = (k_r^2 + k_b^2)^{1/2}$, $k_b = 2\pi n B_\theta/LB - mB_z/rB$, $B = (B_\theta^2 + B_z^2)^{1/2}$, B_θ = magnetic field strength in the azimuthal direction and B_z = magnetic field strength in the axial direction. Further from Ohm's generalized law Eq. (12) we find the following expressions for $\tilde{\vec{v}}$ and $\tilde{j}_\parallel$ neglecting the density and temperature dependence of $\ell n\Lambda$

$$\tilde{\vec{v}}_\perp = \frac{\vec{B}\times\mathrm{grad}\,\tilde{\varphi}}{B^2} - \eta_\perp\frac{\mathrm{grad}\,\tilde{p}}{B^2} + \frac{3\eta_\perp nk\,\mathrm{grad}\,\tilde{T}}{2B^2} \tag{22}$$

$$\tilde{j}_\parallel = \frac{3}{2}\, j_0 \frac{\tilde{T}}{T} - \frac{ik_\parallel}{\eta_\parallel}\,\tilde{\varphi} \tag{23}$$

Here j_0 denotes the zeroth order current density along the lines of force. Thus combining Eqs. (21)-(23) we find to lowest order

$$k_\perp^2\left(\frac{\mu_c k_\perp^2 - i\omega n m_c}{B^2}\right)\tilde{\varphi} + ik_\parallel\left(\frac{3}{2}\, j_0 \frac{\tilde{T}}{T} - i\frac{k_\parallel}{\eta_\parallel}\tilde{\varphi}\right) = 0 \tag{24}$$

Here $k_\parallel = mB_\theta/rB + 2\pi n B_z/LB$. From the heat balance Eq. (20) we can deduce the following expression for $\tilde{T}$ neglecting excitation radiation effects and the weak dependence of $\tilde{T}$ on any density variation. Thus

$$\tilde{T} = \frac{k_b}{(\omega + i\omega_{npT})B} \frac{dT}{dr} \tilde{\varphi} \tag{25}$$

Here

$$\omega_{npT} = \omega_\xi^T + \omega_\lambda + \frac{5}{12}\omega_{np} - \omega_j^T \tag{26}$$

$$\omega_\xi^T = \frac{1}{2(1+\frac{n_n}{2n})} \left[n_n \frac{d(\xi T)}{dT} + \frac{2}{3}\frac{n_n W}{k}\frac{d\xi}{dT} \right] \tag{27}$$

$$\omega_\lambda = \omega_\lambda^i + \omega_\lambda^n \tag{28}$$

$$\omega_\lambda^{i(n)} = \frac{k_\perp^2}{3(1 + \frac{n_n}{2n})} \lambda_{\perp(n)} \tag{29}$$

Here we have considered ionization effects associated with temperature variations only due to the strong temperature dependence of the ionization rate, see Eq. (4) in the parameter range of interest

$$\omega_{np} = \frac{2nkTn_\parallel k_\perp^2}{B^2} \tag{30}$$

$$\omega_j^T = \frac{1}{2(1+\frac{n_n}{2n})} \frac{n_\parallel j_\parallel^2}{nkT} \tag{31}$$

Combining Eqs. (24) and (25) we find after simplification and normalization in terms of characteristic frequencies the following dispersion relation

$$(\omega + i\omega_\mu)(\omega + i\omega_{npT}) - \omega_j\omega^*_{ci} + i\omega_\chi(\omega + i\omega_{npT}) = 0 \qquad (32)$$

Here

$$\omega_\mu = \frac{\mu_c k_\perp^2}{nm_c} \qquad (33) \qquad\qquad \omega_j = \frac{3}{2}\frac{k_\parallel j_\parallel}{ne} \qquad (34)$$

$$\omega^*_{ci} = \frac{mk_b}{m_c k_\perp^2}\frac{d(\ell nT)}{dr}\,\omega_{ci} \qquad (35) \qquad\qquad \omega_\chi = \frac{\omega_A^2}{\omega_r} \qquad (36)$$

$$\omega_A = k_\parallel \frac{B}{(\mu_o nm_c)^{1/2}} \qquad (37) \qquad\qquad \omega_r = \frac{\eta_\parallel k_\perp^2}{\mu_o} \qquad (38)$$

Proceeding in a similar way as in Ref. [4-6] we arrive at the following stability condition

$$\omega_j\omega^*_{ci} + (\omega_\mu + \omega_\chi)(\omega_\lambda + \frac{5}{12}\,\omega_{np} + \omega_\zeta^T - \omega_j^T) > 0 \qquad (39)$$

2.3.2. Discussion of Results

The stability criterion for current driven modes given by inequality (39) is similar in structure to the criterion derived in the adiabatic approximation [4-6] i.e.

$$\omega_j' \omega_{ci}'^{*} + (\omega_\mu + \omega_\chi)\omega_{np}' > 0 \quad (40)$$

Here

$$\omega_j' = \frac{3(\gamma-1)k_{\parallel} j_{\parallel}}{2\,ne} \quad (41) \qquad \omega_{ci}'^{*} = \frac{mk_b}{m_c k_\perp^2} \frac{d(\ln n)}{dr}\,\omega_{ci} \quad (42)$$

$$\omega_{np}' = \frac{2nkT\eta_\perp k_\perp^2}{B^2} f(\gamma) \quad (43)$$

Here $f(\gamma) = [\gamma - 3(\gamma-1)/4]$ and γ = ratio between specific heats, $\gamma = 1 + d(\ln T)/d(\ln n)$. The first term on the left hand side of inequality (40) $\omega_j \omega_{ci}^{*}$ is associated with the mechanisms driving the modes of interest, while the last term $(\omega_\mu + \omega_\chi)\omega_{np}'$ is connected with stabilizing mechanisms. For further discussion of the physical effects associated with the different terms we refer to previous work [4-6]. As is evident from inspection of inequalities (39) and (40) terms associated with effects already present in the adiabatic approximation are modified by certain factors essentially depending only on γ in the present theory. Furthermore additional stabilizing and destabilizing effects arise in a more general stability analysis. As can be seen from inequality (39) additional stabilizing effects arise due to a joint viscous-heat conduction effect proportional to $\omega_\mu \omega_\lambda$ and a joint resistive-heat conduction effect proportional to $\omega_\chi \omega_\lambda$. Furthermore ionization effects associated with ω_ξ^T corresponds typically to stabilizing effects. Also these effects arise due to a coupling between ionization and viscous and resistive effects proportional $\omega_\mu\omega_\xi^T$ and $\omega_\chi\omega_\xi^T$ respectively. Moreover ohmic heating effects, associated with the currents driving the modes of interest corresponds, in a very similar way as heat conduction and ionization, to a destabilizing effect proportional to $\omega_\mu\omega_j^T$ and $\omega_\chi\omega_j^T$. For numerical data typical of boundary layers of cold mantle systems the net result of heat conduction, ionization and ohmic heating corresponds usually to an additional stabilizing effect. The final stability condition can now be found from the inequality (40) after optimization with respect to $k_\parallel$ and $k_\perp$. Thus

$$j_\parallel < \frac{4neB}{3\varkappa_{ci}} \left(\omega_\lambda + \frac{5}{12}\omega_{np} + \omega_\xi^T - \omega_j^T\right)\left[\frac{\omega_\mu}{\mu_0 n m_c \omega_r}\right]^{1/2}_{k_\perp = \frac{\sqrt{2}}{x_b}\pi} \tag{44}$$

Here x_b = boundary layer thickness. In the present analysis we have assumed that density gradient effects are unimportant. This assumption is justified since the modes considered are essentially driven by the spatial variation in resistivity and this variation is to a very good approximation due to a temperature gradient. A very weak density dependence enters through the Coulomb logaritm $\ell n\Lambda$.

In a comparison between the adiabatic theory and the general theory presented we will only consider care when $d(\ell nT)/d(\ell nn)$ is strictly positive (i.e. $\gamma>1$). This is what one normally expects under cold mantle conditions. For cases where $d(\ell nT)/d(\ell nn) << 1$ order unity (i.e. $\gamma \approx 1$ and $f(\gamma) \approx 1$) one finds that the critical current density given by Eq. (44) is somewhat larger than the critical current derived in the adiabatic approximation. The discrepancy is however not too large, usually less than a factor of five or so. Therefore the results derived in the adiabatic approximation remains valid in order of magnitude estimates for cases when $d(\ell nT)/d(\ell nn) << 1$.

In the opposite limit i.e. $d(\ell nT)/d(\ell nn) >> 1$ (i.e. $\gamma >> 1$ and $f(\gamma) >> 1$ we find that the critical current density given by Eq. (44) is considerably less than expected from the adiabatic theory. Consequently the adiabatic theory given by overestimates the stabilizing effects in the limit d(lnT)/d(lnn)>>1. This is essentially due to the fact that all stabilizing terms in the adiabatic theory involving ω_{np} scale proportional to $f(\gamma)$ and $f(\gamma)$ goes to infinity as d(lnT)/d(lnn) becomes very large. This is of course unrealistic on physical grounds.

2.3.3. Magnetic Curvature Driven Modes

We now proceede in a similar way as in Sec. 2.3.1. also taking into account effects associated with the curvature of the lines of force. We thus find that the quasi-neutrality condition Eq. (10) can be expressed as

$$ik_{\parallel}\tilde{j} + \frac{\mu_c k_\perp^2 - i\omega n m_c}{B^2}\,\mathrm{grad}(\vec{B}\times\tilde{\vec{v}}) - i\,\frac{2k_b}{B^2}\frac{dB}{dr}\,\tilde{p} = 0 \qquad (45)$$

From the particle and heat balances Eqs. (9), (20) we find for arbitrary values of d(lnT)/d(lnn)

$$(\omega + i\omega_{np})\tilde{n} = i\left(\frac{kk_\perp^2 n}{2\,B^2} - \omega_j^n \frac{dn}{dT}\right)\tilde{T} - \frac{k_b}{B}\frac{dn}{dr}\,\tilde{\varphi} \qquad (46)$$

$$i\left(\frac{10kk_\perp^2 n_\perp}{3\,B^2} + \omega_j^n\right)\tilde{n} + (\omega + i\omega_{npT})\,\tilde{T} = \frac{k_b}{B}\frac{dT}{dr}\,\tilde{\varphi} \qquad (47)$$

Here

$$\omega_j^n = \frac{2}{3}\,\omega_j^T \qquad (48)$$

After a straight forward elimination process we can now express $\tilde{n}$ and $\tilde{T}$ in terms of $\tilde{\varphi}$. Now inserting these expressions into the quasi-neutrality condition Eq. (45) and using the expressions for $\tilde{v}_\perp$ and $\tilde{j}_\perp$ given by Eqs. (22)-(23) neglecting effects associated with currents parallel to the lines of force in the steady state in the expression for $\tilde{j}_\parallel$ we arrive at the following dispersion relation after normalization in terms of characteristic frequencies

$$\omega^3 + iA_1\omega^2 + A_2\omega + iA_3 = 0 \qquad (49)$$

Here

$$A_1 = \omega_{\mu\chi} + \omega_{\eta p} + \omega_{\eta pT} \qquad (50)$$

$$A_2 = \omega_{gn}^2 + \omega_{gT}^2 - \omega_{\eta p}\omega_{\eta pT} + \omega_{\eta}^2 -$$

$$- \omega_{\mu\chi}(\omega_{\eta p} + \omega_{\eta pT}) + \omega_j^n(\frac{1}{4}\,\omega_{\eta p} - \omega_\xi^n) \qquad (51)$$

$$A_3 = \omega_{gn}^2\omega_{\eta pT} + \omega_{gT}^2\omega_{\eta p} - \omega_{\mu\chi}(\omega_{\eta p}\omega_{\eta pT} - \omega_{\eta}^2) -$$

$$- \omega_{gn}^2\omega_\beta - \omega_{gT}^2\omega_\gamma - \omega_{gn}^2\omega_j^n + \omega_{\mu\chi}\omega_j^n\,(\frac{1}{4}\,\omega_{\eta p} - \omega_\xi^n) \qquad (52)$$

Here

$$\omega_\eta^2 = \frac{5}{12}\,\omega_{\eta p}^2 - \frac{5}{3}\,\omega_\xi^n\,\eta p \qquad (53)$$

$$\omega_\beta = \left(\frac{n^2 k k_\perp^2 \eta_\perp}{2B^2} - n\eta_n \frac{d\xi}{dT}\right) \frac{d(\ell nT)}{d(\ell nn)} \qquad (54)$$

$$\omega_\gamma = \frac{10T^2 k k_\perp^2 \eta_\perp}{3\ \ B^2}\,\frac{d(\ell nn)}{d(\ell nT)} \qquad (55)$$

$$\omega_{gn(T)}^2 = \frac{4k_b e\omega_{n(T)}}{m_c k_\perp^2}\,\frac{dB}{dr} \qquad (56)$$

$$\omega_{n(T)} = \frac{k_b kT}{neB}\,\frac{dn}{dr}\,\frac{d(\ell nT)}{d(\ell nn)} \qquad (57)$$

$$\omega_j^n = \frac{2}{3}\,\omega_j^T \qquad (58)$$

$$\omega_{\mu\chi} = \omega_\mu + \omega_\chi \qquad (59)$$

$$\omega_\xi^n = \eta_n T\,\frac{d\xi}{dT} \qquad (60)$$

The stability threshold conditions corresponding to the dispersion relation Eq. (49) are now given by the following conditions [3-6] $A_1 > 0$, $A_3 < 0$ and $A_3 - A_1A_2 > 0$. Using the definitions of A_1, A_2 and A_3 we arrive at the following equivalent condition

$$\omega_{gn}^2(\omega_\lambda + \omega_\xi^T - \omega_j^T - \omega_j^n - \frac{5}{4}\omega_{np}) + \omega_{gT}^2(\omega_\xi^n + \frac{3}{4}\omega_{np}) -$$

$$- \omega_{\mu\chi}\omega_{np}(\omega_\lambda + \omega_\xi^T + \frac{5}{3}\omega_\xi^n - \frac{1}{4}\omega_j^n - \omega_j^T) - \omega_{\mu\chi}\omega_j^n\omega_\xi^n < 0 \qquad (61)$$

$$\omega_{gn}^2(\omega_{\mu\chi} + \frac{8}{3}\omega_{np} + \omega_j^n) + \omega_{gT}^2(\omega_{\mu\chi} - \omega_j^T + \omega_\xi^T + \omega_\lambda +$$

$$+ \frac{2}{3}\omega_{np} - \omega_\xi^n) - (\omega_{np} + \omega_{npT})\left[\omega_{np}(\omega_\lambda + \omega_\xi^T - \omega_j^T + \frac{5}{3}\omega_\xi^n) -\right.$$

$$\left. - \omega_j^n(\frac{1}{4}\omega_{np} - \omega_\xi^n) + \omega_{\mu\chi}^2 + \omega_{\mu\chi}\omega_{np} + \omega_{\mu\chi}\omega_{npT}\right] < 0 \qquad (62)$$

2.3.4. Discussion of Results

Using the adiabatic approximation the dispersion relation becomes a second order equation in ω and the stability criterion is given by [1,3]

$$\omega_{gn}'^2 - \omega_\mu\omega_{np} - \omega_\chi\omega_{np}' < 0 \qquad (63)$$

assuming $\omega_{\eta p} >> \omega_{\mu p}$. Here

$$\omega_{gn}'^{2} = \gamma\omega_{gn}^{2} \qquad (64) \qquad\qquad \omega_{\mu p} = \frac{\gamma k T \mu_c k_\perp^4}{n e^2 B^2} \qquad (65)$$

For a detailed interpretation of the various terms in the inequality (63) we refer to previous work [1,3]. Now in a more general theory considering the particle and heat balance equation separately the dispersion relation becomes a third order equation. Consequently not only are the different branches of dispersion relation modified but also a new branch is introduced. It turns out that two conditions corresponding to the inequalities (61) and (62) have to be satisfied simultaneously in order to achieve stability in this latter case. The inequality (61) corresponds to ordinary density gradient driven flute modes. A similar condition can be derived in the limit when all temperature variations are neglected. This condition then becomes somewhat modified when temperature gradient effects are introduced. The inequality (62) corresponds in a similar way to ordinary temperature gradient driven flute modes. A similar condition can be derived when all density variations are neglected. It is however important to note that this does not necessarily imply that inequality (61) becomes the relevant stability condition in the limit $|d(\ell nT)/d(\ell nn)| << 1$ and inequality (62) the relevant stability condition in the limit $|d(\ell nT)/d(\ell nn)| >> 1$. Let us now first consider the case corresponding to $|d(\ell nT)/d(\ell nn)| << 1$. Then one can actually show, just noticing that typically $\omega_\lambda, \omega_{\mu\lambda} \gg \omega_{\eta p}, \omega_j^T, \omega_\zeta^T, \omega_\xi^n$, that the most restrictive condition is given by the inequality (61) which can be rewritten as

$$\underset{①}{\omega_{gn}^2} - \underset{②}{\omega_{\eta p}(\omega_\mu + \omega_\chi)} \underset{③}{\frac{\omega_\lambda + \omega_\xi^T + \frac{5}{3}\omega_\xi^n - \frac{1}{4}\omega_j^n - \omega_j^T}{\omega_\lambda + \omega_\zeta^T - \omega_j^T - \omega_j^n - \frac{5}{4}\omega_{\eta p}}} -$$

$$- \underset{④}{(\omega_\mu + \omega_\chi)} \underset{⑤}{\frac{\omega_j^n \omega_\xi^n}{\omega_\lambda + \omega_\zeta^T - \omega_j^T - \omega_j^n - \frac{5}{4}\omega_{\eta p}}} < 0 \qquad (66)$$

Here the term labelled (3) is typically of order unity and the term labelled (5) is typically much less than ω_{np}. Thus the inequality (66) becomes almost identical to inequality (63) for $\gamma = 1$. Heat conduction, ionization and ohmic heating affects thus the stability properties very weakly if at all in this case. Consequently the stability criterion derived in the adiabatic approximation remains valid to a very good approximation in the limit $|d(\ell nT)/d(\ell nn)| \ll 1$. This is of course exactly what one would expect.

Now turning to the opposite limit i.e. $|d(\ell nT)/d(\ell nn)| \gg 1$ the situation becomes quite different. One could imagine cases when either of the inequalities (61) or (62) becomes the relevant stability criterion. Let us first consider the case that inequality (62) yields the most restrictive condition. Thus

$$\underbrace{\omega_{gT}^2}_{(1)} - \underbrace{\frac{\omega_{np} + \omega_{npT}}{\omega_{\mu\chi} - \omega_\xi^n + \frac{1}{4}\omega_{np} + \omega_{npT}}}_{(2)} \underbrace{\Big[\omega_{np}(\omega_\lambda + \omega_\xi^T - \frac{7}{6}\omega_j^T + \frac{5}{3}\omega_\xi^n) + \omega_j^n\omega_\xi^n + \omega_{\mu\chi}^2 + \omega_{\mu\chi}\omega_{np} + \omega_{\mu\chi}\omega_{npT}\Big]}_{(3)} < 0 \qquad (67)$$

Let us consider the representative case when the term labelled (2) is approximately unity and varies only slowly with the parameters ω_{np}, ω_{npT}, etc. For this case one easily convinces oneself that the main differences as compared to the adiabatic theory are that heat conduction and ionization in combination with particle diffusion viscosity and resistivity introduce stabilizing effects whereas ohmic heating in a similar way usually corresponds to a destabilizing effect i.e. when $\frac{7}{6}\omega_{np} + \omega_{\mu\chi} > \frac{2}{3}\omega_\xi^n$.

For the case that the most restrictive condition is given by the inequality (61) one finds

$$\omega_{gT}^2 - (\omega_\mu + \omega_\chi)\left[\omega_{\eta p}\,\frac{\omega_\lambda + \omega_\xi^T + \frac{5}{3}\,\omega_\xi^n - \frac{7}{6}\,\omega_j^T}{\omega_\xi^n + \frac{3}{4}\,\omega_{\eta p}} + \right.$$

$$\left. + \omega_j^n\,\frac{\omega_\xi^n}{\omega_\xi^n + \frac{3}{4}\,\omega_{\eta p}}\right] < 0 \qquad (68)$$

In this case one can draw the same conclusions as in the previous case concerning heat conduction. However ionization corresponds to either a stabilizing or destabilizing effect depending on the relative sizes of the different terms in the inequality (68). Joule heating has also either a stabilization or destabilizing effect depending on whether $\omega_{\eta p}$ is less or larger than $\frac{4}{7}\omega_\xi^n$. It has been verified by numerical computations that the conclusions drawn in the limit $|d(\ln T)/d(\ln n)| \gg 1$ also applies for arbitrary values of $d(\ln n)/d(\ln T)$. The different effects associated with heat conduction, ionization and Joule heating become of course less and less pronounced in the limit $|d(\ln T)/d(\ln n)| \ll 1$. Now turning to a comparison with the stability conditions derived in the adiabatic theory in the limit $|d(\ln T)/d(\ln n)| \gg 1$ it is concluded that the adiabatic theory grossly overestimates the stabilizing effects. This is essentially due to the fact that all terms associated with the resistive-pressure effects, i.e. $\omega_{\eta p}$ goes to infinity as $|d(\ln T)/d(\ln n)| \gg 1$ i.e. $\gamma \gg 1$ and $f(\gamma) \gg 1$. Note that $\omega_{\eta p}$ scales proportional to $f(\gamma)$ in the adiabatic theory.

Finally we illustrate the threshold conditions in stability diagrams for data typical of cold blanket boundary layers i.e. $10^{20}\,m^{-3} < n < 10^{22}\,m^{-3}$, $10^4\,K < T < 10^6\,K$ $0.1\,T < B < 10\,T$, and $x_b \gtrsim 10^{-2}\,m$.

We have assumed the following scaling of neutral gas density with density, magnetic field strength and temperature [8]

$$n_n = \frac{2}{3}\,\frac{n^3}{k_B^2 B^2}\,h \qquad (69)$$

where $k_B = (2/\Omega m \omega_{in} w_{in})^{1/2}$ and h a factor simulating anomalous or neoclassical effects in internal plasma regions. We have chosen $h = 10^2$. For the numerical data chosen it turns out that the relevant stability criterion in the general theory is given by inequality (61) independently of the value of $d(\ln n)/d(\ln T)$. In Figs. 1-3 we have illustrated the stability domains in n-B space according to the different theories for various values of the curvature of the lines of force for the following cases: Fig.1 $d(\ln n)/dr = 10^2\ m^{-1}$, $d(\ln T)/dr = 0$, Fig.2 $d(\ln T)/dr = 10^2\ m^{-1}$, $d(\ln n) = 1\ m^{-1}$ and Fig.3 $d(\ln n)/dr = d(\ln T)/dr = 10^2\ m^{-1}$. As can be seen from the diagrams the adiabatic theory predicts slightly less favourable or similar stability properties in the limit $d(\ln T)/d(\ln n) \ll 1$ than the more general theory. In the limit $d(\ln T)/d(\ln n) \gg 1$ the adiabatic theory predicts unrealistically favourable stability properties. Furthermore the stability properties typically improves with increasing density and decreasing magnetic field strength. In Figs 4-5 we have illustrated the stability domains in n-B space for various values of the temperature in the boundary regions for the cases that $d(\ln n)/dr = 10^2\ m^{-1}$, $d(\ln T)/dr = 0$ and $d(\ln n)/dr = 0$, $d(\ln T)/dr = 10^2\ m^{-1}$ according to the general theory. The stability properties typically improves with decreasing temperature. Finally in Figs. 6-7 we keep the magnetic curvature and temperature in the boundary region fixed while varying the boundary layer thickness for density and temperature gradient driven modes respectively. It is found that the stability improves with decreasing width of the boundary region.

3. Conclusions and Summary

We have considered the stability of cold plasma boundary layers for two particular types of electrostatic modes, namely current and magnetic curvature driven modes in a more general approach than in previous analysis. In particular the adiabatic approximation has been relaxed. Several important new effects are introduced and effects also present in the adiabatic theory analysis are modified particularly in the limit $|d(\ell n T)/d(\ell n n)| \gg 1$.

Let us first discuss modes driven by a current along the lines of force and a resistivity gradient i.e. a temperature gradient assuming classical resistivity. In this case it is found that in addition to effects already considered in the adiabatic theory heat conduction and ionization introduces stabilizing effects whereas ohmic heating introduces a destabilizing effect. The net impact corresponds typically to an additional stabilizing effect. Now also taking into account the fact that effects already considered in the adiabatic theory are modified in the general theory the following conclusions can be drawn. The general theory typically predicts more favourable stability properties in the limit $|d(\ell n T)/d(\ell n n)| \ll 1$. Estimates of characteristic quantities, threshold current density, etc, in the two theories differs however typically at most by an order of magnitude. In the limit $d(\ell n T)/d(\ell n n)| \gg 1$ the situation becomes quite different. The adiabatic theory grossly overestimates the stabilizing effects.

Considering magnetic curvature driven modes the situation becomes somewhat more complicated. In the general theory presented one arrives at two conditions, as compared to one condition in the adiabatic theory, which have to be satisfied in order to get stability. In the limit $|d(\ell n T)/d(\ell n n)| \ll 1$ one can show that inequality (61) typically becomes the relevant stability condition whereas in the limit $|d(\ell n T)/d(\ell n n)| \gg 1$ either of the inequalities (61) or (62) becomes the relevant stability condition. The main physical differences in the more general theory presented as compared to the adiabatic theory are that additional stabilizing and destabilizing effects arise due to heat conduction, ionization and ohmic heating effects. For arbitrary values of $d(\ell n n)/d(\ell n T)$ heat conduction typically corresponds to a stabilizing effect whereas ionization and ohmic heating corresponds to either a stabilizing or destabilizing effect depending on the relative sizes of various terms in the stability criteria . Furthermore effects already considered in the adiabatic theory are modified in a more general theory. In a comparison between the two theories considered it is concluded that in the limit $|d(\ell nT)/d(\ell nn)| << 1$ the final stability criteria are almost identical. From numerical computations it has been found that the adiabatic theory predicts somewhat less favourable stability properties than the general theory. The discrepance is, however, in most cases unimportant. In the limit $|d(\ell nT)/d(\ell nn)| >> 1$ the situation becomes quite different i.e. the adiabatic theory overestimates the stabilizing effects. The conclusions drawn concerning the stability properties for magnetic curvature driven modes are thus similar to current driven modes. One may speculate that the conclusions drawn from comparison between the adiabatic and more general theory applies also for other types of electrostatic modes previously discussed in connection with cold blanket boundary layers.

As a final remark the stability properties typically deteriorate for increasing temperature in the boundary region as well as for increasing boundary layer width and magnetic field strength. The stability properties typically improve for increasing ion densities. This applies for current driven as well as magnetic curvature driven modes for arbitrary values of $|d(\ell nn)/d(\ell nT)|$. These conclusions are thus similar to the conclusions drawn using the adiabatic theory.

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Stockholm, March 1, 1978

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6. Legends to Figures

Fig.1. Stability diagrams for density gradient driven flute modes for different values of the curvature of the lines of force. The regimes above the solid and broken lines correspond to stable regions. Also shown are contours of constant ratio between neutral and charged particle densities.

Fig.2. Stability diagrams for temperature driven flute modes, otherwise same as in Fig.1.

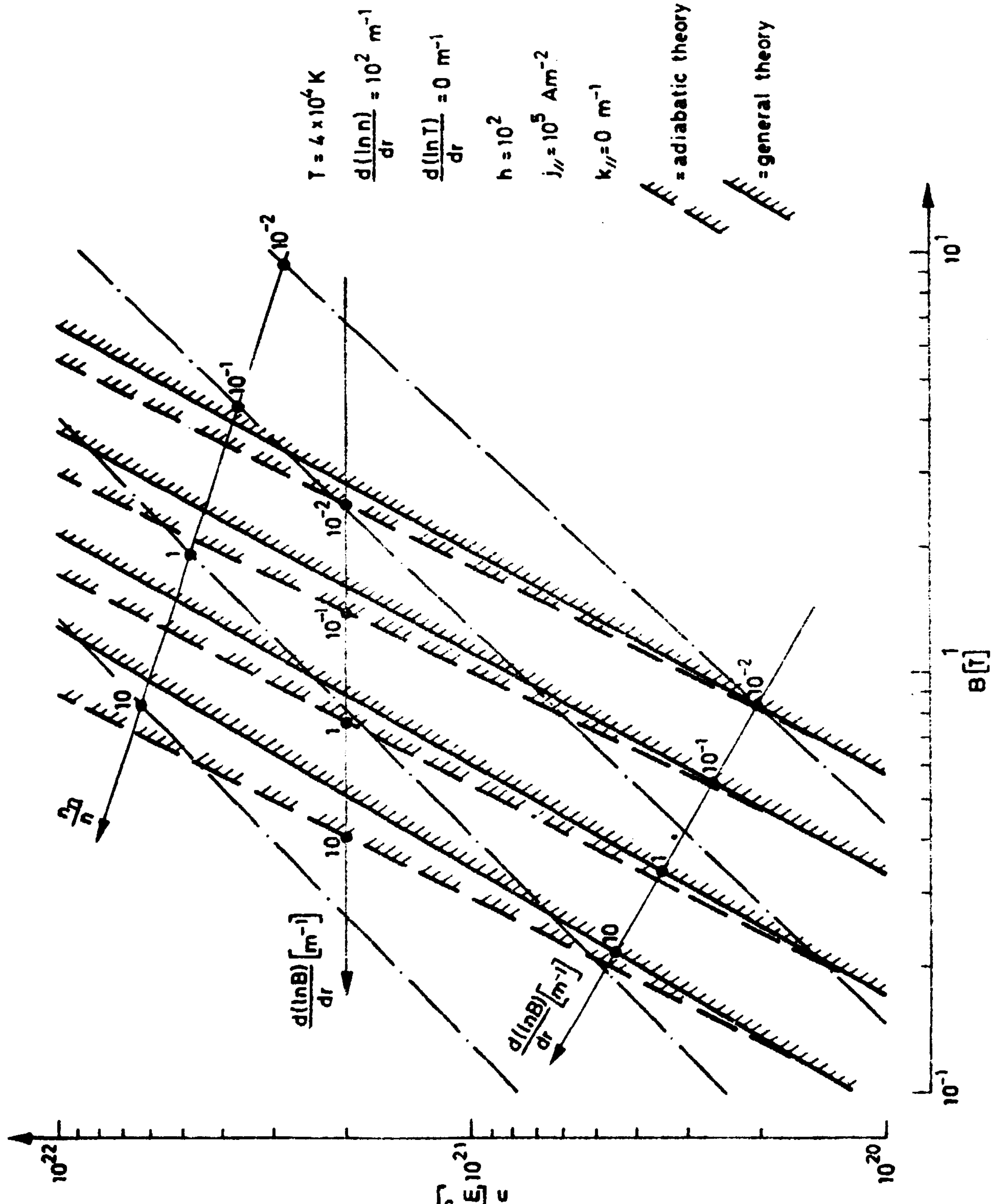
Fig.3. Stability diagrams for pressure driven flute modes, otherwise same as in Fig.1.

Fig.4. Stability diagrams for density gradient driven flute modes for different values of the boundary region temperature. Also shown are contours of constant β, i.e. ratio between plasma energy and magnetic field energy. The regions above the solid lines correspond to stable regions.

Fig.5. Stability diagrams for temperature gradient driven flute modes. Otherwise same as in Fig.4.

Fig.6. Stability diagrams for density gradient driven flute modes for different values of the boundary layer thickness. Also shown are contours of constant ratio between neutral and charged particle densities. The regions above the solid broken lines correspond to stable regions.

Fig.7. Stability diagrams for temperature gradient driven flute modes. Otherwise same as in Fig.6.



T = 4 × 10^4 K
d(ln n)/dr = 10^2 m^-1
d(ln T)/dr = 0 m^-1
h = 10^2
j_// = 10^5 Am^-2
k_// = 0 m^-1
= adiabatic theory
= general theory
Δn/n
d(lnB)/dr [m^-1]
n [m^-3]
B [T]
10^-1
1
10^1
10^20
10^21
10^22
10
1
10^-1
10^-2

Fig. 1

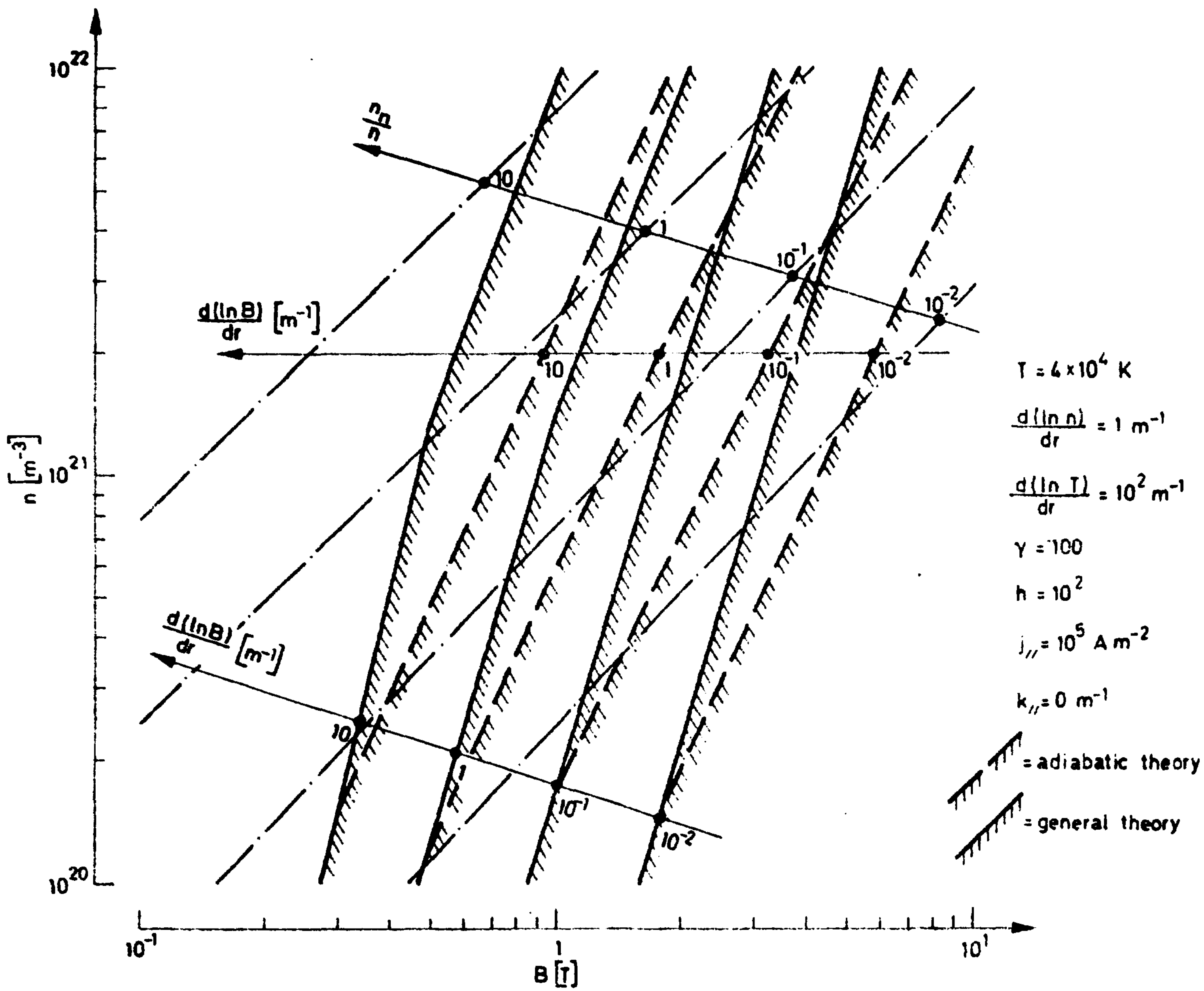
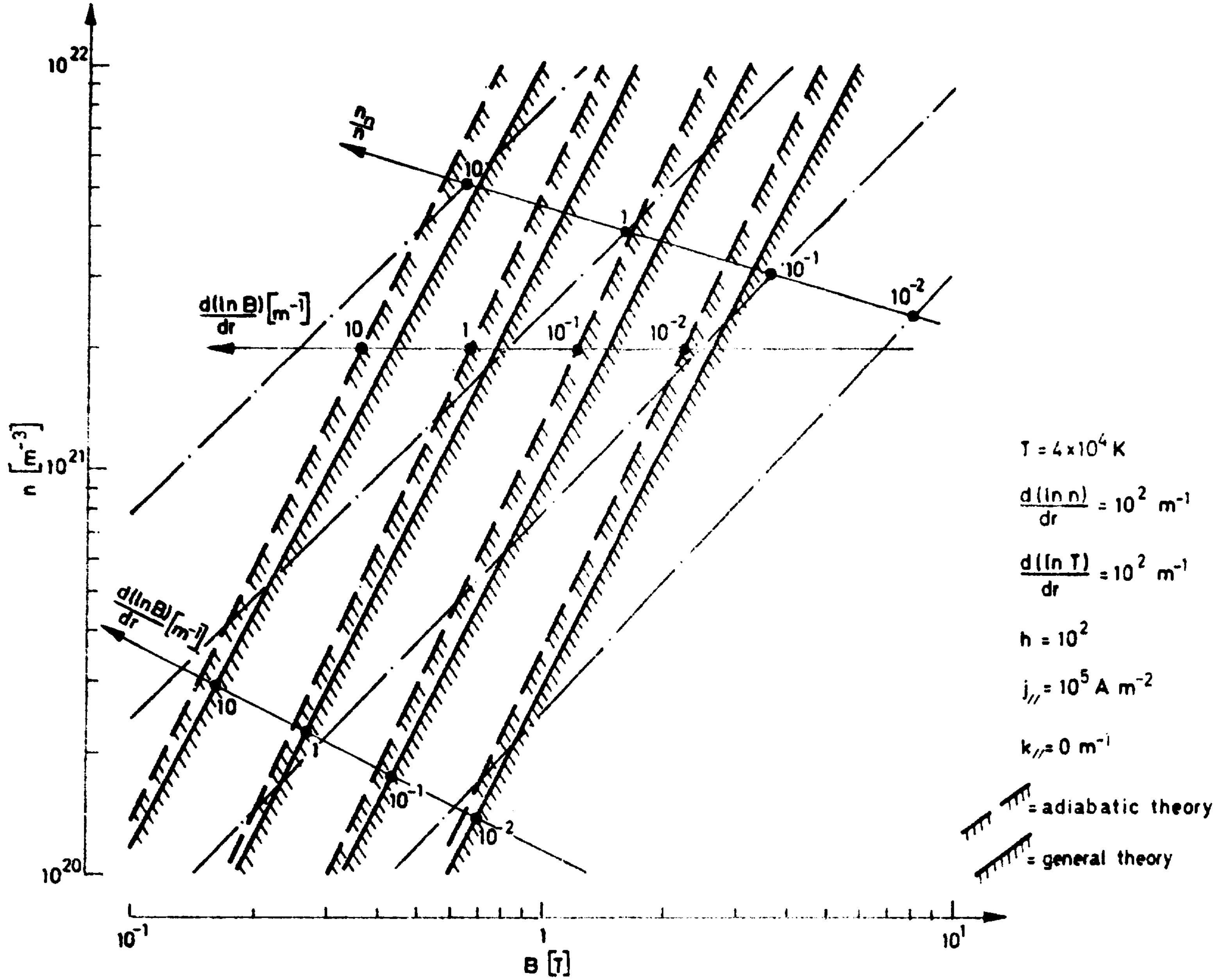
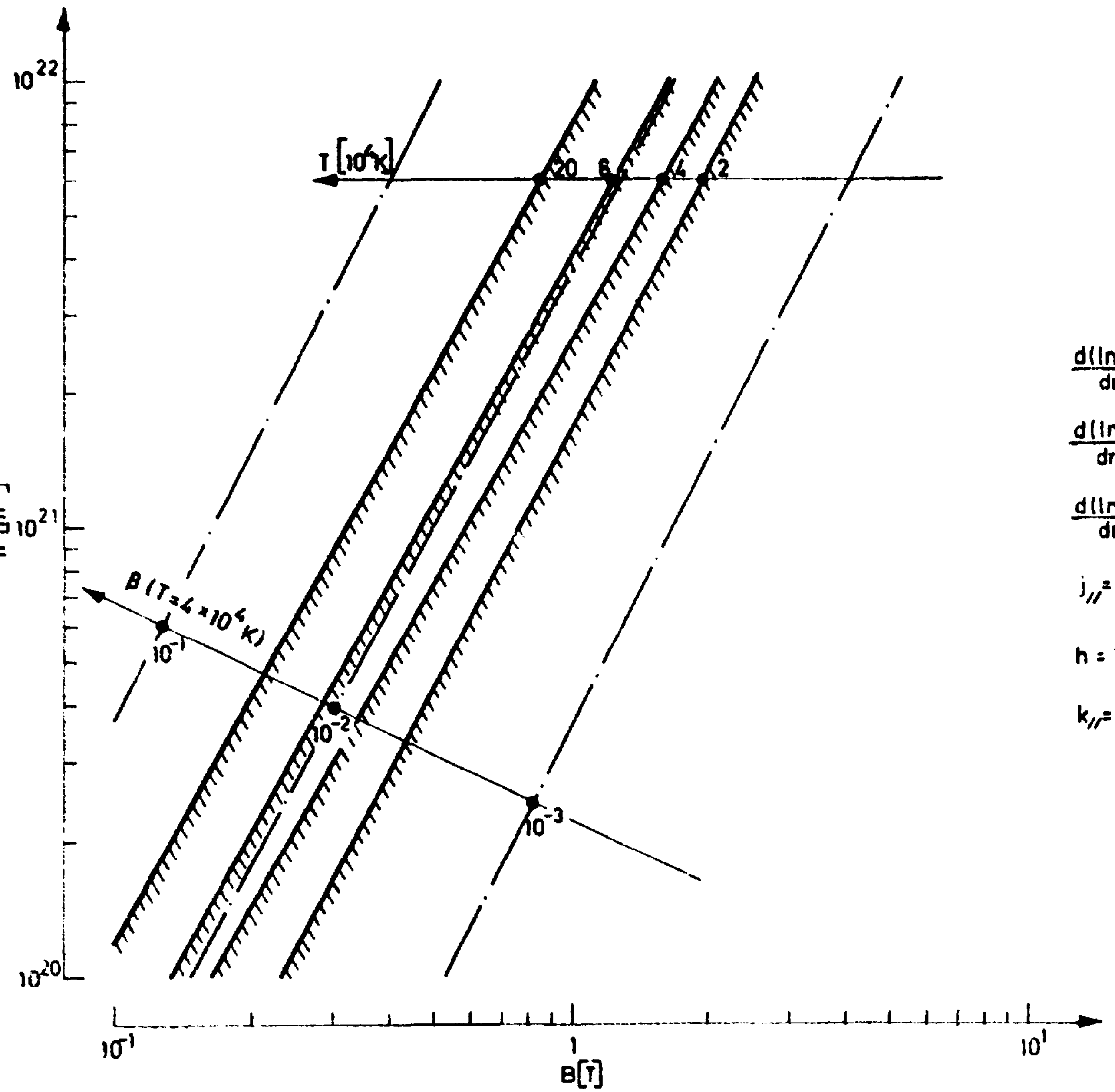

$n\,[m^{-3}]$
10^{22}
10^{21}
10^{20}
$B\,[T]$
10^{-1}
1
10^{1}
$\frac{n_n}{n}$
$\frac{d(\ln B)}{dr}\,[m^{-1}]$
$\frac{d(\ln B)}{dr}\,[m^{-1}]$
10
1
10^{-1}
10^{-2}
$T = 4\times10^{4}$ K
$\frac{d(\ln n)}{dr} = 1\ m^{-1}$
$\frac{d(\ln T)}{dr} = 10^{2}\ m^{-1}$
$\gamma = 100$
$h = 10^{2}$
$j_{//} = 10^{5}\ A\,m^{-2}$
$k_{//} = 0\ m^{-1}$
= adiabatic theory
= general theory

Fig. 2



$\frac{n_n}{n}$
10
1
10^{-1}
10^{-2}
$\frac{d(\ln B)}{dr}$ [m^{-1}]
10
1
10^{-1}
10^{-2}
$\frac{d(\ln B)}{dr}$ [m^{-1}]
10
1
10^{-1}
10^{-2}
10^{22}
10^{21}
10^{20}
n [m^{-3}]
10^{-1}
1
10^{1}
B [T]
$T = 4 \times 10^4$ K
$\frac{d(\ln n)}{dr} = 10^2$ m^{-1}
$\frac{d(\ln T)}{dr} = 10^2$ m^{-1}
$h = 10^2$
$j_{//} = 10^5$ A m^{-2}
$k_{//} = 0$ m^{-1}
= adiabatic theory
= general theory

Fig. 3



$\frac{d(\ln n)}{dr} = 10^2\ m^{-1}$

$\frac{d(\ln T)}{dr} = 0\ m^{-1}$

$\frac{d(\ln B)}{dr} = 1\ m^{-1}$

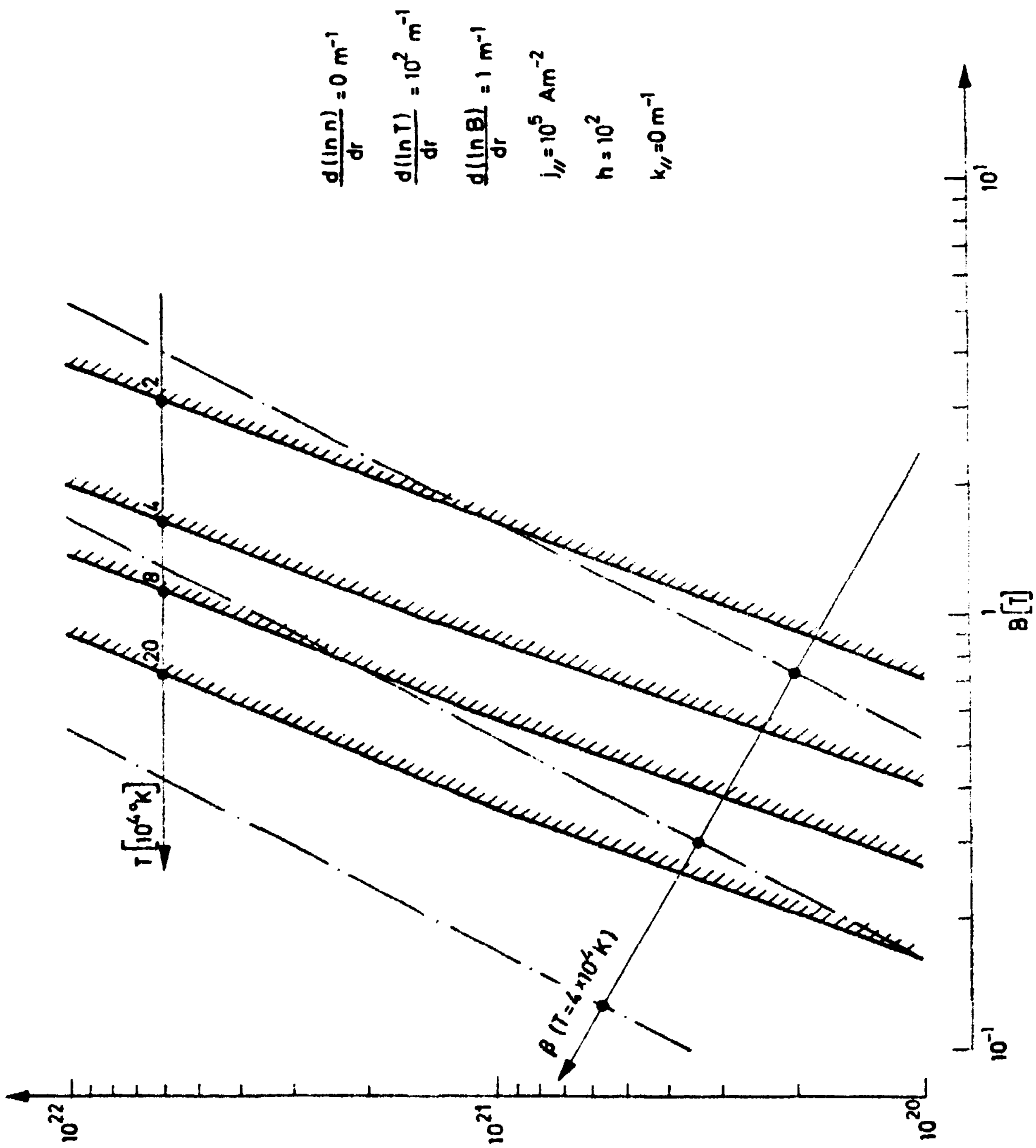
$j_{//} = 10^5\ Am^{-2}$

$h = 10^2$

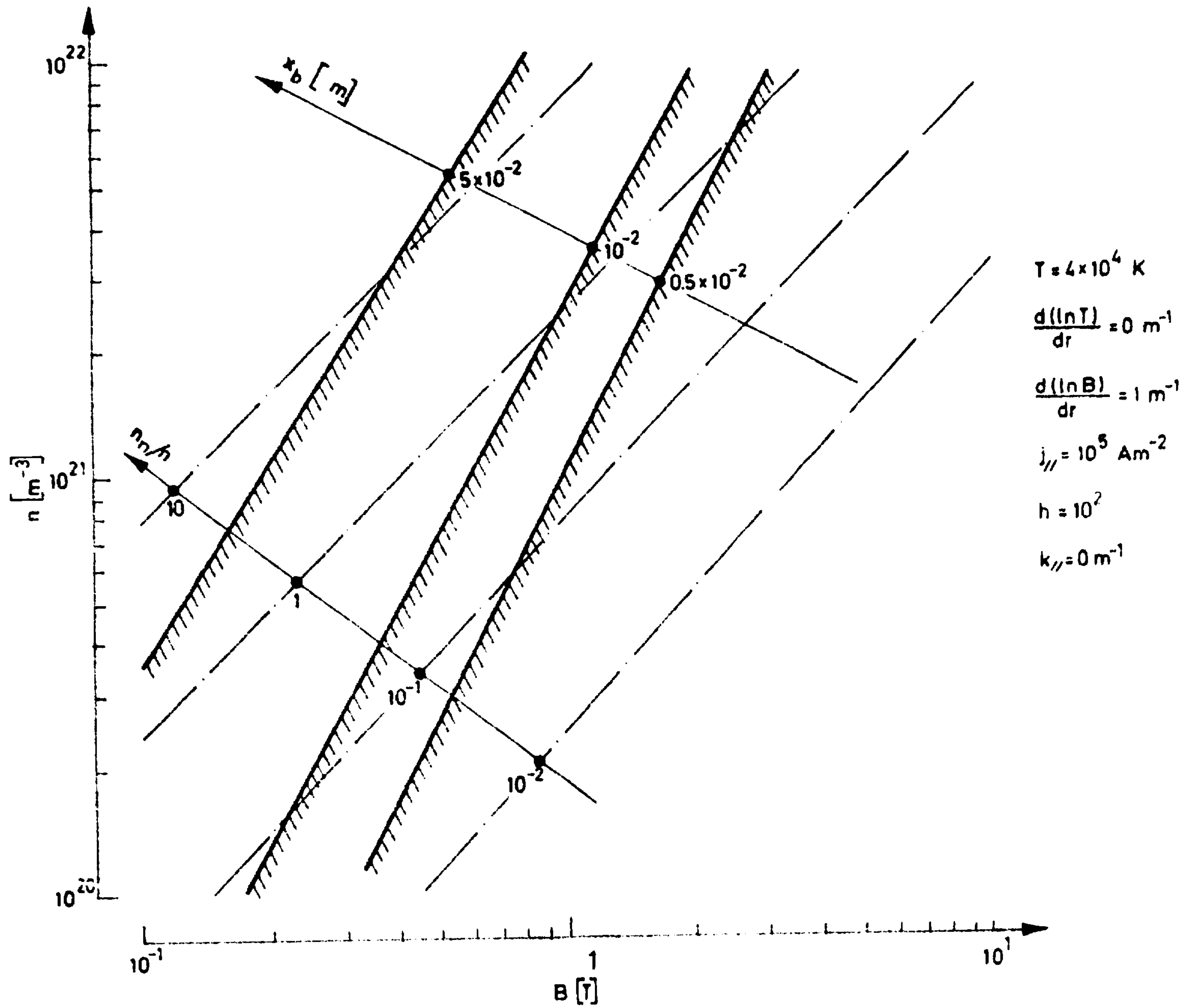
$k_{//} = 0\ m^{-1}$

Fig. 4

Fig. 5

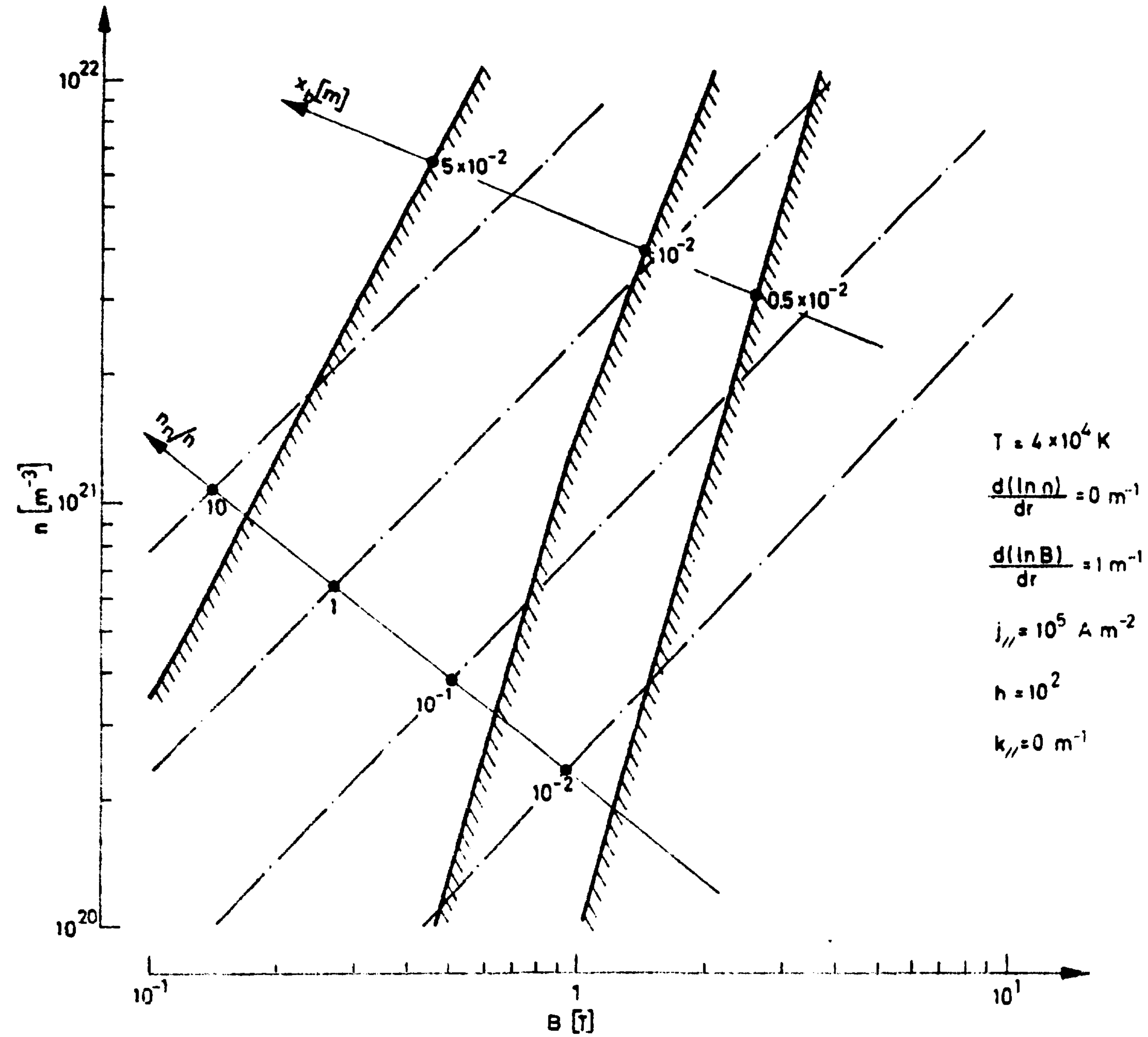


d(ln n)/dr = 0 m⁻¹
d(ln T)/dr = 10² m⁻¹
d(ln B)/dr = 1 m⁻¹
j// = 10⁵ Am⁻²
h = 10²
k// = 0 m⁻¹
T [10⁴ °K]
2
4
8
20
β (T = 4 × 10⁴ K)
n [m⁻³]
10²²
10²¹
10²⁰
10⁻¹
1
10¹
B [T]



10^{22}
10^{21}
10^{20}
n [m^{-3}]
10^{-1}
1
10^{1}
B [T]
x_b [m]
5×10^{-2}
10^{-2}
0.5×10^{-2}
n_n/h
10
1
10^{-1}
10^{-2}
$T = 4\times10^{4}$ K
$\frac{d(\ln T)}{dr} = 0$ m^{-1}
$\frac{d(\ln B)}{dr} = 1$ m^{-1}
$j_{//} = 10^{5}$ Am^{-2}
$h = 10^{2}$
$k_{//} = 0$ m^{-1}

Fig. 6



x_p [m]
5×10^{-2}
10^{-2}
0.5×10^{-2}
n_n/h
10
1
10^{-1}
10^{-2}
10^{22}
10^{21}
10^{20}
n $[m^{-3}]$
10^{-1}
1
10^{1}
B [T]
$T = 4\times10^{4}$ K
$\frac{d(\ln n)}{dr} = 0\ m^{-1}$
$\frac{d(\ln B)}{dr} = 1\ m^{-1}$
$j_{//} = 10^{5}\ A\ m^{-2}$
$h = 10^{2}$
$k_{//} = 0\ m^{-1}$

Fig. 7

TRITA-PFU-78-03

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NON-ADIABATIC STABILITY ANALYSIS OF CURRENT AND MAGNETIC CURVATURE DRIVEN MODES IN COLD PLASMAS PENETRATED BY NEUTRAL GAS

D. Ohlsson, August 1978, 27 p. in English

Previous stability theories concerning electrostatic current and magnetic curvature driven modes in cold plasma mantle boundary layers are generalized. In particular the commonly used adiabatic approximation is relaxed. In the general theory presented important new effects associated with heat conduction, ionization and ohmic heating are found. In combination with viscosity and resistivity these effects introduce additional stabilizing as well as destabilizing effects. Furthermore the present theory typically predicts similar stability properties as the adiabatic theory in the limit $|d(\ln T)/d(\ln n)| \ll 1$. However in the limit $|d(\ln T)/d(\ln n)| \gg 1$ the general theory predicts less favourable stability properties. One may speculate that these conclusions also apply to more general types of electrostatic modes associated with density and temperature gradients in cold plasma mantle boundary layers.

Key words: Cold mantle, boundary layers, stability, electrostatic modes, plasma-neutral gas interaction current driven modes, curvature driven modes, adiabatic approximation.