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ORNL/TM-6772

Wall Effects on the Absorption of Electron Cyclotron Waves in an EBT Plasma

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OPERATED BY UNION CARBIDE CORPORATION · FOR THE DEPARTMENT OF ENERGY

ORNL/TM-6772
Dist. Category UC-20 f, g

Contract No. W-7405-eng-26

FUSION ENERGY DIVISION

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Date Published: March, 1979

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DEPARTMENT OF ENERGY

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ABSTRACT

The absorption of electron cyclotron waves propagating along an externally applied magnetic field in a uniform plasma surrounded by a cylindrical metallic cavity wall is studied. In the model, the cavity wall, the vacuum-plasma interface, and the effects of finite electron temperature are considered, and the dispersion relation for the wave propagation is derived. The results are then applied to the ELMO Bumpy Torus (EBT-I) plasma, and the propagation characteristics are computed. The wave absorption in the ordinary mode is found to be a result of the wall effects, which cannot be predicted with the infinite plasma theory. The loaded quality factor, Q_L, is also estimated from the model to be about 12, which is in good agreement with the experimentally observed value.

1. INTRODUCTION

The propagation and absorption characteristics of the electron cyclotron waves propagating in a bounded plasma are investigated. In this preliminary work, the wave propagation is assumed to be in the direction of a constant external magnetic field. This subject has a particular relation to the ELMO Bumpy Torus (EBT) because of the utilization of intense microwaves in the device for heating the electrons through cyclotron damping. Here the primary interest is to see the effects of the metallic cavity wall on the cyclotron absorption, especially on the ordinary mode, even though the wavelength is much shorter than the scale dimension of the cavity plasma.

The theory of electromagnetic wave propagation in a bounded plasma in the direction of an applied magnetic field has been studied in great detail (see, for example, Ref. 1). However, the calculations are mainly based on the cold plasma assumption, i.e., $\partial\varepsilon_{ij}/\partial k_i = 0$, where ε_{ij} and k_i are the components of the dielectric tensor, $\underline{\underline{\varepsilon}}$, and the propagation vector, $\underline{k}$, respectively. But this assumption cannot adequately treat the important problem of the cyclotron wave damping in a bounded plasma. Therefore, in this study the cold plasma dielectric tensor will be modified with the finite temperature corrections.

The infinite plasma theory predicts that the ordinary mode is not completely absorbed unless one includes the finite Larmor radius effects.[2] Even with this correction only modest absorption may be observed. As shown in Sect. 3, introducing the plasma-wall boundary conditions into the calculations of a bounded plasma permits a considerable absorption for the ordinary wave.

Before we move on to the bounded plasma calculations, we briefly review the cyclotron wave absorption in an infinite plasma.

2. INFINITE PLASMA

The following assumptions are made:

(1) The collisionless Maxwellian plasma is homogeneous, and the external uniform magnetic field, $\underline{B}_o$, is along the z-axis.

(2) The electron cyclotron frequency, $\Omega(>0)$, is larger than the electron plasma frequency, ω_p.

(3) The frequency of the propagating wave, ω, is close to the electron cyclotron frequency.

(4) The propagation is almost along the external magnetic field, i.e., $k_z^2 \gg k_\perp^2$, where k_z and $k_\perp$ (or k_x) are the components of the wave vector parallel and perpendicular to $\underline{B}_o$, respectively.

(5) The ion contribution is ignored, since ω is much larger than the ion cyclotron frequency.

Considering the first-order finite Larmor radius effects, we may give the components of the dielectric tensor as[2]

$$\varepsilon_{11} = \varepsilon_{22} \equiv \varepsilon_1 = 1 - \frac{\omega_p^2}{2\omega(\omega + \Omega)} + \frac{\omega_p^2}{2\omega k_z \alpha} Z(\xi) ,$$

$$\varepsilon_{12} = -\varepsilon_{21} \equiv i\varepsilon = -i \left[\frac{\omega_p^2}{2\omega(\omega + \Omega)} + \frac{\omega_p^2}{2\omega k_z \alpha} Z(\xi) \right] ,$$

$$\varepsilon_{13} = \varepsilon_{31} = - \frac{\omega_p^2}{4\omega\Omega k_z} k_\perp \frac{dZ(\xi)}{d\xi} ,$$

$$\varepsilon_{23} = -\varepsilon_{32} = i\varepsilon_{13} ,$$

$$\varepsilon_{33} \equiv \varepsilon_3 = 1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_p^2}{4\omega k_z \alpha} k_\perp^2 \rho^2 \xi \frac{dZ(\xi)}{d\xi} ,$$

where $Z(\xi)$ is the plasma dispersion function with $\xi = (\omega - \Omega)/k_z\alpha$, $\alpha^2 \equiv 2T_e/m_e$, and $\rho \equiv \alpha/\Omega$. Here, T_e is the temperature, m_e is the mass, and ρ is the Larmor radius of the electrons.

The nonzero elements of $\underline{\underline{\varepsilon}}(\omega,\underline{k})$ for the electron cyclotron waves, $\omega \cong \Omega$, may be rewritten as

$$\varepsilon_1 \cong 1 + \sigma \ , \tag{1}$$

$$\varepsilon \cong -\sigma \ , \tag{2}$$

$$\varepsilon_3 \cong 1 \ , \tag{3}$$

with

$$\sigma = i\sqrt{\pi}\,\frac{\omega_p^2}{2\omega k_z\alpha} \ . \tag{4}$$

Here, it is assumed that $\omega_p^2/4\Omega^2 \ll 1$ and $|\sigma| \gg 1$.

In order to estimate the absorption coefficient of the wave, i.e., Im k_z, we start with the vector wave equation,

$$\underline{\eta} \times (\underline{\eta} \times \underline{E}) + \underline{\underline{\varepsilon}}\cdot\underline{E} = 0 \ , \tag{5}$$

where $\underline{E}$ is the electric field of the wave and $|\underline{\eta}| = |c\underline{k}/\omega|$ is the refractive index. Equation (5) can also be written in the form

$$\underline{\underline{\Lambda}} \cdot \underline{E} = \begin{bmatrix} \varepsilon_1 - \eta_z^2 & i\varepsilon & \eta_z\eta_x \\ -i\varepsilon & \varepsilon_1 - \eta^2 & 0 \\ \eta_x\eta_z & 0 & \varepsilon_3 - \eta_x^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$

($\eta_x \equiv ck_x/\omega$, $\eta_z \equiv ck_z/\omega$), which yields to the dispersion relation by

$$\det\,[\underline{\underline{\Lambda}}] = 0$$

or

$$\eta_z^4 + [(\varepsilon_1 + \varepsilon_3)\eta_x^2 - 2\varepsilon_1\varepsilon_3]\eta_z^2/\varepsilon_3 + (\varepsilon_1^2 - \varepsilon^2)$$

$$- (\varepsilon_1^2 - \varepsilon^2 + \varepsilon_1\varepsilon_3 - \varepsilon_1\eta_x^2)\eta_x^2/\varepsilon_3 = 0 \ .$$

Using Eqs. (1)-(3), we obtain

$$\eta_z^4 - \tilde{A}\eta_z^2 + \tilde{C} = 0 \ , \qquad (6)$$

where

$$\tilde{A} = 2(1 + \sigma) - \eta_x^2(2 + \sigma)$$

and

$$\tilde{C} = [(1 + \sigma)\eta_x^4 + (1 + 2\sigma) - (2 + 3\sigma)\eta_x^2] \ .$$

The solution of Eq. (6) gives two distinct modes with k_{z_1} and k_{z_2}, the extraordinary mode and the ordinary mode.

For the extraordinary mode,

$$k_{z_1}^2 = (1 + 2\sigma)k^2 - (1 + \sigma)k_x^2 \ . \qquad (7)$$

Here,

$$k_{z_1}^2 \gtrsim k^2 = (\omega/c)^2 \ .$$

Using Eq. (4), we obtain the absorption coefficient,

$$\mathrm{Im}\ k_{z_1} \cong \frac{1}{2}\left[\sqrt{\pi}\,\frac{\omega_p^2}{2\omega\alpha}\,(2k^2 - k_x^2)\right]^{1/2}$$

(here $k_x^2 > 0$, and it is given), which becomes maximum if the wave propagates along the external magnetic field.

For the ordinary mode,

$$k_{z_2}^2 = k^2 - k_x^2 \lesssim k^2 \ . \qquad (8)$$

From Eq. (8), we see that under the assumptions which are made for the plasma, the ordinary mode experiences almost no absorption, i.e., $\mathrm{Im}\ k_{z_2} \doteq 0$, and propagates only if $k^2 > k_x^2$. Some absorption may be seen if the finite temperature effects are considered in the dispersion relation, but a substantial wave absorption is not expected.

3. BOUNDED PLASMA

We now study the same problem for the bounded plasma surrounded with a perfectly conducting wall, with the following additional assumption: a long cylindrical cavity with radius R_o is uniformly filled by the plasma up to a radius of R_p (i.e., a "square density" profile), where R_o may be greater than or equal to R_p. We should mention that the large aspect ratio nature of the EBT device enables us to ignore the toroidal geometry effects in calculations.

Taking the space and time dependence of the field quantities $\underline{E}$ and $\underline{B}$ to be of the form

$$\underline{E}, \underline{B} = \underline{E}(r), \underline{B}(r) e^{i(k_z z + m\phi - \omega t)} ,$$

with the azimuthal wave number, m, in the cylindrical geometry, we find that the Maxwell equations yield

$$\begin{bmatrix} -ik\varepsilon_1 & k\varepsilon & 0 & ik_z \\ -k\varepsilon & -ik\varepsilon_1 & -ik_z & 0 \\ 0 & -ik_z & -ik & 0 \\ ik_z & 0 & 0 & -ik \end{bmatrix} \begin{bmatrix} E_r \\ E_\phi \\ B_r \\ B_\phi \end{bmatrix} = \begin{bmatrix} imB_z/r \\ -\dfrac{\partial B_z}{\partial r} \\ -imE_z/r \\ \dfrac{\partial E_z}{\partial r} \end{bmatrix} . \tag{9}$$

In addition to Eq. (9), we have

$$B_\phi + r \frac{\partial B_\phi}{\partial r} - imB_r = -ikr\varepsilon_3 E_z \tag{10}$$

and

$$E_\phi + r \frac{\partial E_\phi}{\partial r} - imE_r = ikrB_z . \tag{11}$$

With the use of Eqs. (9)-(11), the following coupled relations for E_z and B_z are easily obtained:

$$\Delta_\perp E_z + aE_z = bB_z \ , \tag{12}$$

and

$$\Delta_\perp B_z + cB_z = dE_z \ , \tag{13}$$

where

$$\begin{aligned} a &= \varepsilon_3(k^2\varepsilon_1 - k_z^2)/\varepsilon_1 \ , \\ b &= i\varepsilon k k_z/\varepsilon_1 \ , \\ c &= [(\varepsilon_1^2 - \varepsilon^2)k^2/\varepsilon_1 - k_z] \ , \\ d &= -i\varepsilon\varepsilon_3 k k_z/\varepsilon_1 \ , \end{aligned} \tag{14}$$

and

$$\Delta_\perp \equiv \nabla^2 - \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \left(\frac{m}{r}\right)^2 \ .$$

Examination of Eqs. (12) and (13) shows that pure TE (transverse electric or H-wave, i.e., $E_z = 0$, $B_z \neq 0$) and TM (transverse magnetic or E-wave, i.e., $B_z = 0$, $E_z \neq 0$) modes do not exist in the system but are coupled to each other. Therefore, the plasma contains both E_z and B_z, which indicates that hybrid modes are present rather than pure TE or TM modes.

The coupled equations may be solved for E_z (or B_z) by seeking a solution in the form

$$\Delta_\perp E_z = -k_\perp^2 E_z \ , \tag{15}$$

which leads to

$$(k_\perp^4 - 2pk_\perp^2 + q)E_z = 0$$

or

$$k_{\perp_{1,2}}^2 = p \pm (p^2 - q)^{1/2} , \tag{16}$$

where

$$2p = (a + c) = \left(\frac{\varepsilon_1^2 - \varepsilon^2}{\varepsilon_1} + \varepsilon_3\right) k^2 - \left(\frac{\varepsilon_3}{\varepsilon_1} + 1\right) k_z^2$$

and

$$q = (ac - bd) = \frac{\varepsilon_3}{\varepsilon_1} [(\varepsilon_1 k^2 - k_z^2)^2 - (\varepsilon k^2)^2] \equiv \frac{\varepsilon_3}{\varepsilon_1} \Delta .$$

For the values of $k_{\perp_{1,2}}$, the solution of Eq. (15) is given in terms of the Bessel, $J_m(x)$, and the Neumann, $N_m(x)$, functions provided that E_z (or B_z) is a bounded function at $r = 0$. Hence

$$E_z(r) = AJ_m(k_{\perp_1} r) + BJ_m(k_{\perp_2} r) , \tag{17}$$

$$B_z(r) = \frac{A}{b} (a - k_{\perp_1}^2) J_m(k_{\perp_1} r) + \frac{B}{b} (a - k_{\perp_2}^2) J_m(k_{\perp_2} r) . \tag{18}$$

The rest of the components of the field are easily obtained with the aid of Eqs. (10) and (11).

In general, there is a vacuum gap between the plasma and the cavity wall. Therefore, we must obtain $\underline{E}$ and $\underline{B}$ in the vacuum in order to satisfy wall-vacuum and vacuum-plasma boundary conditions. In the vacuum defined by $R_p \leqslant r \leqslant R_o$, the dielectric tensor is unity, i.e., $\varepsilon_1 = \varepsilon_3 = 1$ and $\varepsilon = 0$. Hence from Eq. (14), we find

$a = k^2 - k_z^2$, $b = d = 0$, and $c = a$.

Therefore,

$$k_{\perp_1}^2 = k_{\perp_2}^2 \equiv k_\perp^2 = k^2 - k_z^2 , \tag{19}$$

and the longitudinal field components become

$$E_z(r) = CJ_m(k_\perp r) + DN_m(k_\perp r) \tag{20}$$

and

$$B_z(r) = FJ_m(k_\perp r) + GN_m(k_\perp r) . \tag{21}$$

The rest of the elements of the field are expressed through E_z and B_z by means of the relations

$$E_\phi = - \frac{ik}{k_\perp^2} \frac{\partial B_z}{\partial r} - \frac{mk_z}{rk_\perp^2} E_z \tag{22}$$

and

$$B_\phi = \frac{ik}{k_\perp^2} \frac{\partial E_z}{\partial r} - \frac{mk_z}{rk_\perp^2} B_z . \tag{23}$$

From the boundary condition, which requires that all the tangential components of the electric field on the perfectly conducting wall must vanish,[3]

$$E_z(r = R_o) = E_\phi(r = R_o) = 0 ,$$

and

$$C/D = N_m(k_\perp R_o)/J_m(k_\perp R_o)$$

and

$$F/G = -N_m'(k_\perp R_o)/J_m'(k_\perp R_o) \ .$$

In the plasma (where $0 \leq r \leq R_p$), on the other hand, we have

$$E_z(r) = AJ_m(k_{\perp_1} r) + BJ_m(k_{\perp_2} r) \ ,$$

$$E_\phi(r) = -AH_1 k_{\perp_1} J_m'(k_{\perp_1} r) - BH_2 k_{\perp_2} J_m'(k_{\perp_2} r)$$

$$- \frac{AP_3}{r} J_m(k_{\perp_1} r) - \frac{BP_4}{r} J_m(k_{\perp_2} r) \ ,$$

$$B_z(r) = AP_1 J_m(k_{\perp_1} r) + BP_2 J_m(k_{\perp_2} r) \ ,$$

and

$$B_\phi(r) = ASk_{\perp_1} J_m'(k_{\perp_1} r) + BTk_{\perp_2} J_m'(k_{\perp_2} r)$$

$$+ \frac{AK}{r} J_m(k_{\perp_1} r) + \frac{BL}{r} J_m(k_{\perp_2} r) \ ,$$

where

$$P_1 = M/b \ , \ P_2 = N/b \ , \ P_3 = \frac{mk^2\varepsilon_1}{k_z \Delta} M \ , \ P_4 = \frac{mk^2\varepsilon_1}{k_z \Delta} N \ ,$$

$$M = (a - k_{\perp_1}^2) \ , \ N = (a - k_{\perp_2}^2) \ ,$$

$$H_1 = \frac{Q}{k_z \varepsilon \Delta} \ , \ H_2 = \frac{P}{k_z \varepsilon \Delta} \ ,$$

$$Q = k^2 k_z^2 \varepsilon^2 + \varepsilon_1 (k^2 \varepsilon_1 - k_z^2) M \ ,$$

$$P = k^2 k_z^2 \varepsilon^2 + \varepsilon_1 (k^2 \varepsilon_1 - k_z^2) N ,$$

$$S = (\Delta_{44} - Mk^2 k_z \varepsilon / b)/\Delta ,$$

$$T = (\Delta_{44} - Nk^2 k_z \varepsilon / b)/\Delta ,$$

$$K = imk_z^2 [1 + i(k^2 \varepsilon_1 - k_z^2) M / k_z b]/\Delta ,$$

$$L = imk_z^2 [1 + i(k^2 \varepsilon_1 - k_z^2) N / k_z b]/\Delta ,$$

$$\Delta_{44} = ik\varepsilon_1 (k^2 \varepsilon_1 - k_z^2) - ik^3 \varepsilon^2 ,$$

and all the derivatives are with respect to the argument of the functions.

Again from the boundary condition, which requires that the tangential components of the fields be continuous across the plasma-vacuum boundary,[3]

$$[E_z] = [E_\phi] = [B_z] = [B_\phi] = 0 ,$$

with

$$[A] = (A^{pl} - A^{vac})_{r=R_p} ,$$

the constants A, B, C, and D can be eliminated from the equations. This process immediately leads to the dispersion relation of the wave:

$$\begin{aligned} &\{T_1 T_9 - T_7 [J_m(\lambda_1) - T_1 T_8 / T_6]\} [J_m(\lambda_2)(T_3 T_6 + T_1 T_2 P_2) \\ &\quad - T_1 T_5 T_6] = [T_1 T_4 T_6 - J_m(\lambda_1)(T_3 T_6 + T_1 T_2 P_1)] \qquad (24) \\ &\quad \times [J_m(\lambda_2)(T_7 - T_1 T_8 P_2 / T_6) - T_1 T_{10}] , \end{aligned}$$

where

$$T_1 = [J_m(\kappa) - J_m(\mu)N_m(\kappa)/N_m(\mu)]$$

$$T_2 = \frac{ik}{k_\perp} [J_m'(\kappa) - J_m'(\mu)N_m'(\kappa)/N_m'(\mu)]$$

$$T_3 = mk_z T_1/R_p k_\perp^2 ,$$

$$T_4 = H_1 k_{\perp_1} J_m'(\lambda_1) + P_3 J_m(\lambda_1)/R_p ,$$

$$T_5 = H_2 k_{\perp_2} J_m'(\lambda_2) + P_4 J_m(\lambda_2)/R_p ,$$

$$T_6 = J_m(\kappa) - J_m'(\mu)N_m(\kappa)/N_m'(\mu) ,$$

$$T_7 = \frac{ik}{k_\perp} [J_m'(\kappa) - J_m(\mu)N_m'(\kappa)/N_m(\mu)] ,$$

$$T_8 = mk_z T_6/R_p k_\perp^2 ,$$

$$T_9 = Sk_{\perp_1} J_m'(\lambda_1) + KJ_m(\lambda_1)/R_p ,$$

$$T_{10} = Tk_{\perp_2} J_m'(\lambda_2) + LJ_m(\lambda_2)/R_p ,$$

$$\lambda_1 = k_{\perp_1} R_p , \lambda_2 = k_{\perp_2} R_p ,$$

and

$$\kappa = k_\perp R_p , \mu = k_\perp R_o .$$

The dispersion relation, Eq. (24), completely determines the propagation characteristics of the wave in a cylindrical cavity filled to a radius R_p with the plasma.

From now on, for simplicity, the axisymmetric modes, $m = 0$, are considered. Equation (24) then reduces to

$$[T_1T_9 - T_7J_o(\lambda_1)][T_2P_2J_o(\lambda_2) - T_5T_6]$$

$$= [T_4T_6 - T_2P_1J_o(\lambda_1)][T_7J_o(\lambda_2) - T_1T_{10}] \; ;$$

this can be rewritten as

$$T_5J_o(\lambda_1) - T_4J_o(\lambda_2) + \frac{1}{T_6T_7}\{T_1T_2[T_9P_2J_o(\lambda_2) - T_{10}P_1J_o(\lambda_1)] - T_1T_6(T_5T_9 - T_4T_{10}) - T_2T_7J_o(\lambda_1)J_o(\lambda_2)(P_2 - P_1)\} = 0 \; . \tag{25}$$

Here, we have two possibilities to consider; either the plasma fills the cavity up to the wall, which means $R_p = R_o$, or there may be a small vacuum gap between the plasma and the wall, i.e., $R_p - R_o \cong 0$. These two cases are examined separately.

3.1 NO VACUUM GAP

Since $R_p = R_o$, then $T_1 = T_2 = 0$. The dispersion relation simply becomes

$$D(\omega,k_z) \equiv T_5J_o(\lambda_1) - T_4J_o(\lambda_2) = 0$$

or

$$\frac{J_1(\lambda_1)}{J_o(\lambda_1)} - \frac{P}{Q}\,\frac{\lambda_2}{\lambda_1}\,\frac{J_1(\lambda_2)}{J_o(\lambda_2)} = 0 \; . \tag{26}$$

3.2 SMALL VACUUM GAP

In this case, we assume $(R_o - R_p)/R_p \ll 1$ such that T_1 and T_2 are very small quantities; therefore $T_1T_2 \cong 0$. Thus, Eq. (25) may be rewritten as

$$D(\omega,k_z) + \frac{T_1}{T_7}(T_4T_{10} - T_5T_9)$$

$$+ \frac{T_2}{T_6} J_o(\lambda_1)J_o(\lambda_2)(P_2 - P_1) = 0 \; . \tag{27}$$

It should be noted that the dispersion relation which we have obtained is in a general form. That is to say, so far we have not yet used any specific values for ε_1, ε, or ε_3. For the plasma model adopted, the components of the dielectric tensor are given by Eqs. (1)-(3). Using these in Eq. (27), we get

$$F(\lambda_1) + z^2F(\lambda_2) - \frac{F_1}{F_4}\frac{\lambda_2}{(1+\sigma)}\frac{(kR_p)^2}{\lambda_1^2}$$

$$\times F(\lambda_1)F(\lambda_2)\left\{[1 - z^2 + \sigma(1+\sigma)] + z^2\frac{\lambda_1^2}{\lambda_2^2}\right.$$

$$\left.\times [1 + \sigma^2 - z^2(1+\sigma)]\right\} + \frac{F_2}{F_3}\frac{(1+z^2)}{\lambda_2} = 0 \; , \tag{28}$$

where

$$F(x) \equiv \frac{J_1(x)}{xJ_o(x)} \; ,$$

$$F_1 \equiv T_1(m=0) \; , \; F_2 \equiv i\,\frac{k_\perp}{k}\,T_2(m=0) \; ,$$

$$F_3 \equiv T_6(m = 0), \quad F_4 \equiv i\,\frac{k_\perp}{k}\,T_7(m = 0) \,,$$

$$z = k_z/k \,, \quad \lambda_1 = R_p k_{\perp_1} \,, \lambda_2 = R_p k_{\perp_2} \,,$$

$$\kappa = R_p k_\perp \,, \text{ and } \mu = k_\perp R_o \,.$$

Here $k_{\perp_{1,2}}$ are given by Eq. (16), explicitly:

$$k_{\perp_1}^2/k^2 = (1 + 2\sigma - z^2)/(1 + \sigma) \,, \tag{29}$$

$$k_{\perp_2}^2/k^2 = 1 - z^2 \,. \tag{30}$$

4. CHARACTERISTICS OF THE WAVE

Simultaneous solution of Eqs. (28), (29), and (30) for z completely determines the propagation characteristics of the cyclotron waves in the bounded plasma. For example:

(1) The propagation constant of the wave = (ω/c) Re (z).

(2) The absorption coefficient of the wave = (ω/c) Im (z).

(3) The cutoff frequency, ω_c, of the wave can be obtained by setting $z \to 0$ (i.e., $k_z \to 0$) in the dispersion relation. For $k_z \to 0$, the nonzero elements of the dielectric tensor are

$$\varepsilon_{11} = 1 - \frac{\omega_p^2}{\omega^2 - \Omega^2} = \varepsilon_1 \ ,$$

$$\varepsilon_{12} = i \frac{\Omega}{\omega} \frac{\omega_p^2}{\omega^2 - \Omega^2} = i\varepsilon \ ,$$

$$\varepsilon_{33} = 1 - \frac{\omega_p^2}{\omega^2} = \varepsilon_3 \ .$$

At the resonance frequency, $\omega \cong \Omega$, $\varepsilon_1^2 - \varepsilon^2 \cong 0$. From Eq. (14), we then obtain $a = k^2\varepsilon_3$ and $b = c = d = 0$. Thus $k_{\perp_1}^2 = a = k^2\varepsilon_3$ and $k_{\perp_2}^2 = 0$. Again for simplicity, we compute the cutoff frequency for $R_p = R_o$. Thus from Eq. (26), we obtain

$$F(\lambda_1) = \frac{J_1(\lambda_1)}{\lambda_1 J_o(\lambda_1)} = 0 \ ,$$

$J_1(\lambda_1) = 0$, which yields

$$R_p^2 k^2 \varepsilon_3 = \nu_{1n}^2 \ , \qquad (31)$$

where ν_{1n} is the nth zero of the first-order Bessel function. Using ε_3 in Eq. (31), we find the cutoff frequency to be

$$\omega_{cn}^2 = \omega_p^2 + \left(\frac{c\nu_{1n}}{R_p}\right)^2 .$$

Therefore, in order to have wave propagation in the plasma, the microwave frequency, ω, should satisfy the relation

$$\omega^2 > \omega_{cn}^2 .$$

5. THE LOADED QUALITY FACTOR, Q_L, OF THE CAVITY

Having obtained the propagation constant and the absorption coefficient of the wave from the dispersion relation, we may estimate the quality factor of the cavity. This experimentally observable quantity is defined by

$$Q_L = \omega \frac{W}{P_{ab}} . \tag{32}$$

Here, W is the stored wave energy and P_{ab} is the absorbed power in the cavity. Noting that

$$P_{ab} = -\frac{dW}{dt} \cong 2v_g W \, \mathrm{Im}\, k_z$$

and using this in Eq. (32), we get[4]

$$Q_L \cong \frac{\omega}{2v_g \, \mathrm{Im}\, k_z} .$$

Furthermore, we may replace the group velocity, v_g, by $(c^2 \, \mathrm{Re}\, k_z/\omega)$ in the relation

$$Q_L \cong \frac{(\omega/c)^2}{2 \, \mathrm{Re}\, k_z \, \mathrm{Im}\, k_z} . \tag{33}$$

The assumption made here is that the unloaded quality factor, Q_{wall}, is large enough since the cavity walls are considered to be perfectly conducting (i.e., no skin effects). Otherwise,

$$\frac{1}{Q_L} = \frac{1}{Q_{wall}} + \frac{1}{Q_{plasma}}$$

must be used.

It is also seen from Eq. (32) that the lower value of Q_L is desirable from the point of more power absorption in the plasma.

6. APPLICATION TO EBT-I AND CONCLUSION

We now estimate the wave characteristics, such as Re k_z, Im k_z, and Q_L, for EBT-I. The computation is carried out for $R_p \neq R_o$, which means that a vacuum gap is present between the cavity and the plasma. The dispersion relation which characterizes the propagation in this case is Eq. (28). This is a complicated equation for obtaining an analytical expression for z. Therefore, a numerical solution is sought. In the computation, we use the following characteristic EBT-I plasma parameters.[5]

$$T_e = 500 \text{ eV} .$$

$$\omega_p/\omega = 0.7 .$$

$$\omega/2\pi = 18 \text{ GHz} .$$

$$R_p = 12 \text{ cm} .$$

$$R_o = 15 \text{ cm} .$$

Having solved Eqs. (28), (29), and (30) simultaneously, we obtain

$$\text{Re } k_z = 2.822 \quad 1/\text{cm} ,$$

$$\text{Im } k_z = 0.1856 \quad 1/\text{cm} ,$$

which leads to

$$k_z^2 c^2/\omega^2 = 0.56 < 1 .$$

This mode may then be called an ordinary wave because of the resemblance to the definition in the infinite plasma case for parallel propagation [see Eq. (8)].

Now, knowing the propagation constant and the absorption coefficient of the wave, we may estimate the loaded quality factor, Q_L, of the cavity.

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From Eq. (33) we have

$$Q_L \simeq \frac{(1.2\pi)^2}{2 \times 2.822 \times 0.1856} = 13.56 \ ,$$

which is very close to the experimentally observed Q_L (see Ref. 6).

In conclusion, we have studied the absorption of electron cyclotron waves in a uniform plasma surrounded with a cylindrical wall. The effects on the plasma propagation due to the presence of a vacuum gap between the wall and plasma have been considered, and the dispersion relation, Eq. (28), has been obtained. However, in the case of $R_p = R_o$, the simplified version of the dispersion relation,

$$F(\lambda_1) + z^2 F(\lambda_2) = 0$$

may well serve the purpose (i.e., to estimate some of the characteristic parameters of the electron cyclotron waves in a cylindrical cavity) better than the infinite plasma formulation. An expression for the estimation of the loaded quality factor, Q_L, is also given in terms of the wave propagation characteristics.

REFERENCES

1. W. P. Allis et al., *Waves in Anisotropic Plasma*, M.I.T. Press, Cambridge, Massachusetts, 1963.
2. A. I. Akhiezer et al., *Plasma Electrodynamics*, vol. I, Pergamon Press, Oxford, 1975.
3. T. H. Stix, *The Theory of Plasma Waves*, McGraw-Hill, New York, 1962.
4. J. C. Slater, *Microwave Electronics*, Dover Publishing, Inc., New York, 1969.
5. R. A. Dandl et al., *Summary of EBT-I Experimental Results*, ORNL/TM-6457, Oak Ridge, Tennessee (1978).
6. H. O. Eason (Oak Ridge National Laboratory), private communication, December 1978.

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