JAMES COOK UNIVERSITY OF NORTH QUEENSLAND

ELASTIC SCATTERING OF LOW ENERGY γ-RAYS
II. THEORETICAL CROSS SECTIONS FOR SCATTERING OF ^{152}Eu γRAYS BY Pb

by

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Physics Department,
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Abstract

Theoretical cross sections for the elastic scattering of 245, 344, 444, 779, 964, 1086, 1112 and 1408 keV γ rays by Pb are obtained for scattering angles up to 150°. Three sets of Rayleigh scattering amplitudes have been computed using (i) the calculations of Johnson and Cheng, (ii) the K shell calculations of Brown and co-workers supplemented by form factor amplitudes for higher shells, and (iii) form factor amplitudes for all shells. Nuclear Thomson amplitudes have been included for all energies and, for 1408 keV, Delbrück scattering based upon the calculations of Papatzacos and Mork has been included. Nuclear resonance scattering is shown to be negligible for all energies. The differences between the cross sections calculated using the various sets of Rayleigh amplitudes are generally around 10% which is about the estimated error in the cross sections; cross sections using the Johnson-Cheng Rayleigh amplitudes are generally lower than those using Brown-form factor Rayleigh amplitudes and, for the higher photon energies and medium to large scattering angles, the cross sections using pure form factor Rayleigh amplitudes are larger than those using the other Rayleigh amplitudes. Inclusion of Delbrück scattering at 1408 keV lowers the differential cross sections for scattering angles less than 75°, the maximum reduction being 7.5 mb at 15°, and raises the differential cross sections for scattering angles greater than 75° by 0.03 to 0.04 mb.

1. Introduction

In an earlier report (Whittingham 1978, hereafter referred to as I) the theory of elastic scattering of low energy γ rays (below 5 MeV say) was discussed in some detail, the emphasis being on the formalisms used by different authors and the nature of any fundamental approximations built in to current theory. In this report we will discuss existing numerical calculations based upon the theories reviewed in I and attempt to predict the theoretical cross sections for the elastic scattering of the γ rays of $^{152}_{63}Eu$ by a Pb target.

The amplitude for elastic scattering of low energy γ rays is the coherent sum of Rayleigh (R), nuclear Thomson (T), Delbrück (D) and nuclear resonance (N) scattering amplitudes. Here R, T, D and N denote scattering amplitudes in units of the classical electron radius $r_e = e^2/4\pi\varepsilon_0 mc^2 = 2.818 \times 10^{-15}$ m. Thus the total scattering amplitude for the elastic scattering of photons of energy $\hbar\omega$ through an angle θ is

$$f(\omega,\theta) = r_e\ (R + T + D + N)\ . \qquad (1.1)$$

Simple formulae exist for nuclear Thomson and nuclear resonance scattering which allow the direct computation of the corresponding scattering amplitudes for any photon energy and scattering angle. However in the case of Rayleigh and Delbrück scattering no simple accurate formulae exist and "accurate" calculations are available only for selected photon energies, scatterers and scattering angles. Hence we must interpolate and/or modify these calculations to the energies and scatterer of our interest.

2. Rayleigh scattering.

2.1 Exact calculations

The most accurate calculations of Rayleigh scattering are those of Johnson and Cheng (1976) and Kissel and Pratt (1977a). The binding effects of the nuclear field on the initial, intermediate and final electron states are accounted for completely by the evaluation of the second order S-matrix in the Furry or Bound Interaction Picture (I,§ 2.1). The initial and final photon fields are expanded in a multipole series (I,§ 3.3). The Rayleigh amplitude for [illegible]ttering of a photon from polarization state $\varepsilon^{(i)}$ to polarization [illegible] $^{(f)}$ is expressed in terms of the components of the polarization [illegible] ($\varepsilon_{\parallel}^{(i,f)}$) and perpendicular ($\varepsilon_{\perp}^{(i,f)}$) to the scatt[illegible] (I, equations 3.58, 3.73)

$$R(\omega,\theta) = R^{\parallel}(\omega,\theta) + R^{\perp}(\omega,\theta) \qquad (2.1)$$

$$= m\,[\mathcal{M}_{\parallel}(\omega,\theta)\,\varepsilon_{\parallel}^{(f)*}\varepsilon_{\parallel}^{(i)} + \mathcal{M}_{\perp}(\omega,\theta)\,\varepsilon_{\perp}^{(f)*}\varepsilon_{\perp}^{(i)}] \tag{2.2}$$

where

$$\left\{\begin{matrix}\mathcal{M}_{\parallel}(\omega,\theta)\\ \mathcal{M}_{\perp}(\omega,\theta)\end{matrix}\right\} = \frac{1}{2}\sum_{J=1}^{\infty}(2J+1)\,\{[P_J^o(\cos\theta) - \frac{P_J^2(\cos\theta)}{J(J+1)}]\left\{\begin{matrix}X_J^1(\omega)\\ X_J^o(\omega)\end{matrix}\right\}$$
$$+ \frac{1}{2}[P_{J+1}^o(\cos\theta) + P_{J-1}^o(\cos\theta) + \frac{P_{J+1}^2(\cos\theta) + P_{J-1}^2(\cos\theta)}{J(J+1)}]\left\{\begin{matrix}X_J^o(\omega)\\ X_J^1(\omega)\end{matrix}\right\}\}. \tag{2.3}$$

$P_n^m(\cos\theta)$ are the normalized associated Legendre functions, J and λ denote the photon angular momentum and multipole parity respectively and $X_J^\lambda(\omega)$ are the multipole amplitudes

$$X_J^o(\omega) = \frac{1}{J(J+1)}\sum_{n_i\kappa_i\kappa}(\kappa+\kappa_i)^2\,(2j+1)(2j_i+1)\,\Lambda_{-\kappa_i J\kappa}\,[R_{J\kappa}^o(\omega) + R_{J\kappa}^o(-\omega)]$$

$$X_J^1(\omega) = \frac{J+1}{J}\sum_{n_i\kappa_i\kappa}(2j+1)(2j_i+1)\,\Lambda_{\kappa_i J\kappa}[R_{J\kappa}^1(\omega) + R_{J\kappa}^1(-\omega)] \tag{2.4}$$

κ (and κ_i) take on all non-zero integer values and specify the total and orbital angular momentum quantum numbers of the intermediate (and initial/final) electron states via, for example,

$$j = |\kappa| - \frac{1}{2}\ , \quad \ell = j \pm \frac{1}{2} = \begin{cases}\kappa & \kappa > 0\\ -\kappa-1 & \kappa < 0\end{cases} \tag{2.5}$$

The Λ coefficients are

$$\Lambda_{\kappa_i J\kappa} = \begin{cases}\dfrac{C(j_i,J,j;\frac{1}{2},0)^2}{2j+1} & , \ \ell_i + \ell + J = \text{even}\\ 0 & , \ \ell_i + \ell + J = \text{odd}\end{cases} \tag{2.6}$$

where $C(j_i,J,j;\frac{1}{2},0)$ is a Clebsch-Gordon coefficient. The radial integrals

$$R_{J\kappa}^\lambda(\hbar\omega) = \int_0^\infty dr\,[S_{J\kappa}^\lambda(\hbar\omega,r)\;K_J^\lambda(\hbar\omega,r) + T_{J\kappa}^\lambda(\hbar\omega,r)\;L_J^\lambda(\hbar\omega,r)] \tag{2.7}$$

are construc[illegible] ns $S(\hbar\omega,r)$ and $T(\hbar\omega,r)$ of the inhomogeneous radial Dir[illegible]

$$\begin{pmatrix} \frac{d}{dr} + \frac{\kappa}{r} & -[E_i \pm \hbar\omega + mc^2 - V(r)]/\hbar c \\ [E_i \pm \hbar\omega - mc^2 - V(r)]/\hbar c & \frac{d}{dr} - \frac{\kappa}{r} \end{pmatrix} \begin{pmatrix} S^{\lambda}_{J\kappa}(\pm\omega,r) \\ T^{\lambda}_{J\kappa}(\pm\omega,r) \end{pmatrix} = \begin{pmatrix} K^{\lambda}_{J}(\pm\omega,r) \\ -L^{\lambda}_{J}(\pm\omega,r) \end{pmatrix} \qquad (2.8)$$

The inhomogeneous terms are

$$\begin{pmatrix} K^{0}_{J}(\pm\omega,r) \\ L^{0}_{J}(\pm\omega,r) \end{pmatrix} = j_J(\pm\omega r) \begin{pmatrix} f_{\kappa_i E_i}(r) \\ g_{\kappa_i E_i}(r) \end{pmatrix} ,$$

$$\begin{pmatrix} K^{1}_{J}(\pm\omega,r) \\ L^{1}_{J}(\pm\omega,r) \end{pmatrix} = \left[j_J(\pm\omega r) \mp \frac{\omega r j_{J+1}(\pm\omega r)}{J+1} \right] \begin{pmatrix} g_{\kappa_i E_i}(r) \\ f_{\kappa_i E_i}(r) \end{pmatrix} \pm \frac{\omega r j_J(\pm\omega r)}{J+1} \begin{pmatrix} f_{\kappa_i E_i}(r) \\ -g_{\kappa_i E_i}(r) \end{pmatrix} \qquad (2.9)$$

where $j_J(\omega r)$ is the spherical Bessel function of order J, and $f_{\kappa_i E_i}(r)$ and $g_{\kappa_i E_i}(r)$ are the radial wavefunctions of the initial/final atomic electron of energy E_i which are generally taken to be Dirac Hartree Fock Slater (DHFS) wavefunctions. The sum over κ in (2.4) includes all intermediate states permitted by the non-vanishing of the A coefficients (2.6), and the $(n_i\kappa_i)$ summation is just a summation over atomic subshells.

These recent "accurate" calculations therefore require, for a given photon energy $\hbar\omega$ and scatterer Z, the numerical solution of (2.8) for each atomic subshell and value of J, and the evaluation of the multipole summation over J. Johnson and Cheng (1976) have performed these calculations for the scattering of 145,279,412,662 and 889 keV γ rays by Pb and the scattering of 279 and 662 keV γ rays by Ta,Nd, Sn,Mo and Zn. For Pb the K,L and M shells are included for 145 and 279 keV but K and L shells only are used for the higher energies. For all cases the amplitudes are given for scattering angles $0^\circ[30^\circ]\ 150^\circ$. However the results for $\theta = 0^\circ$ are of mathematical interest only as the higher omitted shells dominate the zero degrees Rayleigh amplitude.

For Pb a comparison of Johnson and Cheng's calculations with the recent experim[illegible] of Schumacher (1969) and Schumacher Smend and B[illegible] show discrepancies of 12% at 145 keV and at 889 keV. The greatest source of error in these "accurate" calculations is the truncation error of the sum over atomic shells, ranging from 10% for Pb to 20% for the lightest element considered (Zn). Details are not yet available of the calculations of Kissel and Pratt (1977b) but in their preliminary report (1977a) they state that where their calculations overlap with those of Johnson and Cheng they always find agreement.

K shell amplitudes based on the second order S-matrix in the Furry Picture were first calculated by Brown and co-workers for scattering by Hg (Z = 80) of photons of energy 0.32 mc^2 (163 keV) (Brown, Peierls and Woodward 1954; Brenner, Brown and Woodward, 1954), 0.64 mc^2 (327 keV) (Brown and Mayers 1956), and 1.28 mc^2 (654 keV) and 2.56 mc^2 (1308 keV) (Brown and Mayers 1957). Later Cornille and Chapdelaine (1959) used the theory of Brown et al to calculate the scattering of 5.12mc^2 (2616 keV) rays by Hg. For 0.32 mc^2 and 0.64 mc^2 the amplitudes are given as expansions in terms of the Legendre polynomials P^0_ℓ (cos θ) and P^2_ℓ(cos θ) and can be evaluated for any scattering angle. However for 1.28 mc^2 and 2.56 mc^2 numerical results only are presented for $\theta = 0^o[15^o] 180^o$ and in the case of 5.12 mc^2, $\theta = 0^o [10^o] 180^o$. Collectively these calculations will be referred to as the Brown K shell amplitudes and be denoted $^{BR}R_K$.

An important finding of Kissel and Pratt (1977a) is that the 1308 keV calculations of Brown and Mayers are incorrect by a factor of approximately two for $\theta \geq 90^o$, thus resolving the discrepancies between theory and the experiments at 1.33 MeV of Dixon and Storey (1968), Hardie, de Vries and Chiang (1971), Basavaraju and Kane (1970) and Schumacher, Smend and Borchert (1973).

[illegible] Approximate calculations

Prior to the calculations of Johnson and Cheng only the Brown K shell amplitudes were available and several schemes were used to supplement these amplitudes to obtain total atom Rayleigh scattering. Also, as the Brown K shell amplitudes are available for a much wider energy range than that covered by Johnson and Cheng, we still have use for these schemes.

The schemes are based on the form factor approximations of the [illegible] scattering [illegible] (., equations 3.166, 3.167)

$$[illegible] = \cos\theta\, F_{[illegible]}(q)$$

$$[illegible] = - F_{[illegible]}(q) \qquad [illegible]$$

where the form factor for the ith atomic shell ($i = K, L, M, \ldots$) is

$$F_i(\Delta) = 2|\kappa_i| \int d^3x \, \phi_i^{\dagger}(x) \, e^{i\Delta\cdot x} \, \phi_i(x) \, f(r) \; . \qquad (2.11)$$

Here $x = (r, \Omega)$, $\Delta = k_i - k_f$ is the change in wave vector of the photon during the scattering process, ie

$$\Delta = |\Delta| = 2k \sin \theta/2 \qquad (2.12)$$

and $2|\kappa_i|$ is the number of electrons in the atomic shell i with wave function $\phi_i(x)$. The two common choices for $f(r)$ are

$$f(r) = 1,$$

$$F_i(\Delta) \to f_i(\Delta) = 2|\kappa_i| \int d^3x \, \phi_i^{\dagger}(x) \, e^{i\Delta\cdot x} \, \phi_i(x) \qquad (2.13)$$

and

$$f(r) = mc^2/[E_i + V(r)] \; ,$$

$$F_i(\Delta) \to g_i(\Delta) = 2|\kappa_i| \int d^3x \phi_i^{\dagger}(x) \, \frac{mc^2 e^{i\Delta\cdot x}}{E_i + V(r)} \, \phi_i(x) \; . \qquad (2.14)$$

$f_i(\Delta)$ and $g_i(\Delta)$ are called the (Franz) form factor and corrected or modified form factor respectively.

The form factor approximation should only be used in the non-relativistic domain (1, §3.5)

$$\hbar\omega \ll mc^2, \; \varepsilon_i \equiv mc^2 - E_i \ll mc^2 \qquad (2.15)$$

where ε_i is the binding energy of the atomic electron. Clearly these conditions are not satisfied for the scattering of 1 MeV; rays by an element such as Pb where $\varepsilon_K(Pb) = 88$ keV and $\varepsilon_L(Pb) = 15$ keV. However the form factor approximation continues to be used as it is possible to evaluate (2.13) and (2.14) for any photon energy, scattering angle and scatterer electron state. The evaluation of the f and g form factors will be discussed in §2.3.

Before discussing the techniques for obtaining total atom Rayleigh scattering we recall that, for the choice of photon circular polarization vectors

$$\varepsilon_{\pm} = \mp \frac{1}{\sqrt{2}} \left(\varepsilon_{\parallel} \pm i \, \varepsilon_{\perp} \right) \qquad (2.16)$$

the relationship between the "spin flip" and "non spin flip" amplitudes R^{SF} and R^{NSF} respectively and the previously introduced amplitudes $R^{\perp}$ and $R^{\parallel}$ is (1, equation 3.32)

$$R^{NSF} = \tfrac{1}{2}(R^{\perp} + R^{\parallel}) \; , \; R^{SF} = \tfrac{1}{2}(R^{\perp} - R^{\parallel}) . \qquad (2.17)$$

By spin flip (or non spin flip) we mean change (or no change) in state of circular polarization during the scattering. With (2.17) the form factor approximation (2.10) is then

$$R_i^{NSF}(\omega,0) = -\frac{1}{2}(1+\cos\theta)\,F_i(\underset{\sim}{\Delta})$$
$$R_i^{SF}(\omega,0) = -\frac{1}{2}(1-\cos\theta)\,F_i(\underset{\sim}{\Delta}). \qquad (2.18)$$

Several authors (Brenner, Brown and Woodward 1954; Brown and Mayers 1956,1957; Cornille and Chapdelaine 1959; Dixon and Storey 1968; Schumacher 1969; Schumacher, Smend and Brochert 1973; Papatzacos and Mork 1975a) use the photon circular polarization vectors

$$\epsilon'_{\pm} = \frac{1}{\sqrt{2}}(\epsilon_{/\!/} \pm i\,\epsilon_{\perp}) \qquad (2.19)$$

for which we have

$$R^{NSF'} = \frac{1}{2}(R^{\perp} + R^{/\!/}),\; R^{SF} = \frac{1}{2}(R^{/\!/} - R^{\perp}). \qquad (2.20)$$

Thus the two conventions lead to spin flip amplitudes of opposite sign. In addition to this point of confusion we must change the signs of all amplitudes calculated from the equations of Brown and co-workers to ensure (Papatzacos and Mork 1975b; Schumacher, Smend and Borchert 1976) that the imaginary parts of the Rayleigh amplitudes are in agreement with the optical theorem. As Brown and co-workers use the convention (2.19) then the nett effect is that we must change the sign of their NSF amplitudes and leave unchanged the sign of their SF amplitudes when computing our total Rayleigh scattering amplitudes.

The total atom Rayleigh scattering amplitudes are

$$R^{NSF} = R_K^{NSF} + R_L^{NSF} + R_M^{NSF} + \ldots.$$
$$R^{SF} = R_K^{SF} + R_L^{SF} + R_M^{SF} + \ldots \qquad (2.21)$$

With the choice $$R_K^{NSF/SF} = BR_{R_K}^{NSF/SF}, \qquad (2.22)$$

Schumacher, Smend and Borchert (1973) suggest the following "rules' for obtaining the higher shell contributions:

(i)
$$R_L^{NSF} = BR_{R_K}^{NSF}\, f_L/f_K \qquad \hbar\omega < 200\text{ keV}$$
$$R_L^{NSF} = BR_{R_K}^{NSF}\, g_L/g_K \qquad \hbar\omega > 200\text{ keV}$$
$$R_L^{SF} = BR_{R_K}^{SF}\, f_L/f_K \qquad (2.23)$$

(ii) $$R^{NSF}_{M,N,..} = -\frac{1}{2}(1+\cos\theta)\ f_{M,N,....}$$

$$R^{SF}_{M,N,..} = -\frac{1}{2}(1-\cos\theta)\ f_{M,N,...} \qquad (2.24)$$

(iii) Omit $R^{SF}_{M,N}$ for $\hbar\omega \leq 412$ keV

(iv) Omit R^{NSF}_{L} and higher shell contributions for $\hbar\omega \geq 662$ keV.

Before presenting the arguments in favour of these rules we note that, as ${}^{BR}R_K^{NSF/SF}$ both have real and imaginary components, we are assuming in (2.23) that the real (dispersive) and imaginary (absorptive) parts of the Rayleigh amplitudes can be transferred from the K to L shell in like manner (Hardie, De Vries and Chiang 1971). However as the imaginary component contributes only 3% to the cross section at 145 keV and is negligible at higher energies, no significant error results.

Justification of the above rules is based upon the observations:

(i) at 154 keV the real part of ${}^{BR}R_K$ agrees well with the Franz form factor amplitude, ie

$$\mathrm{Re}[{}^{BR}R_K^{NSF/SF}] = -\frac{1}{2}(1 \pm \cos\theta)\ f_K, \qquad (2.25)$$

whereas for the higher energies (327,654,1308 keV) the real part of the NSF amplitude is better approximated by the corrected form factor amplitude provided the momentum transfer $\hbar\Delta$ is less than $(2\alpha Z)$ mc (Brown and Mayers 1957), ie

$$\mathrm{Re}[{}^{BR}R_K^{SF}] = -\frac{1}{2}(1-\cos\theta)\ f_K$$

$$\mathrm{Re}[{}^{BR}R_K^{NSF}] = -\frac{1}{2}(1+\cos\theta)\ g_K, \qquad (2.26)$$

(ii) the difference between f and g form factors is negligible for M,N and higher shells;

(iii) M and N shell contributions become negligible as the γ ray energy increases;

(iv) R_L^{NSF} requires the use of the corrected form factor for $\hbar\omega > 200$ keV but the condition $\hbar\Delta > (2\alpha Z)$mc, ie

$$\hbar\omega \sin\theta/2 < (\alpha Z)\ mc^2, \qquad (2.27)$$

then prevents the use of g for large momentum transfers. For Pb (2.27) becomes $\hbar\omega \sin\theta/2 < 306$ keV which, for 662 keV and 1308 keV γ rays implies scattering angles less than 55° and 27° respectively. For $\hbar\omega \geq 662$ keV it is therefore argued that the error in using g is greater than the error incurred in neglecting R_L^{NSF}.

Kissel and Pratt (1977a) comment that for high photon energies ($\hbar\omega > 10\ \epsilon_K$) the modified form factor amplitude may be used to predict total atom Rayleigh amplitudes to better than 5% accuracy provided $\hbar\omega < 1mc$ whereas for lower energies ($\epsilon_K \leq \hbar\omega < 10\epsilon_K$) the modified form factor may still be used but only for shells higher than the K shell. As Kissel and Pratt make no suggestion that rules (iii) and (iv) are needed to achieve agreement between the supplemented form factor calculations and their accurate calculations, it is possible that rules (iii) and (iv) were needed by Schumacher et al because these authors based their calculations on the now known to be inaccurate 1308 keV calculations of Brown and Mayers. Consequentially we shall use in this report as one model for calculating Rayleigh amplitudes the Brown K shell amplitudes, "Brown" L shell amplitudes calculated from ${}^{BR}R_K^{NSF/SF}$ via (2.23), and M and N shell form factor amplitudes taken to be

$$R_{M,N}^{NSF} = -\frac{1}{2}(1+\cos\theta)\, g_{M,N}$$

$$R_{M,N}^{SF} = -\frac{1}{2}(1-\cos\theta)\, f_{M,N} \quad . \qquad (2.28)$$

We shall refer to this model as the Brown-form factor or BR-FF model. In applying this model we will not use the Brown and Mayers 1308 keV calculations for $\theta \geq 90^{\circ}$.

Finally we mention the pure form factor (FF) model

$$R_{K,L,M,N}^{NSF} = -\frac{1}{2}(1+\cos\theta)\, f_{K,L,M,N}\ , \quad \hbar\omega < 200\,\text{keV}$$

$$R_{K,L,M,N}^{NSF} = -\frac{1}{2}(1+\cos\theta)\, g_{K,L,M,N}\ , \quad \hbar\omega > 200\,\text{keV}$$

$$R_{K,L,M,N}^{SF} = -\frac{1}{2}(1-\cos\theta)\, f_{K,L,M,N} \qquad (2.29)$$

If g were replaced by f we would have

$$R_i^{SF}(\theta=0) = 0, \quad R_i^{NSF}(\theta=0) = -2|\kappa_i|, \qquad (2.30)$$

that is for an atom with Z electrons the total Rayleigh amplitude is $-Z$. The measurements of Hauser and Mussgnug (1966) for $\theta = 0.5^{\circ}$ indicate the differential cross section approaches the $\theta = 0$ f form factor result $(Zr_e)^2$. The calculations of Johnson and Cheng indicate that the K + L + M relativistic amplitudes at $\theta = 0^{\circ}$ are 97% of the f form factor result. Thus the relativistic corrections to the total atom Rayleigh amplitudes at $\theta = 0^{\circ}$ are quite small.

2.3 Evaluation of form factors

We have seen in §2.2 that the f and g form factors are needed to estimate the L,M, and N shell Rayleigh amplitudes for energies not covered by the calculations of Johnson and Cheng (and also to estimate higher shell corrections to Johnson and Cheng's calculations). In §2.4 the form factors will be used to interpolate existing calculations with respect to atomic number and photon energy. For these reasons we will briefly discuss the evaluation of the f and g form factors.

We first recall that for a central potential the Schrödinger atomic wave functions have the form

$$\phi_{n\ell m_\ell m_s}(r,\theta,\phi,\zeta) = R_{n\ell}(r)\, Y_\ell^{m_\ell}(\theta,\phi)\, \chi_{m_s}(\zeta) \qquad (2.31)$$

whereas the Dirac atomic wave functions are

$$\phi_{n\kappa\mu}(r,\theta,\phi,\zeta) = \begin{pmatrix} g_{n,\kappa}(r)\, \chi_\kappa^\mu(\theta,\phi,\zeta) \\ if_{n,\kappa}(r)\, \chi_{-\kappa}^\mu(\theta,\phi,\zeta) \end{pmatrix} . \qquad (2.32)$$

Here $\chi_{m_s}(\zeta)$ are the two component Pauli spinors and the spinor spherical harmonics are

$$\chi_\kappa^\mu(\theta,\phi,\zeta) = \sum_{m_s} C(\ell,\tfrac{1}{2},j;\mu-m_s,m_s)\, Y_\ell^{\mu-m_s}(\theta,\phi)\, \chi_{m_s}(\zeta) . \qquad (2.33)$$

As $f(r) \approx 0 (\frac{v}{c})\, g(r)$ then $f(r)$ vanishes in the non-relativistic limit and $g(r)$ becomes $R(r)$. $f(r)$ and $g(r)$ are called the small and large amplitude Dirac radial functions.

If we choose $\phi_i(\underset{\sim}{x})$ to be the Schrödinger Coulomb wavefunction for a K electron, ie

$$R_{10}(r) = (Z/a_o)^{3/2}\, 2\, e^{-Zr/a_o} , \qquad (2.34)$$

where a_o is the Bohr radius, then we have

$$f_K(Q) = (1+Q^2)^{-2} \qquad (2.35)$$

where

$$Q = \frac{\hbar\Delta}{2\alpha Z} = \frac{\hbar\omega}{mc^2}\, \frac{\sin\theta/2}{\alpha Z} . \qquad (2.36)$$

Bethe has evaluated the f form factor taking $\phi_i(\underset{\sim}{x})$ to be the large amplitude Dirac Coulomb wavefunction for a K electron, that is (Rose 1961, p.179)

$$g_{1,-1}(r) = \frac{(2\alpha Z)^{\gamma_1+\frac{1}{2}}}{[2\Gamma(2\gamma_1+1)]^{\frac{1}{2}}}\, (\gamma_1+1)^{\frac{1}{2}}\, (rmc/\hbar)^{\gamma_1-1}\, e^{-\lambda r} \qquad (2.37)$$

where

$$\gamma_1 = [1-(\alpha Z)^2]^{\frac{1}{2}}, \quad \lambda = \alpha Z\, mc/\hbar = Z/a_o . \qquad (2.38)$$

He obtains (Levinger 1952)

$$f_K(Q) = \frac{\sin(2\gamma_1 \tan^{-1} Q)}{\gamma_1 Q (1+Q^2)^{\gamma_1}} . \qquad (2.39)$$

Smend and Schumacher (1974a) have evaluated the f and g form factors using full Dirac Coulomb wavefunctions for K,L,M and N electrons. They obtain

$$\begin{pmatrix} f(\Delta) \\ g(\Delta) \end{pmatrix} = 2|\kappa|\frac{D}{\Delta} \sum_{\nu=0}^{2(n-1)} a_\nu \begin{pmatrix} I_\nu \\ K_\nu \end{pmatrix} \quad (2.40)$$

where D and a_ν are messy constants not reproduced here, n is the principal quantum number and

$$I_\nu = \Gamma(2\gamma+\nu) \frac{\sin(2\gamma+\nu) \tan^{-1}(\Delta N/2\alpha Z)}{[(2\alpha Z/N)^2 + \Delta^2]^{\gamma+\nu/2}} , \quad (2.41)$$

$$K_\nu = \frac{mc^2}{E_n} \left(\frac{\alpha Zmc^2}{E_n}\right)^{2\gamma+\nu} \Gamma(2\gamma+1+\nu)$$

$$\times \text{Im} \{\exp [(\frac{2\alpha Z}{N} - i\Delta) \frac{\alpha Zmc^2}{E_n}] \ \Gamma(-2\gamma-\nu, [\frac{2\alpha Z}{N} - i\Delta]\frac{\alpha Zmc^2}{E_n})\}$$

(2.42)

Here

$$\gamma = [\kappa^2 - (\alpha Z)^2]^{\frac{1}{2}} ,$$

$$N = [n^2 + 2n\gamma + 2\kappa^2 - 2(n-\gamma)|\kappa|]^{\frac{1}{2}} \quad (2.43)$$

and $\Gamma(a,z)$ is the incomplete gamma function. I_ν obviously reduces to (2.39) for the K shell. A computer program to evaluate (2.40) has been written by Smend and Schumacher (1974b) and is available from the CPC Program Library, Queens University of Belfast, as program AAGY (and correction AAGY000A).

Smend and Schumacher (1974a) have numerically evaluated the f and g form factors using DHFS one electron wavefunctions. However they point out that such elaborate numerical calculations can be avoided by replacing the Coulomb energy levels E_n in (2.40) by $mc^2 - \varepsilon_n$ where ε_n are empirical binding energies (eg Lotz 1970), and by replacing the atomic number Z by the effective atomic number $Z_{eff} = Z - \sigma$. Excellent agreement with the DHFS results was obtained using the screening constants

$$\sigma_K = 0.45 , \quad \sigma_L = 2.5, \quad (2.44a)$$

together with the Slater values

$$\sigma_{M_{1,2,3}} = 11.25, \ \sigma_{M_{4,5}} = 21.15, \ \sigma_{N_{1,2,3}} = 27.75 \quad (2.44b)$$

In this report all form factor calculations have been performed using the computer program AAGY with experimental binding energies and the screening constants given in (2.44). For small angle scattering of low energy γ rays we will also need the Slater values

$$\sigma_{N_{4,5}} = 39.15, \quad \sigma_{N_{6,7}} = 50.55 \quad (2.44c)$$

2.4 Results for ^{152}Eu γ rays

2.4.1 Nature of the problem

In this section we will discuss the methods of interpolating existing calculations to the scattering of $^{152}_{63}$Eu γ rays (245,344,444, 779,964,1086,1112,1408 keV) by a Pb scatterer. Following Schumacher (1969), Hardie, de Vries and Chiang (1971) and Schumacher, Smend and Borchert (1973), we first interpolate with respect to energy the existing amplitudes for given Z and θ to obtain amplitudes at our desired ^{152}Eu energies. For each of these ^{152}Eu energies the interpolated amplitudes are then transferred to Z = 82. Total Rayleigh amplitudes and ultimately differential cross sections for the scattering of ^{152}Eu γ rays by Pb through $\theta = 0^{o}[15^{o}]\ 90^{o}\ [30^{o}]\ 150^{o}$ are then computed and plotted against θ so that results can be obtained for desired values of θ.

2.4.2 Interpolation of Johnson and Cheng's results

Johnson and Cheng (1976) present results for Z = 82 so that we need only interpolate with respect to photon energy. For Z = 82 the Johnson and Cheng calculations include K,L and M shells for 145 and 279 keV but only K and L shells for 412, 662 and 889 keV. For $\theta = 0^{o}$ the omitted shells dominate and the only consistent approach is to use the f form factor amplitudes

$$R_i\ (\theta = 0) = R_i\ (\theta = 0) = -2|\kappa_i| \qquad (2.45)$$

for all of the atomic shells. For $\theta \geq 30^{o}$ we have added M shell form factor amplitudes (2.28) to the 412, 662 and 889 keV results before performing the interpolation. However the M shell contribution was less than 5% except for 412 keV at 30^{o} where it was ≈ 15%.

Graphical interpolation using linear and semi-logarithmic paper was used. As the amplitudes varies over two to four orders of magnitude and in most cases were almost linear on semilogarithmic paper, the values obtained by semilogarithmic interpolation are used. Johnson-Cheng Rayleigh amplitudes for scattering of ^{152}Eu γ rays by Pb - denoted $JC_R^{NSF/SF}$ - are given in table 2.

2.4.3. Interpolation of Brown K shell amplitudes with respect to γ ray energy

Schumacher (1969) and Hardie, de Vries and Chiang (1971) note that, for fixed θ, the Brown K shell amplitudes exhibit an almost linear energy dependence on a semilogarithmic plot. Alternatively Hardie et all suggested the form factor procedure

$$^{BR}R_K^{NSF}(\hbar\omega_2,\theta) = {}^{BR}R_K^{NSF}(\hbar\omega_1,\theta)\; g_K(\hbar\omega_2,\theta)/g_K(\hbar\omega_1,\theta)$$

$$^{BR}R_K^{SF}(\hbar\omega_2,\theta) = {}^{BR}R_K^{SF}(\hbar\omega_1,\theta)\; f_K(\hbar\omega_2,\theta)/f_K(\hbar\omega_1,\theta) \qquad (2.46)$$

with g_K replaced by f_K for $\hbar\omega <$ 200 keV. The two methods agreed to within 1% for interpolation to slightly higher energies (eg $\hbar\omega_1$ = 1.31 Mev, $\hbar\omega_2$ = 1.33 MeV).

Schumacher, Smend and Borchert (1973) interpolated the ratios

$$\lambda_K^{NSF} = -\frac{1}{2}(1+\cos\theta)\; g_K/\, \mathrm{Re}\; {}^{BR}R_K^{NSF}$$

$$\lambda_K^{SF} = -\frac{1}{2}(1-\cos\theta)\; f_K/\mathrm{Re}\; {}^{BR}R_K^{SF} \qquad (2.47)$$

with g_K replaced by f_K for $\hbar\omega$ < 200 keV. They found that results obtained by direct interpolation of $^{BR}R_K^{NSF/SF}$ and those obtained by interpolation of $\lambda_K^{NSF/SF}$ agreed very closely.

In this report we used linear and semilogarithmic graphical interpolation of the Brown K amplitudes with the latter being more reliable. Brown K shell amplitudes for scattering of ^{152}Eu γ rays by Hg based upon semilogarithmic interpolation are shown in table 1. Note that, in light of the comments of Kissel and Pratt we do not use the 1308 keV $\theta \geq 90^{\circ}$ calculations of Brown and Mayers.

2.4.4 Interpolation of Brown K shell amplitudes with respect to atomic number

Accurate information concerning the Z dependence of Rayleigh scattering is not available and most workers (Schumacher 1969; Hardie, de Vries and Chiang 1971; Schumacher, Smend and Borchert 1973) assume the Z dependence indicated by the f and g form factors. Thus for each ^{152}Eu γ ray energy and scattering angle we use

$$^{BR}R_K^{NSF}(\mathrm{Pb}) = {}^{BR}R_K^{NSF}(\mathrm{Hg})\; g_K(\mathrm{Pb})/g_K(\mathrm{Hg})$$

$$^{BR}R_K^{SF}(\mathrm{Pb}) = {}^{BR}R_K^{SF}(\mathrm{Hg})\; f_K(\mathrm{Pb})/f_K(\mathrm{Hg}) \qquad (2.48)$$

Again g_K is replaced by f_K for $\hbar\omega$ < 200 keV.

Brown K shell amplitudes for scattering of ^{152}Eu γ rays by Pb are given in table 2.

2.4.5 Total atom Brown form factor Rayleigh amplitudes

"Brown" L shell amplitudes were computed from the Brown K shell amplitudes ${}^{BR}R_{K}^{NSF/SF}$ (Pb) via (2.23), ie

$${}^{BR}R_{L}^{NSF}(Pb) = {}^{BR}R_{K}^{NSF}(Pb)\ g_{L}(Pb)/g_{K}(Pb)$$

$${}^{BR}R_{L}^{SF}(Pb) = {}^{BR}R_{K}^{SF}(Pb)\ f_{L}(Pb)/f_{K}(Pb)\,. \tag{2.49}$$

M and N form factor amplitudes

$${}^{FF}R_{M,N}^{NSF}(Pb) = -\frac{1}{2}\,(1 + \cos\theta)\ g_{M,N}(Pb)$$

$${}^{FF}R_{M,N}^{SF}(Pb) = -\frac{1}{2}\,(1 - \cos\theta)\ f_{M,N}(Pb) \tag{2.50}$$

were then added to produce total atom Brown - form factor Rayleigh amplitudes

$${}^{BR-FF}R^{NSF/SF}(Pb) = {}^{BR}R_{K}^{NSF/SF}(Pb) + {}^{BR}R_{L}^{NSF/SF}(Pb) + {}^{FF}R_{M}^{NSF/SF}(Pb) + {}^{FF}R_{N}^{NSF/SF}(Pb). \tag{2.51}$$

These calculations are presented in table 2 together with Johnson-Cheng and pure form factor results.

2.4.6 Accuracy of interpolated Rayleigh amplitudes

The errors in our Rayleigh amplitudes are due to (i) the errors in the calculations of Johnson and Cheng and of Brown and co-workers, (ii) the errors incurred in using form factor amplitudes for scattering by atomic shells not included in the calculations of (i), and (iii) the errors associated with the process of interpolation. The errors in (i) are negligible compared with those of (ii) and (iii).

To assess the errors of type (ii) we note that the largest amplitudes, and therefore the largest absolute errors, occur for small angle scattering of low energy γ rays. For example at $\theta = 0^{o}$ the Johnson and Cheng amplitudes for K+L+M shell scattering of 145 and 279 keV γ rays are approximately 97% of the form factor amplitudes. Much larger relative errors occur in the small amplitudes associated with large angle scattering of high energy γ rays. For example the Johnson and Cheng spin flip amplitudes for K + L shell scattering through 150^{o} of 662 and 889 keV γ rays are 77% and 72% of the corresponding form factor amplitudes, and the much smaller non spin flip amplitudes are only 64% and 8% of the form factor amplitudes. These comments, taken with those of Kissel and Pratt mentioned in §2.2, suggest that the errors incurred in using form factor amplitudes for those angles and photon energies where the momentum transfer is small should be $\lesssim 5\%$. Fortunately for those cases where the momentum transfer is large the contributions from M and N shells are small and the large errors incurred in using the form factor approximations for these M and N shells should not lead to large errors in the total scattering amplitude (provided the large K + L shell contribution has been computed from the equations of Johnson and Cheng).

In the case of errors of type (iii), the largest absolute interpolation errors occur in the regions of greatest variation of scattering amplitudes with photon energy, ie for small angle scattering of high energy γ rays, and were $\leq 6\%$. Some larger percentage errors did occur in the interpolation of very small amplitudes ($< 10^{-2}\ r_e$) but these errors will not lead to significant errors in the total scattering amplitude.

We may conclude that the total error in our calculated Rayleigh amplitudes is probably less than 10% .

3. Nuclear Thomson scattering

The amplitudes for nuclear Thomson scattering are given by the energy-independent expressions (I, equations 4.9 and 4.10)

$$T^{NSF}(\omega,\theta) = -\frac{Z^2 m}{M}(1+\cos\theta),$$

$$T^{SF}(\omega,\theta) = -\frac{Z^2 m}{M}(1-\cos\theta). \qquad (3.1)$$

Here Ze and M are the nuclear charge and mass respectively.

Nuclear Thomson scattering amplitudes for Pb are given in table 3 and are 5% or more of the Rayleigh amplitudes of table 2 for $\theta > 75^\circ$(779 keV), $\theta > 30^\circ$(964 keV), $\theta > 15^\circ$(1086 keV) and $\theta \gtrsim 15^\circ$ (1408 keV). For photon energies above 1 MeV and for $\theta \gtrsim 90^\circ$ the spin flip nuclear Thomson amplitude is of the same order or larger than the corresponding Rayleigh amplitude.

4. Delbrück scattering

Delbrück amplitudes based upon the first Born approximation for the nuclear Coulomb field have been computed by Papatzacos and Mork (1975a) and are given in the form

$$D^{\perp}(\omega,\theta) = (\alpha Z)^2\, f_D(\omega,\theta)$$

$$D^{\parallel}(\omega,\theta) = (\alpha Z)^2\, [f_D(\omega,\theta)\cos\theta - g_D(\omega,\theta)\sin^2\theta] \qquad (4.1)$$

where $f_D(\omega,\theta)$ and $g_D(\omega,\theta)$ are, in general, complex functions of ω and θ.

For $\hbar\omega < 2mc^2$ $f_D(\omega,\theta)$ and $g_D(\omega,\theta)$ are real, ie

$$\mathrm{Im}\, D^{\perp}(\omega,\theta) = \mathrm{Im}\, D^{\parallel}(\omega,\theta) = 0, \qquad \hbar\omega < 2mc^2 \qquad (4.2)$$

and an expansion in terms of $\hbar\omega/mc^2$ yields

$$f_D(\omega,\theta) = (\frac{\hbar\omega}{mc^2})^2 (59 + 14 \cos\theta)/(32 \times 72),$$

$$g_D(\omega,\theta) = -14(\frac{\hbar\omega}{mc^2})^2/(32 \times 72) . \qquad (4.3)$$

We can estimate the contribution of Delbrück scattering to the elastic scattering of photons of energies below $2mc^2$ by considering the maximum values of the Delbrück amplitudes, ie by considering the $\theta = 0^o$ amplitudes

$$D^{\perp}(\omega,0) = D^{\parallel}(\omega,0) = (\alpha Z)^2 \frac{72}{32x72} (\frac{\hbar\omega}{mc^2})^2 \quad , \hbar\omega < 2mc^2$$

$$= D^{NSF}(\omega,0) \quad . \qquad (4.4)$$

For Pb we have

$$D^{NSF}(\omega,0) = 1.135 \times 10^{-2}(\frac{\hbar\omega}{mc^2})^2 . \qquad (4.5)$$

For photons energies of 245, 779 and 964 keV $D^{NSF}(\omega,0)$ is equal to 0.0026 r_e, 0.026 r_e and 0.040 r_e respectively. We therefore conclude that Delbrück scattering is negligible for energies $\lesssim$ 1MeV.

The Delbrück contribution at 1.408 MeV can be estimated by use of the Papatzacos and Mork 1.33 MeV amplitudes (see table 4).
The error incurred in not transferring these 1.33 MeV amplitudes to 1.408 MeV should not be significant, the ratios for linear and quadratic energy dependence being 1.06 and 1.12 respectively.

5. Nuclear resonance scattering

For a heavy spherical nucleus the amplitude for nuclear resonance scattering can be represented by a single Lorentzian centred on energy E_1 and having half width Γ_1. The nuclear resonance scattering amplitude is then (I, equation 6.8)

$$f_N(\omega,\theta) = \underset{\sim}{\varepsilon}^{(f)*}.\underset{\sim}{\varepsilon}^{(i)} \frac{\sigma_1\Gamma_1}{4\pi\hbar c} (\hbar\omega)^2 \frac{E_1^2-(\hbar\omega)^2+i\hbar\omega\Gamma_1}{[E_1^2-(\hbar\omega)^2]^2+(\hbar\omega\Gamma_1)^2} . \qquad (5.1)$$

Amplitudes for nuclear resonance scattering by Pb of 245, 779, 1086 and 1408 keV γ rays have been computed using the parameters (Veyssiere et al 1970)

$$E_1(Pb) = 13.42 \text{ MeV} \quad , \Gamma_1(Pb) = 4.05 \text{ MeV}, \sigma_1(Pb) = 640 \text{ mb} \qquad (5.2)$$

and are presented in table 5. These results can be expressed conveniently by noting that both nuclear resonance and nuclear Thomson amplitudes contain the same angular factor $\underset{\sim}{\varepsilon}^{(f)*}.\underset{\sim}{\varepsilon}^{(i)}$. Hence by dividing the nuclear resonance amplitudes by the nuclear Thomson amplitude for Pb

$$f_T(\omega,\theta) = -\ 1.765 \times 10^{-2}\ r_e\ \underset{\sim}{\varepsilon}^{(f)*}.\ \underset{\sim}{\varepsilon}^{(i)} \qquad (5.3)$$

we obtain the angle-independent ratios f_N/f_T of table 5. In all cases the nuclear resonance amplitudes are less than 3% of the nuclear Thomson amplitudes and will consequently be ignored.

6. Differential cross sections

With the neglect of nuclear resonance scattering the total amplitudes (in units of r_e) for elastic scattering of photons are

$$A^{NSF/SF} = R^{NSF/SF} + T^{NSF/SF} + D^{NSF/SF} . \qquad (6.1)$$

To obtain the differential cross section for scattering of unpolarized photons we must average $r_e^2\ |A|^2$ over initial, and sum over final, photon polarizations. If we use photon polarization vectors parallel and perpendicular to the scattering plane then, as discussed in §3.2 of I, changes of polarization during the scattering process are forbidden by parity considerations and the differential cross section for scattering of unpolarized photons is simply

$$\frac{d\sigma}{d\Omega} = r_e^2\ \frac{1}{2}\ (|A^{\parallel}|^2 + |A^{\perp}|^2). \qquad (6.2)$$

Recalling the relations

$$A^{NSF} = \frac{1}{2}\ (A^{\perp} + A^{\parallel}), \quad A^{SF} = \frac{1}{2}\ (A^{\perp} - A^{\parallel}) \qquad (6.3)$$

then (6.2) becomes

$$\frac{d\sigma}{d\Omega} = r_e^2\ (|A^{NSF}|^2 + |A^{SF}|^2). \qquad (6.4)$$

For 245, 344, 444, 779 and 964 keV differential cross sections are given in table 6 for Rayleigh plus nuclear Thomson scattering by Pb for three models of Rayleigh scattering; viz Johnson-Cheng $[d\sigma^{JC}/d\Omega]$, Brown (K+L) plus form factor (M+N) $[d\sigma^{BR-FF}/d\Omega]$ and form factor (K+L+M+N) $[d\sigma^{FF}/d\Omega]$. For 1086, 1112 and 1408 keV the differential cross sections $d\sigma^{BR-FF}/d\Omega$ and $d\sigma^{FF}/d\Omega$ are given in table 6. In addition, for 1408 keV, the differential cross sections $d\sigma_D^{BR-FF}/d\Omega$ and $d\sigma_D^{FF}/d\Omega$, which include Delbruck scattering, are given.

Finally, for each photon energy, the angular distributions of the computed differential cross sections are presented in figures 1 to 8. Differential cross sections at desired scattering angles up to 150^{o} can be directly read from these graphs.

In summary we see that for most cases the differences between the cross sections computed using the various models for Rayleigh scattering are of the order of the errors ($\leq$ 10%) in the cross sections. In general the Johnson-Cheng cross sections are lower than the Brown-form factor and pure form factor cross sections and, for higher energies and medium to large scattering angles, the pure form factor cross sections are higher than the Brown-form factor and Johnson-Cheng cross sections. For 1408 keV the inclusion of Delbruck scattering lowers the differential cross sections for $\theta < 75^{o}$ by a maximum of 7.5 mb at 15^{o} and raises the cross sections for $\theta \geq 75^{o}$ by 0.03 to 0.04 mb.

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TABLE 1. BROWN K SHELL RAYLEIGH AMPLITUDES FOR Hg(Units r_e)

		245 keV	334 keV	444 keV	779 keV	964 keV	1086 keV	1112 keV	1408 keV
0	NSF	-1.891+0.483i	-1.842+0.277i	-1.797+0.186i	-1.728+0.086i	-1.712+0.067i	-1.705+0.058i	-1.702+0.056i	-1.698+0.044i
	SF	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	NSF	-1.835+0.475i	-1.732+0.258i	-1.642+0.164i	-1.381+0.070i	-1.234+0.051i	-1.135+0.042i	-1.118+0.041i	-0.892+0.028i
	SF	-0.030+0.015i	-0.034+0.011i	-0.034+0.008i	-0.031+0.006i	-0.029+0.005i	-0.027+0.005i	-0.026+0.004i	-0.022+0.004i
30	NSF	-1.625+0.381i	-1.466+0.209i	-1.268+0.127i	-0.765+0.037i	-0.550+0.024i	-0.442+0.019i	-0.422+0.018i	-0.242+0.012i
	SF	-0.131+0.057i	-0.125+0.037i	-0.115+0.026i	-0.078+0.014i	-0.061+0.011i	-0.051+0.010i	-0.049+0.010i	-0.032+0.007i
45	NSF	-1.335+0.298i	-1.108+0.153i	-0.865+0.082i	-0.346+0.022i	-0.203+0.014i	-0.142+0.011i	-0.130+0.011i	-0.057+0.008i
	SF	-0.264+0.107i	-0.235+0.068i	-0.195+0.044i	-0.096+0.016i	-0.065+0.012i	-0.051+0.009i	-0.049+0.009i	-0.028+0.006i
60	NSF	-1.010+0.193i	-0.770+0.094i	-0.548+0.052i	-0.168+0.017i	-0.070+0.011i	-0.039+0.009i	-0.034+0.009i	-0.009+0.006i
	SF	-0.409+0.158i	-0.333+0.092i	-0.248+0.051i	-0.099+0.015i	-0.064+0.009i	-0.048+0.007i	-0.046+0.007i	-0.024+0.005i
75	NSF	-0.760+0.100i	-0.505+0.052i	-0.305+0.031i	-0.082+0.012i	-0.040+0.009i	-0.020+0.007i	-0.017+0.007i	+0.003+0.005i
	SF	-0.530+0.195i	-0.407+0.100i	-0.274+0.049i	-0.094+0.012i	-0.057+0.007i	-0.042+0.006i	-0.040+0.005i	-0.021+0.004i
90	NSF	-0.525+0.040i	-0.323+0.022i	-0.177+0.015i	-0.037+0.008i	-0.016+0.006i	-0.006+0.006i	-0.004+0.005i	+0.006+0.004i
	SF	-0.640+0.205i	-0.450+0.099i	-0.282+0.045i	-0.087+0.009i	-0.053+0.005i	-0.039+0.004i	-0.037+0.004i	-0.018+0.003i
120	NSF	-0.190+0.003i	-0.103-0.004i	-0.051-0.009i	-0.008-0.008i	-0.003-0.004i	-0.001-0.001i	0.0	+0.002+0.003i
	SF	-0.738+0.205i	-0.487+0.082i	-0.291+0.036i	-0.067+0.007i	-0.035+0.004i	-0.024+0.003i	-0.023+0.003i	-0.011+0.002i
150	NSF	-0.038-0.003i	-0.019-0.004i	-0.009-0.004i	-0.002-0.002i	-0.001-0.001i	0.0	0.0	-0.001+0.001i
	SF	-0.785+0.175i	-0.495+0.070i	-0.270+0.030i	-0.062+0.006i	-0.032+0.004i	-0.023+0.003i	-0.021+0.003i	-0.011+0.002i

TABLE 2. RAYLEIGH AMPLITUDES FOR Pb (UNITS r_e)

245 keV

		BROWN K	BROWN L	BROWN (K+L) plus FORM (M+N)	JOHNSON-CHENG	FORM (K+L+M+N)
0	NSF	-1.876+0.479i	-7[illegible].504+1.916i	-81.38+2.395i	(-82.00)	-82.0[illegible]
	SF	0.0	0.0	0.0	0.0	0.0
15	NSF	-1.823+0.472i	-6.676+1.729i	-1[illegible].35+2.201i		-11.42
	SF	-0.030+0.015i	-0.097+0.050i	-0.197+0.065i		- 0.213
30	NSF	-1.619+0.380i	-3.392+0.796i	-4.937+1.176i	-4.550+0.481i	- 4.428
	SF	-0.131+0.057i	-0.251+0.109i	-0.381+0.166i	-0.360+0.069i	- 0.365
45	NSF	-1.337+0.298i	-1.200+0.267i	-2.897+0.565i		- 2.670
	SF	-0.266+0.108i	-0.229+0.093i	-0.566+0.201i		- 0.545
60	NSF	-1.017+0.194i	-0.297+0.057i	-1.433+0.251i	-1.320+0.256i	- 1.358
	SF	-0.415+0.160i	-0.135+0.052i	-0.599+0.212i	-0.550+0.182i	- 0.573
75	NSF	-0.770+0.101i	-0.042+0.006i	-0.826+0.107i		- 0.766
	SF	-0.541+0.199i	-0.060+0.022i	-0.618+0.221i		- 0.605
90	NSF	-0.535+0.041i	+0.010-0.001i	-0.521+0.040i	-0.495+0.097i	- 0.488
	SF	-0.657+0.210i	-0.028+0.009i	-0.690+0.219i	-0.650+0.245i	- 0.677
120	NSF	-0.195+0.008i	+0.004+0.0i	-0.190+0.008i	-0.191+0.035i	- 0.197
	SF	-0.764+0.212i	-0.026+0.007i	-0.797+0.219i	-0.785+0.253i	- 0.843
150	NSF	-0.039+0.003i	0.0	-0.040+0.003i	-0.048+0.005i	- 0.047
	SF	-0.817+0.182i	-0.041+0.009i	-0.871+0.191i	-0.843+0.228i	- 0.951

TABLE 2 (Contd) RAYLEIGH AMPLITUDES FOR Pb (UNITS r_e)

334 keV

		BROWN K	BROWN L	BROWN (K+L) plus FORM (M+N)	JOHNSON-CHENG	FORM (K+L+M+N)
0	NSF	-1.828+0.275i	-7.312+1.100i	-81.14+1.375i	(-82.00)	-82.00
	SF	0.0	0.0	0.0	0.0	0.0
15	NSF	-1.722+0.257i	-5.327+0.795i	-7.096+1.052i		-6.580
	SF	-0.034+0.011i	-0.095+0.029i	-0.132+0.040i		-0.128
30	NSF	-1.466+0.209i	-1.647+0.235i	-3.548+0.444i	-3.060+0.255i	-3.329
	SF	-0.126+0.037i	-0.134+0.040i	-0.295+0.077i	-0.260+0.047i	-0.280
45	NSF	-1.117+0.154i	-0.247+0.034i	-1.462+0.188i		-1.398
	SF	-0.239+0.069i	-0.063+0.018i	-0.324+0.087i		-0.308
60	NSF	-0.784+0.096i	+0.007-0.001i	-0.774+0.095i	-0.798+0.105i	-0.764
	SF	-0.341+0.094i	-0.018+0.005i	-0.362+0.099i	-0.363+0.115i	-0.351
75	NSF	-0.518+0.053i	+0.013-0.001i	-0.502+0.052i		-0.515
	SF	-0.421+0.103i	-0.013+0.003i	-0.437+0.106i		-0.431
90	NSF	-0.334+0.023i	-0.001+0.0i	-0.337+0.023i	-0.321+0.031i	-0.350
	SF	-0.469+0.103i	-0.024+0.005i	-0.500+0.108i	-0.469+0.125i	-0.509
120	NSF	-0.108+0.004i	-0.006+0.0i	-0.116+0.004i	-0.109+0.006i	-0.134
	SF	-0.513+0.086i	-0.046+0.008i	-0.573+0.094i	-0.514+0.108i	-0.611
150	NSF	-0.020+0.005i	-0.002+0.0i	-0.023-0.005i	-0.025+0.002i	-0.030
	SF	-0.525+0.074i	-0.057+0.008i	-0.599+0.082i	-0.535+0.091i	-0.655

TABLE 2 (contd) RAYLEIGH AMPLITUDES FOR Pb (UNITS r_e)

444 keV

		BROWN K	BROWN L	BROWN (K+L) plus FORM (M+N)	JOHNSON-CHENG	FORM (K+L+M+N)
0	NSF	-1.783+0.185i	-7.132+0.740i	-80.92+0.925i	(-82.00)	-82.00
	SF	0.0	0.0	0.0	0.0	0.0
15	NSF	-1.635+0.163i	-3.856+0.384i	-5.155+0.547i		-4.874
	SF	-0.034+0.003i	-0.074+0.018i	-0.103+0.026i		-0.097
30	NSF	-1.275+0.128i	-0.516+0.052i	-1.997+0.180i	-2.305+0.147i	-1.932
	SF	-0.117+0.027i	-0.050+0.011i	-0.185+0.038i	-0.165+0.035i	-0.172
45	NSF	-0.881+0.084i	+0.012-0.001i	-0.865+0.083i		-0.852
	SF	-0.200+0.045i	-0.009+0.002i	-0.211+0.047i		-0.202
60	NSF	-0.565+0.054i	+0.006-0.001i	-0.559+0.053i	-0.526+0.049i	-0.562
	SF	-0.257+0.053i	-0.010+0.002i	-0.270+0.055i	-0.256+0.068i	-0.270
75	NSF	-0.318+0.032i	-0.012+0.001i	-0.335+0.033i		-0.373
	SF	-0.287+0.051i	-0.022+0.004i	-0.316+0.055i		-0.328
90	NSF	-0.186+0.016i	-0.041+0.001i	-0.205+0.017i	-0.199+0.011i	-0.233
	SF	-0.298+0.048i	-0.031+0.005i	-0.338+0.053i	-0.321+0.063i	-0.365
120	NSF	-0.054-0.009i	-0.006-0.001i	-0.062-0.010i	-0.063+0.002i	-0.077
	SF	-0.312+0.039i	-0.041+0.005i	-0.364+0.044i	-0.333+0.047i	-0.395
150	NSF	-0.009-0.005i	-0.001+0.0i	-0.011-0.005i	-0.014+0.001i	-0.016
	SF	-0.291+0.032i	-0.041+0.005i	-0.343+0.037i	-0.327+0.036i	-0.400

TABLE 2 (contd) RAYLEIGH AMPLITUDES FOR Pb (UNITS r_e)

779 keV

		BROWN K	BROWN L	BROWN (K+L) plus FROM (M+N)	JOHNSON-CHENG	FORM (K+L+M+N)
0	NSF	-1.715+0.085i	-6.860+0.340i	-80.58+0.425i	(-82.00)	-82.00
	SF	0.0	0.0	0.0	0.0	0.0
15	NSF	-1.385+0.070i	-0.939+0.047i	-2.677+0.117i		-2.639
	SF	-0.032+0.006i	-0.021+0.004i	-0.060+0.010i		-0.055
30	NSF	-0.785+0.038i	-0.020-0.001i	-0.760+0.037i	-0.723+0.050i	-0.771
	SF	-0.081+0.015i	-0.002+0.0i	-0.084+0.015i	-0.078+0.020i	-0.079
45	NSF	-0.363+0.023i	-0.023+0.002i	-0.394+0.025i		-0.435
	SF	-0.101+0.017i	-0.010+0.002i	-0.114+0.019i		-0.115
60	NSF	-0.179+0.018i	-0.021+0.002i	-0.206+0.020i	-0.199+0.014i	-0.223
	SF	-0.106+0.016i	-0.014+0.002i	-0.124+0.018i	-0.121+0.023i	-0.129
75	NSF	-0.088+0.013i	-0.012+0.002i	-0.104+0.015i		-0.110
	SF	-0.102+0.013i	-0.015+0.002i	-0.121+0.015i		-0.129
90	NSF	-0.041+0.009i	-0.006+0.001i	-0.049+0.010i	-0.039+0.007i	-0.055
	SF	-0.095+0.010i	-0.015+0.002i	-0.113+0.012i	-0.103+0.015i	-0.125
120	NSF	-0.009-0.009i	-0.001-0.001i	-0.010-0.010i	-0.006+0.004i	-0.013
	SF	-0.074+0.008i	-0.011+0.001i	-0.088+0.009i	-0.088+0.010i	-0.115
150	NSF	-0.002-0.003i	0.0 -0.004i	-0.002-0.003i	-0.001+0.001i	-0.002
	SF	-0.069+0.007i	-0.011+0.001i	-0.083+0.008i	-0.081+0.007i	-0.108

TABLE 2(contd) RAYLEIGH AMPLITUDES FOR Pb (UNITS r_e)

964 keV

		BROWN K	BROWN L	BROWN(K+L) plus FORM (M+N)	JOHNSON-CHENG	FORM (K+L+M+N)
0	NSF	-1.699+0.066i	-6.796+0.264i	-80.50+0.330i	(-82.00)	-82.00
	SF	0.0	0.0	0.0	0.0	0.0
15	NSF	-1.244+0.051i	-0.305+0.013i	-1.678+0.064i		-1.669
	SF	-0.029+0.005i	-0.008+0.001i	-0.040+0.006i		-0.037
30	NSF	-0.571+0.025i	-0.010+0.0i	-0.586+0.025i	-0.642+0.034i	-0.613
	SF	-0.064+0.012i	-0.004+0.001i	-0.068+0.012i	-0.075+0.015i	-0.065
45	NSF	-0.216+0.015i	-0.023+0.002i	-0.246+0.017i		-0.287
	SF	-0.070+0.012i	-0.009+0.002i	-0.081+0.014i		-0.083
60	NSF	-0.076+0.012i	-0.011+0.002i	-0.090+0.014i	-0.101+0.011i	-0.126
	SF	-0.070+0.010i	-0.010+0.001i	-0.082+0.011i	-0.071+0.014i	-0.084
75	NSF	-0.044+0.010i	-0.007+0.002i	-0.053+0.012i		-0.056
	SF	-0.063+0.008i	-0.010+0.001i	-0.074+0.009i		-0.080
90	NSF	-0.018+0.007i	-0.003+0.001i	-0.021+0.008i	-0.015+0.007i	-0.025
	SF	-0.059+0.006i	-0.009+0.001i	-0.070+0.007i	-0.059+0.009i	-0.075
120	NSF	-0.003+0.004i	-0.001+0.001i	-0.004+0.005i	-0.001+0.003i	-0.005
	SF	-0.039+0.005i	-0.006+0.001i	-0.047+0.005i	-0.049+0.005i	-0.066
150	NSF	-0.001-0.001i	0.0	-0.001-0.001i	+0.001+0.001i	-0.001
	SF	-0.037+0.004i	-0.006+0.001i	-0.044+0.005i	-0.044+0.004i	-0.062

TABLE 2(contd) RAYLEIGH AMPLITUDES FOR Pb(UNITS r_e)

1086 keV

		BROWN K	BROWN L	BROWN (K+L) plus FORM (M+N)	FORM (K+L+M+N)
0	NSF	-1.692+0.057i	-6.768+0.228i	-80.46+0.285i	-82.00
	SF	0.0	0.0	0.0	0.0
15	NSF	-1.148+0.042i	-0.111+0.004i	-1.303+0.046i	-1.306
	SF	-0.027+0.005i	-0.004+0.001i	-0.032+0.005i	-0.030
30	NSF	-0.462+0.020i	-0.0[illegible]3+0.001i	-0.494+0.021i	-0.522
	SF	-0.054+0.010i	-0.005+0.001i	-0.059+0.011i	-0.057
45	NSF	-0.152+0.012i	-0.019+0.002i	-0.177+0.014i	-0.216
	SF	-0.055+0.010i	-0.008+0.001i	-0.065+0.011i	-0.067
60	NSF	-0.043+0.010i	-0.006+0.002i	-0.051+0.012i	-0.087
	SF	-0.053+0.008i	-0.008+0.001i	-0.063+0.009i	-0.065
75	NSF	-0.022+0.008i	-0.004+0.001i	-0.027+0.010i	-0.036
	SF	-0.047+0.006i	-0.007+0.001i	-0.056+0.007i	-0.060
90	NSF	-0.006+0.006i	-0.001+0.001i	-0.007+0.007i	-0.015
	SF	-0.044+0.005i	-0.007+0.001i	-0.052+0.005i	-0.055
120	NSF	-0.001-0.001i	0.0	-0.001-0.001i	-0.003
	SF	-0.028+0.004i	-0.004+0.001i	-0.033+0.004i	-0.048
150	NSF	0.0	0.0	0.0	0.0
	SF	-0.026+0.003i	-0.004+0.001i	-0.031+0.004i	-0.045

TABLE 2 (contd). RAYLEIGH AMPLITUDES FOR Pb (UNITS r_e)

1112 keV

		BROWN K	BROWN L	BROWN (K+L) plus FORM (M+N)	FORM (K+L+M+N)
0	NSF	-1.689+0.056i	-6.756+0.224i	-80.45+0.280i	-82.00
	SF	0.0	0.0	0.0	0.0
15	NSF	-1.132+0.042i	-0.084+0.003i	-1.248+0.045i	-1.250
	SF	-0.027+0.004i	-0.004+0.001i	-0.031+0.005i	-0.029
30	NSF	-0.442+0.019i	-0.024+0.001i	-0.475+0.020i	-0.503
		-0.052-0.100i	-0.005+0.009i	-0.057+0.109i	-0.055
45	NSF	-0.140+0.012i	-0.018+0.002i	-0.163+0.014i	-0.203
	SF	-0.052+0.010i	-0.007+0.001i	-0.062+0.011i	-0.064
60	NSF	-0.037+0.010i	-0.006+0.002i	-0.045+0.011i	-0.080
	SF	-0.050+0.007i	-0.008+0.001i	-0.060+0.008i	-0.061
75	NSF	-0.019+0.008i	-0.003+0.001i	-0.023+0.009i	-0.033
	SF	-0.044+0.006i	-0.007+0.001i	-0.053+0.007i	-0.056
90	NSF	-0.004+0.006i	-0.001+0.001i	-0.005+0.007i	-0.014
	SF	-0.041+0.004i	-0.006+0.001i	-0.049+0.005i	-0.052
120	NSF	0.0	0.0	0.0	-0.003
	SF	-0.025+0.004i	-0.004+0.001i	-0.030+0.004i	-0.045
150	NSF	0.0	0.0	0.0	0.0
	SF	-0.024+0.003i	-0.004+0.001i	-0.029+0.004i	-0.042

TABLE 2 (contd). RAYLEIGH AMPLITUDES FOR Pb (UNITS r_e)

1408 keV

		BROWN K	BROWN L	BROWN (K+L) plus FORM (M+N)	FORM (K+L+M+N)
0	NSF	-1.689+0.044i	-6.756+0.176i	-80.45+0.220i	-82.00
	SF	0.0	0.0	0.0	0.0
15	NSF	-0.911+0.029i	+0.025-0.001i	-0.877+0.028i	-0.894
	SF	-0.023+0.004i	-0.001+0.0i	-0.023+0.004i	-0.022
30	NSF	-0.257+0.013i	-0.027+0.001i	-0.292+0.014i	-0.323
	SF	-0.034+0.008i	-0.004+0.001i	-0.040+0.009i	-0.039
45	NSF	-0.062+0.009i	-0.009+0.001i	-0.074+0.010i	-0.101
	SF	-0.031+0.007i	-0.005+0.001i	-0.037+0.008i	-0.039
60	NSF	-0.010+0.007i	-0.002+0.001i	-0.013+0.008i	-0.033
	SF	-0.027+0.005i	-0.004+0.001i	-0.032+0.006i	-0.035
75	NSF	+0.003+0.006i	+0.001+0.001i	+0.004+0.007i	-0.012
	SF	-0.024+0.004i	-0.004+0.001i	+0.028+0.005i	-0.031
90	NSF	+0.007+0.005i	-0.001+0.001i	+0.008+0.006i	-0.004
	SF	-0.020+0.003i	-0.003+0.001i	-0.024+0.004i	-0.028
120	NSF	+0.003+0.004i	+0.001+0.001i	+0.004+0.004i	-0.001
	SF	-0.013+0.003i	-0.002+0.0i	-0.016+0.003i	-0.024
150	NSF	+0.002+0.002i	+0.001+0.001i	+0.002+0.002i	0.0
	SF	-0.012+0.002i	-0.002+0.0i	-0.015+0.003i	-0.022

TABLE 3. NUCLEAR THOMSON AMPLITUDES FOR Pb (UNITS r_e)

	T^{NSF}	T^{SF}
0	-0.0353	0.0
15	-0.0350	-0.0006
30	-0.0329	-0.0024
45	-0.0 301	-0.0052
60	-0.0265	-0.0088
75	-0.0222	-0.0131
90	-0.0176	-0.0176
120	-0.0088	-0.0265
150	-0.0024	-0.0329

TABLE 4. 1.33 MeV DELBRUCK AMPLITUDES FOR Pb(UNITS r_e)

	D^{NSF}	D^{SF}
0	0.0863	0.0
15	0.0528	-0.0023
30	0.0319	-0.0032
45	0.0190	-0.0039
60	0.0124	-0.0038
75	0.0072	-0.0043
90	0.0047	-0.0047
120	0.0014	-0.0050
150	0.0005	-0.0045

TABLE 5. NUCLEAR RESONANCE AMPLITUDES FOR Pb

$$N(\omega,\theta) = \underset{\sim}{\varepsilon}^{(f)*}\cdot\underset{\sim}{\varepsilon}^{(i)} \; \mathcal{N}(\omega,\theta)$$

$\hbar\omega$ (keV)	Re$\mathcal{N}$(r_e)	Im $\mathcal{N}$(r_e)	Re (N)/T	Im(N)/T
245	1.236×10^{-5}	1.135×10^{-6}	7.00×10^{-4}	6.40×10^{-5}
779	1.253×10^{-4}	3.630×10^{-6}	7.10×10^{-3}	2.06×10^{-4}
1086	2.441×10^{-4}	5.092×10^{-6}	1.38×10^{-2}	2.89×10^{-4}
1408	4.124×10^{-4}	6.657×10^{-6}	2.34×10^{-2}	3.77×10^{-4}

TABLE 6. DIFFERENTIAL CROSS SECTIONS FOR Pb (UNITS mb sr^{-1})

(NUMBERS IN PARENTHESES REPRESENT POWERS OF 10)

$\hbar\omega$ (keV)	$d\sigma$	0	15	30	45	60	75	90	120	150
245	$d\sigma^{JC}$	5.34(5)		1.70(3)		1.77(2)		6.19(1)	6.07(1)	6.53(1)
	$d\sigma^{BR-FF}$	5.27(5)	1.26(4)	2.09(3)	7.35(2)	2.07(2)	9.35(1)	6.69(1)	6.09(1)	6.79(1)
	$d\sigma^{FF}$	5.34(5)	1.04(4)	1.59(3)	6.03(2)	1.79(2)	7.96(1)	5.87(1)	6.35(1)	7.71(1)
334	$d\sigma^{JC}$	5.34(5)		7.71(2)		6.70(1)		2.93(1)	2.53(1)	2.63(1)
	$d\sigma^{BR-FF}$	5.23(5)	4.13(3)	1.04(3)	1.89(2)	6.34(1)	3.50(1)	3.23(1)	3.05(1)	3.23(1)
	$d\sigma^{FF}$	5.34(5)	3.48(3)	9.04(2)	1.70(2)	6.00(1)	3.86(1)	3.28(1)	3.39(1)	3.77(1)
444	$d\sigma^{JC}$	5.34(5)		3.34(2)		3.04(1)		1.32(1)	1.09(1)	1.04(1)
	$d\sigma^{BR-FF}$	5.20(5)	2.16(3)	3.33(2)	6.80(1)	3.39(1)	1.90(1)	1.43(1)	1.27(1)	1.14(1)
	$d\sigma^{FF}$	5.34(5)	1.91(3)	3.09(2)	6.52(1)	3.37(1)	2.16(1)	1.67(1)	1.47(1)	1.49(1)
779	$d\sigma^{JC}$	5.34(5)		4.61(1)		5.46		1.43	1.08	1.04
	$d\sigma^{BR-FF}$	5.16(5)	5.85(2)	5.07(1)	1.55(1)	5.78	2.72	1.74	1.09	1.08
	$d\sigma^{FF}$	5.34(5)	5.68(2)	5.19(1)	1.83(1)	6.48	2.98	2.05	1.64	1.58
964	$d\sigma^{JC}$	5.34(5)		3.68(1)		1.83		5.68(-1)	4.69(-1)	4.72(-1)
	$d\sigma^{BR-FF}$	5.15(5)	2.34(2)	3.09(1)	6.68	1.77	1.07	7.45(-1)	4.52(-1)	4.74(-1)
	$d\sigma^{FF}$	5.34(5)	2.31(2)	3.35(1)	8.60	2.56	1.17	8.34(-1)	7.02(-1)	7.17(-1)
1086	$d\sigma^{BR-FF}$	5.15(5)	1.42(2)	2.24(1)	3.82	9.13(-1)	5.81(-1)	4.45(-1)	2.95(-1)	3.27(-1)
	$d\sigma^{FF}$	5.34(5)	1.43(2)	2.47(1)	5.22	1.47	6.90(-1)	5.10(-1)	4.58(-1)	4.83(-1)
1112	$d\sigma^{BR-FF}$	5.14(5)	1.31(2)	2.17(1)	3.34	8.04(-1)	5.37(-1)	4.04(-1)	2.66(-1)	3.07(-1)
	$d\sigma^{FF}$	5.34(5)	1.31(2)	2.13(1)	4.69	1.30	6.18(-1)	4.70(-1)	4.23(-1)	4.47(-1)
1408	$d\sigma^{BR-FF}$	5.14(5)	6.62(1)	8.55	1.01	2.69(-1)	1.65(-1)	1.51(-1)	1.51(-1)	1.84(-1)
	$d\sigma^{FF}$	5.34(5)	6.86(1)	1.02(1)	1.52	4.40(-1)	2.46(-1)	2.07(-1)	2.15(-1)	2.41(-1)
	$d\sigma_D^{BR-FF}$	5.13(5)	5.87(1)	7.00	7.55(-1)	2.31(-1)	1.76(-1)	1.81(-1)	1.86(-1)	2.24(-1)
	$d\sigma_D^{FF}$	5.33(5)	6.10(1)	8.49	1.18	3.66(-1)	2.41(-1)	2.30(-1)	2.56(-1)	2.89(-1)

FIGURE CAPTIONS

Fig. 1 Differential cross sections (mb sr^{-1}) for elastic scattering of 245 keV γ rays by Pb as a function of scattering angle

——— Differential cross sections $d\sigma^{BR-FF}/d\Omega$ computed using Nuclear Thomson and Brown-form factor Rayleigh scattering amplitudes.

—·— Differential cross sections $d\sigma^{FF}/d\Omega$ computed using Nuclear Thomson and pure form factor Rayleigh scattering amplitudes.

– – – Differential cross sections $d\sigma^{JC}/d\Omega$ computed using Nuclear Thomson and Johnson-Cheng Rayleigh scattering amplitudes.

Fig. 2 As for Fig. 1 but for 334 keV γ rays

Fig. 3 As for Fig. 1 but for 444 keV γ rays

Fig. 4 As for Fig. 1 but for 779 keV γ rays

Fig. 5 As for Fig. 1 but for 964 keV γ rays

Fig. 6 Differential cross sections (mb sr^{-1}) for elastic scattering of 1086 keV γ rays by Pb as a function of scattering angle

——— $d\sigma^{BR-FF}/d\Omega$

—·— $d\sigma^{FF}/d\Omega$

Fig. 7 As for Fig. 6 but for 1112 keV γ rays

Fig. 8 Differential cross sections (mb sr^{-1}) for elastic scattering of 1408 keV γ rays by Pb as a function of scattering angle

——— $d\sigma^{BR-FF}/d\Omega$

—·— $d\sigma^{FF}/d\Omega$

– – – Differential cross sections $d\sigma_D^{BR-FF}/d\Omega$ computed using Nuclear Thomson, Delbrück and Brown- form factor Rayleigh scattering amplitudes

—··— Differential cross sections $d\sigma_D^{FF}/d\Omega$ computed using Nuclear Thomson, Delbruck and pure form factor Rayleigh scattering amplitudes.

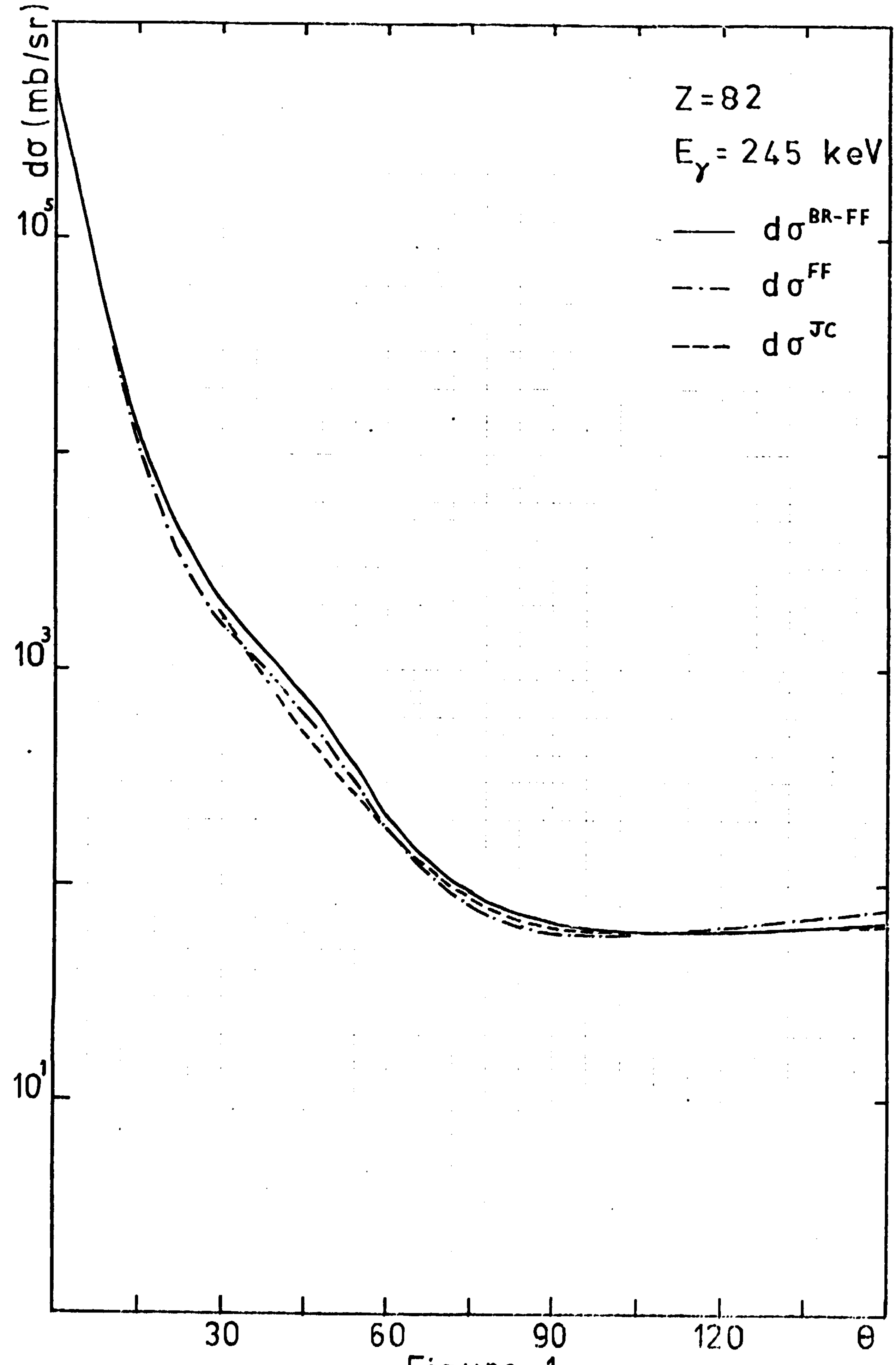
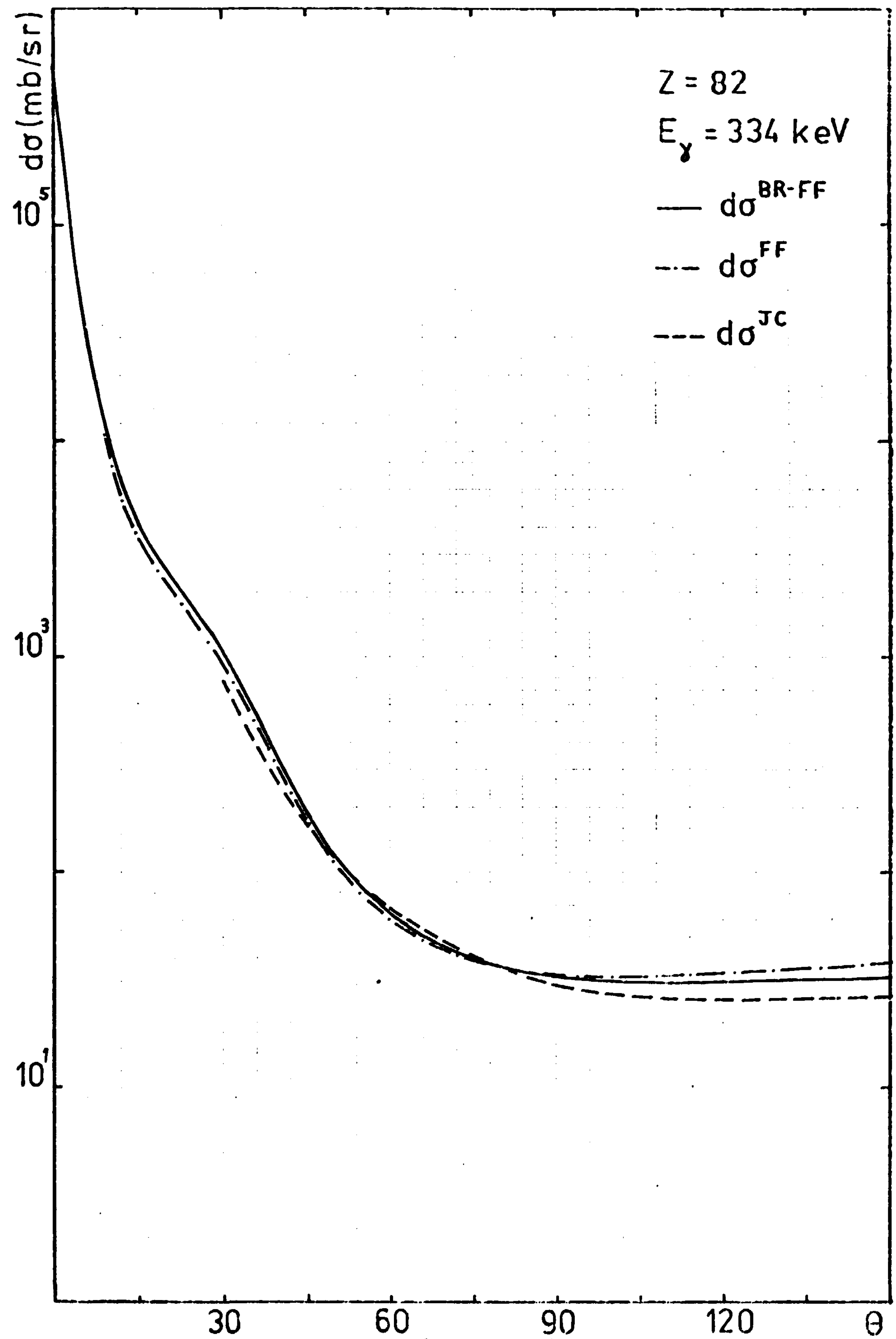
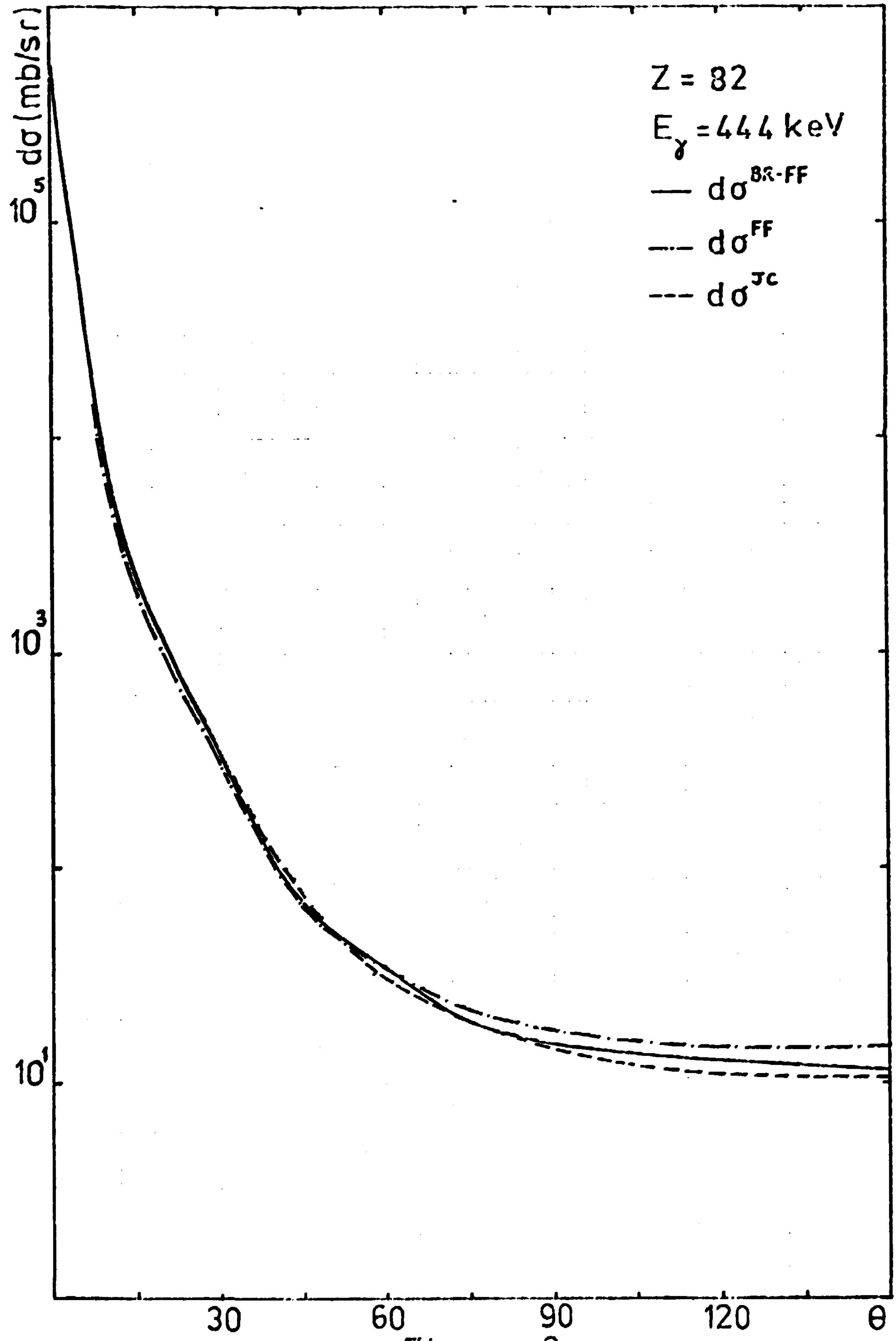
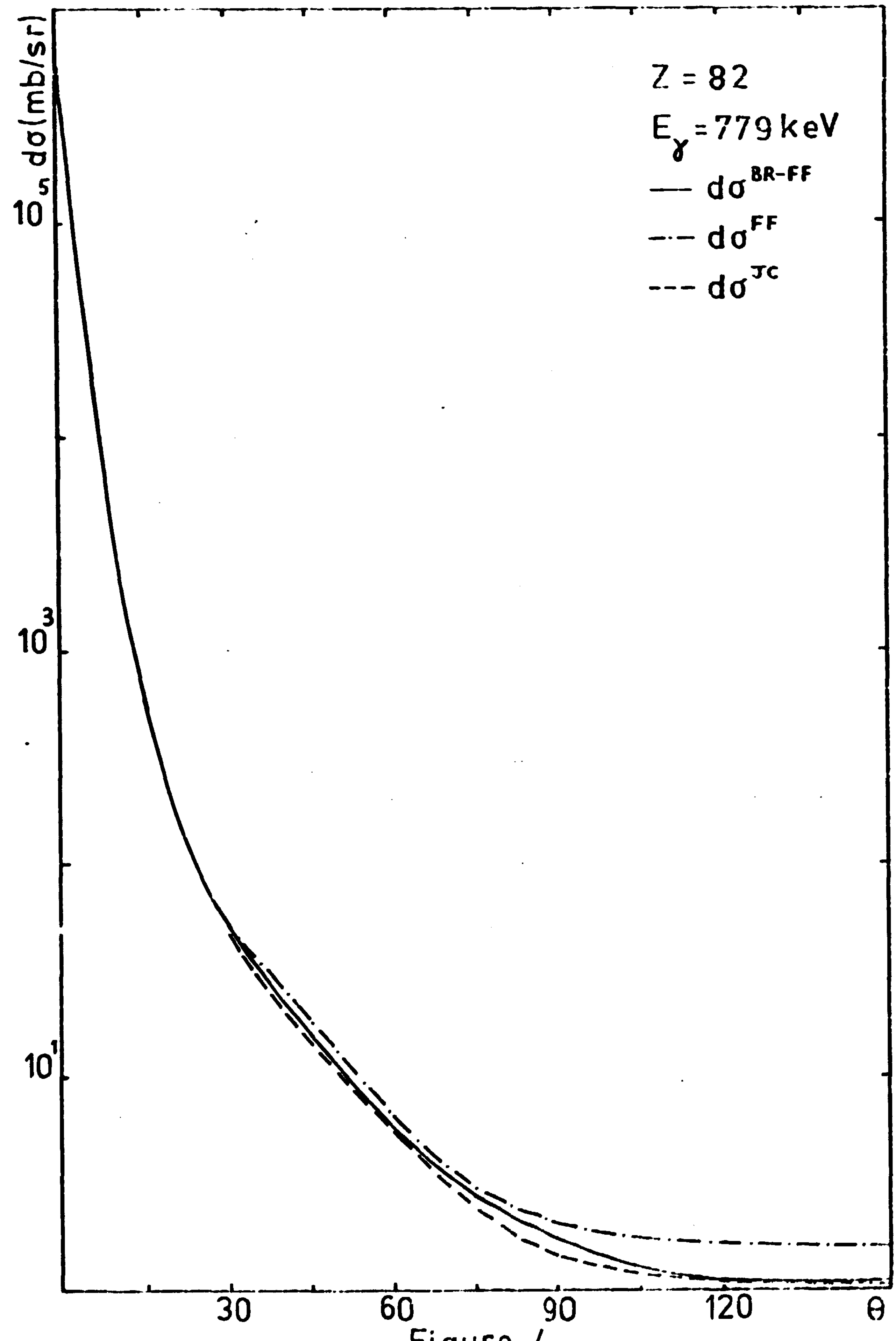
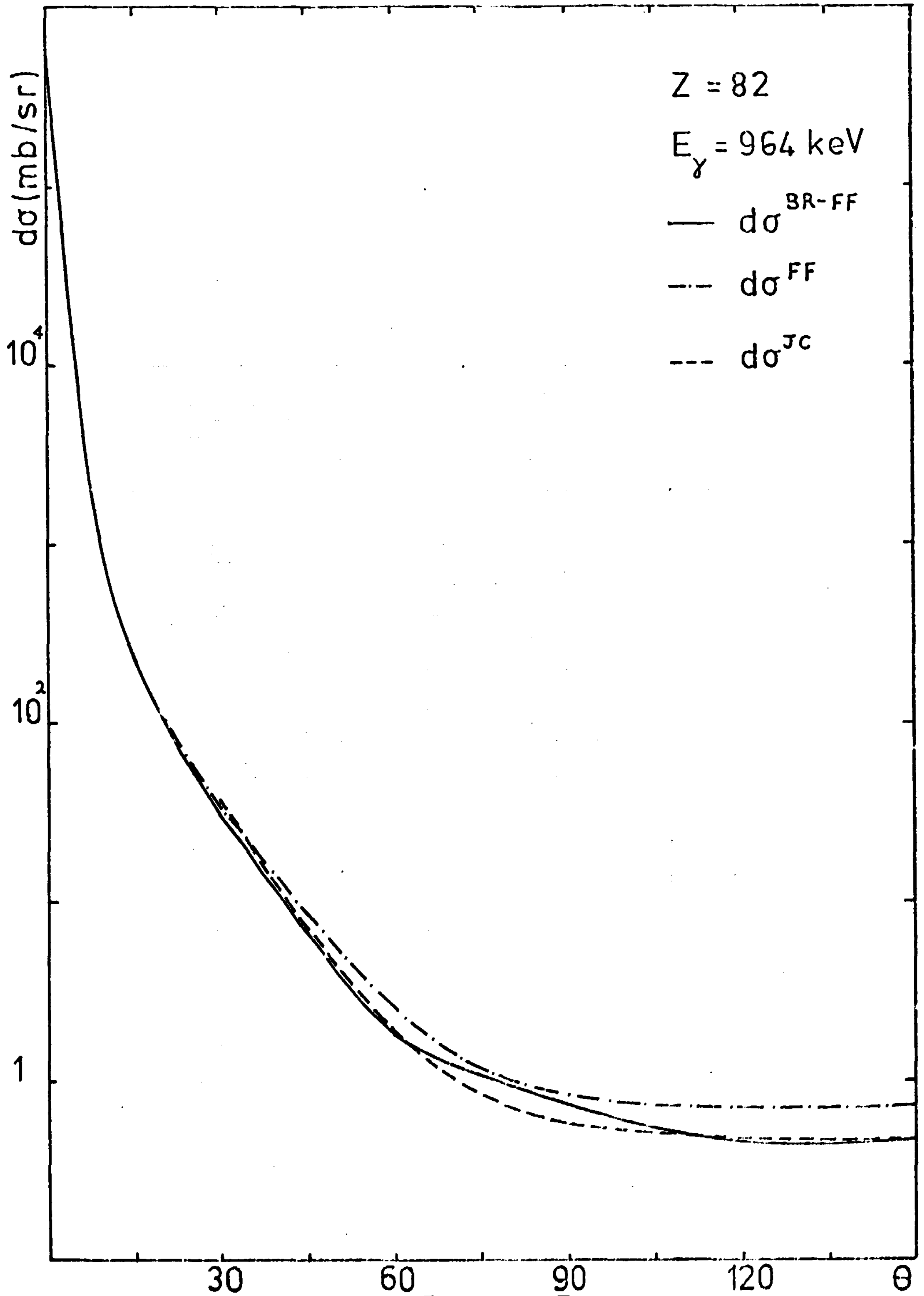

dσ (mb/sr)
10^5
10^3
10^1
Z=82
E_γ = 245 keV
$d\sigma^{BR-FF}$
$d\sigma^{FF}$
$d\sigma^{JC}$
30
60
90
120
θ

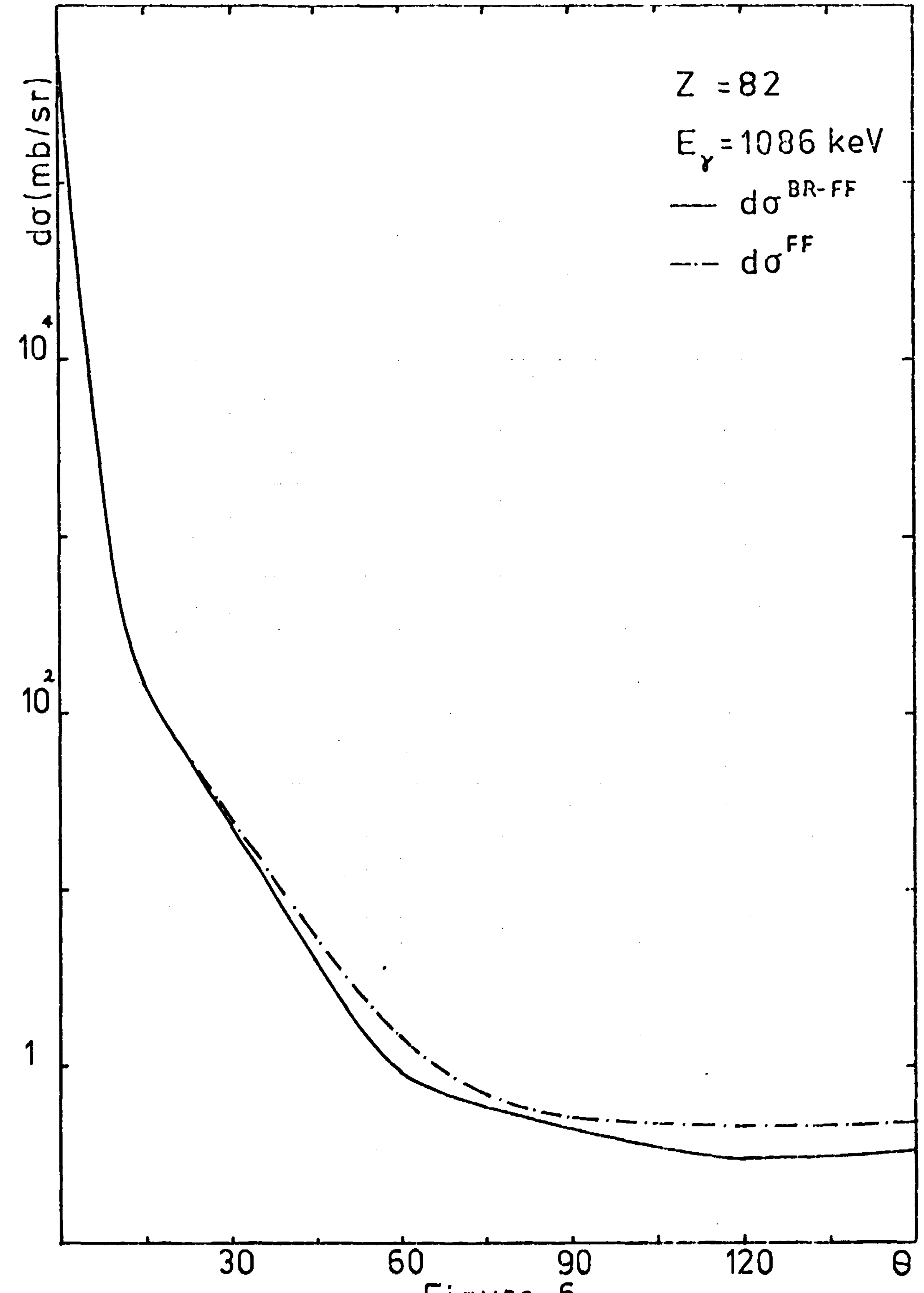
Figure 1


dσ(mb/sr)
10^5
10^3
10^1
Z = 82
E_γ = 334 keV
$d\sigma^{BR-FF}$
$d\sigma^{FF}$
$d\sigma^{JC}$
30
60
90
120
θ
Figure 2


10^5
10^3
10^1
$d\sigma$ (mb/sr)
Z = 82
E_γ = 444 keV
$d\sigma^{BR\text{-}FF}$
$d\sigma^{FF}$
$d\sigma^{JC}$
30
60
90
120
θ
Figure 3


dσ(mb/sr)
10^5
10^3
10^1
Z = 82
E_γ = 779 keV
— $d\sigma^{BR-FF}$
-·- $d\sigma^{FF}$
--- $d\sigma^{JC}$
30
60
90
120
θ
Figure 4


dσ(mb/sr)
Z = 82
E_γ = 964 keV
dσ^BR-FF
dσ^FF
dσ^JC
10^4
10^2
1
30
60
90
120
θ
Figure 5



dσ(mb/sr)
10⁴
10²
1
Z = 82
E_γ = 1086 keV
— $d\sigma^{BR-FF}$
-·- $d\sigma^{FF}$
30
60
90
120
θ

Figure 6

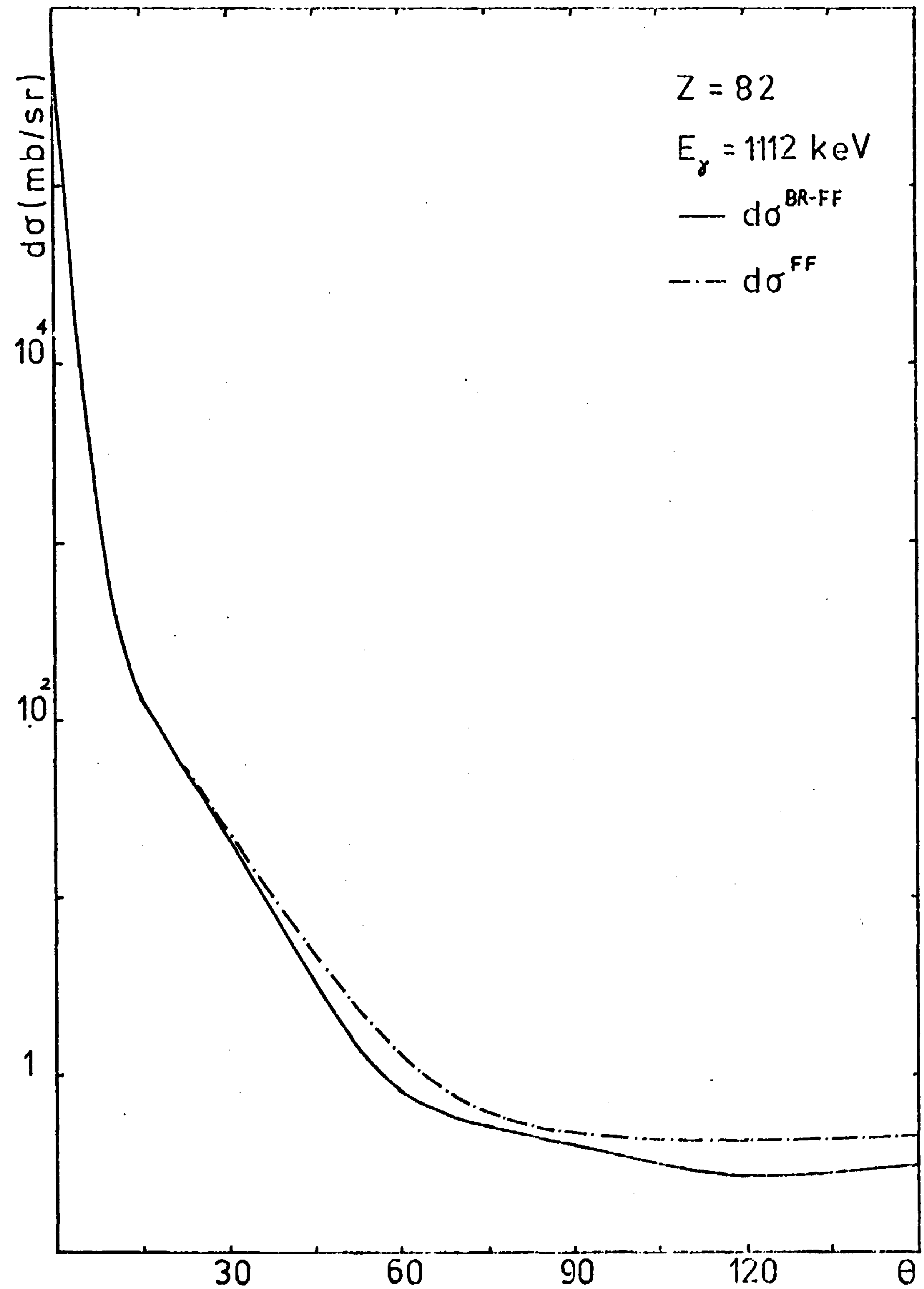


Figure 7

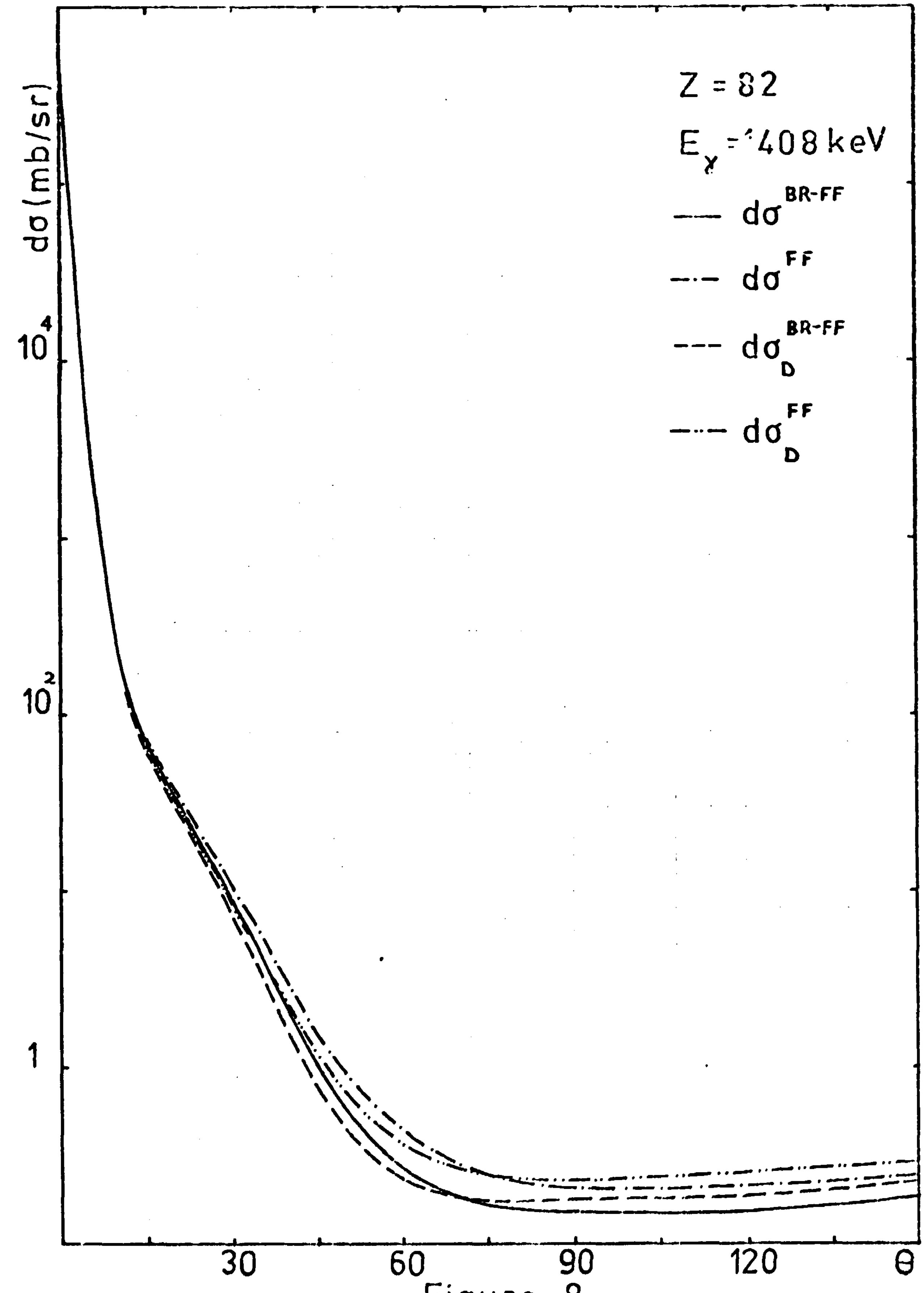

dσ(mb/sr)
10^4
10^2
1
Z = 82
E_γ = 1408 keV
$d\sigma^{BR-FF}$
$d\sigma^{FF}$
$d\sigma_D^{BR-FF}$
$d\sigma_D^{FF}$
30
60
90
120
θ

Figure 8

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