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Abstract

Presented paper deals with one of the factors influencing the accuracy and speed of thermal calculation of fast reactors. This factor consists in the method of stating the sources of heat. The distribution of heat sources in the core and other inner parts of the fast reactor is described - by the method of least squares - in the form of regression polynomials. Relations are derived of outputs of both individual components of fuel elements, and whole inner parts of the reactor. A comparison was performed of various methods of solution for obtained source integrals. The optimum integrating method was found.

Introduction

Thermal calculation is one of the basic factors underlying a reactor design. The choice of the method of calculation is necessarily a compromise between two competitive demands: maximum possible accuracy of the calculation and, on the other hand, the speed of calculation and available memory of the computer.

The way of how to state the heat sources in the active core and other inner elements of a reactor is a factor by which the speed as well as the accuracy and memory demands are essentially influenced.

In the general case - i.e. when heat distribution along the height of the active core is not cosinusoidal - we get as one of the results of the physical calculation a two-dimensional field of heat sources. It is necessary to use these data in the proper form as an input for thermal calculation. The field of sources which was obtained from physical calculation was not continual and this is a discrete one.

We obtain values of heat generation only for singular points of the computing network. The density of network is limited by computer possibilities.

For the thermal calculation it is possible to state the field of sources, obtained from physical calculation, by two dofferent ways:

1) As a table of values - heat sources, these values are function of two independent variables - a radial and an axial coordinate of the point of com-

puting network. An advantage of this kind of stating is a simple preparation of input values for the thermo-energetical calculation. Disadvantages are as follows:

Discontinuous field of heat sources, i.e. the knowledge of values of heat development only in the cross points of the computing network. Among these points we know neither the values of heat development, nor any analytical expression of them.

Considering that in the general case a division into computing intervals which is suitable for the thermo-energetical calculation is not the same as the density of computing network used for physical computation, it is necessary for thermo-energetical calculation to compute the values of heat development between the points of computing network (used for physical calculation) by means of interpolation or extrapolation. In such a way an inaccuracy caused by interpolation is introduced into the calculation and first of all a considerable part of a computer memory is loaded by the field of the values of heat sources.

It is necessary to keep these values in the computer memory for the whole time of the calculation.

2) The second way of how it is possible to state heat sources for thermo-energetical calculation from physical calculation is as follows:

Using the method of least squares, the field of heat sources will be regressed by a two-demensional polynomial. The polynomial is a function of two independent variables - a radial and an axial coordinate; these variables state the position of particular points of the source field in relation to choosen origin of the coordinate system.

In such a way we get an analytical relation, by which the value of heat development in any point of the considered region of reactor is expressed. As a considered region of the reactor we took such a part for which a least squares regression was performed.

By the aid of the regression polynomials it is possible to derive relations for the calculation of thermal output of the single computation element as well as for all the inner parts of the reactor. These relations we may solve relatively simply either analytically or numerically.

To find a proper degree of both independent variables of the regression polynomials for particular regions of the reactor is rather difficult. It is a disadvantage of this second method of stating the field of sources for thermo-energetical calculation. After getting some experience with choosing the degree of both variables the calculation of coefficients of regression polynomials is relatively simple and rapid.

2. Regression method used:

For a regression of the field of sources was used a least squares method, where the regression polynomial was chosen in the form as follows:

$$q(r,z) = \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} c_{ij} \cdot r^{j-1} \cdot z^{i-1} \qquad /1/$$

Written in full, it is the following expression:

$$\begin{aligned} q(r,z) = {} & C_{1,1} + C_{1,2}\cdot r + C_{1,3}\cdot r^{2} + \cdots\cdots + C_{1,m}\cdot r^{m-1} + C_{1,m+1}\cdot r^{m} + \\ & + C_{2,1}\cdot z + C_{2,2}\cdot r\cdot z + C_{2,3}\cdot r^{2}\cdot z + \cdots\cdots + C_{2,m}\cdot r^{m-1}\cdot z + C_{2,m+1}\cdot r^{m}\cdot z + \\ & \quad \vdots \qquad\quad \vdots \qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \qquad\qquad \vdots \\ & + C_{n,1}\cdot z^{n-1} + C_{n,2}\cdot r\cdot z^{n-1} + C_{n,3}\cdot r^{2}\cdot z^{n-1} + \cdots\cdots + C_{n,m}\cdot r^{m-1}\cdot z^{n-1} + C_{n,m+1}\cdot r^{m}\cdot z^{n-1} + \\ & + C_{n+1,1}\cdot z^{n} + C_{n+1,2}\cdot r\cdot z^{n} + C_{n+1,3}\cdot r^{2}\cdot z^{n} + \cdots\cdots + C_{n+1,m}\cdot r^{m-1}\cdot z^{n} + C_{n+1,m+1}\cdot r^{m}\cdot z^{n} \end{aligned}$$

/2/

Now we sign the given field of sources(which is result of physical calculation) $\bar{q}$ (r, z), the number of cross points of the computing network in the radial direction k, in the axial direction l, and, finally, we choose the degree of polynomial (2) as n = 4 and m = 3.

The aim of the method of least squares is to fulfil the following condition:

$$S = \sum_{i=1}^{k} \sum_{j=1}^{l} \left[\bar{q}(r,z) - q(r,z) \right]^{2} = min \qquad /3/$$

To fulfil this, it is necessary to perform partial derivatives of the sum S consecutively with respect to coeffitients:

$$\frac{\partial S}{\partial c_{1,1}}, \frac{\partial S}{\partial c_{1,2}}, \cdots\cdots, \frac{\partial S}{\partial c_{5,4}}$$

If we not set these partial derivatives equal to zero, we get, after modification, the following set (4) of linear algebraic equations (see appendix 1).

This set may be expressed schematically:

$$\begin{array}{c|c|c|c|c|c|l}
\frac{\partial S}{\partial c_{1,j}} \sim & A & B & C & D & E & = \left\{ \text{right side} \right\} \\
\hline
\frac{\partial S}{\partial c_{2,j}} \sim & B & C & D & E & F & = \left\{ \text{—— " ——} \right\}_2 \\
\hline
\frac{\partial S}{\partial c_{3,j}} \sim & C & D & E & F & G & = \left\{ \text{—— " ——} \right\}_3 \\
\hline
\frac{\partial S}{\partial c_{4,j}} \sim & D & E & F & G & H & = \left\{ \text{—— " ——} \right\}_4 \\
\hline
\frac{\partial S}{\partial c_{5,j}} \sim & E & F & G & H & I & = \left\{ \text{—— " ——} \right\}_5
\end{array} \qquad /5/$$

It is evident that it is a matrix symmetrical along the main diagonal and particular determinants may be expressed as

$$A = \begin{vmatrix}
c_{1,1} \cdot X_{111} + c_{1,2} \cdot X_{121} + c_{1,3} \cdot X_{131} + c_{1,4} \cdot X_{141} \\
c_{1,1} \cdot X_{211} + c_{1,2} \cdot X_{221} + c_{1,3} \cdot X_{231} + c_{1,4} \cdot X_{241} \\
c_{1,1} \cdot X_{311} + c_{1,2} \cdot X_{321} + c_{1,3} \cdot X_{331} + c_{1,4} \cdot X_{341} \\
c_{1,1} \cdot X_{411} + c_{1,2} \cdot X_{421} + c_{1,3} \cdot X_{431} + c_{1,4} \cdot X_{441}
\end{vmatrix} \qquad /6/$$

By the solution of the above mentioned set of equations we obtain the coefficients $c_{i,j}$ of the regression polynomial (1).

Solution of the whole problem was programmed in the ALGOL language,especially for HE 803B computer. The last version of this programm (working name UNISQAUT/A3) is elaborated in such a way that in general it is possible to choose arbitrary degree for both independent variables of regression polynomial (1). (n and m are arbitrary, natural numerals, their values are limited only by the memory of computer). UNISQUAUT/A3 is so arranged, that degrees m and n are searched automatically to fulfil the condition

$$\frac{q_{i,j}(r,z) - \bar{q}_{i,j}(r,z)}{\bar{q}_{i,j}(r,z)} \cdot 100 < R \qquad /7/$$

where R is a stated relative error in %.

3. General relation for the thermal output of the computing element of fuel assembly.

3.1 Deriving the relation

Active core of the fast liquid natrium cooled reactor consists of hexagonal fuel assemblies (see fig. 1). To derive a general relation for a power output of the computing element with lenght z, the following simplification was adopted: hexagonal fuel assembly was replaced theoretically by a cylindrical one of the same area. The cylindrical assembly has a radius R and a distance from an active core axis x (see fig. 2).

If we now circumscribe a circle of a radius r along a center O, cross points with an original circle give us an arc with the lenght l, for which holds relation /8/:

$$l = \frac{\pi \cdot r \cdot \alpha}{90} \qquad /8/$$

The angle α we may express with the aid of cosine law:

$$\alpha = \arc\cos \frac{x^2 + r^2 - R^2}{2\, x \cdot r} \qquad /9/$$

After substitution this expression into the equation /8/ we get the final expression for l independent on α :

$$l = \frac{\pi \cdot r}{90} \cdot \arc\cos \frac{x^2 + r^2 - R^2}{2\, x \cdot r} \qquad /10/$$

Total thermal output of the general computing element of the height $\Delta z = z_2 - z_1$ with the circular cross-section of a radius R is stated by a relation:

$$Q_{el} = \int_{z_1}^{z_2} \int_{x-R}^{x+R} l \cdot q(r,z) \cdot dr \cdot dz \qquad /11/$$

Inserting /10/ into /11/, we obtain:

$$Q_{el} = \frac{\pi}{90} \cdot \int_{z_1}^{z_2} \int_{x-R}^{x+R} r \cdot \arc\cos \frac{x^2 + r^2 - R^2}{2 x r} \cdot q(r,z) \cdot dr \cdot dz \qquad /12/$$

If we now put relation /1/, /12/, after an arrangement we may write symbolic relation for total thermal output of a general fuel assembly element in the form:

$$Q_{el} = \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} Q_{el}^{i,j} \qquad /13/$$

The first member of the preceding sum /13/ may be written also in the following shape:

$$Q_{el}^{1,1} = c_{1,1} \cdot \frac{\pi}{90} \cdot \int_{z_1}^{z_2} \int_{x-R}^{x+R} r \cdot \arccos \frac{x^2+r^2-R^2}{2\,x\,r}\, dr \cdot dz \qquad /14/$$

and the second as

$$Q_{el}^{1,2} = c_{1,2} \cdot \frac{\pi}{90} \int_{z_1}^{z_2} \int_{x-R}^{x+R} r^2 \cdot \arccos \frac{x^2+r^2-R^2}{2\,x\,r}\, dr \cdot dz \qquad /15/$$

Other members of the relation /13/ have a similar form.

3.2 Solution of the general relation for thermal power output of the element.

3.2.1 Analytical solution.

The solution of general relation /13/ is essentially a solution of the following system of integrals:

$$I_{1,j} = \int_{x-R}^{x+R} r^j \cdot \arccos \frac{x^2+r^2-R^2}{2\,x\,r}\, dr \qquad /16/$$

where $j = 1 \div /m + 1/$

3.2.1.1 Solution of relation /16/ for $j = 1$

We use a substitution

$$a = x^2 - R^2 \qquad /17/$$

$$b = 2x$$

The indefinite integral $I_{1,1}$ we solve by per-partes method and we obtain this relation:

$$I'_{1,1} = \frac{r^2}{2} \arccos\left(\frac{a+r^2}{b \cdot r}\right) + \int \frac{r^2}{2} \cdot \frac{(r^2-a)}{r \cdot \sqrt{b^2 \cdot r^2 - (a+r^2)^2}} \cdot dr \qquad /18/$$

Denoting

$$I^{(1)}_{1,1} = \int \frac{r^2}{2} \cdot \frac{(r^2-a)}{r \cdot \sqrt{b^2 \cdot r^2 - (a+r^2)^2}} \, dr \qquad /19/$$

this integral may be solved by means of substitution

$$t = r^2 \qquad /20/$$

Thus, relation /19/ acquires the form:

$$I^{(1)}_{1,1} = \frac{1}{4} \cdot \int \frac{(t-a)}{\sqrt{[-t^2 - t \cdot (2a-b^2) - a^2]}} \cdot dt \qquad /21/$$

It is more advantageous to write out the preceding integral as the difference of two integrals:

$$I_{1,1}^{(1)} = \frac{1}{4} \cdot \left\{ I_{1,1}^{(2)} - a \cdot I_{1,1}^{(3)} \right\} \qquad /22/$$

where

$$I_{1,1}^{(2)} = \int \frac{t \cdot dt}{\sqrt{Z}} \qquad /23/$$

$$I_{1,1}^{(3)} = \int \frac{dt}{\sqrt{Z}} \qquad /24/$$

where $Z = -t^2 - t \,.\, /2a - b^2/ - a^2$ /25/

Considering that the determinant of quadratic equation is $\Delta = -16R^2x^2 < 0$ as well as the coefficient of the quadratic member of eq. /25/ is negative (according to /1/), the integral /24/ has the following solution

$$I_{1,1}^{(3)} = -\arcsin \frac{x^2 + R^2 - r^2}{2Rx} \qquad /26/$$

The integral /23/ has a solution only if the condition $Z > 0$ is fulfilled for the whole range of values of t. When we put the substitution /20/ into the expression /25/, we get for Z the following relation:

$$Z = 2\cdot[r^2x^2+R^2(r^2+x^2)]-(r^4+x^4+R^4) \qquad /27/$$

We shall now find out how is fulfilled the above mentioned condition of $Z > 0$ for the following values of the variable r:

1/ for $r = x - R$ /lower integration limit/ $Z = 0$

2/ for $r = x + R$ /upper integration limit/ $Z = 0$

3/ for $r = x$ /assembly axis/ $Z = 4R^2x^2 - R^4 > 0$

From the preceding analysis it is clear, there is no solution of the integral $I_{1,1}^{/2/}$ for both integrating limits. For $r = /x-R, x+R/$, according to /1/, the integral $I_{1,1}^{/2/}$ has the following solution:

$$I_{1,1}^{(2)} = -\left\{\sqrt{2\cdot[r^2x^2+R^2(r^2+x^2)]-(r^4+x^4+R^4)}+(x^2+R^2)\arcsin\frac{x^2+R^2-r^2}{2Rx}\right\} \qquad /28/$$

If we put relations /26/ and /28/ into the equation /22/ we obtain resulting relation for the integral $I_{1,1}^{/1/}$:

$$I_{1,1}^{(1)} = -\frac{1}{4}\left\{\sqrt{2\cdot[r^2x^2+R^2(r^2+x^2)]-(r^4+x^4+R^4)}+2R^2\arcsin\frac{x^2+R^2-r^2}{2Rx}\right\} \qquad /29/$$

The solutin of integral $I_{1,1}$ we get by substituting relation /29/ into relation /18/:

$$I_{1,1}=\frac{1}{4}\left\{2r^2\arccos\frac{x^2+r^2-R^2}{2xr}-\sqrt{2\cdot[r^2x^2+R^2(r^2+x^2)]-(r^4+x^4+R^4)}-\right.$$

$$\left.-2R^2\cdot\arcsin\left(\frac{x^2+R^2-r^2}{2Rx}\right)\right\} \qquad /30/$$

As a control that the general relation /30/ is correct, we perform the solution of a determined integral

$$F_0=\lim_{C\to 0}\frac{\pi}{90}\int_{x-R+C}^{x+R-C} r\cdot\arccos\frac{x^2+r^2-R^2}{2x\cdot r}\,dr=\pi R^2 \qquad /31/$$

The result obtained agrees with a geometrical meaning of determined integral $I_{1,1}$ - i.e with the area of the circular cross section of fuel assembly with radius R.

3.2.1.2 The solution of relation /16/ for j = 2

The final aim of the analytical solution of set of integrals is to find a general relation for j = 1 + (n=1). We try to find out the analytical solution of relation /16/ for j = 2:

$$I_{1,2}=\int r^2\cdot\arccos\frac{x^2+r^2-R^2}{2xr}\,dr \qquad /32/$$

This integral we solve similarly as $I_{1,1}$ - i.e. by per partes method (see section 3.2.1.1):

$$I_{1,2} = \frac{r^3}{3} \arccos\left(\frac{a+r^2}{b \cdot r}\right) + I_{1,2}^{(1)} \qquad /33/$$

$$I_{1,2}^{(1)} = \frac{1}{3} \cdot \int \frac{r^2 (r^2 - a)}{\sqrt{b^2 \cdot r^2 - (a+r^2)^2}} dr \qquad /34/$$

The solution of the integral /34/ is reduced to the solution of this type of integrals:

$$I = \int \frac{r^p}{\sqrt{b^2 \cdot r^2 - (a+r^2)^2}} \cdot dr \qquad /35/$$

This integral for instance for p = 4 we re-arrange by substitution /20/

$$I = \frac{1}{2} \cdot \int \frac{t^{\frac{3}{2}}}{Z^{\frac{1}{2}}} \cdot dt \qquad /36/$$

where Z is given by relation /25/.

In /2/ is published a solution of the integral of type

$$\bar{I} = \int \frac{t^p \cdot dt}{Z^q} = - \frac{t^{p-1}}{(2q-p-1) \cdot A \cdot Z^{q-1}} +$$

$$+ \frac{(p-1) \cdot C}{(2q-p-1) \cdot A} \cdot \int \frac{t^{p-2} \cdot dt}{Z^q} - \frac{(q-p) \cdot B}{(2q-p-1) \cdot A} \int \frac{t^{p-1} \cdot dt}{Z^q} \qquad /37/$$

where $p \neq 2q - 1$, $Z = A \cdot t^2 + Bt + C$.

The purpose of the solution of relation /37/ is to reduce P to a value of $p = 1$, eventually $p = 2$, for which is known an analytical solution (see for instance /2/).

In regard of a fact that $p = \dfrac{3}{2}$, $q = \dfrac{1}{2}$

it is not possible to use for the integral /36/ the solution of the same type as for /37/.

The integral /36/ does not lead to a solution of the integral $\int \dfrac{d\,T}{Z}$ (i.e. for $p = 1$).

3.2.1.3 Discussion of the analytical solution

From the preceding section may be seen that it is evidently not possible to find a common analytical solution of the set of integrals /16/.

We were successful only in finding the first member of sum /13/ (by inserting /31/ into /14/):

$$Q_{el}^{1,1} = c_{1,1} \cdot \pi \cdot R^2 \cdot (z_2 - z_1) \qquad /38/$$

We failed to find an analytical solution of the whole relation /13/.

3.2.2. Numerical solution

Considering that the analytical solution of the set of integrals /16/ was not found, an optimum method of numerical integration was searched.

For the numerical solution was the function arc cos $\frac{x^2+r^2-R^2}{2xr}$ approximated by polynomial according to /3/:

$$\text{arc cos } z = \sqrt{1-z}\cdot \sum_{i=0}^{7} c_i \cdot z^i + \varepsilon(z) \qquad /39/$$

where

$$z = \frac{x^2+r^2-R^2}{2x\cdot r}$$

c_i - see /3/

$$|\varepsilon(z) \leqq 2\cdot 10^{-8}| \text{ for } 0 \leqq z \leqq 1$$

The following numerical methods were compared: common trapezoidal formula, common Simpson's formula (both taken from /4/), Hrubec's formula (taken from /5/) and Gausse's quadratical formula (according to /4/ and /3/). Provided the stated accuracy of integration is constant the minimum computing time was considered as a criterion of the advantage of method (on ME 8C33 computer).

In the following sections Hrubec's formulae are briefly described, trapezoidal and Simpson's formulae are not introduced here (see /4/).

3.2.2.1 Hrubec's formula

A determined integral $\int_a^b f(x) \cdot dx$ is transformed into the integral

$$\frac{b-a}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f \cdot \left(\frac{b-a}{2} \sin\varphi + \frac{b+a}{2} \right) \cos\varphi \cdot d\varphi \qquad /10/$$

This integral is approximated by universal formula

$$I_n = \frac{b-a}{4n} \sum_{j=0}^{n} d_{jn} \cdot \Big[f \cdot \left(\frac{b+a}{2} + \frac{b-a}{2} \sin \frac{\pi j}{2n} \right) + f \left(\frac{b+a}{2} - \frac{b-a}{2} \sin \frac{\pi j}{2n} \right) \qquad /11/$$

where

$$d_{jn} = \left(2 - \delta_{0j} - \delta_{jn} \right) \cdot \left(1 + 2 \cdot \sum_{k=1}^{n} \frac{(-1)^k}{1-4k^2} \cdot \cos \frac{\pi j k}{n} \right)$$

δ_{ik} - Kronecker's δ

/12/

The approximations for 4, 8, 12, 16, 20 to 512 dividing points are calculated step by step. If the relative difference between consecutive approximations is less than stated number $\mathcal{E}$ and if this situation occurs second time, the calculation is finished.

In /6/ and /7/ is shown that this formula is asymptotically-optimal for broad scale of Hilbertorn's spaces of 2π - periodical functions.

3.2.2.2 Gausse's cuadratical formula

According to /4/ holds the equation:

$$\int_a^b f(x)\cdot dx = \frac{b-a}{2}\cdot\sum_{i=1}^{n} A_i\cdot f(x_i) \qquad /43/$$

where

$$x_i = \frac{b+a}{2} + \frac{b-a}{2}\cdot t_i \quad (i = 1, 2, \dots, n) \qquad /44/$$

t_i - zero points of the Legender's polynomial.

Values of t_i and of coefficients A_i were taken for calculation from /3/.

3.2.2.3 The result of comparison of different cuadratical formulae

To compare the effectiveness of all 4 methods of numerical integrations, the calculation of relation /16/ was performed for some values of stated accuracy and for the whole real range of j and x.

From 4 compared methods of numerical integrations Gausse's quadratical formula emerges as an optimum. For the same value of relative error it is 40 times faster than Simpson's formula and 3 times faster than Hrubec's formula. An additional advantage of Gausse's formula is the fact that for the whole real range of values x and j (see relation /16/) the same number of dividing points of Gause's quadratical formula is enough for a preset accuracy of calculation. It is advantageous in regard of an occupation of computer memory by coefficients t_i and A_i.

Dependence of a number of Gausse's formula dividing points on the stated accuracy of integration is on fig. 3.

On fig. 4 is plotted the time necessary for numerical integration of the relation /16/ by Gausse's formula versus the stated accuracy of integration. (Fig. 4. is related to the computer ME 603B).

3.2.3 The result of solution of relation /13/

The numerical integration by Gausse's quadratical formula have proved to be an optimum method of solution of common relation /13/ for thermal output of fuel assembly element. Numerical solution of the relation /13/ consists in repeated numerical integration of the relation /16/ for the whole considered range of values j.

4. Common relation for the total thermal output of any inner part of the reactor.

There are two ways how to calculate the total thermal output of the active core:

a) by summation of thermal outputs from individual computing elements, i.e. by cyclic application of the relation /13/. This kind of calculation is very tedious, especially for the great number of elements.

b) directly by calculation according to the common general relation for total thermal output of arbitrary inner part of reactor.

Derivation of common relation

Let us take a general part of an active core with radii R_1 and R_2 and height $\Delta z = z_2 - z_1$ (see fig. 5). The distribution of heat sources along radius and height of this part is stated by the regression polynomial (1). If we now suppose a constant distribution of the heat sources along the range, the total heat generation in [illegible] ring is statet by the relation:

$$dQ = 2\pi r \cdot dr \cdot dz \cdot q(r, z) \qquad /45/$$

So the total heat generation in the whole part of the active core we get by double integration of the relation /45/

$$Q_{(R_1, R_2, \Delta z)} = 2\pi \cdot \int_{z_1}^{z_2} \int_{R_1}^{R_2} q(r, z) \cdot r \cdot dr \cdot dz \qquad /46/$$

Then we put an expression /1/ into the preceding relation /46/ and after a double integrating we obtain wanted general relation:

$$Q_{(R_1,R_2,\Delta z)} = 2\pi \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} C_{i,j} \cdot \left(\frac{R_2^{j+1} - R_1^{j+1}}{j+1} \right) \cdot \left(\frac{z_2^{i} - z_1^{i}}{i} \right) \qquad /47/$$

Relation /47/ allows the simple and fast calculation of a total amount of heat developed in any cylindrical inner part of the reactor.

5. Re-calculation of core thermal output to non-nominal output level

Let us take again an arbitrary part of the active core, whose distribution of the heat sources is given by relation /1/. Total thermal output is then given by relation /46/ or /47/.

Thermal output of this part(both total output, and that of any element of this part) is to be re-calculated to other, non-nominal output level in the ratio

$$\bar{q}_1 = \frac{q_1^{max}}{q_0^{max}},$$

where q_0^{max} and q_1^{max} are the original and the new (non-nominal) values of the maximum specific thermal output of the active core, respectively.

Then it holds:

$$\bar{q}(r,z) = \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} \bar{C}_{ij} \cdot r^{j-1} \cdot z^{i-1} \qquad /48/$$

where

$$\bar{c}_{i,j} = \bar{q}_1 \cdot c_{i,j} \qquad /49/$$

Analogically we obtain the relations for the thermal output of the element and the total thermal output of the part of active core for non-nominal thermal output level - we substitute original coefficients c_{ij} in the relations /14/, /15/ and /47/ by coefficients $\bar{c}_{i,j}$. Coefficients $\bar{c}_{i,j}$ we compute by relation /49/.

6. Summary

In the preceding chapters general relations were derived for calculation of the thermal output of arbitrary computing element of fuel assembly, for total thermal output of the arbitrary part of core or breeding zone, as well as relations for re-computing of heat sources to non-nominal thermal output level. In all cases the heat sources distribution was expressed by regression polynomial obtained by least squares method.

Derived relations allow - with the aid of an computer - relatively simple and fast calculation of the thermal outputs for thermal calculation. Regression method used - the least squares method - allows to transform the result of physical calculation (the heat sources distribution) into the thermal calculation with sufficient accuracy.

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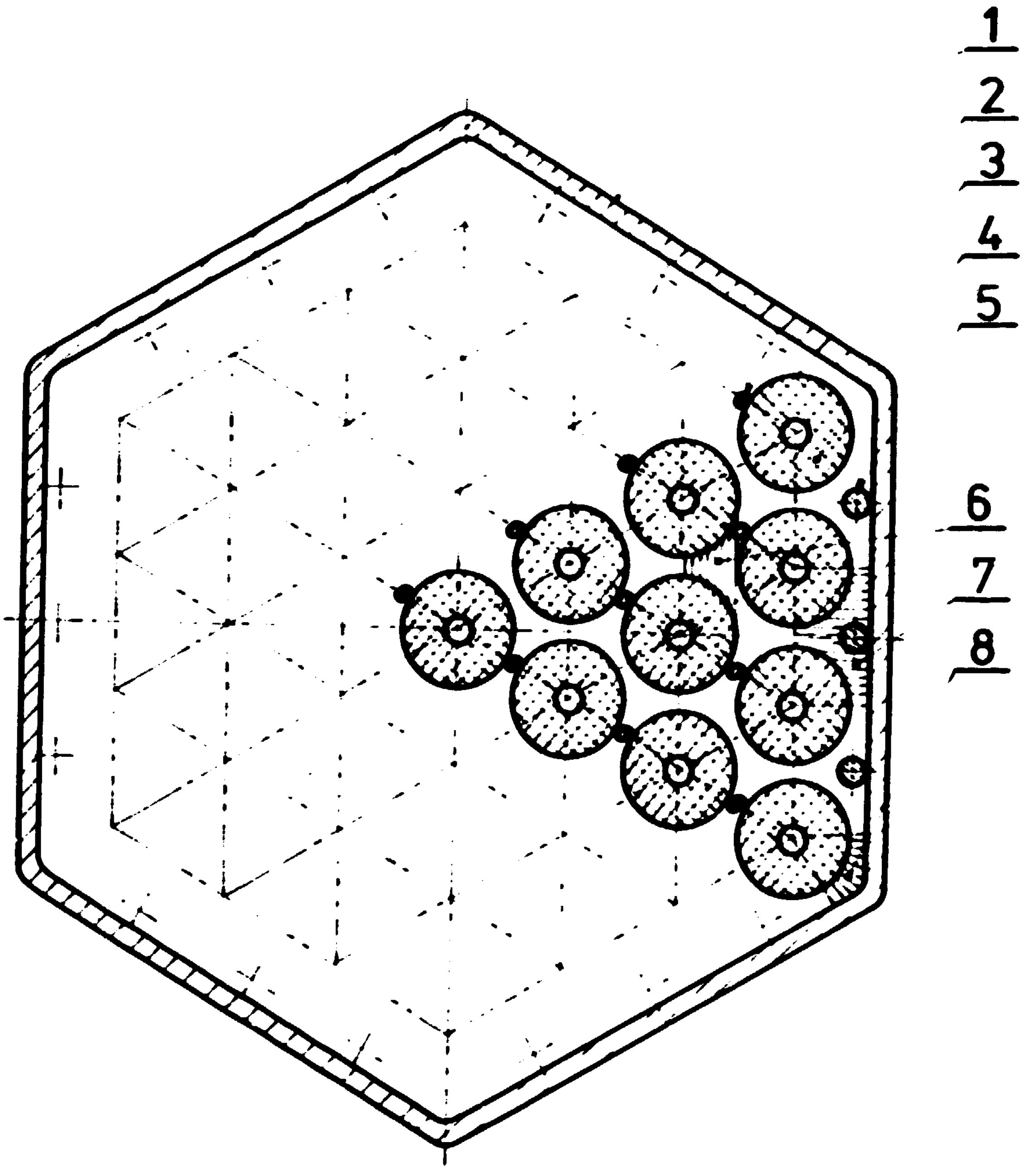
/7/ Aplikace matematiky 11, page 459

Fig:1



1 - [illegible]ual fuel box
2 - [illegible] spacing wire
3 - cladding
4 - fuel (UO_2, PuO_2)
5 - wire
6 - central
7 - peripheral
8 - angular

} subchannel

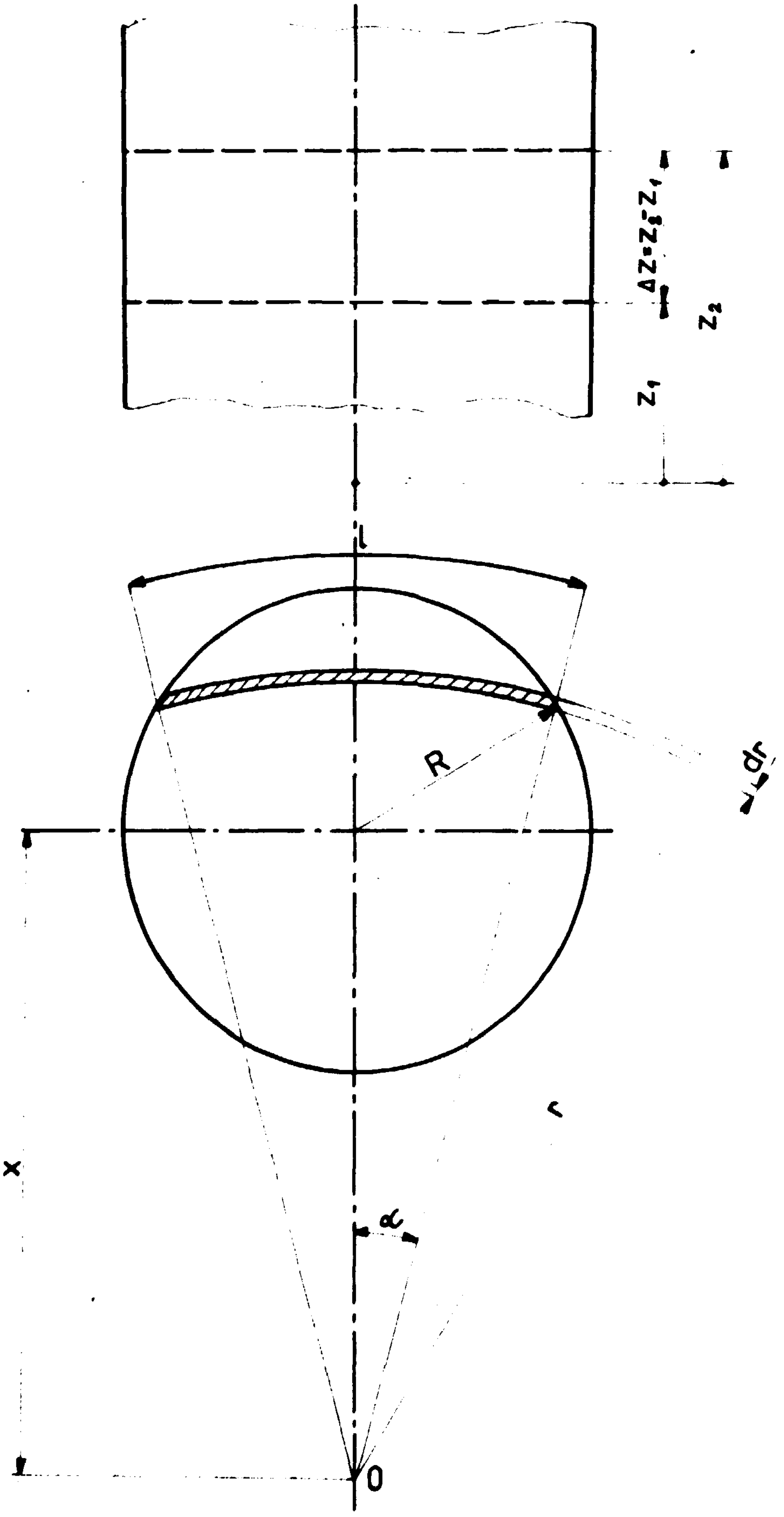


Fig:2

Fig: 3

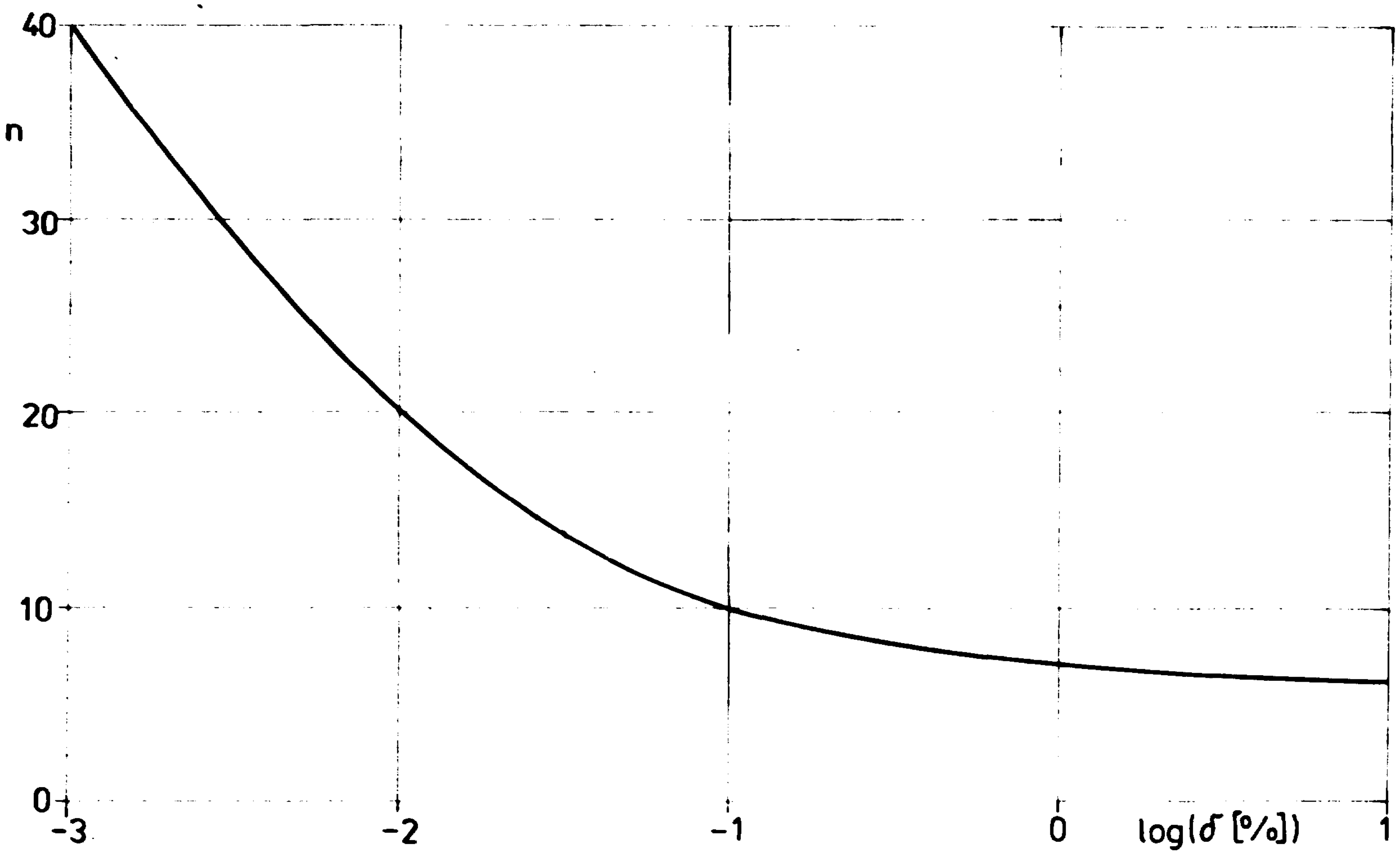


Fig:4

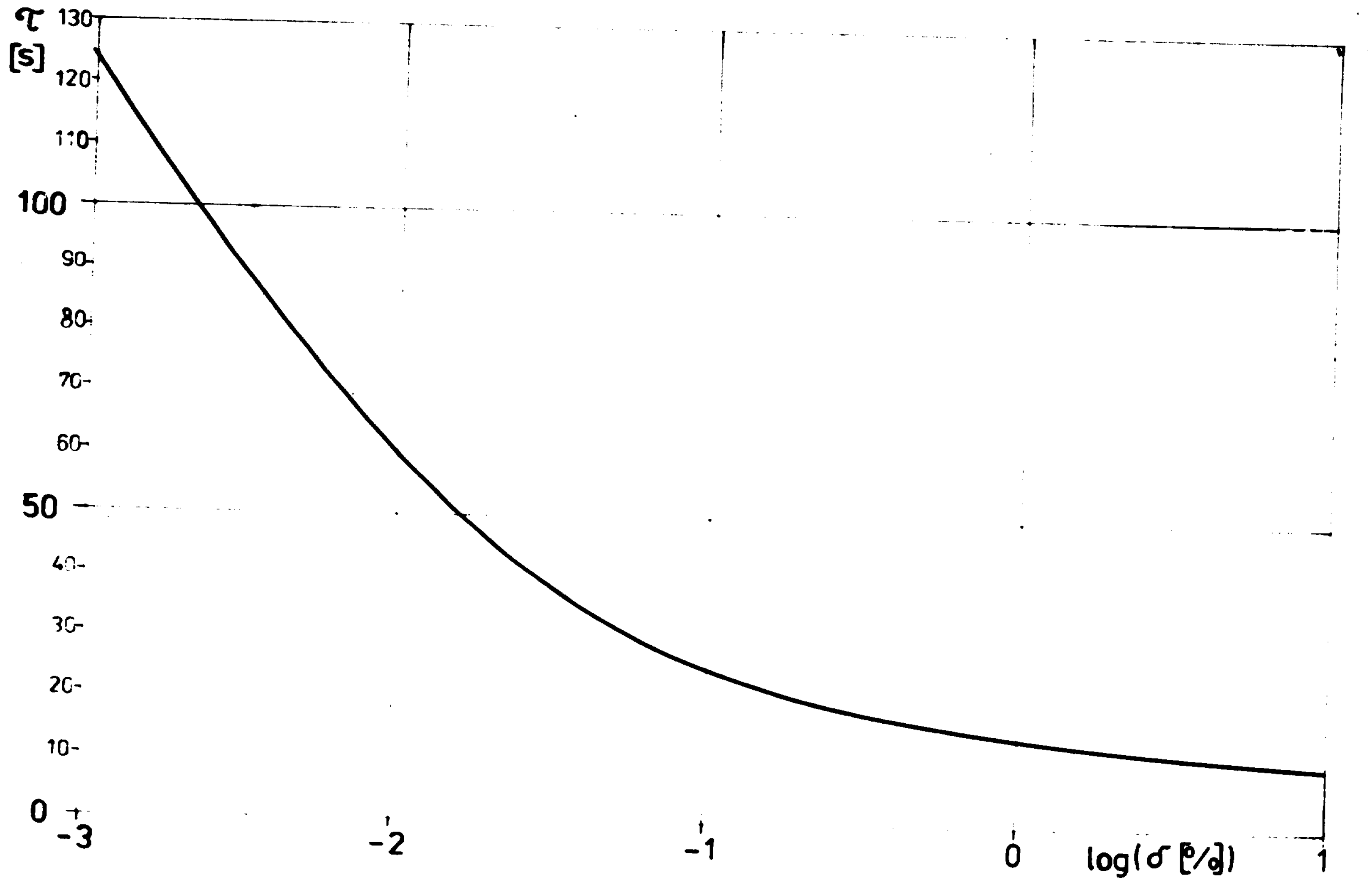


Fig. 5: