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Plasma Flow Driven by

Fusion-Generated Alpha Particles

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Synopsis

The confinement of fusion-generated alpha particles will affect the transports of the background plasma particles by the momentum transfer from the energetic alphas. The ions tend to migrate towards the center of plasma (i.e. fuel injection) and electrons towards the plasma periphery. This means the existence of a mechanism which enable to pump out the ashes in the fuel plasma because of the momentum conservation of whole plasma particles.

The orbit confinement of fusion-generated alpha particles is known to be an important requisite for magnetic confinement systems to operate as steady fusion reactors. Extensive study is performed recently in order to have a design criterion for magnetic systems as an absolute container of alpha particles[1].

For the absolute confinement of alpha particle the product aB_0 has to be larger than a certain constant which depends on the magnetic configuration, where a and B_0 represent the radius of plasma cross section and the strength of magnetic field respectively.

There is a good reason to believe that the source of the alpha particles in magnetic containers is located near elliptic magnetic axis, since the temperature profile in the plasma may take a maximum value on the axis. In this case the distribution function of alpha particle can be far from isotropic near the periphery of the confined plasma, even if the alpha particles are isotropically created in the source region. This means that the alpha particles could act as if the high energy helium beam was injected[2] in the direction of diamagnetic current flow near the periphery of plasma.

The purpose of the present paper is to point out an effect of such a localized alpha particle source to the confined, background plasma particles in some practical magnetic geometries. The background ions and electrons are assumed to be MHD fluids with cylindrical symmetry and the source of the alpha particles is assumed, for simplicity, to be located just on the axis of symmetry, the z axis. We use the cylindrical coordinate system, (r, θ, z), throughout in the discussions.

The plasma electrons and ions including impurities receive momentum through collisions with alpha particles. This momentum transfer causes the ions and the electrons to drift across the magnetic field. The drift velocity is given by

$$v_j = - \frac{\vec{B} \times \vec{P}_j}{Ze\, n_j B^2} , \qquad (1)$$

where $\vec{P}_j$ represents the momentum transfer per unit time from alpha particles to the plasma particles, n the density and the suffix j corresponds to the species which receive the momentum transfer. The notations Z and e are the charge number and the unit charge and $\vec{B}$ the magnetic field vector, i.e. $\vec{B}=\vec{B}(0, B_\theta(r), B_z(r))$.

Since the energy of alpha particles is sufficiently higher than the background plasma particles, the effect of momentum transfer on the motion of the alpha particles themselves may be neglected. Then, in the cylindrical symmetric system we have three constants of motion for a single alpha particle.

$$\begin{aligned} &M u_z + 2eA_z = \text{constant} \equiv C_1 , \\ &(M u_\theta + 2eA_\theta) r = \text{constant} \equiv C_2 , \qquad (2) \\ &\frac{M}{2}(u_r{}^2 + u_\theta{}^2 + u_z{}^2) = \text{constant} \equiv C_3 , \end{aligned}$$

where the electric field is assumed to be sufficiently weak for alpha particle, and $\vec{u}(u_r, u_\theta, u_z)$ is the velocity vector of alpha particle, M its mass, A_θ and A_z the θ and the z components of the vector potential, i.e.

$$B_\theta = -\frac{\partial A_z}{\partial r} \qquad \text{and} \qquad B_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) \quad .$$

By the assumption that the source of the alpha particles is located just on the z axis, the constants of motion become

$$C_1 = M\,u_0 \cos\nu + 2e\,A_{z0} \quad , \tag{3}$$

$$C_2 = 0 \tag{4}$$

and $$C_3 = \frac{M}{2}u_0^2 \quad , \tag{5}$$

where u_0 is the velocity of the alpha particle and ν is the initial pitch angle between the z axis and the initial velocity vector of the alpha particle, i.e. $0 \leqq \nu \leqq \pi$, and A_{z0} is the value of vector potential at r=0. Then from (2) together with (3), (4) and (5) we have

$$u_r^2 + (\frac{2e}{M}A_\theta)^2 + \{u_0\cos\nu + \frac{2e}{M}(A_{z0} - A_z)\}^2 = u_0^2. \tag{6-a}$$

The region of possible particle positions is estimated by (6-a) for given pitch angle ν:

$$u_0^2 - (\frac{2e}{M}A_\theta)^2 - \{u_0\cos\nu + \frac{2e}{M}(A_{z0} - A_z)\}^2 \geqq 0 \ . \tag{7}$$

We are now able to consider a specific practical configuration. An example we give is the mathematically simplest MHD equilibrium which satisfy

$$B_z\frac{dB_z}{dr} + \frac{B_\theta}{r}\frac{d}{dr}(rB_\theta) + \mu_0\frac{dP}{dr} = 0, \tag{8}$$

where P represent the plasma pressure and μ_0 the magnetic permeability of vacuum. This means that we neglect the contributions from alpha particles to the magnetic field.

We choose

$$P = P_0 \left(1 - \frac{r^2}{a^2}\right) \qquad (9)$$

and

$$B_z = B_0 = \text{constant} .$$

Then from (8) and (9) we have

$$B_\theta = \left(\frac{\beta}{2}\right)^{1/2} B_0 \frac{r}{a} , \qquad (10)$$

where $\beta \equiv 2\mu_0 \, P_0/B_0{}^2$.

This example corresponds to a cylindrical tokamak if $\beta << 1$. We focus our attention here to the case of $\beta << 1$. The vector potential A_θ and A_z, then, become

$$A_\theta = \frac{B_0}{2} r$$

and (11)

$$A_z = - \left(\frac{\beta}{2}\right)^{1/2} \frac{B_0 r^2}{2a} .$$

Obviously, the maximum attainable distance of the alpha particle orbit from the z axis must be less than the plasma column radius, a, in order to confine absolutely all the alpha particles produced on the z axis in the configuration. It turns out in this case that the pitch angle giving largest attainable radius varies from $\pi/2 \le \nu < \pi$. We find that if T_c is the critical value of $(u_0/a\Omega)^2$ required to contain all the alpha particle produced

$$T_c = \frac{1}{4} , \qquad (12)$$

where $\Omega \equiv 2eB_0/M$.

For absolute confinement of all the alpha particle produced, we must have $(u_0/a\Omega)^2 \leq T_c$. This means, in this case,

$$a \geqq \frac{2Mu_0}{eB_0} \quad . \tag{13-a}$$

In the present consideration the radius of the plasma column is assumed to be chosen that the marginal condition for the absolute confinement of alpha particle is satisfied.

$$a = \frac{2Mu_0}{eB_0} \quad . \tag{13-b}$$

By solving the inequality (7) with respect to cos ν, we see the initial pitch angle of the alpha particles which are able to arrive at the positon $r \leqq a$.

$$0 \leq \nu_1 \leq \nu \leq \nu_2 \leqq \pi \quad , \tag{14}$$

where

$$\cos\nu_1 = \frac{2e}{Mu_0}\left(A_z - A_{z_0}\right) + \left\{1 - \left(\frac{2eA_\theta}{Mu_0}\right)^2\right\}^{1/2} \tag{15}$$

and

$$\cos\nu_2 = \frac{2e}{Mu_0}\left(A_z - A_{z_0}\right) - \left\{1 - \left(\frac{2eA_\theta}{Mu_0}\right)^2\right\}^{1/2} \quad . \tag{16}$$

We define the average velocity , $\vec{w} = \vec{w}(w_r, w_\theta, w_z)$, of the alpha particles at the position r as

$$w_\theta = \frac{1}{4\pi}\int_{\nu_1}^{\nu_2} u_\theta \sin\nu \, d\nu \tag{17-a}$$

$$w_z = \frac{1}{4\pi}\int_{\nu_1}^{\nu_2} u_z \sin\nu \, d\nu \tag{18-a}$$

and by axisymmetry .

$$w_r = 0 \quad . \tag{19}$$

Straightforward integration leads to the explicit expression for the average velocity of alpha particle.

$$w_\theta = -\frac{e}{\pi M} A_\theta \cdot \{ 1 - (\frac{2eA_\theta}{M u_0})^2 \}^{1/2} , \tag{17-b}$$

$$w_z = 0 \quad . \tag{18-b}$$

We are in a position to estimate the term of the momentum trasfer, $\vec{P}_j$, from alpha particles to plasma particles in (1). We shall assume[3)]

$$\vec{P}_j = M N \zeta_j \vec{w} \quad , \tag{20}$$

where N is the density of the alpha particle and the constant ζ_j the effective rate of collision with alpha particles. From (1) and (20) together with (17-b), (18-b) and (19) a formula for the drift velocity, $\vec{v}_j = \vec{v}_j(v_j, 0, 0)$, in the cylindrical tokamak plasma becomes

$$v_j = -\frac{\zeta_j}{nZ}(\frac{N}{n_j}) \frac{B_z}{B^2} A_\theta \{1 - (\frac{2eA_\theta}{Mu_0})^2\}^{1/2} , \tag{21-a}$$

$$\simeq -\frac{\zeta_j}{\pi Z} (\frac{N}{n_j}) \frac{A_\theta}{B_z} \{1 - (\frac{2eA_\theta}{Mu_0})^2\}^{1/2} \quad . \tag{21-b}$$

Using (9), (11) and (13-b), the formula (21-b) is rewritten as

$$v_j = -\frac{\zeta_j}{2\pi Z} (\frac{N}{n_j}) r \{1 - (\frac{r}{a})^2\}^{1/2} \quad . \tag{21-c}$$

The drift velocity v_j, then, takes its peak value at

$$r = \frac{a}{\sqrt{2}} \quad . \tag{22}$$

The peak velocity becomes

$$v_j = - \frac{\zeta_j}{4\pi Z} \left(\frac{N}{n_j}\right) \frac{u_0}{\Omega} \quad . \tag{23}$$

Thus, we see that the drift velocity is positive for electrons and negative for ions. For $N/n_j \simeq 10^{-2}$, $n_j \simeq 10^{14}$ cm^{-3} and $B_0 \simeq 1$ Tesla, low Z impurities ($Z \simeq 3 \sim 4$) can drift into the fuel plasma with the velocity of order $v_j \simeq$ 1cm/sec.

We conclude therefore that the confinement of the fusion-generated alpha particles may introduce a new transport of the background plasma. The quantitative value of the flow velocity for each species depends on the electron temperature of the plasma.

By this new transport we may emphasize that there are some possibilities of new phenomenon in the fuel plasma as follows:

i) Enhanced impurity concentration unless an efficient divertor is invented.

ii) Fuel penetration could be enhanced by the alpha particles.

iii) Plasma will charge up positively by the outward drift of the electrons. This will affect microscopic instabilities (such as drift instability) and the related turbulence.

iv) Alpha particles will be automatically pumped out because of the momentum conservation law.

More precise analysis using Fokker-Planck equation will be necessary in order to have a definite answer. We leave it a future study.

References

1) K. Ikuta and C.G. Gimblett; Nuclear Fusion 18 (1978) 451.

2) I. Ohkawa; Kakuyugo-Kenkyu 32 (1974) 1.

3) See for example A.A. Vedenov; in Theory of Turbulent Plasma (Iliffe Books LTD, London) p.98