The Measurement of Magnetic Moments of Nuclear States of High Angular Momentum

G. Goldring

The Weizmann Institute of Science

Rehovot

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With the advent of heavy ion accelerators, new regions of nuclear spectroscopy have opened up for investigation, and new phenomena and regularities have been discovered, relating mostly to states of high angular momentum. The most striking of these is probably the "back bending" exhibited by many rotational nuclei - the abrupt change in the moment of inertia at a certain energy of excitation. The interpretation of these phenomena is still far from complete and it is generally believed that the determination of the g-factors of the relevant nuclear levels will be of critical importance in the elucidation of the nature of the processes involved, similar to the role played by systematics of magnetic moments in the now classical nuclear spectroscopy and nuclear structure studies.

There are a number of problems associated with such measurements. First, as the relevant nuclear states are usually at high excitation energies, the mean lives of those states tend to be short - of the order of a few picoseconds or less - and very high magnetic fields are required for the measurements, 50 Megagauss or more. Such fields are available only as hyperfine fields of electrons on other charged particles that are moving in close proximity to the nucleus. Second - the large angular momentum of those nuclear states in itself poses some difficult problems. In the sequel we shall discuss these problems as well as means for their circumvention.

We shall consider here for the most part nuclei that recoil from a nuclear reaction at high speed and move in either vacuum or gas. The environment of these nuclei are the isolated ions with which

they are associated. It has been established that the hyperfine interaction with such ions is primarily magnetic, and in fact no trace of an electric quadrupole interaction could be found even in extreme cases where large nuclear quadrupole moments were involved.

We consider first the symmetry of the nuclear and the ionic systems:

The nuclei under observation usually have a symmetry axis along the impinging particle beam, e.g. if de-excitation gamma rays are observed either directly or in coincidence with backscattered particles or reaction products detected in an annular counter. There are, however, exceptions : particle detection systems that are not cylindrically symmetric around the beam axis.

The ions usually have a symmetry axis along the recoil direction, but there are exceptions to that too- e.g. if the foil is not normal to the recoil axis (the so-called "tilted foil" geometry)[1].

We see that in most cases the nuclear and ionic symmetry axis coincide. The hyperfine interaction then does not affect the symmetry of the γ radiation emitted from the perturbed state, the emission pattern will be only attenuated. We shall examine in the following a special but representative case with an isotropic ionic environment - and we shall see that if the angular distribution of the de-excitation γ's is initially given by:

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$$W(\theta) = \sum_k A_k P_k (\cos\theta)$$

then the hyperfine interaction will transform it into:

$$W_p(\theta) = \sum_k A_k G_k P_k (\cos\theta)$$

Where G_k are the attenuation coefficients. For small perturbations or small precession angles, ωt , of the nuclei in the hyperfine fields, the attenuation coefficients will be shown to be of the form:

$$G_k = 1 - \alpha_k^2 \; ; \quad \alpha_k \propto \omega t$$

This is evidently a measurement of second order in the precession angle and is, specifically, insensitive to the sign of the nuclear g-factor.

If the nuclear and ionic systems do not have identical symmetry axes and if in addition one of the systems is polarized (e.g. in "tilted foils") then the hyperfine interaction will have a polarized component. We shall later examine such a case in the special condition of orthogonal nuclear and ionic symmetry axes. For small perturbations we shall then find the perturbed distribution of γ's to be of the form:

$$W_p(\theta) \sim W(\theta-\beta) \sim W(\theta)\left\{1-\beta\frac{1}{W}\frac{dW}{d\theta}\right\} \; ; \; \beta \propto \omega t$$

This is a measurement of first order in the precession angle and is obviously a more powerful and revealing measurement than symmetric attenuation, but until now the scope of such measurements (in the

context of isolated ions) was very restricted due to the limited polarization that has been achieved so far.

The hyperfine field

The Hamiltonian for the M1 interaction of a nucleus with a bound electron can be written in the non relativistic limit as[2]:

$$H_{M1} = 2\mu_B \{\frac{1}{r^3}(\vec{\ell},\vec{\mu}) - \frac{1}{r^3}[(\vec{s},\vec{\mu}) - \frac{3(\vec{s},\vec{r})(\vec{\mu},\vec{r})}{r^2}] + \frac{8\pi}{3}(\vec{s},\vec{\mu})\,\delta^{(3)}(\vec{r})\} \qquad (1)$$

here $\vec{\mu}$ is the nuclear magnetic moment operator and all other parameters refer to the electron.

We write:

$$H_{M1} = -(\vec{\mu},\vec{H})$$

defining the field operator $\vec{H}$.

$\vec{\mu}$ and $\vec{H}$ operate in the spaces I, M_I and J, M_J respectively. According to the Wigner-Eckart theorem the matrix elements of such vector operators depend on the angular momentum variables in a purely geometric way and their individual character is revealed only in a factor of proportionality, the reduced matrix element. In particular, for the matrix elements of the scalar product of two vector operators $\vec{T},\vec{U}$ in the spaces I, J respectively, with I, J adding to a total angular momentum F, M_F, we have [3]:

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$$(I\ J\ F'M_{F'}|(\vec{T},\vec{U})|I\ J\ F\ M_F)=(-)^{I+J+F}\,\delta_{F'F}\delta_{M_{F'},M_F}\begin{Bmatrix} F & I & J \\ I & I & J \end{Bmatrix}\times$$

$$\times\ (I\ ||\ \vec{T}\ ||I)\ (J\ ||\ \vec{U}\ ||\ J)$$

and we can therefore write:

$$H_{M1} = a\ (\vec{I},\vec{J}) \qquad (2)$$

$$\text{with:}\quad a = -\frac{(I\ ||\ \vec{\mu}\ ||\ I)(J\ ||\ \vec{H}\ ||\ J)}{(I\ ||\ \vec{I}\ ||\ I)(J\ ||\ \vec{J}\ ||\ J)}$$

We have: $\dfrac{(I\ ||\ \vec{\mu}\ ||\ I)}{(I\ ||\ \vec{I}\ ||\ I)} = \dfrac{(I,\ M_I = I\ |\ \mu_z\ |\ I,I)}{(I,\ I\ |\ I_z\ |\ I,I)} = \dfrac{\mu}{I}$

due to the definition of the magnetic moment μ and the matrix elements of I_z.

In analogy to μ, the hyperfine field $H(o)$ is defined by:

$$H(o) = -(J,\ J\ |\ H_z\ |\ J,J)$$

and therefore: $$-\frac{(J\ ||\ \vec{H}\ ||\ J)}{(J\ ||\ \vec{J}\ ||\ J)} = \frac{H(o)}{J}$$

and:

$$a = \frac{\mu}{I}\,\frac{H(o)}{J} = \mu_N g\,\frac{H(o)}{J} \qquad (2a)$$

due to the definition of the nuclear g factor as:

$$\mu = \mu_N \, g \, I$$

H(o) is computed in ref.2) for hydrogen-like ions as :

$$H(o) = K \frac{Z^3}{n^3 (\ell+1/2)(j+1)} \quad ; \quad K = 2 \frac{\mu_B}{a_o^3} = 0.125 \text{ Megagauss}; \quad a_o = \hbar^2/me^2 \qquad (3)$$

in particular for s electrons (ℓ=o, j= 1/2):

$$H(o) = K' \frac{Z^3}{n^3} \quad ; \quad K' = \frac{8}{3} \frac{\mu_B}{a_o^3} = 0.167 \text{ Megagauss} \qquad (3a)$$

The Hamiltonian (2) commutes with : F = I + J

$$[H_{M1}, \vec{F}^2] = o$$

$$[H_{M1}, F_z] = o$$

We have: $\vec{F}^2 = \vec{I}^2 + \vec{J}^2 + 2(\vec{I},\vec{J})$

and therefore:

$$H_{M1} = \frac{a}{2} \{\vec{F}^2 - \vec{I}^2 - \vec{J}^2\}$$

and the eigenvalues of H_{M1} are given by:

$$E_F = \frac{a}{2} \{F(F+1) - I(I+1) - J(J+1)\}$$

The effect of the interaction (2) on the nuclear ensemble is given by the equation of motion:

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$$i\hbar\dot{\rho} = [H_{M1}, \rho]$$

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for the nuclear density matrix ρ in the space of M_I:

$$\rho \equiv \rho_{M\,M'} \; ; \; M \equiv M_I$$

If the nucl ...s; $a_o = \hbar^2/me^2$ (3)

If the nuclear system has a symmetry axis z, $I_z = M$ is a good quantum number, and if α_M is the relative number of nuclei in the state M then:

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$\rho_{M\,M'} = \delta_{M}$

$\rho_{M\,M'} = \delta_{M\,M'}\,\alpha_M \; ; \; \sum_M \alpha_M = 1$

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the equation of motion is solved formally by:

$$\rho(t) = e^{-iH_{M1}t/\hbar}\,\rho(o)\,e^{iH_{M1}t/\hbar}$$

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and transforming to the diagonal scheme F, M_F:

$\rho_{M\,M'}($

$$\rho_{M\,M'}(t) = \sum_{\bar{M}\,\bar{M}'\;F\,F'} \langle M|F\rangle e^{-i\,E_F t/\hbar} \langle F|\bar{M}\rangle \rho_{\bar{M}\,\bar{M}'}(o) \langle \bar{M}'|F'\rangle e^{i\,E_{F'}t/\hbar} \langle F'|M'\rangle$$

$= \sum_{F\,F'} e$

$$= \sum_{F\,F'} e^{-i\omega_{FF'}t}\,\chi_{F\,F'}$$

$$\omega_{F\,F'} = \frac{a}{2\hbar}\,\{F(F+1) - F'(F'+1)\} \qquad (4)$$

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introducing the hyperfine frequencies $\omega_{F\,F'}$ and the perturbation parameters $\chi_{F\,F'}$.

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For given I, J the number of $\omega_{F\,F'}$ is given by:

$$\#(\omega_{F\,F'}) = \begin{cases} J(2J+1) & \text{if } J < I \\ I(2I+1) & \text{if } J > I \end{cases} \qquad (5)$$

...emble is given by

for J =

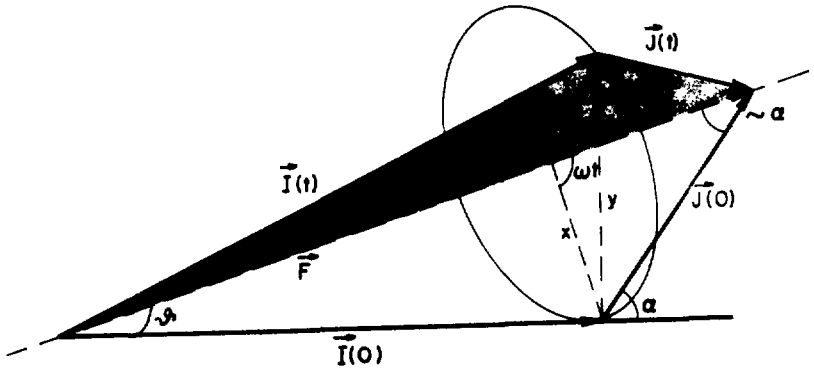
for J = 1/2 there is only one $\omega_{F\,F'}$:

$$\omega_{I+1/2,I-1/2} = \frac{a}{2\pi}(2I+1) = \omega' \qquad (6)$$

The expression for $\rho_{M\,M'}(t)$ above is calculated in more detail in general and in some specific cases in, e.g. refs. 4) and 5). We shall not follow those calculations here but rather consider some simple situations and simplified calculations following the vector coupling model, in order to exhibit the main features of the interaction process.

Approximation for HFI within an isotropic ionic ensemble for small J/I

Consider the coupling scheme in the figure: $\vec{I} + \vec{J} = \vec{F}$, where we assume for the sake of simplicity and ease of calculation that $J/I \ll 1$, The vector $\vec{F}$ is a constant of motion and the motion of the vector $\vec{I}$ will therefore proceed as a turning of the triangle $\vec{I}$, $\vec{J}$, $\vec{F}$ around $\vec{F}$.



We read from the figure:

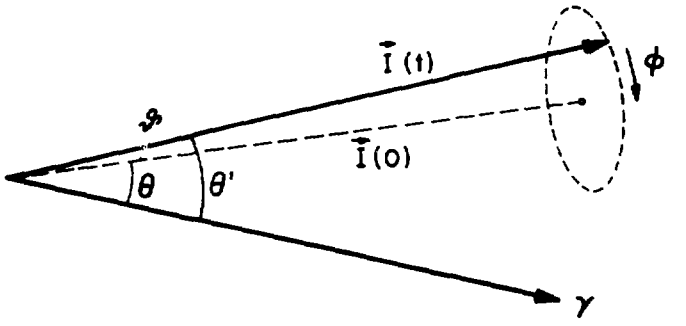
$$x \sim J \sin\alpha$$

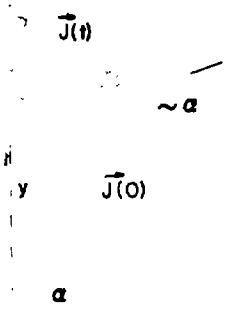
$$y = 2x \sin\frac{\omega t}{2} \sim 2J \sin\alpha \sin\frac{\omega t}{2}$$

$$\vartheta \sim \frac{y}{I} \sim \frac{2J}{I} \sin\alpha \sin\frac{\omega t}{2} = \frac{2J}{I} \sin\alpha \sqrt{\frac{1}{2}(1-\cos\omega t)}$$

We assume $\vec{J}$ to be distributed uniformly in space, and we therefore have to average over its direction, integrating over $d\phi \sin\alpha \, d\alpha$. Let us integrate first over $d\phi$, the azimuthal angle. This will move $\vec{I}$ from its original position to a cone around $\vec{I}(o)$ with opening angle ϑ. In order to evaluate the effect of this transformation on the angular distribution $W(\theta') = \sum_k A_k P_k(\cos\theta')$, we write:

$$P_k(\cos\theta') = \frac{4\pi}{2k+1} \sum_q Y_{kq}(\theta\,\phi')\, Y_{kq}(\vartheta,\phi)^*$$





and:

$$\frac{1}{2\pi}\int_0^{2\pi} P_k(\cos\theta')d\phi = \frac{4\pi}{2k+1}\; Y_{ko}(\theta)\; Y_{ko}(\nu) = P_k(\cos\theta)P_k(\cos\nu)$$

We get:

$$W(\theta,t,\alpha) = \frac{1}{2\pi}\int_0^{2\pi} W(\theta')d\phi = \Sigma\; A_k\; G_k\;(t,\alpha)P_k\;(\cos\theta)$$

with:

$$G_k(t,\alpha) = P_k(\cos\nu)$$

We wish to evaluate this attenuation coefficient for small ν. $P_k(\cos\nu)$ is a solution of the differential equation:

$$(1-x^2)\frac{d^2P_k(x)}{dx^2} - 2x\frac{d\,P_k(x)}{dx} + k(k+1)P_k(x) = o$$

$$x = \cos\nu$$

set: $u = 1 - x$, then:

$$(2-u)u\frac{d^2P_k(u)}{du^2} + 2(1-u)\frac{d\,P_k(u)}{du} + k(k+1)P_k(u) = o$$

for $u \to o$: $2\frac{d\,P_k(u)}{du} + k(k+1)\,P_k(u) = o$

and: $P_k(u) = e^{-\frac{k(k+1)}{2}u} = 1 - \frac{k(k+1)}{2}u = 1 - \frac{k(k+1)}{4}\nu^2$

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$$G_k(t,\alpha) = 1 - k(k+1)\left(\frac{J}{I}\right)^2 \frac{\sin^2\alpha}{2} \quad (1-\cos\omega t)$$

integrating now over $\sin\alpha\, d\alpha$ we get:

$$W(\theta,t) = \frac{1}{2}\int_0^{\pi} W(\theta,t,\alpha)\sin\alpha\, d\alpha = \sum_k A_k\, G_k(t)\, P_k(\cos\theta)$$

$$G_k(t) = \frac{1}{2}\int_0^{\pi} G_k(t,\alpha)\sin\alpha\, d\alpha = 1 - \frac{k(k+1)}{3}\left(\frac{J}{I}\right)^2 (1-\cos\omega t) \qquad (6)$$

and the"integral attenuation coefficient" G_k, for:

$$W(\theta) = \frac{1}{\tau}\int_0^{\infty} W(\theta,t)e^{-t/\tau}dt$$

becomes:

$$G_k = \frac{1}{\tau}\int_0^{\infty} G_k(t)e^{-t/\tau}dt = 1 - \frac{k(k+1)}{3}\left(\frac{J}{I}\right)^2 \frac{(\omega\tau)^2}{1+(\omega\tau)^2} \qquad (6a)$$

The attenuation coefficients $G_k(t)$ are seen to go through a "breathing" motion as t increases, whereas the integral attenuation coefficint G_k decreases gradually and steadily with τ. Both coefficients attain a minimum value:

$$G_k(t)_{min} = 1 - \frac{2}{3}k(k+1)\left(\frac{J}{I}\right)^2 ; \quad \omega t = (2n+1)\pi \qquad (7)$$

$$G_{k\,min} = 1 - \frac{k(k+1)}{3}\left(\frac{J}{I}\right)^2 ; \quad \omega\tau \to \infty \quad (\text{"hard core"}) \qquad (7a)$$

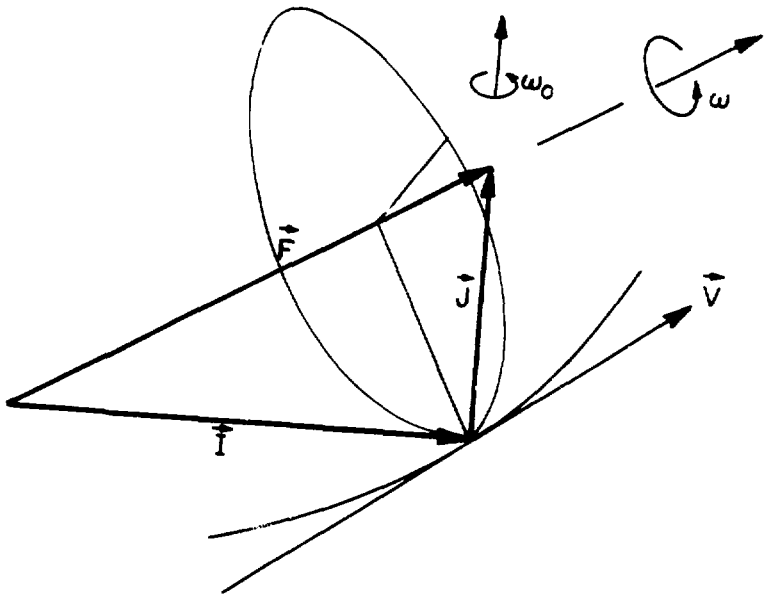
The origin of this limitation can be seen clearly in our vector construction. It is obvious from (7), (7a) that the range of variability of $G_k(t)$, G_k becomes more and more restricted as $\frac{I}{J}$ increases. This is the core of the problem of observing motions induced by hyperfine interaction in nuclear states of high spin.

Let us now examin the limit of (6a) for small perturbations:

$$G_k \underset{\omega\tau\to 0}{\to} 1 - \frac{k(k+1)}{3} \left(\frac{J}{I}\right)^2 (\omega\tau)^2 = 1 - \frac{k(k+1)}{3} (\omega_0\tau)^2;\ \omega_0 = \frac{J}{I}\omega \qquad (8)$$

introducing the frequency ω_0.

In order to examine the significance of ω and ω_0 more closely we consider the triangle $\vec{I}$, $\vec{J}$, $\vec{F}$ in a special case: $\vec{I} \perp \vec{J}$, representing in a sense the "mean" position of $\vec{J}$ relative to $\vec{I}$.



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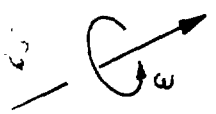

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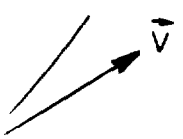

The hyperfine field, initially in the direction of $\vec{J}$, acts on $\vec{I}$ so as to move its tip with a "velocity" v , and as the triangle $\vec{I}$, $\vec{J}$,$\vec{F}$ turns we have (for J/I <<1): $v = \omega J$.

If we consider a hypothetical case where the hyperfine field is <u>fixed</u> along $\vec{J}$, the motion would be a precession around $\vec{J}$ with an angular frequency ω_o' and:

$$v = \omega_o' I$$

we have: $\omega_o' = \frac{J}{I}\omega = \omega_o$

ω_o is, therefore, the angular frequency associated with a fixed magnetic field in the direction J, and it is reasonable to assume:

$$\omega_o = \frac{\mu_N g H(o)}{\hbar}$$

ω is the angular frequency around $\vec{F}$ and it increases as I/J increases and the circle of turning becomes tighter:

$$\omega = \frac{I}{J}\omega_o = \frac{I}{J}\quad\frac{\mu_N g H(o)}{\hbar}$$

The relations we have derived here are confirmed in rigourous quantum mechanical calculations which yield for the limit considered in (7a):

$$G_{k\,\min} \to 1 - \frac{k(k+1)}{3}\,\frac{4J(J+1)}{(2I+1)^2} \qquad (9)$$

$$J/I \to 0$$
$$k/I \to 0$$

and

$$G_k \to 1 - \frac{k(k+1)}{3}\,(\omega_J \tau)^2$$
$$\omega_{FF}\,\tau \to 0$$

$$\omega_J = \sqrt{\frac{J+1}{J}}\;\omega_o \quad ; \quad \omega_o = \frac{\mu_N\, g\, H(o)}{\hbar} \qquad (10)$$

We see that for small $\omega\tau$ the motion can be viewed as a precession (properly averaged over field orientation) in a fixed field with the "classical" frequency ω_o. Only as $\omega\tau$ becomes appreciable is the coupling "turned on" and the motion acquires the characteristics of the spinning $\vec{I}, \vec{J}, \vec{F}$ triangle, with a frequency ω or ω_{FF}, that depends on I and J.

The relation (8) or (10) is the key to I-independent measurements. In the form in which the relation is stated this is not at all obvious since then clearly $G_k \sim 1$, but we shall see later that it is possible to build up an appreciable perturbation in many steps of small precessions, each of which is sufficiently small to guarantee I-independence.

We note, finally, that the limiting relation (9), together with (6), (6a) acquires general validity in the special case: J = 1/2:

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$$G_k(t) = 1 - \frac{k(k+1)}{(2I+1)^2}\,(1 - \cos \omega' t) \qquad \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{for } J = 1/2 \qquad (11)$$

$$G_k = 1 - \frac{k(k+1)}{(2I+1)^2}\,\frac{(\omega'\tau)^2}{1+(\omega'\tau)^2} \qquad (11a)$$

$$\omega' = \frac{a}{2\pi} = \frac{2I+1}{2J}\,\frac{\mu_N\, g\, H(o)}{\hbar} = (2I+1)\,\omega_o \qquad (11b)$$

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Polarized HFI for small ωt

This is again a situation where the field can be presumed to be fixed and we anticipate a precession with frequency ω_o . this case a coherent precession)around $\vec{J}$.

We can derive the motion directly from (2) by replacing the operator $\vec{J}$ by its (constant) expectation value $\langle\vec{J}\rangle$ which we take to be in the z direction:

$$H_{M1} = a(\vec{I}, \vec{J}) = a(\vec{I}, \langle\vec{J}\rangle) = a\, I_z \langle H_z \rangle$$

the polarization p_z is given by $\frac{\langle J_z \rangle}{J}$, and therefore:

$$H_{M1} = a\, J\, p_z\, I_z = \mu_N\, g\, H(o)\, p_z\, I_z$$

This is the Hamiltonian for motion in a fixed field H_z : $H_z = H(o)\, p_z$, and the angular distribution is given by:

$$W_p(\vec{\nu}, t) = W(\vec{\nu} - \omega_p t) \;; \quad \omega_p = \frac{\mu_N\, g\, H(o)}{\hbar}\, p_z \qquad (12)$$

we note that the condition for the applicability of (12) is $\omega t \ll 1$, and not $\omega_p t \ll 1$.

(11) The relation (12) for ω_p again confirms our interpretation of ω_o as a precession frequency around $\vec{J}$, appropriate for small ωt.

(11a) We finally give here a short resumé of the various hyperfine frequencies that we have dealt with:

(11b) around $\vec{F}$: $\omega_{F\,F'}$ or ω as an approximation

$$\omega' = \omega_{I + 1/2,\ I - 1/2} = (2I+1)\omega_o \qquad \text{for } J = 1/2$$

around $\vec{J}$ (significant for small $\omega_{F\,F'}t$, ωt):

ω_J or ω_o as an approximation.

Specific Ionic environments

The ionic environment is usually formed by stripping in the target foil. The degree of ionization is usually adequately represented by the "matching principle" of Niels Bohr: all the electrons are effectively stripped for which the mean orbital velocity is smaller than the ionic translational velocity v; or, more precisely, for which: $v > \sqrt{\frac{2E_I}{m}}$, E_I being the binding energy. We also have to know the state of excitation of the ions. Experience has shown that this can be reasonably well established by means of a "matching width" which is associated with the matching condition. The "matching width" can be deduced from $\phi_i(v)$ curves of the relative frequency (12) of the ion of degree of ionization i vs velocity, most of which have a logarithmic width of about a factor of two in velocity FWHM or a factor of four in ion energy FWHM, or again a factor of two in energy HWHM.

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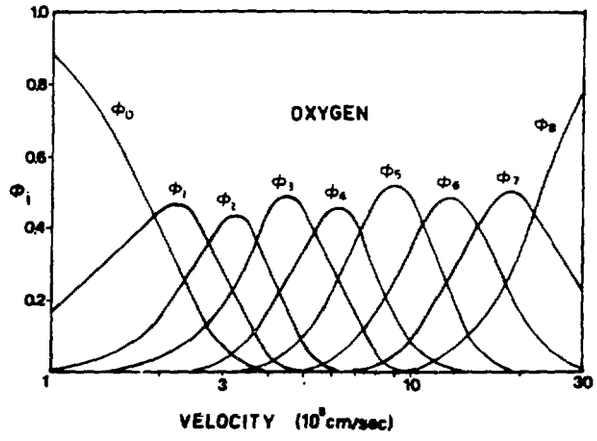

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We therefore postulate that the ions will be formed in all levels of excitation between the ground state E_o and, roughly, $E_o/2$.

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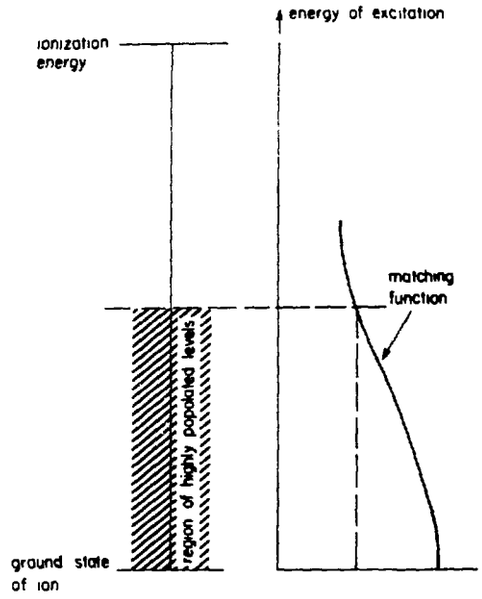

We consider, now, the main quantum number n:

$$E_I = E_n \propto \frac{1}{n^2}$$

For ions formed in the hydrogen-like state of ionization, the first excited state has a binding energy $\frac{1}{4}$ times the ground state binding energy and is well outside our postulated region of excitation. We therefore expect such ions to be formed predominantly in their ground state. This is indeed found to be the case and one finds in such ions a dominant unique hyperfine frequency ω', as given by (11b). Time dependant measurements for such ions were conducted with plunger instruments and they are very well described by the relation (11). These are inherently the cleanest and most accurate hyperfine perturbation measurements. They are restricted only by the limited range of applicability due to the very high recoil energies required, energies that on the one hand tax the technical limits of existing accelerators, and on the other hand lead to difficulties in the detection and analysis of the de-excitation γs in a regime of high excitation energy.

In considering now more complex ions we find that He-like ions are again expected to be preferentially in their ground state and therefore only weakly perturbed compared to H-like ions.

For L_i-like ions the ratio of the binding energy of the first excited to ground state is close to $\frac{1}{2}$ and we therefore expect a dominant n=2 population with more admixture of higher n than found in H-like ions. There are however, now close 2s and a 2p levels, both of which are expected to be

populated, with a total of four hyperfine frequencies.

For Be-like and all the way to F-like ions the n=2 shell is still expected to be dominant, with relatively little excitation to $n \geq 3$. However, for all such ions there are a great many levels within the n = 2 shell and all of these are copiously excited. Moreover, many of these levels have $\ell > 0$ and have therefore a large number of hyperfine frequencies associated with them according to (5), and the frequency spectrum is quite complex.

For ions with ground states in the n = 3 shell, excitation of the n = 4 shell is significant and the spectrum evidently gets progressively more complex for higher electron number.

In most of these ions it is difficult to compute the hyperfine interaction from first principles due to both the complexity of the spectrum and the difficulty in computing the relevant hyperfine fields.

In such ions one would like to attempt a different approach: to carry out measurements involving a state of known g-factor and use it as a direct calibration of the hyperfine field. It is, however, quite obvious that this is not at all a straightforward procedure because, due to the complexity of the spectrum, the hyperfine field does not factor out. Special experimental arrangements are required to furnish a factorization of the perturbation into a nuclear and an ionic part. We shall describe one such method in the sequel.

Recoil into gas: collisions

We now consider ions recoiling into gas and experiencing a series of collisions with the gas molecules. In between the collisions the interaction between the ion and the nucleus in its center is the same as in recoil in vacuum, but during the collisions the ion interacts with the gas molecules and, specifically, its angular momentum may be changed. We shall assume in the

following that every collision wipes out the memory of the preceding history of the ion and reorients its angular momentum vector in a completely random way. The correlation time over which the memory of the ion persists is the average time between collisions and will be denoted by τ_c.

It may be thought that the addition of gas collisions to an already complex process will only complicate it further. We will, however, see that the gas collisions introduce a number of salutary modifications and simplifications. Two of these can be readily perceived: as the ionic angular momentum changes in every collision, the $\vec{I}$, $\vec{J}$, $\vec{F}$ triangle is, as it were, reconstructed after each collision, and the hard core limitation on the movement of $\vec{I}$ is relaxed: as the collisions proceed, the vectors $\vec{F}$ and $\vec{I}$ will diffuse outward from their initial orientation. In the course of the collisions the ion will also fluctuate among various states, so that every nucleus will sample essentially the whole gamut of hyperfine frequencies, and instead of different nuclei experiencing different fields we now have all nuclei in an identical, albeit complex, environment.

Due to the loss of memory in the collisions the perturbation process will appear the same whatever starting time we chose. If we consider the coefficients A_k as function of time we have:

$$A_k(t_1+t_2) = A_k(t_1)\, G_k(t_2) \qquad \text{for } t_1,\ t_2 >> \tau_c$$

$$A_k(t_1) = A_k(o)\, G_k(t_1)$$

but: $$A_k(t_1+t_2) = A_k(o)\, G_k(t_1+t_2)$$

and therefore: $G_k(t_1+t_2) = G_k(t_1)\, G_k(t_2)$

which leads to: $G_k(t) = e^{-\lambda_k t}$

$$\text{and } G_k = \frac{1}{\tau}\int_0^\infty G_k(t) e^{-t/\tau} dt = \frac{1}{1+\lambda_k \tau}$$

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We also have:

$$G_k \underset{\lambda_k \tau \to o}{\to} 1 - \lambda_k \tau \tag{13}$$

In order to evaluate the parameters λ_k we consider the angular motion of the vector $\vec{I}$. This is an orderly precession in between collisions of average angle $\omega_J \tau_c$ but in every collision the direction of motion is changed in a random manner and we therefore have a random walk in two dimensions of unit step: $d_o = \omega_J \tau_c$ and a number of steps: $n \approx \frac{\tau}{\tau_c}$ during the time τ.

The mean square of the distance traversed in such a walk is given by: $\overline{d^2} = d_o^2 n$.

In a directed walk we have instead: $\overline{d^2} = d^2 = d_o^2 n^2$. We now see that if $\omega_{FF}, \tau_c << 1$ we can modify the relation (10) in a way appropriate to a random walk perturbation:

$$(\omega_J \tau)^2 = (\omega_J \tau_c)^2 (\frac{\tau}{\tau_c})^2 \to (\omega_J \tau_c)^2 \frac{\tau}{\tau_c} = \omega_J^2 \tau_c \tau$$

$$G_k = 1 - \frac{k(k+1)}{3} \omega_J^2 \tau_c \tau$$

comparing this expression with (13) above we can evaluate λ_k:

$$\lambda_k = \frac{k(k+1)}{3} \omega_J^2 \tau_c$$

and:

$$G_k(t) = e^{-\frac{k(k+1)}{3}\ \omega_J^{\,2}\ \tau_c t} \qquad t/\tau_c \gg 1 \qquad (14)$$

$$G_k = \frac{1}{1+\frac{k(k+1)}{3}\ \omega_J^{\,2}\ \tau_c\ \tau} \qquad \tau/\tau_c \gg 1 \qquad (14a)$$

$\omega_{FF'}\ \tau_c \ll 1$

(13)

These are the Abragam Pound relations for the specific case of hyperfine M1 perturbation. We note that (14) (14a) do not depend on I or J and we perceive that this is so because we have a diffusion process in which the unit step is sufficiently small. However, the I dependence appears in the condition of sufficient smallness:

$$\omega_{FF'}\tau_c \ll 1$$

as we have seen $\omega_{FF'} \sim \omega \sim \frac{I}{J}\ \omega_o \sim \frac{I}{J}\ \omega_J$ and the above condition can be replaced by:

$$(\omega_J \tau_c)\ \frac{I}{J} \ll 1 \qquad (14b)$$

If we require a finite perturbation in (14) then this relation leads to:

$$\omega_J\ \frac{J}{I}\ t > 1$$

We see that we can employ a given hyperfine field for the perturbation of levels either of short lifetimes or of high spin.

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We now consider a complex hyperfine frequency spectrum and assume first that there are two frequencies ω_1, ω_2 with weights A_1, A_2 respectively. We then have:

$$G_k(t) = e^{-\lambda_1 t_{11}} e^{-\lambda_2 t_{21}} e^{-\lambda_1 t_{12}} e^{-\lambda_2 t_{22}} e^{-\lambda_1 t_{13}} \cdots$$

where t_{ij} is the time associated with the frequency i, at its j-th occurrence.

$$\text{we have : } \left(\sum_j t_{1j}\right) / \left(\sum_j t_{2j}\right) = A_1/A_2$$

and:

$$G_k(t) = e^{-(A_1\lambda_1 + A_2\lambda_2)t} \quad ; \quad A_1 + A_2 = 1$$

or more generally:

$$G_k(t) = e^{-\int A(\omega_J)\omega_J^2 \, d\omega_J \frac{k(k+1)}{3} \tau_c t} \quad ; \int A(\omega_J)\, d\omega_J = 1$$

$$G_k(t) = e^{-\overline{\omega_J^2} \frac{k(k+1)}{3} \tau_c t} = e^{-\frac{k(k+1)}{3} \left(\frac{\mu_N}{\hbar}\right)^2 \left(\overline{\frac{J+1}{J} H(o)^2}\right) \tau_c g^2 t} \qquad (15)$$

We see that within the Abragam Pound regime we get a complete factorization of the nuclear and the ionic parameters even with a complex interaction frequency spectrum.

Another important simplification associated with the Abragam Pound regime relates to cascade transitions, such as are frequently encountered in the decay of high spin states. If t_i is the time at which the l vel i decayed to the level i + 1 and if $t_i \gg \tau_c$ for all i then:

$$G_k(t) = G_k(t_1)\, G_k(t_2) \ldots$$

$$G_k(t) = e^{-\frac{k(k+1)}{3}\left(\frac{\mu_N}{\hbar}\right)^2 \left(\frac{\overline{J+1}}{J}\, H(o)^2\right)\tau_c \sum_{i=1}^{n(t)} g_i^2\, t_i}$$

The decay times t_i will usually have to be averaged over, and it is therefore just as convenient to consider g as a continuous variable and:

$$G_k(t) = e^{-\frac{k(k+1)}{3}\left(\frac{\mu_N}{\hbar}\right)^2 \left(\frac{\overline{J+1}}{J}\, H(o)^2\right)\tau_c \int_o^t g^2(t')dt'} \qquad (16)$$

The collision time τ_c is given as function of pressure and ion velocity by:

$$\tau_c(p,v) = \frac{\ell(p)}{v} = \frac{1}{N\sigma(v)\, v} \qquad (17)$$

where N is the number of gas molecules per unit volume, ℓ is the mean free path and $\sigma(v)$ the collision cross-section.

For not too deep shells and for simple gases σ is of the order of $\approx 5\ 10^{-16}$ cm^2. This yields a correlation time of $\approx 10^{-13}$ for v = 0.01 c and at normal temperature and pressure.

We have: $\frac{N(p)}{p} = \frac{N(p_o)}{p_o}$ and:

$$\tau_c(p,v) = \frac{p_o}{N(p_o)\ \sigma(v)}\ \frac{1}{pv}$$

We can therefore write (15) as:

$$G_k(t, p, v) = e^{-\frac{k(k+1)}{3} \Xi g^2 t/(pv)} \tag{18}$$

with $\Xi = \frac{\mu_N^2}{\hbar^2} \left(\frac{\overline{J+1}}{J} H(o)^2 \right) \frac{p_o}{N(p_o)\,\sigma(v)}$

and $\Xi \equiv \Xi(v,Z)$

with the value given above for σ, with $\sqrt{\frac{J+1}{J}} H(o) = 50$ Megagauss and with p measured in atmospheres and v in units of c, we have for Ξ :

$$\Xi \sim 1.5\ 10^8\ s^{-1}$$

The dependence of Ξ on v and Z is a consequence of its dependence on the state of ionization. This, however, is a smooth and not very fast function of the variables and the dependence of Ξ on these variables is, therefore, monotonic and not very steep.

Experimentally, the functional dependence of G_k in (18) is traced out either as function of time or as function of pressure.

In a cascade:

$$-\frac{d}{dt} \log G_k(t, p, v) = \frac{k(k+1)}{3\,pv} \Xi\, [g(t)]^2$$

What if the conditions for the applicability of the Abragam Pound relations are not fulfilled? For any given ionic state an exact calculation can be performed free of any restrictions, with the aid of the Scherer-Blume theory[6]. For low values of τ_c this theory deviates from the Abragam Pound relation, and the attenuation is dependent on I and J, as is to be expected. Outside of the Abragam-Pound regime the mean ionic angular momentum becomes another important ionic parameter.

For numerical estimates it is sometimes convenient to use a second order approximation to the Scherer-Blume theory, the first order being the Abragam-Pound relations. To this order the relations (14) (14a) are replaced by:

$$G_k(t) = e^{-\frac{k(k+1)}{3}\ \omega_J^2\ \tau_c t\ K_k} \tag{19}$$

$$G_k = \frac{1}{1+\frac{k(k+1)}{3}\ \omega_J^2\ \tau_c\ \tau\ K_k} \tag{19a}$$

$$\text{with} \quad K_k^{-1} = 1+ (\omega_J\tau_c) \left\{ \frac{(2I+1)^2}{4J(J+1)} + [3k(k+1)-1]\ \frac{(2J-1)(2J+3)}{20J(J+1)} \right\}$$

exhibiting the onset of the I, J dependence as well as a modification of the k dependence from the characteristic k(k+1) Abragam-Pound dependence.

The time dependence of the attenuation is still exponential in this order. However, in a complex spectrum the perturbation cannot any longer be completely factorized into nuclear and ionic components.

Dynamic Magnetic Fields

Another promising method for measuring magnetic moments of high spin states is based on the so-called "transient" or "dynamic" magnetic fields. These are intense fields experienced by fast ions moving in magnetized ferromagnets. The phenomenon was discovered in 1967[7] and it was then tentatively ascribed to the transient capture of polarized electrons into deep ionic orbits. In 1971, a reasonably complete and qualitative theory was presented by Lindhard and Winther[8] which - with a certain ad hoc adjustment - appeared to give a good account of the data then existing.

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In this theory the dynamic field is assumed to arise out of the scattering of polarized electrons on the moving ion. In recent measurements, however, serious drawbacks in the Lindhard-Winther theory came to light[9]. It was found specifically that the field increases with increasing velocity (up to a very high value of the velocity after which it decreases down to zero[10]) whereas the Lindhard-Winther theory predicts a steady decrease in the field vs. velocity. Due largely to the characteristic velocity dependence, which parallels more or less the velocity dependence of the free ion field, notions of polarized capture are again surfacing, and with them - questions about a possible I - dependence of the dynamic field. We shall, however, see shortly that judging by our criteria, such an I-dependence can definitely be excluded in this case.

In a limited range of velocities the dynamic field can be described by the following relation[11]:

$$H(Z,v) \sim a' Z\left(\frac{v}{v_o}\right) \mu_B N_p$$

$v_o = \frac{e^2}{\hbar}$, N_p is the density of polarized electrons, and $a' = 69(6)$. The direction of the field H is the direction of the magnetization.

If the ions pass through a foil of thickness L, during a time t, the precession angle of the nuclei is given by:

$$\Delta\theta = \int_o^t \omega dt = \frac{\mu_N}{\hbar} g \int_o^t H\, dt = g \frac{\mu_N}{\hbar} \frac{a'Z}{v_o} \mu_B N_p \int_o^t v dt'$$

here $\Delta\theta$, g and H are considered as absolute values. The sign of $\Delta\theta$ is determined by the direction of H, given above.

$$\Delta\theta = g \frac{\mu_N}{\hbar} \frac{a'Z}{v_o} \mu_B N_p L$$

and specifically for iron foils, in convenient units:

$$(\Delta\theta/\text{mrad}) \sim 0.34(3) \times g\, Z(L/\text{mg}\cdot\text{cm}^{-2})$$

Here the precession is produced in a sequence of many encounters with the individual steps adding coherently. For a rough estimate of τ_c, the duration of a "unit step" - we use the relation (17) which we expect to be approximately valid also in a solid. We then get: $\tau_c \, v \approx 10^{-4}$ mg·cm^{-2}. The "unit step" of the precession, $\omega\tau_c$, is therefore: $\approx 3\ 10^{-8}\ g\ Z$, and the quantity $\omega\tau_c \frac{I}{J}$ of (14b) will be very small even for extreme values of the parameters.

Experiments

Experiments of both types discussed here are in progress now in various laboratories.

Experiments involving recoil in gas are usually carried out in conjunction with (H I, xn) reactions with the heavy partner in the reaction usually serving as projectile, to attain maximum recoil velocity. The control of the interaction time is achieved by means of a plunger arrangement where the recoils are stopped after a given (and variable) recoil distance. Through the Doppler effect one can distinguish between γ's emitted in flight and at rest, and the latter are those emitted from nuclei which experienced by hyperfine interaction for the entire duration of the flight time to the stopper.

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Measurements with dynamic field are usually carried out following Coulomb excitation, with the recoils passing through a magnetized iron foil and the γ's counted in coincidence with the scattered particles or the recoiling nuclei. One measurement involving dynamic fields and following excitation by a (HI ,xn) reaction is now also in progress.

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