EXCITATION ENERGY OF THE LOWEST 2^+ AND 3^- LEVELS IN ^{32}Mg AND ^{146}Gd

by

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Abstract :

The excitation energy of the lowest 2^+ and 3^- levels are calculated for neutron rich Mg-isotopes as well as for N=82 isotones. The calculations are made by assuming quadrupole-quadrupole and octupole-octupole forces. The quasi-particles energies and occupation numbers are taken from the energy density method.

It is widely accepted that generally in even-even nuclei the lowest 2^+ level lyes below the lowest 3^- state. This is in agreement with simple estimates based on the liquid drop model by describing the low-energy collective states as surface vibrations. Exceptions to this systematic rule arise for particular nuclei corresponding to special shell structures, namely doubly magic nuclei like ^{16}O, ^{40}Ca and ^{208}Pb. In this paper we shall discuss two interesting cases : ^{32}Mg and ^{146}Gd, in connection with recent measurements [1,2].

Experimental results concerning the very neutron rich Na-isotopes [1] suggest the onset of a deformation region at neutron number N≈20, in clear contradiction with naive extrapolations of the magic character of neutron numbers observed for stable nuclei. Hartree-Fock calculations on a deformed basis provide some supports to this finding [3]. Actually this is a first evidence of an earlier claim obtained from the energy density method [4] that for neutron rich nuclei with $10<Z<18$, the neutrons are developing a large major shell from N=8 to N=34, together with the disappearance of the N=20 closure. It has to be noticed that nuclei belonging to this major shell may exhibit unusual features, specially in the middle of the shell, since the subshell space involves single particle levels of different parities, namely the 2s-1d and 1f-2p levels.

In the case of the N=82 isotones, the occurence of a low lying 3^- state is also related to a special subshell structure, as it will be shown in the discussion.

In the energy density method, the ground states are described by assuming spherical nuclear shapes and by chosing, as trial many body wave functions, a product of proton and neutron unprojected BCS-wave functions (the pairing correlations are supposed to be present only in T=1 states). The total energy is then minimized with respect to the single particle orbitals and

their occupation numbers. Consequently, for each nucleus, the energy density method provides us with the quasi-particle energies $\{E_j\}$ and the corresponding occupation number $\{V_j\}$. Here j stands for the usual shell model state quantum numbers : $|j\rangle = |n\ell j\ q\rangle$.

Within this framework, the most natural model for describing low energy excited states consists in considering quasi-particle excitations. In order to keep the computational problem to a reasonable level, the quasi-particles are assumed to interact via a long-range separable multipole force, following earlier works by Yoshida [5] and many others. The interaction hamiltonian between pairs of particles takes the form

$$H_\lambda(\text{long-range}) = -\tfrac{1}{2}\chi_\lambda \sum_{j,m,\mu} (-)^{\lambda-\mu} \langle j_1' m_1' | i^\lambda Y_{\lambda\mu}\, w_\lambda | j_1 m_1 \rangle * \tag{1}$$

$$* \langle j_2' m_2' | i^\lambda Y_{\lambda-\mu}\, w_\lambda | j_2 m_2 \rangle\, a^+_{j_1' m_1'}\, a^+_{j_2' m_2'}\, a_{j_2 m_2}\, a_{j_1 m_1}$$

$$w_\lambda = \left(\frac{m\omega_0}{\hbar}\right)^{\lambda/2} r^\lambda \tag{2}$$

The interaction strength χ_λ will be treated as a phenomenological parameter determined from the excitation energy of known levels. It is assumed to be equal between any proton-proton, neutron-neutron and proton-neutron pairs allowed by angular momentum and parity rules. Although such an assumption does not account properly for the isospin structure of the low energy states, we have verified that is does not affect the results significantly

[6]. By using the Tamm-Dancoff approximation, the eigenvalue problem reduces to the solution of the well-known secular equation

$$F_\lambda = F_\lambda^{(n)} + F_\lambda^{(p)} = \frac{2\lambda+1}{\chi_\lambda} \qquad (3)$$

where

$$F_\lambda^{(q)} = \sum_{j,j'} \frac{[\langle j' \| i^\lambda Y_\lambda \, w_\lambda \| j \rangle (u_{j'} v_j + v_{j'} u_j)]^2}{(E_{j'} + E_j - E_x)(1 + \delta_{jj'})} \delta_{qq'} \qquad (4)$$

Here q stands for the charge state ; E_x denotes the excitation energy. For computational convenience, the matrix elements of the multipole force are calculated by replacing the self-consistent radial wave functions by harmonic oscillator orbitals. The oscillator frequency is chosen in such a way to maximize a weighted sum of overlap integrals [4].

As far as the configuration space is concerned, it is known that multipole forces should be merely used for subshells close to the Fermi surface. On the other hand a too small configuration space render the results too sensitive to the subshell structure. In the present work the deeply bound levels are omitted since with realistic forces they yield matrix elements which are very small or even repulsive [7]. However all the bound and quasi bound orbitals above the Fermi surface have been taken into account, which compensates for the fact that the self-consistent radial wave functions are approximated by harmonic oscillator orbitals (see the discussion concerning the pairing correlations in ref. [4]) It

has to be reminded that the excitation energies are much less sensitive to the size of the configuration space than the transition probabilities which will not be considered here.

In the Mg-isotopes case, because of the lack of systematic data concerning the 3^- levels (specially for the neutron rich isotopes), the calculations have been extended to neighbouring nuclei, from Ne- to S-isotopes. This gives us a better determination of average coupling constants and a useful overview.

The results are displayed on fig.1, which shows the variation of the lowest 2^+ and 3^- energies as function of the neutron number N. The hachured area correspond to the calculated values assuming a small A-dependence for the coupling constants, namely

$$\chi_\lambda = \chi_0^\lambda + \chi_1^\lambda A \qquad (5)$$

which is valid only on a limited region in A.

The parameters are determined by fitting the experimental data. The upper and lower limits on E_x are given by the uncertainties on χ_0^λ.

For the 2^+-state, generally speaking we observe a rather smooth behaviour up to N=18, followed by an increase of about .5-1.0 Mev at N=20 and a sharp decrease at N=22. At this point the quadrupole correlations become very large and the very small or negative E_x obtained for N≥22 indicate clearly the onset of a deformation region, in agreement with the calculation of Campi et al. [3].

The quality of the fit is somewhat better in the S-isotopes. This may be connected to the occurence in the HF-BCS calculation of a noticeable subshell effect at Z=16. Furthermore the raising of the 2^+ is actually observed for ^{36}S.

As far as the 3^- state is concerned, the general behaviour is dominated by the strong decrease of the energy for N>14. It does not seem, however, that the decrease is strong enough to reach a point of crossing with the 2^+ state. This is supported by the actual situation in ^{36}S.

In order to try to get further arguments, the calculations have been repeated for the Mg-isotopes, fitting the coupling constants to the known levels in Mg only. The results are displayed in fig.2, where the lowest poles of F_λ are also shown. It clearly illustrates the particular behaviour of the quasi-particle energies. If in this calculations a crossing is obtained at N=20, the lowest state is not predicted low enough as compared to experiment.

Assuming the Mg-isotopes to become gradually deformed from N=14 will probably give a more consistent description of the observed slow decrease of the 2^+ energy against N, in contrast to what is found in Si- and S-isotopes. In this respect experimental data on neutron rich Ne-isotopes would be very valuable. However, if the Mg-isotopes require the introduction of the deformation degrees of freedom, one may wonder about the validity of our calculations for the 3^- state. Because of the very particular subshell situation involved here, the occurence of low lying negative parity states (or even a permanent octupole deformation) could be expected.

The situation is different in the N=82 isotones. A sufficient amount of experimental data allows a better determination of χ_2 and χ_3, and gives more confidence in the extrapolations. Again a small dependence of χ_λ against A as been introduced following eq.(5), but the uncertainties, or the fluctuation of χ_0^λ and χ_1^λ are less important than in the light nuclei, and can be neglected. The χ_λ are fitted to measured levels in nuclei with $52 \leq Z \leq 62$. The results are plotted in fig.3 from the doubly magic ^{132}Sn to ^{148}Dy. The crossing between the 2^+ and the 3^- levels in ^{146}Gd is well reproduced. This result is sensitive to the way the neighbouring nuclei are fitted. In particular, if the configuration space is restricted to its minimum (5 valence proton subshells), it becomes important to fit ^{142}Nd and ^{144}Sm to get the crossing. Note also that the $3^- - 2^+$ splitting in ^{146}Gd is of the order of $\approx$200 kev theoretically ($\sim$400 kev experimentally). The model used in this work is merely designed to yield general trends rather than small differences of this order.

Qualitatively, the occurence of a low-lying 3^--state for Z=64 and 66 is related to the subshell structure of the proton $50 \leq Z \leq 82$ major shell. In this case, the 1h11/2 level is appearing in the middle of the shell, whereas the 2d3/2 and 3s1/2 levels lye somewhat above the 1h11/2, an ordering which is well predicted by self-condistent calculations[4]. The situation is radically different for the corresponding valence neutron $50 \leq N \leq 82$: here the 1h11/2 orbit is the less bound level. Consequently in the Sn-isotopes no sign of the 3^- deepening is observed.

In conclusion, the energy density method seems a convenient starting point for semi-empirical estimates of the lowest 2^+ and 3^- excitation energies E_x. By using multipole forces for the long range interaction, as long as the coupling constants χ_λ vary slowly with A (or N and Z), the detail behaviour of E_x reflects the variation of quasi-particle energies and occupation numbers, and thus constitutes a test of the self-consistent calculations.

Even if the conclusions are more qualitative than quantitative, the present model yield the main features of E_x as a function of A. If the actual situation is well reproduced in the N=82 isotones, the answer is not so clear in the Mg-isotopes. The rather systematic discrepancy for the 2^+ level (N>14) suggests the onset of a deformation earlier than predicted by Campi et al. [3], but eventually in agreement with the claim that for $Z \simeq 12$ the neutron major shell goes from the 1d5/2 up to the 2p3/2 orbits. In such a case, however, our estimates of the 3^- levels are not valid. At present time the answer is more likely to come from experiments : in any case the present calculations indicate that valuable nuclear structure informations should arise from spectroscopic studies of the neutron rich light nuclei.

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Figure Captions

Fig. 1 Excitation energy of the first 2^+ and 3^- states against the neutron number N. The hachured area correspond to the theoretical estimate (see text). The full circles and triangles indicate the experimental energy of the 2^+ and 3^- respectively ; parentheses are put whenever the spin and parity are still uncertain. In Mg-isotopes the arrows indicate lower limits ; the X denotes the level seen in ^{32}Mg.

Fig. 2 The same as fig.1 for the Mg-isotopes. The thick horizontal lines indicate the position of the lowest pole of equation (4) for the 2^+ and 3^- respectively against the neutron number N.

Fig. 3 Excitation energy of the lowest 2^+ and 3^- states against the proton number Z for the N=82 isotones. The thick lines give the theoretical values (see text). The full circles and triangles indicate the experimental energy of the first 2^+ and 3^-.

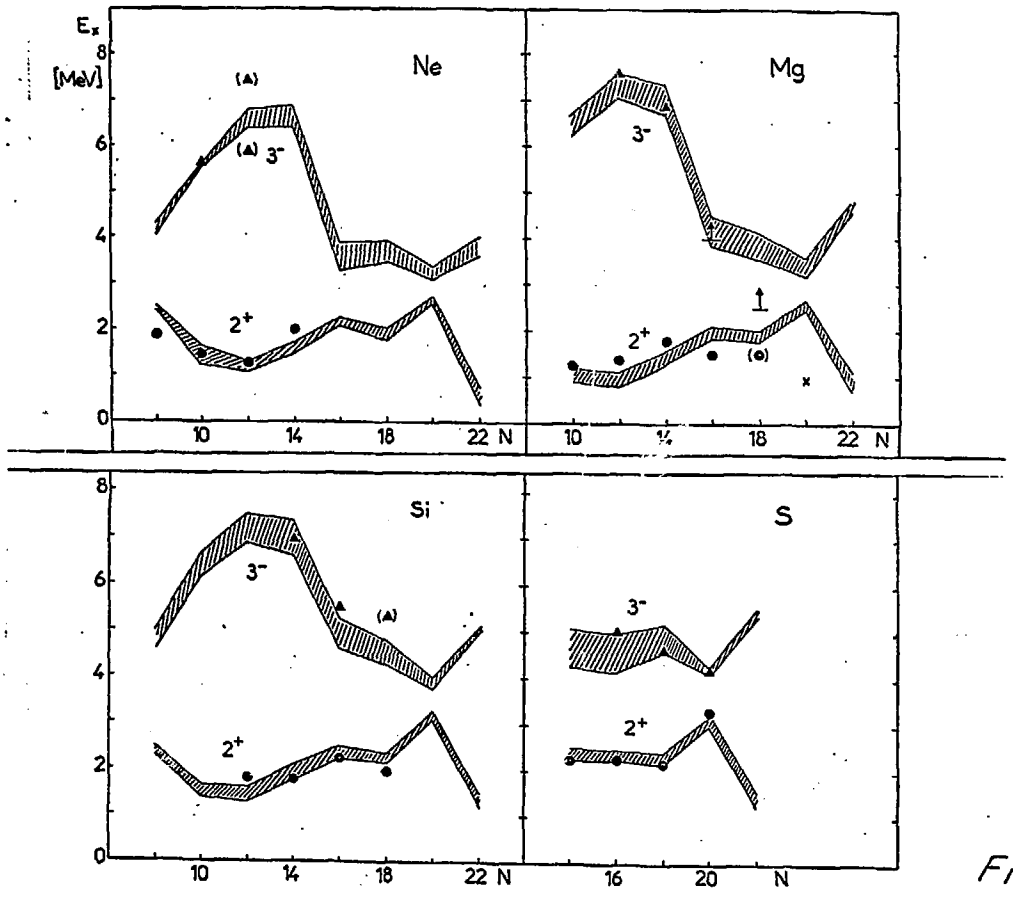

Fig 1

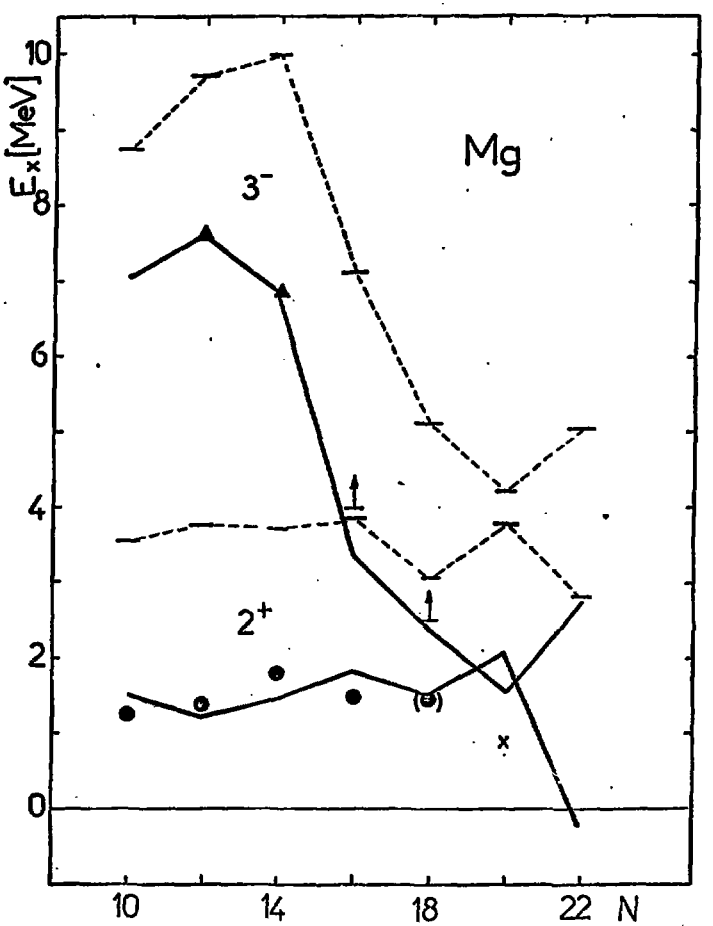

Fig 2

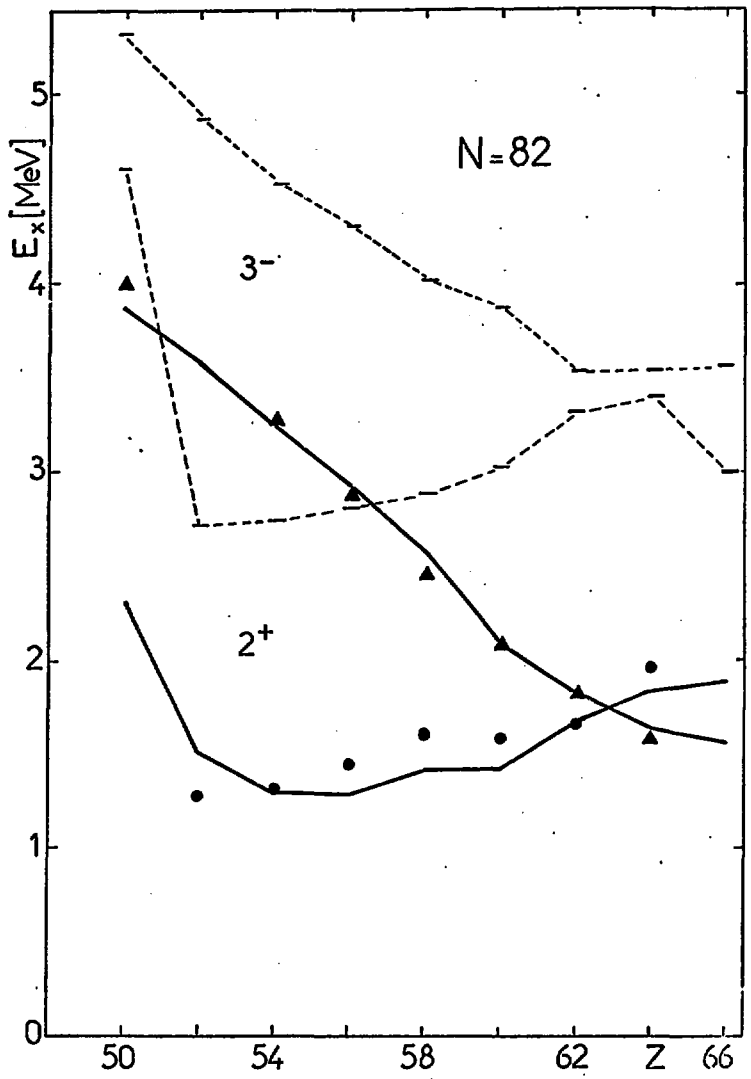

Fig 3