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FLUCTUATION EFFECTS ON AVERAGE CROSS SECTIONS IN COMPOUND, DIRECT AND DOORWAY STATE RESONANCE REACTIONS

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Fluctuation Effects on Average Cross Sections in Compound, Direct and Doorway State Resonance Reactions

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ABSTRACT

The main features of the effects of S-matrix fluctuations on average cross sections are reviewed with emphasis on recent developments on the enhancement of small cross sections and cross sections between directly coupled channels. Examples are given in which the effect can distort the shape of a doorway state resonance so as to reduce its observed width.

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Typically, nuclear reaction amplitudes, or elements of the S-matrix S, fluctuate very strongly with energy. Typically, there are distinct theories which describe the average S-matrix elements $\bar{S}$ and their fluctuations $S^{f\ell} = S - \bar{S}$. The theories that describe $\bar{S}$ are direct interaction theories such as the optical model,[1] coupled channels models,[2] doorway state models,[3] etc.[4] The fluctuations are described by statistical models.[5,6] Amplitude fluctuations cause fluctuations in the observed cross sections.[5] But for many purposes one is interested in the smooth energy averages $\bar{\sigma}$ of such fluctuating cross sections. Since cross sections are bilinear functions of the amplitudes, the average cross sections depend both on the elements of $\bar{S}$ and of $S^{f\ell}$. Often, $\bar{\sigma}$ is determined primarily by $\bar{S}$. But sometimes $S^{f\ell}$ can have a significant effect on $\bar{\sigma}$. I shall discuss some examples.

The average angle integrated cross section between channels a and b in units of $\pi\lambda\!\!\!^{-2}$ is

$$\bar{\sigma}_{ab} = \sigma_{ab}^{dir} + \sigma_{ab}^{f\ell} \tag{1}$$

where

$$\sigma_{ab}^{dir} = |\delta_{ab} - \bar{S}_{ab}|^2 \tag{2}$$

$$\sigma_{ab}^{f\ell} = \overline{|S_{ab}^{f\ell}|^2}. \tag{3}$$

To insure flux conservation, S must be unitary at all energies. However $\bar{S}$ will not be unitary. The resulting unitarity defect in the direct cross section in channel a is the compound nucleus absorption cross section, or the transmission coefficient

$$T_a = 1 - \Sigma_b|\bar{S}_{ab}|^2 = \Sigma_b\sigma_{ab}^{f\ell}. \tag{4}$$

The unitarity of S insures that

$$T_a \leq 1. \quad \text{(average unitarity)} \tag{5}$$

Causality requires that the system be absorptive rather than emissive and that therefore

$$T_a \geq 0 \qquad \text{(average causality).} \tag{6}$$

The origin of the fluctuation effect on the average cross section is easiest seen in the limit of small transmission coefficients when the fluctuations are caused by isolated resonances at energies E_μ with partial widths $\Gamma_{\mu a}$ and total widths $\Gamma_\mu = \Sigma_a \Gamma_{\mu a}$. Then the fluctuation cross section is given by

$$\sigma_{ab}^{f\ell} = \frac{2\pi}{D}\left\langle \frac{\Gamma_{\mu a}\Gamma_{\mu b}}{\Gamma_\mu} \right\rangle \tag{7}$$

where D is the average spacing of the E_μ and $\langle\ \rangle$ is an average over resonances. From Eq. (7)

$$T_a = \frac{2\pi}{D}\langle \Gamma_{\mu a} \rangle , \tag{8}$$

which means that the fluctuation cross section is almost determined by the $\overline{S}$ dependent transmission coefficients alone.

$$\sigma_{ab}^{f\ell} = \left(\frac{T_a T_b}{\Sigma_c T_c} \right) W_{ab} \tag{9}$$

where the factor in the parentheses is the well known Hauser-Feshbach expression and W is the well known width fluctuation correction[7] which can be written[8]

$$W_{ab} = C_{ab} G_{ab} \tag{10}$$

$$C_{ab} = \frac{\langle \Gamma_{\mu a}\Gamma_{\mu b} \rangle}{\langle \Gamma_{\mu a} \rangle \langle \Gamma_{\mu b} \rangle} \tag{11}$$

$$G_{ab} = \frac{\langle \Gamma_{\mu a}\Gamma_{\mu b}/\Gamma_\mu \rangle}{\langle \Gamma_{\mu a}\Gamma_{\mu b} \rangle / \langle \Gamma_\mu \rangle} \tag{12}$$

The factor C_{ab} arises from correlations between the partial widths $\Gamma_{\mu a}$ and $\Gamma_{\mu b}$ and depends therefore on their correlation coefficient ρ_{ab}

$$C_{ab} = 1 + \rho_{ab} \frac{\sqrt{Var(\Gamma_{\mu a})Var(\Gamma_{\mu b})}}{\langle \Gamma_{\mu a} \rangle \langle \Gamma_{\mu b} \rangle} = 1 + \frac{2\ \rho_{ab}}{\sqrt{\nu_a \nu_b}} \tag{13}$$

If $\Gamma_{\mu a}$ is distributed according to the chi-squared distribution with ν_a degrees of freedom, then $Var(\Gamma_{\mu a}) = 2\langle \Gamma_{\mu a} \rangle^2/\nu_a$ and the last expression in Eq. (13) follows. In the absence of direct reactions $\rho_{ab} = \delta_{ab}$ and so the factor C_{ab} produces an enhancement of the average elastic fluctuation cross section by a factor of $1 + 2/\nu$.

In the above mentioned case of isolated resonances and for most independent channels in complex nuclei the value of ν is 1 (Porter-Thomas distribution) and therefore $C_{aa} = 3$ and $C_{ab} = 1$. For large values of the transmission coefficients the situation becomes moderately complicated, but it is known that in the limit of large T the above formulas remain valid with, however, $\nu = 2$.[8,9] How the value of ν changes from 1 to 2 as transmission coefficients increase has been investigated numerically and the results[10,11] have been presented in graphical or algebraic form. In the overlapping resonance limit $C_{aa} = 2$.

The factor G_{ab} depends on the fluctuations of the inverse total width and on its correlations with the channel partial widths. It is easily evaluated in terms of a single integral for the case of no direct reactions. In general $G_{ab} < 1$ and just compensates for the elastic enhancement of C_{aa} by reducing the non-elastic cross sections. Thus in the case of only two competing open channels with equal transmission coefficients $G_{ab} = \nu/(1 + \nu)$ or 1/2 in the isolated resonance limit and 2/3 in the overlapping limit. This results in a maximum elastic enhancement and inelastic reduction of fluctuation cross sections relative to Hauser-Feshbach of 50% for isolated resonances (Fig. 1) and 33% for overlapping resonances.

There are circumstances, however, in which the correction factor can be even greater.[12] This can happen in the case of a reaction involving very weakly absorbed channels which do compete, however, with a small number of strongly absorbed channels. In that case G_{ab} is dominated by the factor

$$\left\langle \Gamma_\mu^{-1} \right\rangle \left\langle \Gamma_\mu \right\rangle = \left(1 - \frac{2}{\nu_t}\right)^{-1} , \quad \nu_t > 2 \tag{14}$$

where ν_t is the degree of freedom that characterizes the fluctuation of the total widths Γ_μ. This factor can become quite large. But of course even if $\nu_t \leq 2$, G_{ab} remains finite and positive. Nevertheless substantial enhancements of weak cross sections are possible as indicated in Fig. 2.

Next we can inquire into the effect of fluctuations on the average cross section in the presence of direct reactions when $\bar{S}$ is no longer diagonal. In that case it is useful to generalize the transmission coefficients T_a into Satchler's hermitean penetration matrix[13]

$$P = 1 - \bar{S}\bar{S}^{+} \tag{15}$$

the diagonal elements of which are just the transmission coefficients.

The method to be used is then to diagonalize the average S-matrix by means of the unitary Engelbrecht-Weidenmüller transformation[14]

$$\bar{S}' = U\, \bar{S}\, U^{tr}, \tag{16}$$

where U^{tr} is the transpose of U, and to calculate the average cross sections from the diagonal transformed average S-matrix $\bar{S}'$, and then to transform these cross sections back by means of the inverse Engelbrecht-Weidenmüller transformation.[10,11]

From Eq. (16) it follows that U diagonalizes the Satchler penetration matrix so that

$$P' = UPU^{-1} \quad \text{is diagonal.} \tag{17}$$

Eq. (17) makes it easy to find U for a given $\bar{S}$, by diagonalizing P. It also tells us what the average unitarity and causality conditions are in the case of direct reactions.[10] Since $P_{aa} = T_a$, we have

$$P_{aa} \leq 1 \qquad \text{(average unitarity)} \tag{18}$$

and since P' must be causal, we see that P cannot have negative eigenvalues and hence

$$P \text{ is positive semidefinite,} \qquad \text{(average causality.)} \tag{19}$$

The case where P has one vanishing eigenvalue is the causality limit and this case is of special interest. In the case where there are two directly coupled channels, but perhaps many other competing channels, the causality limit implies that in the transformed space only one of the two coupled channels is absorbed, hence also only one of the two channels gives rise to amplitude fluctuations. It follows that the fluctuations in the two physical untransformed coupled channels must be strongly correlated. As a result the correlation factor C_{ab} of Eq. (11) will cause an enhancement of the non-elastic cross section between the two coupled channels. This expectation has been confirmed by calculations involving 10 channels with equal transmission coefficients for which pairs of coupled channels have average S-matrices of the form

$$\begin{pmatrix} F & iD \\ iD & -F \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} F & e^{i\theta}D \\ E^{-i\theta}D & F \end{pmatrix} \tag{20}$$

where D and F are real.[10] The results show strong enhancements of the nonelastic cross section along the causality limit F + D = 1. However for values of F and D away from that line the nonelastic enhancement disappears rapidly. On the other hand for average S-matrices of the form

$$\begin{pmatrix} F & D \\ D & -F \end{pmatrix} \tag{21}$$

the causality limit $F^2 + D^2 = 1$ coincides with vanishing absorption in both channels and therefore no strong nonelastic enhancement is expected or found. The results of these calculations are shown in Fig. 3.

We see that significant non-elastic correlation enhancements due to competing direct reactions will occur only in very special circumstances, in particular when only few channels are strongly coupled and the average S-matrix is at or near the causality limit. One type of situation in which one might expect such a special situation to occur is an intermediate resonance of the type encountered in doorway state[3] or analog resonances.[4] In passing through such a resonance, the average S-matrix undergoes large variations which might pass close to the causality limit at some energy, causing a significant distortion of the average cross section at that point.

To investigate this possibility, let us look at the simplest case of a doorway state with purely internal mixing, and with only two channels coupled to the doorway state. To reduce the number of free parameters we also assume that the coupling strengths of the two coupled channels are the same. Then we can write the average S matrix for the two coupled channels as

$$\overline{S}_{ab} = e^{i(\theta_a+\theta_b)}\left(\sqrt{1 - T_B}\,\delta_{ab} - \frac{\frac{1}{2}i\Gamma^{\uparrow}}{E + \frac{1}{2}i\Gamma}\right) \tag{22}$$

where E is the energy measured from the doorway state resonance energy and $\Gamma = \Gamma^{\uparrow} + \Gamma^{\downarrow}$ is the total width. T_B is the background transmission coefficient of each of the two channels far from the doorway state resonance. From (22) we get the penetration matrix

$$P_{ab} = T_B\delta_{ab} - 2e^{i(\theta_a-\theta_b)}\frac{\Gamma^{\uparrow 2}}{E^2 + \frac{1}{4}\Gamma^2}\left(1 - \frac{\Gamma}{\Gamma^{\uparrow}}\sqrt{1 - T_B}\right). \tag{23}$$

This is of the form of Eq. (20). We find the causality limit by setting

$$\det P = T_B\left(T_B - \frac{\Gamma^{\uparrow 2}}{E^2 + \frac{1}{4}\Gamma^2}\left(1 - \frac{\Gamma}{\Gamma^{\uparrow}}\sqrt{1 - T_B}\right)\right) \tag{24}$$

equal to zero which gives

$$E^2 = \frac{\Gamma^{\uparrow}}{T_B}\left(\Gamma^{\uparrow} - \Gamma\sqrt{1 - T_B}\right) - \frac{\Gamma^2}{4} \qquad \text{(causality limit)} \tag{25}$$

which will occur at real energies provided

$$\frac{\Gamma^{\uparrow}}{\Gamma^{\downarrow}} \geq \frac{1 + \sqrt{1 - T_B}}{1 - \sqrt{1 - T_B}} \tag{26}$$

where $\Gamma^{\downarrow} = \Gamma - \Gamma^{\uparrow}$. Thus the causality limit is actually reached when (26) is an equality.

It is now interesting to calculate the magnitude of the distortion of the nonelastic cross section between the two coupled channels of such a doorway state due to the direct enhancement effect of the fluctuation part of the cross section. We do this, first for the case where $T_B = 0.96$ and the causality limit is reached so that according to Eq. (26) $\Gamma^{\uparrow}/\Gamma^{\downarrow} = 1.5$. The resulting nonelastic average cross sections for the case where there are no other competing channels present are plotted in Fig. 4a. Off resonance the fluctuation cross section is reduced relative to the Hauser-Feshbach value because of the usual width fluctuation effect. However as the causality limit is reached at the doorway resonance, a sharp 50% enhancement occurs which brings the cross section back to its Hauser-Feshbach value. The result is an effective narrowing of the resonance by 24%, as measured by its width at half maximum. If we have the same situation but include eight other uncoupled but competing strongly absorbed channels, then off-resonance reduction of the fluctuation cross section effectively disappears and we are left only with a slight enhancement at the resonance where the causality limit is reached. As seen in Fig. 4b the cross section shape is not appreciably affected. Similarly, if we retain only two channels with $T_B = 0.96$, each, but reduce $\Gamma^{\uparrow}/\Gamma^{\downarrow} = 1.0$, we obtain the result of Fig. 4c. The causality limit is no longer reached and so there is only a slight enhancement of about 10% at the resonance, which narrows the resonance shape by only about 12%.

We can conclude that pronounced effects of competing direct reactions upon fluctuation cross sections are relatively rare, being confined to cases of two or at most three coupled channels at or near the causality limit. However when these conditions do prevail the direct effects can be pronounced and can, in particular, distort the shapes of the doorway state resonances, so that they appear narrower than their actual widths.

REFERENCES

1. H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. 96, 448 (1954).
2. Taro Tamura, Rev. Mod. Phys. 37, 679 (1965).
3. H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. 41, 230 (1967).
4. Nuclear Analog States, edited by Donald Robson and John D. Fox (Dowden, Hutchinson & Ross, Stroudsburg, 1976). P. A. Mello and M. Moshinsky, Am. J. Phys. 39, 54 (1971).
5. Statistical Theory of Spectra, edited by C. E. Porter (Academic Press, N.Y., 1965).
6. P. A. Mello, J. Flores, T. A. Brody, J. B. French, S. S. M. Wong, Proc. Int. Conference on the Interactions of Neutrons with Nuclei, Eric Sheldon, editor, CONF-760715-P1 (1976) p. 496.
7. A. M. Lane and J. E. Lynn, Proc. Phys. Soc. (London) A70, 557 (1957). P. A. Moldauer, Phys. Rev. 135, p. 642 (1964).
8. P. A. Moldauer, Phys. Rev. C11, 426 (1975).
9. J. W. Tepel, H. M. Hofmann and H. A. Weidenmüller, Phys. Lett. 49B, 1 (1974).
10. P. A. Moldauer, Phys. Rev. C12, 744 (1975).
11. H. M. Hofmann, J. Richert, J. W. Tepel, and H. A. Weidenmüller, Ann. Phys. (N.Y.) 90, 403 (1975).
12. P. A. Moldauer, Phys. Rev. C14, 764 (1976).
13. G. R. Satchler, Phys. Lett. 7, 55 (1963).
14. C. A. Engelbrecht and H. A. Weidenmüller, Phys. Rev. C8, 859 (1973).

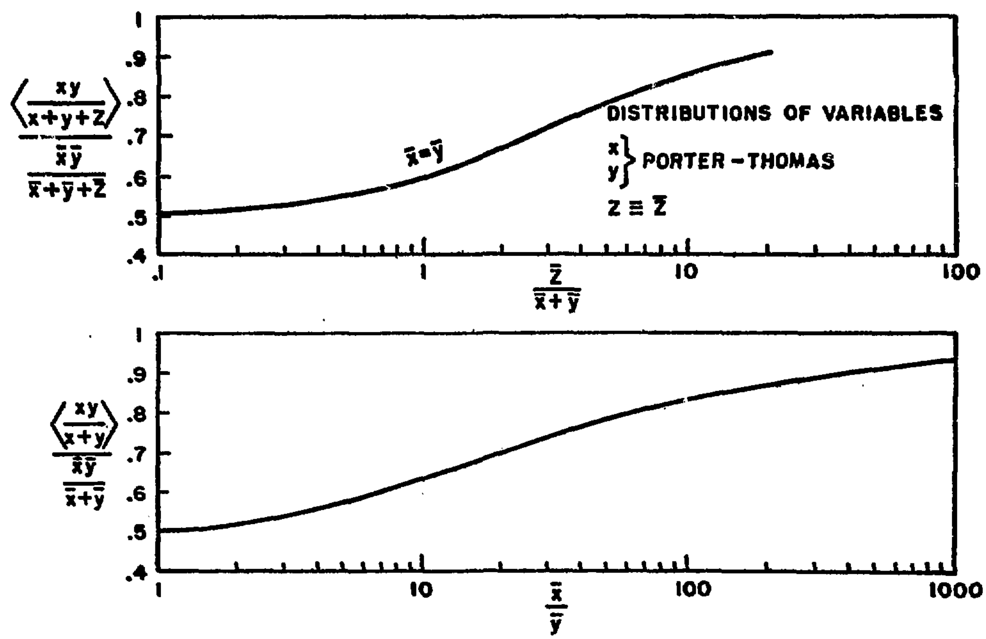

Fig. 1. Reduction of the non-elastic fluctuation cross section due to the fluctuation correction factor G in the limit of isolated resonances.

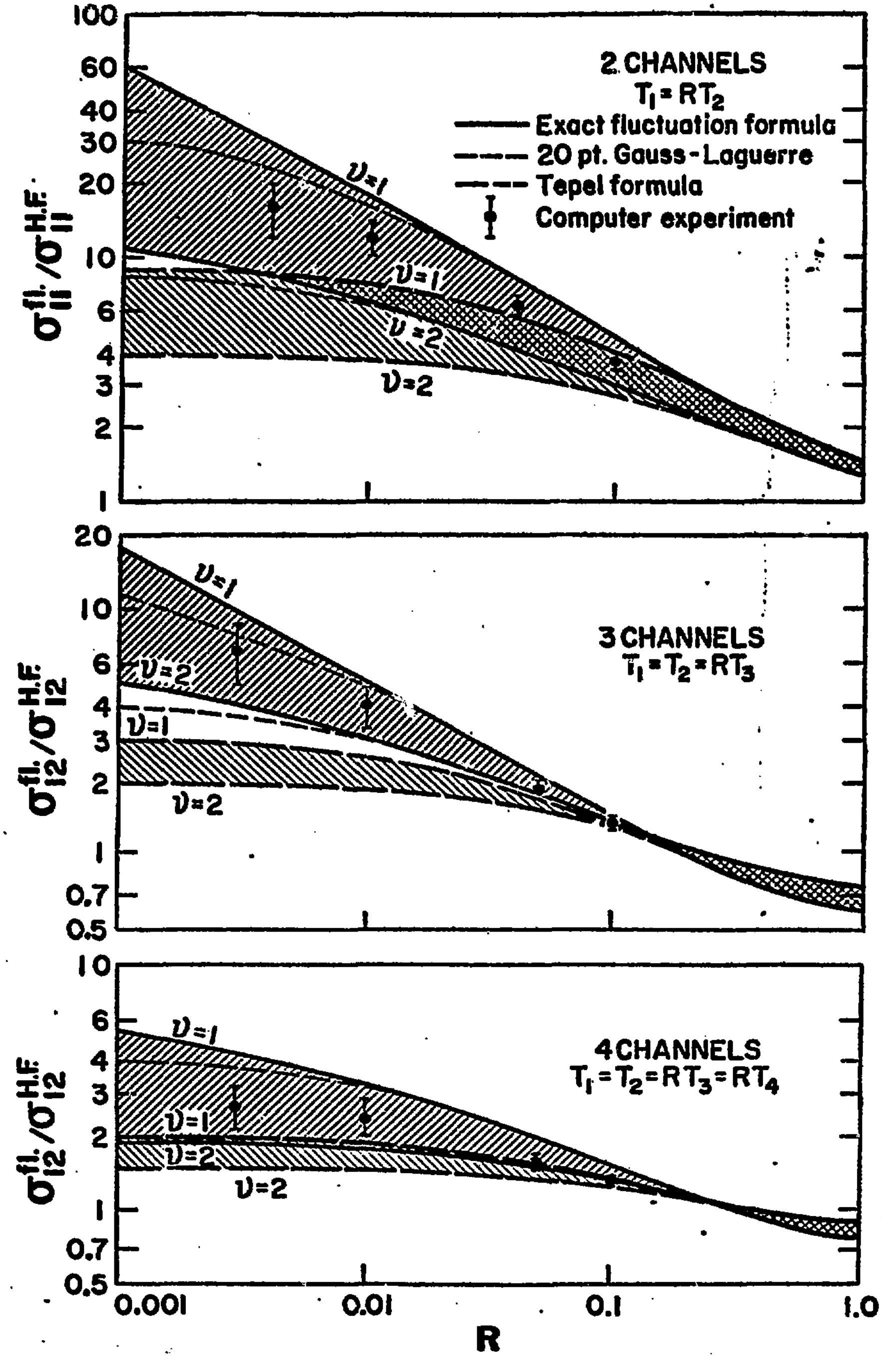

Fig. 2. Fluctuation enhancement factors for small cross sections due to total width fluctuations.

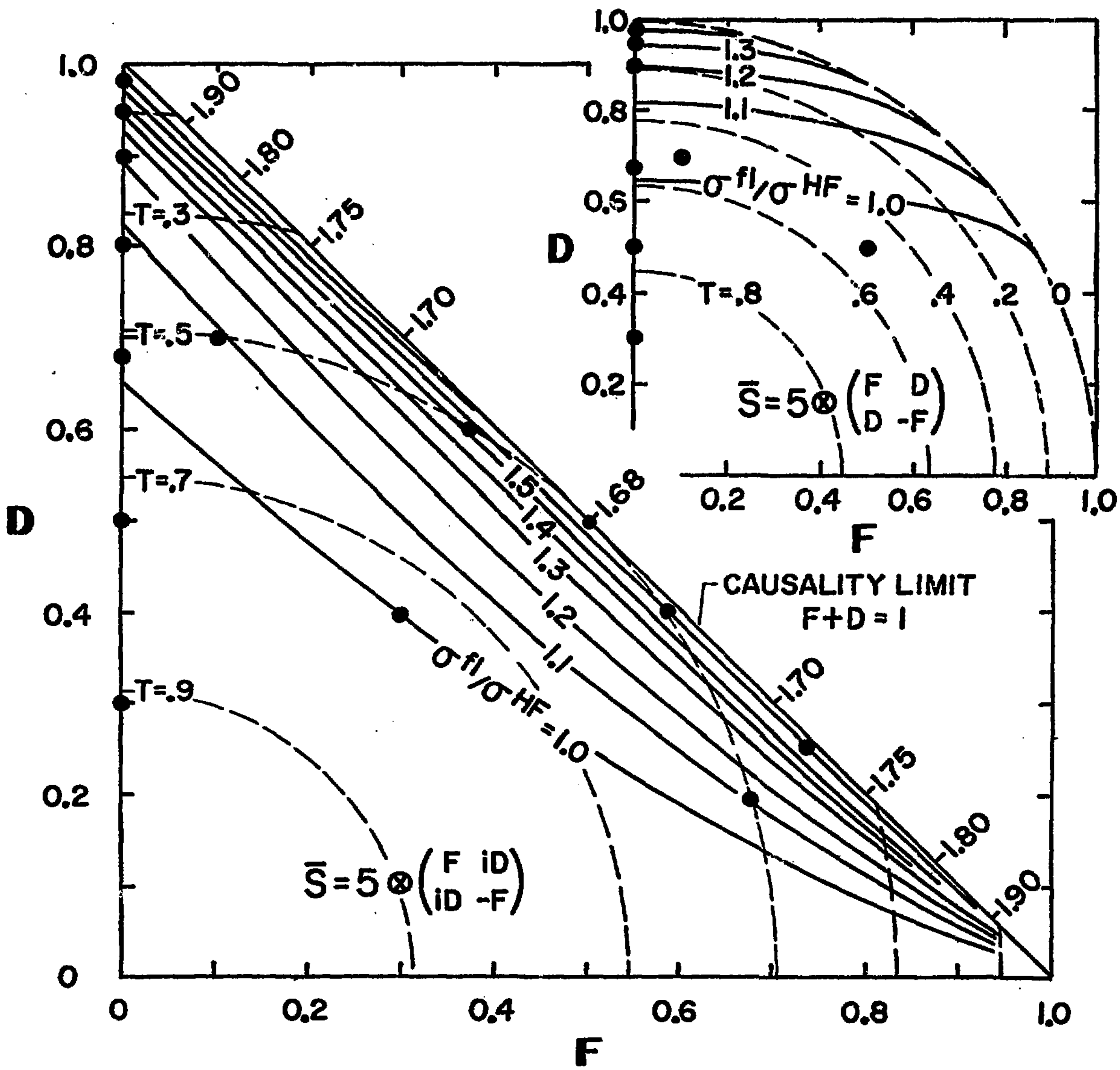

Fig. 3. Fluctuation enhancement of non-elastic cross sections due to completing direct reactions.

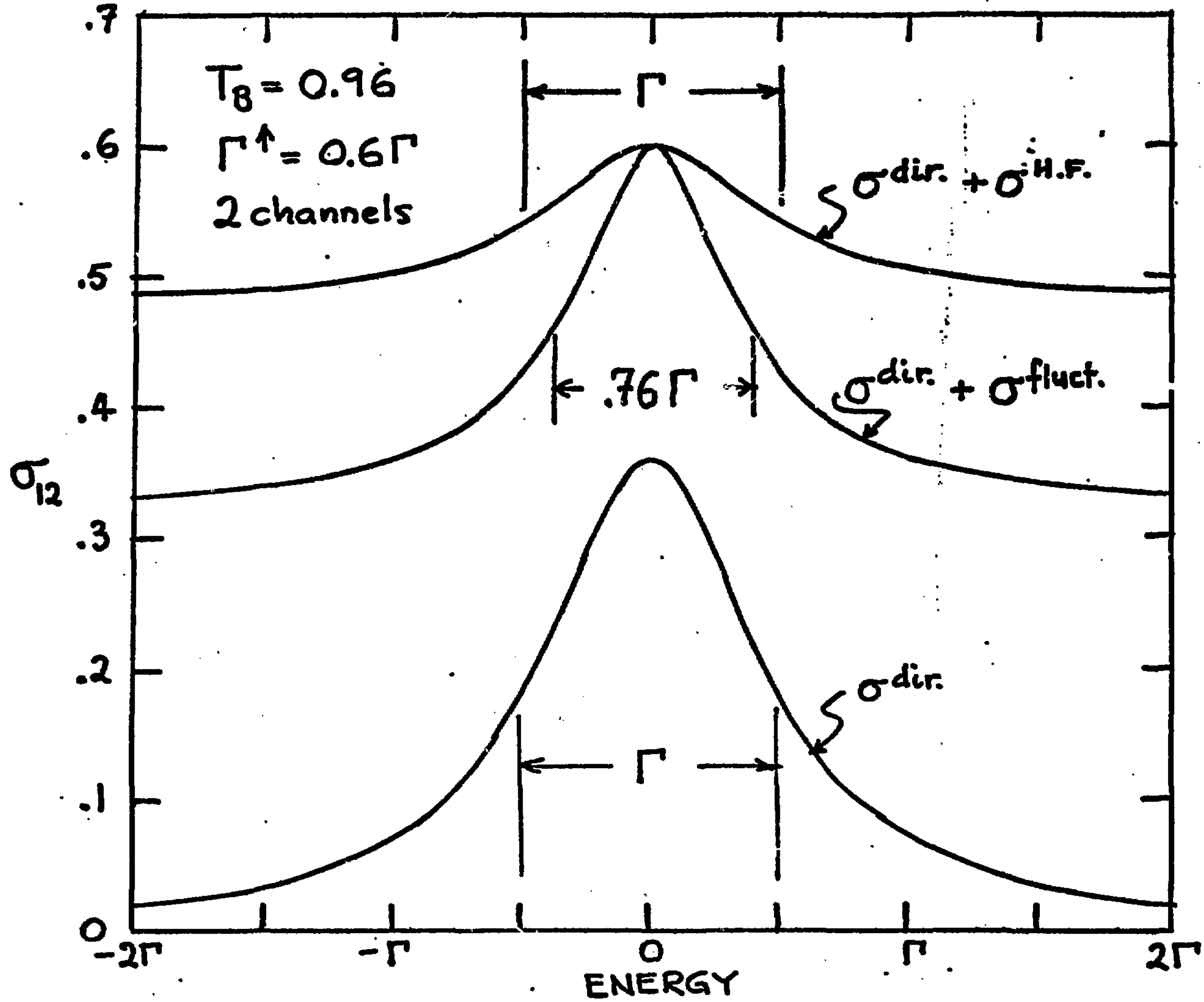

Fig. 4a. Narrowing of a nonelastic doorway state resonance due to the fluctuation correction. Two channel case where the causality limit is reached.

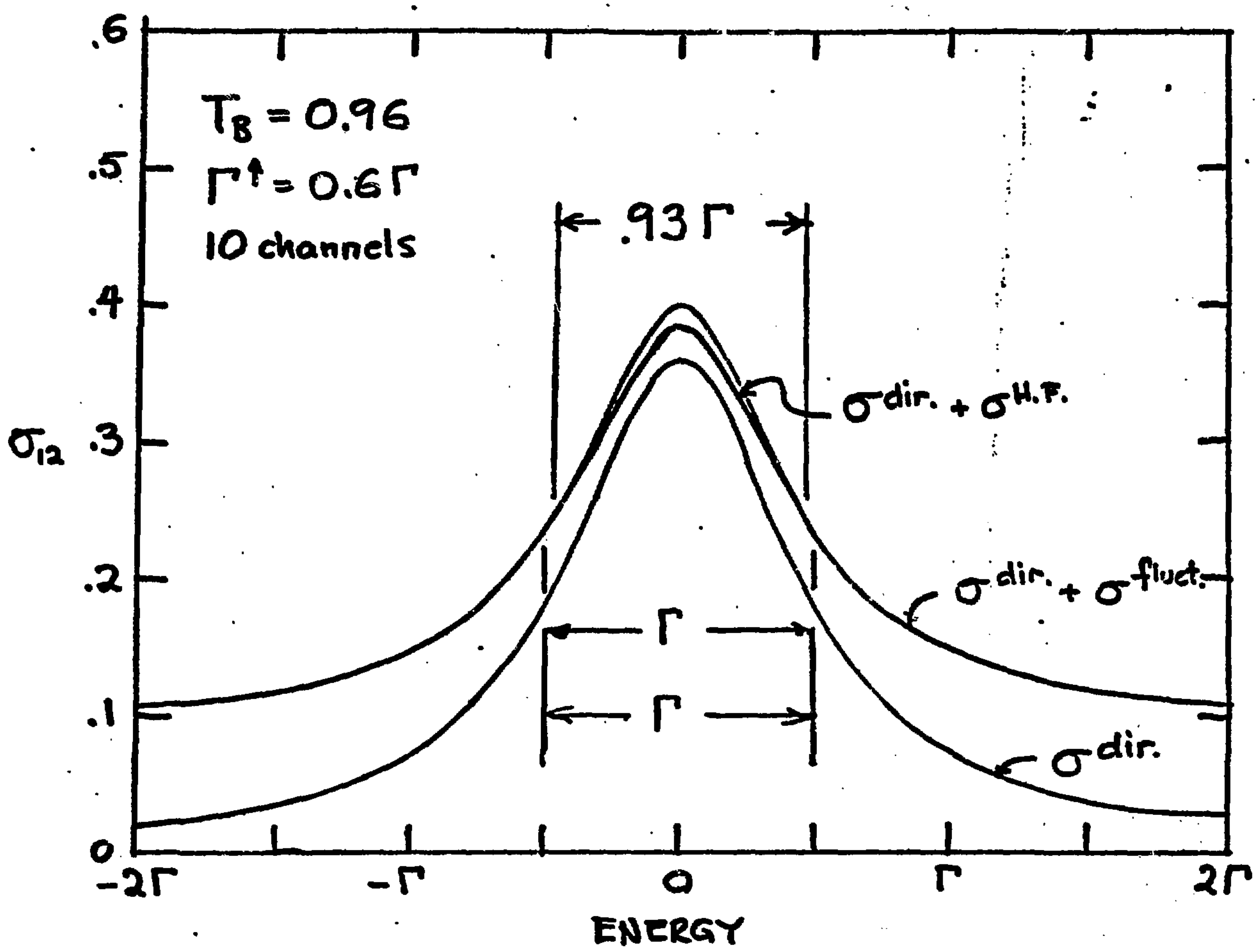

Fig. 4b. Narrowing of a nonelastic doorway state resonance due to the fluctuation correction. The causality limit is reached but competition from 10 channels suppresses the fluctuation correction.

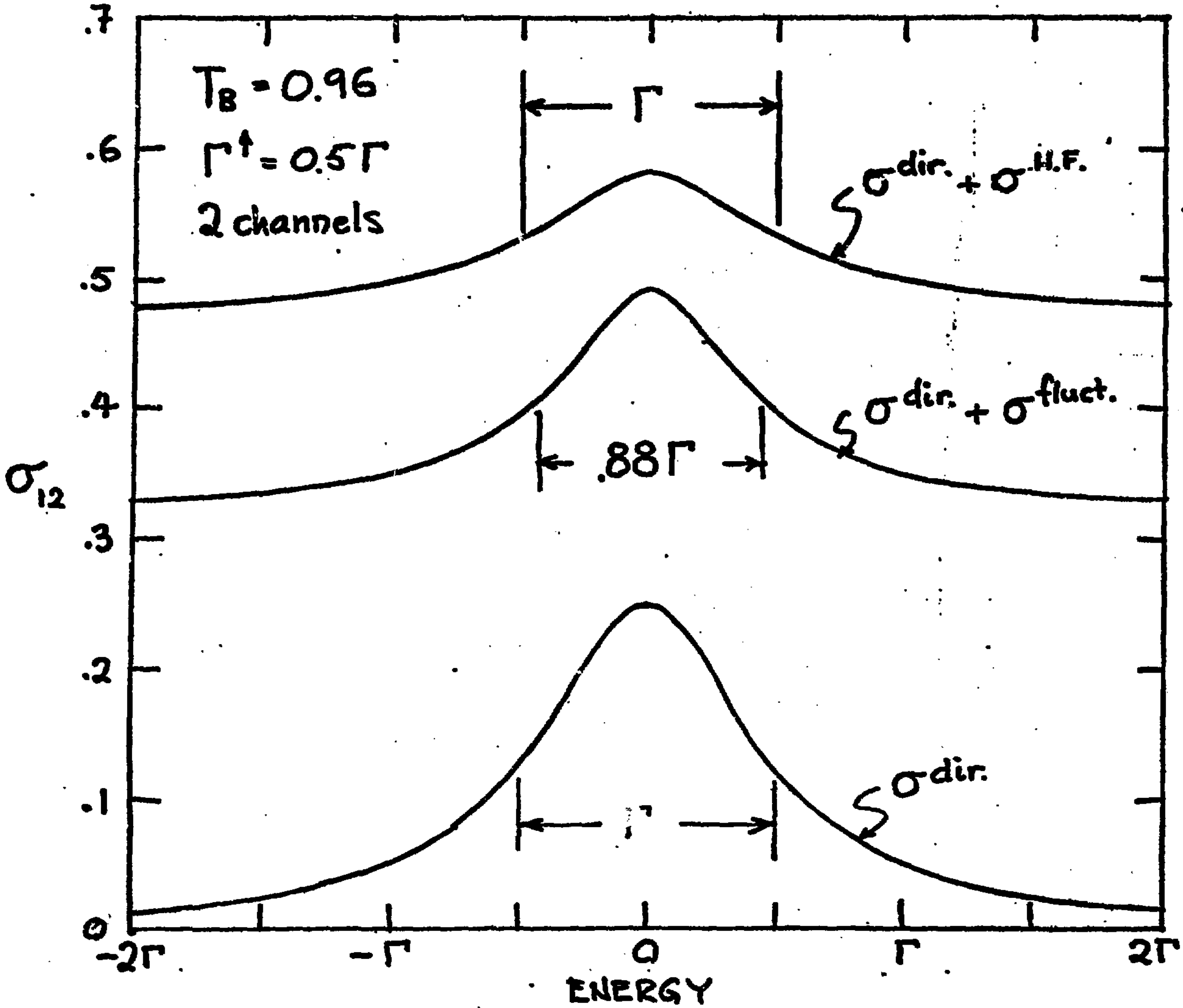

Fig. 4c. Narrowing of a nonelastic doorway state resonance due to the fluctuation correction. Two channel case where the causality limit is not reached.