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LIGHT FRONT WAVE FUNCTION OF COMPOSITE SYSTEM WITH SPIN

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Abstract

The method to construct the relativistic wave function with spin on the light front is developed. The spin structure of the deuteron wave function in relativistic region is found. The calculative methods are illustrated by calculation of elastic pd - scattering cross section. The consideration carried out is equivalent to solution of the problem of taking into account the spins and angular momenta in the parton wave functions in the infinite momentum frame.

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Волновая функция со спином на световом фронте.

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1. Introduction

In the paper [1] the formalism of the wave functions (WF) on the light front describing the relativistic systems consisting of spinless particles and having zero total angular momentum has been developed. Necessity of development the explicitly covariant and practically convenient formalism for description of the nuclei at relativistic relative nucleons momenta and elementary particles in the composite systems is required by contemporary experimental data and was pointed out by I.S.Shapiro [2]. The review of experimental situation in relativistic nuclear physics, of theoretical problems and existing approaches was given in Ref. [3].

WF on the light front is the Fock component of the state vector defined in invariant Schrödinger representation (ISchR). In conventional Schrödinger representation the state vector is defined on the hypersurface $t = 0$, while the state vector in ISchR is defined on the arbitrary flat space-like hyper surface. It is convenient to choose the hypersurface of the light front $\omega x = \omega_0 t - \vec{\omega}\vec{x} = c$, where ω is four- vector: $\omega = (\omega_0, \vec{\omega})$, $\omega^2 = 0$, $\omega_0 > 0$.

Covariant WFs on the light front have the following advantages : they admit the probabilistic interpretation, depend on three- dimensional vectors and simply related with the vertices of special graph technique, which allows to express the processes amplitudes through these WFs. The problems arising when one expresses the amplitudes in terms of WFs on the light front were investigated in Ref.[4]. The graph technique is based on the three- dimensional formulation of the field theory developed by Kadyshevsky [5]. For the case of the light front this graph technique is invariant generalization of the old fashioned perturbation theory in the infinite momentum frame (IMF). WFs on the light front are the covariant generalization of the equal-time Fock components in IMF. Note that WFs in IMF are broadly used in the parton models.

In the present paper we consider WFs on the light front for the systems with spin. Because of relation of WFs on the light front with WFs in IMF the construction of WFs with spin on the light front is automatically solves the problem of taki g into account of spins and angular momenta in the parton WFs. This theoretical problem was pointed out by Feynman [6].

In Sect.2 we briefly remind the basic results of Ref. 1 concerned to WFs on the light front for the case of spinless particles. In Sect.3 we construct the WF with spin. In Sect.4 the graph technique ith spin taken into account is stated, which allows to express the reaction amplitudes through WFs on the light front. The formalism developed is illustrated by example of calculation of the elastic pd- scattering cross section in the framework of the one-nucleon-exchange mechanism.

2. Wave Function of a Spinless System

WF on the light front $\Psi(x_1, x_2, p, \omega)$ is coefficient of expansion of the state vector $\phi(p)$, defined on the hypersurface $\omega x = 0$, in series of states of free particles:

$$\phi(p) = \int \psi(x_1, x_2, p, \omega)\, \varphi^{(+)}(x_1)\, \varphi^{(+)}(x_2)|0\rangle \times \delta(\omega x_1)\, d^4x_1\, \delta(\omega x_2)\, d^4x_2 + \ldots \qquad (1)$$

where

$$\varphi^{(+)}(x) = \int e^{ik(x-\omega(\omega x))} C^{+}(\vec{k}) \frac{d^3k}{\sqrt{2\varepsilon_k}} \quad (2)$$

$C^{+}(\vec{k})$ is the crition operator. The state vector is determined by totality of components corresponding to states with different number of particles. Their contribution in eq.(1) is noted by dots. For simplicity we shall consider below only the two particles component. In the momentum space taking into account eq.(2) we have:

$$\Psi(k_1, k_2, p, \omega) = \int \psi(x_1, x_2, p, \omega) \times \exp(ik_1x_1 + ik_2x_2)\delta(\omega x_1)d^4x_1\delta(\omega x_2)d^4x_2 \quad (3)$$

where $k_1^2 = k_2^2 = m^2,\ p^2 = M^2$.

Let us clear up the WFs properties which result from invariance under translations, Lorentz transformations and rotations.

Let us begin from consideration of translations. We consider that dependence of WF on the "oblique" time (along the ω - direction), analogous to the factor $e^{-i\varepsilon t}$ in nonrlativistic WF $e^{-i\varepsilon t}\psi(\vec{r})$, is extracted from WF $\psi(x_1, x_2, p, \omega)$. Therefore $\psi(x_1, x_2, p, \omega)$ is not changed under translation of the arguments x_1, x_2 along ω. The translations in arbitrary direction μ, orthogonal to ω, give usual phase factor: $\psi(x_1+\mu, x_2+\mu, p, \omega) = e^{-ip\mu}\psi(x_1, x_2, p, \omega)$. On the other hand having substituted WF $\psi(x_1+\mu, x_2+\mu, p, \omega)$ in integral (3) and having replaced the variables $x_{1,2}+\mu = x'_{1,2}$, we obtain the phase factor $\exp(-i(k_1+k_2)\mu)$. Consequently the projections of four-vectors p and (k_1+k_2) on the hypersurface of the light front should be equaled: $p\mu = (k_1+k_2)\mu$. Having introduced the delta-function, which takes into account the equality of these projections:

$$\Psi(k_1, k_2, p, \omega) = \int \psi(k_1, k_2, p, \omega\tau)\, \delta^{(4)}(k_1 + k_2 - p - \omega\tau)\, d\tau \quad (4)$$

we obtain that the translation invariance results in the following relation between the four- momenta in $\psi(k_1, k_2, p, \omega\tau)$:*)

$$k_1 + k_2 = p + \omega\tau \qquad (5)$$

This allows to represent WF $\psi(k_1, k_2, p, \omega\tau)$ in the form of the four- point diagram (Fig.1).

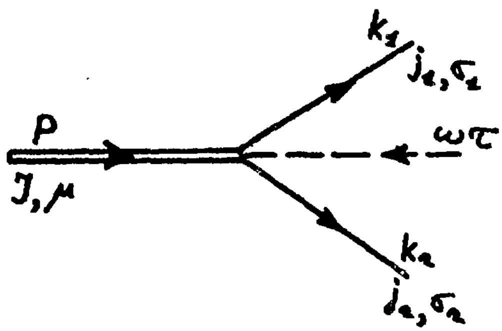

Fig. 1

Meaning the consideration of the case of the particles with spin we show in Fig. 1 the spin indices. For the spinless system WF $\psi(k_1, k_2, p, \omega\tau)$ is invariant under rotation and Lorentz transformation. Therefore in the same

*) Strictly speaking the consideration carried out above is not quite correct since the four- vector ω is "orthogonal to it self" ($\omega\cdot\omega = 0$). Equation (5) can be obtained from consideration of the translation properties of WF defined on the orbitrary flat hypersurface $\lambda x = 0$ $(\lambda = (\lambda_0, \vec{\lambda}),\ \lambda^2 = 1)$. WF on the light front and eq.(5) are obtained in limit $\vec{\lambda} \to \infty$.

manner as any reaction amplitude $1+2 \to 3+4$ the WF depends on the scalar products of the four- vectors. Introducing the variables:

$$s_1 = (k_1+k_2)^2,\ t_1 = (p-k_1)^2,\ u_1 = (p-k_2)^2,$$
$$s_1 + t_1 + u_1 = M^2 + 2m^2 \tag{6}$$

we obtain: $\Psi = \Psi(s_1, t_1)$

It is convenient to introduce the variables which have sense of the particle momenta in c.m.- system of the "reaction" shown in Fig.1. Introducing the notation:

$$Q = \frac{m}{\sqrt{s_1}}(k_1+k_2)$$

we have:

$$\vec{q} = L^{-1}(Q)\vec{k}_1 = \vec{k}_1 - \frac{\vec{Q}}{m}\left[k_{10} - \frac{\vec{k}_1\vec{Q}}{m+\varepsilon(\vec{Q})}\right] \tag{8}$$

$$\vec{n} = L^{-1}(Q)\vec{\omega}/|L^{-1}(Q)\vec{\omega}| = \sqrt{s_1}\,L^{-1}(Q)\vec{\omega}/(\omega p) \tag{9}$$

where: $k_{10} = \varepsilon(\vec{k}_1) = \sqrt{\vec{k}_1^{\,2}+m^2}$, $\varepsilon(\vec{Q}) = \sqrt{\vec{Q}^{\,2}+m^2}$.
$L^{-1}(Q)$ is the Lorentz transformation: $L^{-1}(Q)Q = \mathring{Q}$, where $\mathring{Q} = (m,0,0,0)$ *).

The vector $\vec{q}$ has the sense of momentum of the particle 1 in the c.m.- system of the "reaction" shown in Fig.1, $\vec{n}$ is the unit vector in direction $\vec{\omega}$ in the c.m.- system. Note that the vectors $\vec{q}, \vec{n}$ undergoes only

*) The notations in Eqs.(7-9) are changed in comparison with Ref. [1].

rotations under a Lorentz transformation of the four- vectors $k_1, k_2, p, \omega\tau$:

$$\vec{q}\,' = R(g,Q)\vec{q}\,, \qquad \vec{n}\,' = R(g,Q)\vec{n} \tag{10}$$

where

$$R(g,Q) = L^{-1}(gQ)\,g\,L(Q) \tag{11}$$

Therefore $\vec{q}^{\,2}$ and $\vec{n}\vec{q}$ are invariants. They can be expressed in terms of the invariants s_1, t_1 and u_1 :

$$\vec{q}^{\,2} = \frac{s_1}{4} - m^2$$

$$\vec{n}\vec{q} = \frac{(u_1 - t_1)\sqrt{s_1}}{2(s_1 - M^2)} \tag{12}$$

So, the WF of the spinless system depends on two variables $\vec{q}^{\,2}$ and $\vec{n}\vec{q}$:

$$\Psi = \Psi(\vec{q}^{\,2}, \vec{n}\vec{q}) \tag{13}$$

The physical reason of dependence of the relativistic WF on the variable $\vec{n}$ is impossibility to separate the center-of-mass motion in a system of interacting particles. This impossibility is connected with relativistic retardation of interaction. For the equal- time WF $\Psi(\vec{k}_1, \vec{k}_2, \vec{p})$ this means that after introducing the relativistic relative momentum $\vec{q}$ the WF remains to depend on the total momentum $\vec{p}$ of system: $\Psi = \Psi(\vec{q}, \vec{p})$. In IMF the dependence on the modulus of $\vec{p}$ disappears, but a dependence on the unit vector $\vec{n} = \vec{p}/|\vec{p}|$ remains.

The consideration of the dynamical models shows that the characteristic parameter determining the dependence of a nuclear WF on variable $\vec{n}\vec{q}$ is the nucleon mass. Therefore at $\vec{q}^{\,2} \ll m^2$. the dependence of WF on the variable $\vec{n}\vec{q}$ becomes unessential, and we return to nonrelativistic WF depending on the only variable $\vec{q}$.

The WF on the light front which is not equal- time one in arbitrary system becomes equal- time one in IMF moving along $\vec{\omega}$. Therefore the WF on the light front coincides with the equal- time WF $\Psi(\vec{k}_1, \vec{k}_2, \vec{p})$, defined in IMF, i.e. at $\vec{p} \to \infty$. This WF in IMF is conventially parametrized by means of variables $\vec{R}_\perp$ and x : $\Psi = \Psi(\vec{R}_\perp^{\,2}\ x)$. Here x is the momentum portion of the particle 1 from the infinite momentum of total system, $\vec{R}_\perp$ is the momentum projection of the particle 1, which is orthogonal the infinita momentum:

$$x = (\vec{k}_1 \vec{p})/\vec{p}^{\,2} \tag{14}$$

$$\vec{R}_\perp^{\,2} = \vec{k}_1^{\,2} - (\vec{k}_1 \vec{p})^2/\vec{p}^{\,2} \tag{15}$$

Let us connect the varriables $x, \vec{R}_\perp^{\,2}$ with $\vec{q}^{\,2}, \vec{n}\vec{q}$. For this purpose we firstly consider WF $\Psi(k_1, k_2, p, \lambda\tau)$, where $\lambda^2 = 1$. This WF is equal- time one in the system, where $\vec{\lambda} = 0$. This system moves with velocity $\vec{V}$ = $= \vec{\lambda} c / \sqrt{1 + \vec{\lambda}^{\,2}}$. The variable x , which in this system goes over into expression, given by Eq.(14), has the form:

$$x = \frac{(k_1\lambda)(p\lambda) - (k_1 p)}{(p\lambda)^2 - M^2} \tag{16}$$

One can obtain the transition to equal- time WF in IMF by means of limit $\vec{\lambda} \to \infty$. At $\vec{\lambda} \to \infty$ in Eq.(16) one can neglect the difference between $|\vec{\lambda}|$ and λ_0 .

Having replaced $\lambda \to \omega$, we obtain:

$$x = \frac{\omega k_1}{\omega p} = \frac{1}{2}\left(1 - \frac{\vec{n}\vec{q}}{\varepsilon(\vec{q})}\right) \tag{17}$$

Analogously we find the expression for $\vec{R}_\perp^{\,2}$:

$$\vec{R}_\perp^{\,2} = \vec{q}^{\,2} - (\vec{n}\vec{q})^2 \tag{18}$$

The variables $\vec{q}, \vec{n}$ seems to be more convenient ones than $\vec{R}_\perp, x$ due to two reasons. Firstly, the transition to nonrelativistic limit in WF depending on $\vec{n}$ is realized by rather simple way (by neglecting the WF dependence on $\vec{n}$). In terms of variables $\vec{R}_\perp, x$ the nonrelativistic WF dependends on rather complicated combination of $\vec{R}_\perp$ and x :

$$\vec{q}^{\,2} = \frac{\vec{R}_\perp^{\,2} + m^2}{4x(1-x)} - m^2 \tag{19}$$

which can be obtained by solving the system of Eqs.(17), (18) relative to $\vec{q}^{\,2}$. Secondly, the simple transformation properties of the variables $\vec{q}, \vec{n}$ allows to construct the states with spin by rather simple way.

3. Wave Function for a System With Spin

At first we consider the case of the system with spin and parity $J^P = \frac{1}{2}^+$,which consists of two particles with spins 0^+ and $\frac{1}{2}^+$, and than we shall go over to the general case. We shall work in representation in which the particles have the definite spin projections on the axis z in their rest systems.

The state vector of the system concerned has the form

$$\Phi_\mu(p) = \sum_\sigma \int \Psi_\sigma^\mu(k_1, k_2, p, \omega\tau)\, a_\sigma^+(\vec{k}_1)\, c^+(\vec{k}_2)|0\rangle \times \delta^{(4)}(k_1+k_2-p-\omega\tau)\, d\tau\, \frac{d^3k_1}{\sqrt{2\varepsilon_1}}\, \frac{d^3k_2}{\sqrt{2\varepsilon_2}} + \dots \tag{20}$$

Our starting point in construction of WF with spin is the transformation law of the state with spin. Under the definition of spin the state vector of a system with definite spin is transformed under the transformation of the system-of-reference $x \to x' = g x$ in accordance with the law:

$$\phi_\mu(p) \to \phi'_\mu(gp) = U\phi_\mu(p) = \sum_{\mu'} D^{\frac{1}{2}}_{\mu'\mu}\{R(g,p)\}\,\phi_{\mu'}(gp) \tag{21}$$

where $D^{\frac{1}{2}}_{\mu'\mu}$ is the matrix elements of the rotation group, $R(g,p) = L^{-1}(gp)\, g\, L(p)$.

Let us emphasize that the transformation law (21) is exclusively determined by the group properties of spin. This law does not depend on concrete representation in the field theory and has the same form both in the Schrödinger representation on the hypersurface $t=0$ and in ISchR on the light front. Concrete representation determines the form of generators determining the operator U and the transformation law of Fock components, which follows from eq.(21). The explicit form in ISchR of the generators of Poincare group and group- theoretical problems, concerned to transformation of the state vector in ISchR were investigated by the author of the present paper in collaboration with I.S.Shapiro and will be published in future publications. In the present paper the explicit form of the operator U will not be necessary for us.

Let us find out the transformation law of WF $\Psi_\sigma^\mu(k_1, k_2, p, \omega\tau)$ under Lorentz transformation and rotation of its arguments. The state vector $\phi_\mu(p)$ in

ISchR in different reference systems is defined on the same hypersurface, while the state vector in the Schrödinger representation at $t=0$ in systems A and A' is defined on the different hypersurfaces $t=0$ and $t'=0$. Therefore the transformation law of the state vector in ISchR is purely kinematical one. Consequently, the operator U does not contain the interaction and does not change the number of particles. The states $a^+_\sigma(\vec{k}_1)|0\rangle$ and $c^+(\vec{k}_2)|0\rangle$ under action of operator U are transformed independently. As any states with spins $\frac{1}{2}$ and 0 they are transformed according to the law:

$$U a^+_\sigma(\vec{k}_1)|0\rangle = \sum_{\sigma'} D^{\frac{1}{2}}_{\sigma'\sigma}\{R(g,k_1)\}\, a^+_{\sigma'}(g\vec{k}_1)|0\rangle$$

$$U c^+(\vec{k}_2)|0\rangle = c^+(g\vec{k}_2)|0\rangle \tag{22}$$

From Eqs.(20) and (22) we obtain:

$$U\phi_\mu(p) = \sum \Psi^\mu_\sigma(k_1,k_2,p,g^{-1}\omega\tau)\, D^{\frac{1}{2}}_{\sigma'\sigma}\{R(g,k_1)\}\times$$
$$\times\, a^+_{\sigma'}(g\vec{k}_1)\, c^+(g\vec{k}_2)|0\rangle\, \times \tag{23}$$
$$\times\, \delta^{(4)}(k_1+k_2-p-g^{-1}\omega\tau)\,d\tau\, \frac{d^3k_1}{\sqrt{2\varepsilon_1}}\, \frac{d^3k_2}{\sqrt{2\varepsilon_2}}$$

The quantity $g^{-1}\omega\tau$ in Eq.(23) corresponds to the following fact: if a basis is transformed under transformation g, than the vector components in this basis are transformed by means of g^{-1}. One can obtain this by more formal way, if he consider the transformation law of the field operator (for simplicity we consider the scalar field):

$$\varphi'(x) = \varphi(g^{-1}x) = U^{-1}\varphi(x)U \qquad (24)$$

where $\varphi(x)$ in ISchR is given by Eq.(2). From (24) it is follows:

$$U^{-1}\varphi^{(+)}(x)U = \varphi^{(+)}(g^{-1}x) = \int \exp\{i[x - g\omega(g\omega x)]gk\} C^{+}(g\vec{k}) \frac{d^3k}{\sqrt{2\varepsilon_k}}$$

In other words, the operator U transforms four- vector ω as follows: $U^{-1}\omega U = g\omega$, or: $U\omega U^{-1} = g^{-1}\omega$. This results in $g^{-1}\omega\tau$ in Eq.(23).

Comparing Eqs.(21) and (23), after simple transformation we obtain the transformation law of WF $\Psi^{\mu}_{\sigma}(k_1, k_2, p, \omega\tau)$:

$$\Psi^{\mu}_{\sigma}(gk_1, gk_2, gp, g\omega\tau) =$$
$$= \sum D^{\frac{1}{2}*}_{\mu\mu'}\{R(g,p)\} D^{\frac{1}{2}}_{\sigma\sigma'}\{R(g,k_1)\} \Psi^{\mu'}_{\sigma'}(k_1, k_2, p, \omega\tau) \qquad (25)$$

So, the WF in ISchR is transformed in covariant way. The transformation law (25) coincides with the transformation law of amplitude of the reaction $\frac{1}{2} + 0 \to \frac{1}{2} + 0$. Therefore the problem of construction of WFs with spin is reduced to well known one: this is expansion of the reaction $\frac{1}{2} + 0 \to \frac{1}{2} + 0$ amplitude in invariant amplitudes. This expansion has the form:

$$\Psi^{\mu}_{\sigma} = F_1 \bar{u}^{\sigma}(k_1) u^{\mu}(p) + F_2 \bar{u}^{\sigma}(k_1)(\hat{k}_1 + \hat{k}_2) u^{\mu}(p) \qquad ($$

where $\hat{k}_1 + \hat{k}_2 = (k_1 + k_2)_\alpha \gamma_\alpha$, F_1 and F_2 depends on th invariant variables s_1, t_1 or $\vec{q}^{\,2}, \vec{n}\vec{q}$; $\bar{u}^{\sigma}(k_1), u^{\mu}$, are the spinors. For example, the spinor $u^{\mu}(p)$ has the form

$$u^{\mu}(p) = \begin{pmatrix} \sqrt{\mathcal{E}(\vec{p})+M}\, w^{\mu} \\ \sqrt{\mathcal{E}(\vec{p})-M}\, \frac{(\vec{p}\vec{\sigma})}{|\vec{p}|} w^{\mu} \end{pmatrix}$$

$$w^{\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad w^{-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{27}$$

We remind that a spinor is transformed as follows:

$$u'^{\mu}(gp) = \Lambda(g)u^{\mu}(p) = \sum_{\mu_1} D^{\frac{1}{2}}_{\mu_1\mu}\{R(g,p)\}\, u^{\mu_1}(gp) \tag{28}$$

This law indeed results in the transformation law (25) for WF (26).

It is convenient to transform the expansion (26) into the form, which is more close to nonrelativistic one. For this purpose in construction of the spin structure we shall use the variables $\vec{q}$ and $\vec{n}$. Let us go over from WF Ψ^{μ}_{σ} to $\tilde{\Psi}^{\mu}_{\sigma}$, transformed by means of D - functions depending on the same rotation $R(g,Q) = L^{-1}(gQ)gL(Q)$, which determines the transformation law of the variables $\vec{q}, \vec{n}$ (see Eqs.(10)). Let us show that Ψ^{μ}_{σ} and $\tilde{\Psi}^{\mu}_{\sigma}$ are connected by the formulae:

$$\tilde{\Psi}^{\mu}_{\sigma} = \sum D^{\frac{1}{2}}_{\sigma\sigma_1}\{R(L^{-1}(Q),k_1)\}\, D^{\frac{1}{2}*}_{\mu\mu_1}\{R(L^{-1}(Q),p)\}\, \Psi^{\mu_1}_{\sigma_1} \tag{29}$$

$$\Psi^{\mu}_{\sigma} = \sum D^{\frac{1}{2}*}_{\sigma_1\sigma}\{R(L^{-1}(Q),k_1)\}\, D^{\frac{1}{2}}_{\mu_1\mu}\{R(L^{-1}(Q),p)\}\, \tilde{\Psi}^{\mu_1}_{\sigma_1} \tag{30}$$

where in accordance with definition (11):

$$R(L^{-1}(Q),k_1) = L^{-1}(L^{-1}(Q)k_1)\, L^{-1}(Q)\, L(k_1) \tag{31}$$

$L^{-1}(Q)$ and $L(k_1)$ are as usual the inverse and direct Lorentz transformations with parameters $\vec{Q}$ and $\vec{k}_1$, $L^{-1}(L^{-1}(Q)k_1)$ is the inverse Lorentz transformation with parameters obtained from $\vec{k}_1$ by the Lorentz transformation $L^{-1}(Q)$.

Note that the following equality takes place [7]:

$$D^{\frac{1}{2}}\{R(L^{-1}(Q),k)\} = \frac{(k_0+m)(Q_0+m)-(\vec{\sigma}\vec{k})(\vec{\sigma}\vec{Q})}{[2(k_0+m)(Q_0+m)(k_0Q_0-\vec{k}\vec{Q}+m^2)]^{1/2}} \quad (32)$$

Under the transformation g of the parameters Q and k_1 the quantity $R(L^{-1}(Q),\, k_1)$ is transformed as follows:

$$R(L^{-1}(gQ),gk_1) = R(g,Q)R(L^{-1}(Q),k_1)R^{-1}(g,k_1) \quad (33)$$

In order to verify the formula (33) we write down it in the explicit form:

$$L^{-1}(L^{-1}(gQ)gk_1)L^{-1}(gQ)L(gk_1) =$$

$$= L^{-1}(gQ)gL(Q)L^{-1}(L^{-1}(Q)k_1)L^{-1}(Q)L(k_1)L^{-1}(k_1)gL(gk_1) \quad (34)$$

Meaning that $L^{-1}(gQ)g\vec{k}_1 = R(g,Q)\vec{q}$, $\vec{q} = L^{-1}(Q)\vec{k}_1$, after simple transformations we obtain from Eq.(34):

$$L^{-1}(R(g,Q)\vec{q}) = R(g,Q)L^{-1}(\vec{q})R^{-1}(g,Q)$$

Introducing the notation $R(g,Q) \equiv R$, we have:

$$RL(\vec{q}) = L(R\vec{q})R \quad (35)$$

Correctness of the relation (35) is evident. It follows from this that the identity (33) is correct. From Eqs.(33), (29) and (25) and taking into account the equality $D(R_1 R_2) = D(R_1) D(R_2)$ we obtain the transformation law of the function $\tilde{\Psi}^{\mu}_{\sigma}$:

$$\tilde{\Psi}^{\mu}_{\sigma}(gk_1, gk_2, gp, g\omega\tau) = \\ = \sum D^{\frac{1}{2}*}_{\mu\mu_1}\{R(g,Q)\} D^{\frac{1}{2}}_{\sigma\sigma_1}\{R(g,Q)\} \tilde{\Psi}^{\mu_1}_{\sigma_1}(k_1, k_2, p, \omega\tau) \tag{36}$$

Thus, the function $\tilde{\Psi}^{\mu}_{\sigma}$ is transformed by means of the D - functions depending on the same rotation $R(g,Q)$ that transforms the variables $\vec{q}, \vec{n}$. Therefore it is not difficult to construct in terms of variables $\vec{q}, \vec{n}$ the function $\tilde{\Psi}^{\mu}_{\sigma}$:

$$\tilde{\Psi}^{\mu}_{\sigma}(\vec{q}, \vec{n}) = f_1(\vec{q}^{\,2}, \vec{n}\vec{q})\,\delta_{\mu\sigma} + \\ + f_2(\vec{q}^{\,2}, \vec{n}\vec{q})\, C^{\frac{1}{2}\mu}_{\frac{1}{2}\sigma 1m}\, C^{1m}_{1m_1 1m_2}\, Y^{*}_{1m_1}\left(\frac{\vec{q}}{|\vec{q}|}\right) Y^{*}_{1m_2}(\vec{n}) \tag{37}$$

where C are the Clebsch-Gordan coefficients.

In terms of the Pauli matrices this function is represented in the form:

$$\tilde{\Psi}(\vec{q}, \vec{n}) = f_1(\vec{q}^{\,2}, \vec{n}\vec{q}) + \\ + \frac{i\sqrt{6}}{8\pi} f_2(\vec{q}^{\,2}, \vec{n}\vec{q})\,(\vec{\sigma}[\vec{n}\times\vec{q}])/|\vec{q}| \tag{38}$$

Comparing the expansion (37) and (38) in the system, where $\vec{k}_1 + \vec{k}_2 = 0$ (in this system all the D- functions in Eqs.(29), (30) turns into the unit matrices), we obtain the relations between the coefficients f_1, f_2 and F_1, F_2 :

$$f_1 = N^{\frac{1}{2}}(F_1 + \sqrt{s_1}F_2) + N^{-\frac{1}{2}}(F_1 - \sqrt{s_1}F_2)(\vec{q}\vec{n})/|\vec{q}|$$

$$\frac{\sqrt{6}}{8\pi} f_2 = N^{-\frac{1}{2}}(\sqrt{s_1}F_2 - F_1)\vec{q}^{\,2} \tag{39}$$

where $N = (\sqrt{\vec{q}^{\,2}+m^2}+m)(\sqrt{\vec{q}^{\,2}+M^2}+M)$, $s_1 = (k_1+k_2)^2$, m is the mass of the particle with spin $1/2$.

In nonrelativistic case in WF of the system concerned only the zero angular momentum is admissible. This means that at $|\vec{q}| \ll m$ the coefficient f_1 turns into nonrelativistic S - wave function (and becomes independent on $\vec{n}\vec{q}$), and coefficient f_2 becomes equaled to zero. Of course the coefficient f_2 is not nullified automatically in Eqs.(37) and (38), since the nonrelativistic character of a system is manifested in the dynamical properties of the invariant functions f_1 and f_2, i.e. in character of their dependence on the variables $\vec{q}^{\,2}$ and $\vec{n}\vec{q}$. Coincidence of relativistic WFs on the light front with nonrelativistic ones at $|\vec{q}| \ll m$ is guaranted by the Schrödinger equation to be the nonrelativistic limit of the dynamics determining the relativistic WFs and their dependence on $\vec{n}$. Consequently the functions which contributes only due to dependence on $\vec{n}$ of the spin structures turns at $|\vec{q}| \ll m$ into zero.

It is easy to generalize the method involved of construction of the relativistic WFs with spin in the case of arbitrary spins. For a system with total momentum J consisting of the particles with spins j_1 and j_2 we introduce WF $\tilde{\Psi}^{\mu}_{\sigma_1\sigma_2}$. WF $\tilde{\Psi}^{\mu}_{\sigma_1\sigma_2}$ is connected with WF $\Psi^{\mu}_{\sigma_1\sigma_2}$ which is the coefficient in expansion analogous to Eq.(20) as follows:

$$\tilde{\Psi}^{\mu}_{\sigma_1\sigma_2}=\sum D^{J*}_{\mu\mu'}\{R(L^{-1}(Q),P)\}\, D^{j_1}_{\sigma_1\sigma_1'}\{R(L^{-1}(Q),k_1)\}\times$$
$$\times D^{j_2}_{\sigma_2\sigma_2'}\{R(L^{-1}(Q),k_2)\}\,\Psi^{\mu'}_{\sigma_1'\sigma_2'} \qquad (40)$$

Function $\tilde{\Psi}^{\mu}_{\sigma_1\sigma_2}$ is transformed by means of the D - functions containing the rotation $R(q, Q)$ and is constructed according to well known rules of expansion of nonrelativistic amplitude of reaction $J+0 \to j_1 + j_2$ in invariant amplitudes [8] (with replacement of nonrelativistic relative momenta by $\vec{q}$ and $\vec{n}$). WF $\tilde{\Psi}^{\mu}_{\sigma_1\sigma_2}$ has the form:

$$\tilde{\Psi}^{\mu}_{\sigma_1\sigma_2}=\sum_{j_{12}L\lambda} f(j_{12},L,\lambda)\sum C^{j_{12}\sigma_{12}}_{j_1\sigma_1 j_2\sigma_2}\, C^{J\mu}_{j_{12}\sigma_{12}LM}\times$$

$$\times C^{LM}_{\tau-\lambda,m_1,\tau+\lambda,m_2}\, Y^{*}_{\tau-\lambda,m_1}(\vec{n})\, Y^{*}_{\tau+\lambda,m_2}\left(\frac{\vec{q}}{|\vec{q}|}\right) \qquad (41)$$

where $\tau = \frac{1}{2}\left(L + \frac{1}{2}\left(1 + \pi\pi_1\pi_2(-1)^{L+1}\right)\right)$,

λ has the values from $-\tau$ to τ, π, π_1, π_2 are the internal parities of the particles. The functions $f(j_{12},L,\lambda)$ depend on $\vec{q}^{\,2}$ and $\vec{n}\vec{q}$. The number of the invariant functions $f(j_{12},L,\lambda)$ in the case of one integer and two half-integer spins is: $N=(2J+1)(2j_1+1)(2j_2+1)/2$ (the space parity conservation is taken into account).
If all the spins are integer, then: $N=[(2J+1)(2j_1+1)\times(2j_2+1)+\pi\pi_1\pi_2(-1)^{J+j_1+j_2}]/2$. Thus, for a deuteron we have: N = 6 instead of N = 2 in nonrelativistic case.

From Eq.(41) it is clear that the angular momentum operator in the representation chosen has the form:

$$\hat{\vec{J}} = \frac{1}{i}[\vec{q} \times \frac{\partial}{\partial \vec{q}}] + \frac{1}{i}[\vec{n} \times \frac{\partial}{\partial \vec{n}}] + \hat{\vec{j}}_1 + \hat{\vec{j}}_2 \qquad (42)$$

where $\hat{\vec{j}}_1$ and $\hat{\vec{j}}_2$ are the spin operators of the particles 1 and 2. This expression can be also obtained from the Pauli-Lubansky vector, constructed from the Poincare group generators realazing the transformations of the state vector in ISchR. For the spinless system the WF $\Psi(\vec{q}^{\,2}, \vec{n}\vec{q})$, considered in Sect.2, infact is eigenfunction of the operator (42) corresponding to zero eigenvalue. The action of the operator $[\vec{n} \times \frac{\partial}{\partial \vec{n}}]$ on nonrelativistic WF independent on $\vec{n}$ gives zero, and we return to conventional nonrelativistic expression for total angular momentum.

To construct the N - particles WF $\Psi^{\mu}_{\sigma_1 \dots \sigma_N}$ it is necessary by means of the formula analogous to Eq.(40) to go over from $\Psi^{\mu}_{\sigma_1 \dots \sigma_N}$ to $\tilde{\Psi}^{\mu}_{\sigma_1 \dots \sigma_N}$. WF $\tilde{\Psi}^{\mu}_{\sigma_1 \dots \sigma_N}$ is constructed by means of the formulae obtained by Kolybasov [9] for the nonrelativistic N - particle amplitude expansion in invariant amplitudes. In our case we deal with the amplitude of "reaction" $J + 0 \to j_1 + \dots + j_N$. The nonrelativistic relative momenta in these formulae should be replaced by $\vec{n}$ and $\vec{q}_i = L^{-1}(Q_i)\vec{k}_i$, where $Q_i = m_i(p+\omega\tau)/[(p+\omega\tau)^2]^{1/2}$, $i = 1, \dots, N$.

Instead of the function expansion by means of the Clebsch-Gordan coefficients and spherical functions one can in the case of arbitrary spins write down the direct expansion of WF Ψ in invariant amplitudes in the framework of the bispinor formalism (analogously to Eq. (26)). For this purpose one should use the method of the relativistic amplitudes expansion with arbitrary spins in invariant amplitudes in the bispinor formalism developed by Marinov [10].

The method of construction of the relativistic WFs with spin used here can be applied for expansion of relativistic amplitudes in invariant amplitudes. For this aim from the amplitude $M^{\sigma_3\sigma_4}_{\sigma_1\sigma_2}$ one should go over to the amplitude $\tilde{M}^{\sigma_3\sigma_4}_{\sigma_1\sigma_2}$:

$$\langle p_4\sigma_4, p_3\sigma_3 | M | p_1\sigma_1, p_2\sigma_2 \rangle =$$

$$= \sum D^{j_4 *}_{\sigma_4'\sigma_4}\{R_4\} D^{j_3 *}_{\sigma_3'\sigma_3}\{R_3\} D^{j_2}_{\sigma_2'\sigma_2}\{R_2\} D^{j_1}_{\sigma_1'\sigma_1}\{R_1\} \cdot \tag{43}$$

$$\cdot \langle p_4\sigma_4', p_3\sigma_3' | \tilde{M} | p_1\sigma_1', p_2\sigma_2' \rangle$$

where $R_i = R(L^{-1}(Q_i), p_i)$, $Q_i = m_i(p_1+p_2)/[(p_1+p_2)^2]^{\frac{1}{2}}$.

For construction of $\tilde{M}$ one should use the formulae for nonrelativistic amplitudes [8], analogous to Eqs.(41), with replacemen of nonrelativistic relative momenta by relativistic ones. This method is cloused to one developed in the papers [11]. Generally speaking, the invariant amplitudes of $\tilde{M}$ will have the kinematical singularities. This is illustrated by the Eqs.(39) in which f_1 and f_2 after expression of variables $|\vec{q}|$ and $\vec{n}\vec{q}$ through the variables s_1 and t_1 have the square root singularities. This expansion can be usefull in the problems, in which the existence of the kinematical singularities is not essential.

In nonrelativistic limit the amplitude $\tilde{M}$ does not change its form, only the relativistic relative momenta go over into nonrelativistic ones, and the D - functions turns, into the unit matrices.

Note that the choice of the four- vector Q in Eqs.(40) connecting Ψ and $\tilde{\Psi}$ (or in analogous formulae (43) for connection between M and $\tilde{M}$) is determined by the reasons of convenience. If in Eq.(43) we choose $Q = P_3$ in the D - functions "adjoined" to the indices σ_1', σ_3' and $Q = P_4$ in the D - functions "adjoined" to the indices σ_2', σ_4' we come to the representation used in the paper by Skachkov [12], where the expressions for the amplitudes in the one boson exchange model were found.

In conclusion of this Section we give the expression for the relativistic deuteron WF on the light front:

$$\tilde{\Psi}^{\mu}_{\sigma_1\sigma_2}(\vec{q},\vec{n}) = G_1\, C^{1\mu}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2} +$$

$$+ G_2\, C^{1\sigma}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2}\, C^{1\mu}_{1\sigma 1m}\, C^{1m}_{1m_1 1m_2}\, Y^*_{1m_1}(\vec{n})\, Y^*_{1m_2}\left(\frac{\vec{q}}{|\vec{q}|}\right) +$$

$$+ \sum_{\lambda=-1,0,+1} G_{4+\lambda}\, C^{1\sigma}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2}\, C^{1\mu}_{1\sigma 2m}\, C^{2m}_{1-\lambda, m_1, 1+\lambda, m_2} \times$$

$$\times Y^*_{1-\lambda, m_1}(\vec{n})\, Y^*_{1+\lambda, m_2}\left(\vec{q}/|\vec{q}|\right) +$$

$$+ G_6\, C^{00}_{\frac{1}{2}\sigma_1\frac{1}{2}\sigma_2}\, C^{1\mu}_{1m_1 1m_2}\, Y^*_{1m_1}(\vec{n})\, Y^*_{1m_2}\left(\vec{q}/|\vec{q}|\right) \tag{44}$$

From the Pauli principle it follows:

$$G_{1,3,5,6}(\vec{q}^{\,2}, \vec{n}\vec{q}) = G_{1,3,5,6}(\vec{q}^{\,2}, -\vec{n}\vec{q})$$

$$G_{2,4}(\vec{q}^{\,2}, \vec{n}\vec{q}) = -\,G_{2,4}(\vec{q}^{\,2}, -\vec{n}\vec{q}) \tag{45}$$

The functions G_1 and G_5 survive in the nonrelativistic limit and go over into S and D wave functions.

In the framework of the bispinor formalism the deuteron

WF $\psi^{\alpha}_{\sigma_1\sigma_2}$ has the form:

$$\psi^{\alpha}_{\sigma_1\sigma_2} = F_1 \bar{u}_1 U_c \bar{u}_2 e^-_{\alpha} + F_2 \bar{u}_1 (\gamma_{\alpha} - \frac{p_{\alpha}\hat{p}}{M^2}) U_c \bar{u}_2 + $$
$$+ F_3 \bar{u}_1 \gamma_5 U_c \bar{u}_2 e_{1\alpha} + F_4 \bar{u}_1 \hat{p} U_c \bar{u}_2 e^+_{\alpha} + \qquad (46)$$
$$+ F_5 \bar{u}_1 U_c \bar{u}_2 e^+_{\alpha} + F_6 \bar{u}_1 \hat{p} U_c \bar{u}_2 e^-_{\alpha}$$

Where $U_c = \gamma_2 \gamma_0$ is the Dirac charge conjugation matrix,

$$\bar{u}_{1,2} \equiv \bar{u}^{\sigma_{1,2}}(k_{1,2}),$$

$$e^- = \left(\frac{k_1}{pk_1} - \frac{k_2}{pk_2}\right) \Big/ \left[-\left(\frac{k_1}{pk_1} - \frac{k_2}{pk_2}\right)^2\right]^{\frac{1}{2}},$$

$$e^+ = \left(p - \frac{M^2}{p\omega}\omega - \frac{M^2(\omega e^-)}{p\omega} e^-\right) \Big/ M\left[1 - \frac{M^2(\omega e^-)}{(p\omega)^2}\right]^{1/2},$$

$$e_{1\alpha} = \varepsilon_{\alpha\nu\rho\gamma} \frac{p}{M} e^-_{\rho} e^+_{\gamma},$$

$$(e^-)^2 = (e^+)^2 = (e_1)^2 = -1.$$

Four- vectors $e^-, e^+, e_1, p/M$ are mutually orthogonal. In this representation the WF ψ^{α} is the four- dimensional pseudo-vector (in Eq.(46) the index has the values 0, 1, 2, 3). WF ψ^{α} is satisfied to condition $p_{\alpha}\psi^{\alpha} = 0$. From the Pauli principle it follows that the functions $F_{1,2,3,4}$ are not changed under replacement $\vec{q} \to -\vec{q}$, the functions $F_{5,6}$ change the sign at $\vec{q} \to -\vec{q}$.

The functions F_1 and F_2 survive in the nonrelativistic limit. The space components of the four- vector go over into the well known nonrelativistic deuteron WF:

$$\psi(\vec{q}) = w_1^* \left[\frac{1}{\sqrt{2}} u_S \vec{\sigma} + \frac{1}{2} u_D \left(\frac{3(\vec{\sigma}\vec{q})\vec{q}}{\vec{q}^2} - \vec{\sigma}\right)\right] \sigma_y w_2^* \qquad (47)$$

where u_S and u_D are S - and D - wave functions,

$$u_S=\frac{4\sqrt{2}}{3}|\vec{q}|F_1-2\sqrt{2}\,mF_2\,,\quad u_D=\frac{4}{3}|\vec{q}|F_1\,.$$

4. The Diagram Technique for the Case of the Particles with Spin

The rules of the diagram technique for the particles with spin in ISchR in the plane $\lambda x=0$ $(\lambda^2=1)$ were developed by Kadyshevsky, Mir-Kasimov and Skachkov [13]. These rules appear after transformation to the normal product of the S - matrix, represented in the form:

$$S^{(n)}=(-i)^n\int\widetilde{H}(-\lambda\tau_1)\frac{d\tau_1}{2\pi i(\tau_1-i0)}\,\widetilde{H}(\lambda\tau_1-\lambda\tau_2)\times$$
$$\times\ldots\times\frac{d\tau_{n-1}}{2\pi i(\tau_{n-1}-i0)}\,\widetilde{H}(\lambda\tau_{n-1})\tag{48}$$

where $\widetilde{H}(k)=\int e^{-ikx}\,H_{int}(x)\,d^4x$,

$H_{int}(x)$ is the interaction Hamiltonian in the interaction representation. The expression (48) can be transformed to the form of conventional T - product, therefor the amplitudes obtained from Eq.(48) coincide with Feynman ones. However at the intermediate stages of calculations the vertices parts arise which do not coincide with Feynman ones. They are connected with the WFs introduced above that allows to express the amplitudes through WFs. The rules of the diagram technique obtained from Eq.(48) in the model with Hamiltonian $H=-g\overline{\Psi}\gamma_5\Psi\varphi$ (Ψ and φ are the nucleon and neutral meson fields) are the following [13]:

1. In the given Feynman diagram number all vertices in an arbitrary way and orient each internal line in direction from the larger number to the smaller one. Starting from the

conservation law of the baryon number in every vertex we label the nucleon and antinucleon lines by indices $N, \bar{N}$.

2. Connect the vertices by dashed oriended lines (we call them the spurion lines) in the order of increasing numbers and to each line attribute the four- momentum $\lambda\tau_j$ $(j=1,2..)$.

3. To each dashed line with four- momentum $\lambda\tau_j$ set in correspondence the factor $1/2\pi(\tau_j - i0)$. To each interna_ nucleon and antinucleon lines associate the propagators $(\hat{p}+m)\theta(\lambda p)\delta(p^2-m^2)$ or $(\hat{p}-m)\theta(\lambda p)\times$ $\times\delta(p^2-m^2)$ correspondingly. To each internal meson line set in correspondence the propagator $\theta(\lambda p)\delta(p^2-\mu^2)$

4. To each vertex associate the factor $\delta^{(4)}(...)g\gamma_5/\sqrt{2\pi}$, where the delta function takes into account four- mor^ntum conservation, including the spurion momentum.

5. Integrate over all independent internal variables τ_j and the particle four- momenta in the infinite limits.

6. Repeat the procedure outlined for all $n!$ possible numbering of the vertices, sum the expressions obtained and multiply the result by the factor δ_p/η, where η is the number of permutaticns of the external lines entering in the diagram in symmetric way, δ_p is well known sign factor.

To the external incoming nucleon (antinucleon) lines we assign the spinors $u^\nu(q)$ $(\bar{v}^\nu(q))$, to outgoing ones we assign $\bar{u}^\mu(p)$ $(v^\mu(p))$. All the matrices acting on the spinor indices should be disposed in succession (from left to right) they are met when one moves along the spinor line, the antinucleon lines being passed in the direction of their orientation, and the nucleon lines - in the cpposite direction.

Let us illustrate the rules outlined by example of the πN scattering amplitude, determined by the sum of the diagrams Fig.2.

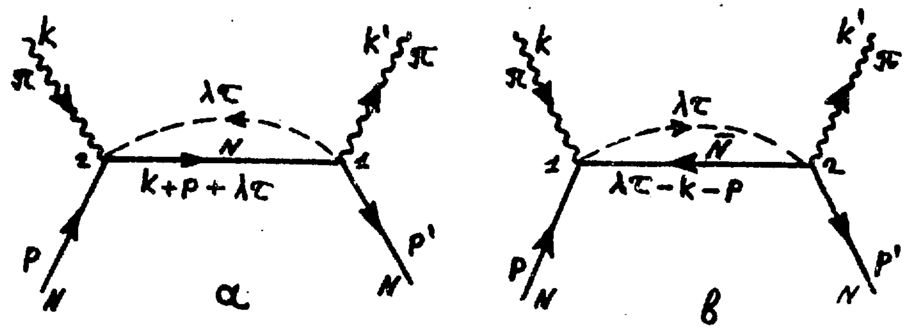

Fig. 2

In example of these diagrams we shall explain the features which arise at transition on the light front. Corresponding amplitudes have the form:

$$F_a = \frac{g^2}{(2\pi)^2}\frac{\bar{u}(p')\gamma_5(\hat{k}+\hat{p}+\hat{\lambda}\tau^- + m)\gamma_5 u(p)}{2\sqrt{[\lambda(k+p)]^2+m^2-(k+p)^2}\ \tau^-} \tag{49}$$

$$F_b = \frac{g^2}{(2\pi)^2}\frac{\bar{u}(p')\gamma_5(\hat{k}+\hat{p}-\hat{\lambda}\tau^+ + m)\gamma_5 u(p)}{2\sqrt{[\lambda(k+p)]^2+m^2-(k+p)^2}\ \tau^+} \tag{50}$$

where $\tau^{\mp} = \mp\lambda(k+p) + \sqrt{[\lambda(k+p)]^2+m^2-(k+p)^2}$

Sum of the amplitudes F_a and F_b coincides with the Feynman amplitude.

From the formulae (49), (50) one can see that the amplitudes obtained by means of the rules (1–6) are the invariantly represented amplitudes of the old fashioned perturbation theory. Thus, in a system where $\vec{\lambda} = 0$, $\lambda_0 = 1$ the denominators in Eqs. (49), (50) obtaines the form:

$$\frac{1}{2\mathcal{E}(\vec{k}+\vec{p})(\mathcal{E}(\vec{k}+\vec{p})-\mathcal{E}(\vec{k})-\mathcal{E}(\vec{p}))}$$

and

$$\frac{1}{2\mathcal{E}(\vec{k}+\vec{p})(\mathcal{E}(\vec{k}+\vec{p})+\mathcal{E}(\vec{k})+\mathcal{E}(\vec{p}))}$$

which coincides with the form of energy denominators of the old fashioned perturbation theory. Therefore the amplitudes in ISchR on the light front which can be obtained in limit $\vec{\lambda} \to \infty$, are the invariantly represented amplitudes of the old fashioned perturbation theory in IMF and can be transformed to the form of the latters by means of simple replacement of variables. This transformation in explicit form is demonstated in the paper [4].

In the limit $\vec{\lambda} \to \infty$ we can neglect the difference between $|\vec{\lambda}|$ and λ_0 and replace $\lambda \to \omega$. After that we obtain:

$$F_a = \frac{g^2}{(2\pi)^2}\frac{\bar{u}(p')\gamma_5(\hat{k}+\hat{p}+\hat{\omega}\tau+m)\gamma_5 u(p)}{m^2-(k+p)^2} \tag{51}$$

$$F_b = \frac{g^2}{(2\pi)^2}\frac{\bar{u}(p')\gamma_5\hat{\omega}\gamma_5 u(p)}{2(p+k)\omega} \tag{52}$$

where $\tau = \dfrac{m^2-(k+p)^2}{2(p+k)\omega}$

So, in the case of the particles with spin the vacuum diagrams contribution (e.g., the diagram Fig.2b) does not equal to zero on the light front. This property in the framework of the old fashioned perturbation theory in IMF has been noted in Refs. [14, 15]. In order to obtain the expression for amplitudes on the light front directly, without preliminary consideration of a theory on the plane $\lambda\chi = 0$, it is necessary to modify the diagram technique rules cited above. *) The diagram technique in the framework of the old fashioned perturbation theory in IMF has been considered in Refs. [14, 15]. Keeping in mind the application of the formalism developed to the nuclear physics problems we restrict ourself by consideration of the mechanisms not including the vacuum diagrams (and some other diagrams) which disappears on the light front in the case of the scalar particles. In order to calculate the amplitudes of these mechanism it is enough to replace $\lambda \to \omega$ in the rules represented above.

The vertices Γ having the same graph representation as WFs (see Fig.1) is connected with WF by means of the formula [1]: $\Psi = \sqrt{\frac{2\pi}{m}}\,\frac{\Gamma}{s_1 - M^2}$. In the expressions for the amplitudes calculated according to the rules (1-6) one should substitute the vertices represented in the bispinor formalism, i.e. represented through the γ - matrices. So, for the deuteron vertex Γ after expressing Γ through WF one should substitute the expression contained in Eq.(46) between the spinors. For going over to WF $\Psi^{\mu}_{\sigma_1\sigma_2}$ we note that the "spin parts" of the propagators, corresponding to a particle and antiparticle can be written in the form:

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*) After completion of this paper I learned about the paper [16] in which the rules of the invariant diagram technique on the light front for the particles with spin have been formulated.

$$(\hat{p}+m)_{\beta\beta'}=\sum_{\sigma=\pm\frac{1}{2}}\bar{u}^{\sigma}_{\beta'}(p)\,u^{\sigma}_{\beta}(p) \tag{53}$$

$$(\hat{p}-m)_{\beta\beta'}=\sum_{\sigma=\pm\frac{1}{2}}V^{\sigma}_{\beta}(p)\,\bar{V}^{\sigma}_{\beta'}(p) \tag{54}$$

Let us remind that because of the presence of the delta-functions $\delta(p^2-m^2)$ in propagators the four- momentum p in Eqs.(53) and (54) lies on the mass shell and $p_0 > 0$.

The spinors contained in the propagators and the spinors corresponding to the extertal lines with the vertices give the matrix elements $\Gamma^{\mu_1\ldots\mu_m}_{\sigma_1\ldots\sigma_n}$ depending on the spin indices, corresponding to the spin projection on the z - axis in the rest systems. One should summarize over the spin indices of the internal particles. Consequently, in this representation the spin parts of the propagators are the unit matrices, In this form the rules of the diagram technique are available to the case of particles with arbitrary spin.

Let us illustrate the stated above by the example of calculation of the elastic $p\alpha$- scattering cross section in the framework of the one-nucleon-exchange mechanism and demonstrate the methods of calculation of the cross sections with spin taken into account. The diagrams corresponding to the one nucleon exchange model are shown in Fig. 3. This process in the framework of ISchR has been considered in Ref. [17], where the qualitative sequences of the deuteron WF dependence on the variable k was studied. The following expression for the cross section (without spin) has been obtained:

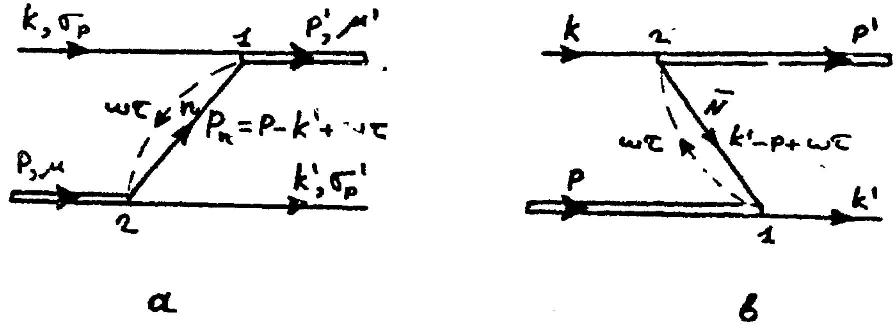

Fig. 3

$$\frac{d\sigma}{d\Omega} = \frac{m^2(m^2-u)^2}{64\pi^2 s(1-\beta)^4}\,\Psi^4(\vec{q}^{\,2}, \vec{n}\vec{q}) \tag{55}$$

where $s = (p+k)^2, \quad u = (p-k')^2,$

$$\beta = \frac{\mathcal{E}_p(\vec{k}) - |\vec{k}|\cos\frac{\theta}{2}}{\mathcal{E}_d(\vec{k}) + |\vec{k}|\cos\frac{\theta}{2}} \tag{55a}$$

$$\vec{q}^{\,2} = \frac{m^2-u}{4(1-\beta)} + \frac{M^2}{4} - m^2 \tag{55b}$$

$$\vec{n}\vec{q} = \mathcal{E}(\vec{q})(1-2\beta) \tag{55c}$$

$\mathcal{E}(\vec{q}) = \sqrt{\vec{q}^{\,2}+m^2}$, m and M are the nucleon and deuteron masses, $|\vec{k}|$ and θ are the momentum and the scattering angle in the center-of-mass system of the reaction.

The formula (55) but without $\vec{n}$ - dependence of the deuteron WF taken into account was also obtained by Kondratyuk and Schevchenko [18].

In the case of particles with spin the contribution of the diagram Fig.3b does not equal to zero. However this diagram contains the vertices corresponding to probability of the state $d\bar{N}$ in a nucleon. This allows to neglect the diagram Fig.3b. In the case of particles with spin the product of the vertex functions $\Gamma_1 \Gamma_2$, corresponding to the vertices 1 and 2 in Fig.3a turns into:

$$\bar{u}_{\alpha}^{\mu'}(p')\,\Gamma_{\beta\gamma}^{\alpha}(1)\,u_{\beta}^{\sigma_p}(k)\,(\hat{p}_n+m)_{\gamma\gamma'}\,\bar{u}_{\beta'}^{\sigma_p'}(k')\,\Gamma_{\beta'\gamma'}^{\alpha'}(2)\,u_{\alpha'}^{\mu}(p)$$

where $p_n = p - k' + \omega\tau$ is the neutron four- momentum. Having represented $(\hat{p}_n + m)$ in the form Eq.(53) and connected Γ with WF we obtain that Ψ^2 in the amplitude for the spinless particles turns into:

$$\sum_{\sigma_n} \Psi^{T\mu'}_{\sigma_p\sigma_n}(1)\,\Psi^{\mu}_{\sigma_p'\sigma_n}(2) \tag{56}$$

The sum over σ_n in Eq.(56) means that the propagator in the representation adopted is proportional to the unit matrix.

WF $\psi^{T\mu}_{\sigma_p\sigma_n}$ is connected with the vertex part reversed in time which describes the process $NN \to d$ and is expressed through $\psi^{\mu}_{\sigma_p\sigma_n}$ by means of the relation:

$$\psi^{T\mu}_{\sigma_p\sigma_n} = (-1)^{\mu+\sigma_p+\sigma_n}\,\psi^{-\mu}_{-\sigma_p,-\sigma_n} \tag{57}$$

The space components of the four- momenta on which WF depends, have opposite signs in right and left hand parts of Eq.(57).

Coming to the function $\tilde{\Psi}^{\mu}_{\sigma_p\sigma_n}$ (see Eq.(40)), which is determined by Eq.(44), suming over the spin projections and taking into account the orthogonality of D - functions, we obtain that the cross section is determined by the expression:

$$\bar{\Psi}^{4} = \frac{1}{6} \sum \tilde{\Psi}^{T\mu'}_{\sigma_p \sigma_n}(\vec{q}\,', \vec{n}\,') D^{\frac{1}{2}}_{\sigma_n \sigma_{n1}}\{R(L^{-1}(Q'), P_n)\} \times$$

$$\times D^{\frac{1}{2}}_{\sigma_{n2} \sigma_{n1}}\{R(L^{-1}(Q), P_n)\} \tilde{\Psi}^{\mu}_{\sigma_p' \sigma_{n2}}(\vec{q}, \vec{n}) \times$$

$$\times \tilde{\Psi}^{T\mu' *}_{\sigma_p \sigma_n'}(\vec{q}\,', \vec{n}\,') D^{\frac{1}{2} *}_{\sigma_n' \sigma_{n1}'}\{R(L^{-1}(Q'), P_n)\} \times$$

$$\times D^{\frac{1}{2}}_{\sigma_{n2}' \sigma_{n1}'}\{R(L^{-1}(Q), P_n)\} \tilde{\Psi}^{\mu *}_{\sigma_p' \sigma_{n2}'}(\vec{q}, \vec{n}) \quad (58)$$

where

$$Q = m(p + \omega\tau)/\sqrt{s_1}\ , \quad Q' = m(p' + \omega\tau)/\sqrt{s_1'}\ ,$$

$$\vec{q} = L^{-1}(Q)\vec{P}_n\ , \quad \vec{q}\,' = L^{-1}(Q')\vec{P}_n\ ,$$

$$\vec{n} = \sqrt{s_1}\, L^{-1}(Q)\vec{\omega}/(\omega p)\ , \quad \vec{n}\,' = \sqrt{s_1'}\, L^{-1}(Q')\vec{\omega}/(\omega p'),$$

$$s_1 = (p + \omega\tau)^2, \quad s_1' = (p' + \omega\tau)^2.$$

At deriving Eq.(58) we took into account that $D_{\sigma, -\sigma'} = (-1)^{\sigma + \sigma'} D^{*}_{-\sigma, \sigma'}$. Using Eq.(57) and relation (40) between Ψ and φ one can easy obtain that $\tilde{\Psi}^{T\mu}_{\sigma_p \sigma_n}$ in Eq.(58) is determined by Eq.(44) but with noncojugated spherical functions. It is convenient to calculate expression (58) in the system where $\vec{P}_n = 0$. In this system the D - functions in Eq.(58) turn into the unit matrices. This can be seen, for example, from Eq.(32).

The expression $\sum \tilde{\Psi}^{\mu *}_{\sigma_p \sigma_n'} \tilde{\Psi}^{\mu}_{\sigma_p \sigma_n}$, which arises in $\bar{\Psi}^{4}$, can be represented in the form:

$$\sum \tilde{\Psi}^{\mu *}_{\sigma_p \sigma_n'}(\vec{q}, \vec{n}) \tilde{\Psi}^{\mu}_{\sigma_p \sigma_n}(\vec{q}, \vec{n}) =$$

$$= A\,\delta_{\sigma_n \sigma_n'} + B\,(\vec{\sigma}[\vec{n} \times \vec{q}\,])_{\sigma_n' \sigma_n} \quad (59)$$

The coefficients A and B are the real functions. Their expressions through $G_1, \dots, G_6$ are given in Appendix. Summation in Eq.(58) is reduced to calculation of the traces of the Pauli matrices. This gives:

$$\bar{\Psi}^4 = \frac{1}{3}\left\{AA' + BB'\left[(\vec{n}\vec{n}')(\vec{q}\vec{q}') - (\vec{n}\vec{q}')(\vec{n}'\vec{q})\right]\right\} \quad (60)$$

where A, B and A', B' depend $\vec{n}, \vec{q}$ and $\vec{n}', \vec{q}'$ correspondingly.

The expression (60) depends on four- vector ω (by means of variables $\vec{q}, \vec{n}, \vec{q}', \vec{n}'$). In Refs. [4, 17] it has been shown that the four- vector ω should be chosen from the condition: $y = y' = \beta$, where $y = \omega k/\omega p'$, $y' = \omega k'/\omega p$, β is given by Eq.(55a). At $y = y'$ the expression $(\vec{n}\vec{n}')(\vec{q}\vec{q}') - (\vec{n}'\vec{q})(\vec{n}\vec{q}')$ is transformed to the form: $\vec{\omega}^2(\vec{Q}\vec{Q}') - (\vec{\omega}\vec{Q})(\vec{\omega}\vec{Q}')$. One can show this using explicit expressions of the variables $\vec{q}, \vec{n}, \vec{q}', \vec{n}'$ through P_n , ω, Q, Q' (see Eq.(8)). Having represented this expression in the invariant form by means of relations $\vec{\omega}^2 = \frac{1}{m^2}(\omega P_n)^2$, $\vec{Q}\vec{Q}' = \frac{1}{m^2}(QP_n)(Q'P_n) - (QQ')$, etc., and taking into account that at $y = y'$ we have $\vec{q}^2 = \vec{q}'^2$, $\vec{n}\vec{q} = \vec{n}'\vec{q}'$, we obtain:

$$\bar{\Psi}^4 = \frac{1}{3}\Big\{A^2(\vec{q}^2, \vec{n}\vec{q}) + B^2(\vec{q}^2, \vec{n}\vec{q})\Big[\beta(1-\beta)m^2 - (1-\beta)M^2 + \frac{1}{2}(1-\beta)^2 S + \frac{1}{2}(1+\beta^2)u\Big]\Big\} \quad (61)$$

Expressions of variables $\vec{q}^2, \vec{n}\vec{q}$ in terms of the energy and the scattering angle are given by Eqs.(55b) and (55c). In nonrelativistic limit the formula (61) turns into the well known expression:

$$\bar{\Psi}^4 = \frac{3}{4}\left(\psi_S^2(\vec{q}) + \psi_D^2(\vec{q})\right)^2$$

where ψ_S, ψ_D are S- and D - wave functions, $\psi_S = G_1$, $\psi_D = G_5/4\pi$. The normalization is follows:

$$\int\left(\psi_S^2(\vec{q}) + \psi_D^2(\vec{q})\right)\frac{d^3 q}{(2\pi)^3} = 1$$

Taking into account of the dependence of $\bar{\Psi}^4$ on $\vec{n}\vec{q}$ results in the qualitative change of behaviour of the pd - scattering cross section v.s. the scattering angle at the angles closed to 180° [16].

5. Conclusion

The construction of WFs with spin on the light front carried out above which is necessary for study of nuclei in the relativistic region solves also the problem of taking into account the spins and angular momenta in the parton WFs. This construction is carried out in terms of other variables than those which are conventially used for parametrization of the parton WFs ($\vec{k}_\perp$ and x). Although one can go over from the variables $\vec{q}, \vec{n}$ to $\vec{k}_\perp, x$, but often pratically there is no necessity in that, since the expressions for amplitudes in terms of invariant variables s and t are obtained by more direct way namely in the framework of the invariant formalism considered above. Thus, in the case of pd - csattering the variables $\vec{q}^{\,2}, \vec{n}\vec{q}$ are simply expressed through the energy and the scattering angle (see Eqs. (55a-c)).

Additional spin structures in relativistic WFs, conneoted with the WF dependence on $\vec{n}$, can result in the qualitative polarization phenomena in nuclear reactions with high momentum transfer. To point out the experiments, crucial relative to relativistic WF $\vec{n}$ dependence, to find and investigate experimentally its character is of considerable interest.

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Appendix

The coefficients A and B, determining the cross section of pd - scattering according to Eq.(61) have the form:

$$A(\vec{q}^{\,2},\vec{n}\vec{q}) = \frac{1}{2}\sum \tilde{\Psi}^{\mu}_{\sigma_p\sigma_n}\tilde{\Psi}^{\mu *}_{\sigma_p\sigma_n} =$$

$$= \frac{3}{2^5\pi^2}\sum_{jL\lambda\lambda' l} f(j,L,\lambda)\, f^*(j,L,\lambda')\times$$

$$\times(-1)^{L+\lambda+\lambda'} N_{r+\lambda} N_{r-\lambda} N_{r+\lambda'} N_{r-\lambda'}\times$$

$$\times C^{l0}_{r-\lambda,0,r-\lambda',0}\, C^{l0}_{r+\lambda,0,r+\lambda',0}\begin{Bmatrix} r-\lambda' & l & r-\lambda \\ r+\lambda & L & r+\lambda' \end{Bmatrix} P_l(\cos\widehat{\vec{n}\vec{q}}) \tag{62}$$

$$(\vec{q}^{\,2}-(\vec{n}\vec{q})^2)\,B(\vec{q}^{\,2},\vec{n}\vec{q}) = i\frac{4\pi}{\sqrt{6}}|\vec{q}|\sum \tilde{\Psi}^{\mu}_{\sigma_p\sigma_n}\tilde{\Psi}^{\mu *}_{\sigma_p\sigma_n'}\times$$

$$\times C^{\frac{1}{2}\sigma_n'}_{\frac{1}{2}\sigma_n 1m}\, C^{1m}_{1m_1 1m_2}\, Y_{1m_1}\!\left(\frac{\vec{q}}{|\vec{q}|}\right) Y_{1m_2}(\vec{n}) =$$

$$= i\frac{3^2}{2^4\pi^2}|\vec{q}|\sum f(j,L,\lambda)\, f^*(j',L',\lambda')(-1)^{j+j'+L+a'}\times$$

$$\times N_j N_{j'} N_L N_{L'} N_{r+\lambda} N_{r'+\lambda'} N_{r-\lambda} N_{r'-\lambda'} N_a N_{a'}\times$$

$$\times C^{a0}_{r-\lambda,0,r'-\lambda',0}\, C^{a'0}_{r+\lambda,0,r'+\lambda',0}\begin{Bmatrix} 1 & j & j' \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix}\begin{Bmatrix} L & L' & 1 \\ j' & j & 1 \end{Bmatrix}\times$$

$$\times\begin{Bmatrix} a' & r+\lambda & r'+\lambda' \\ 1 & L & L' \\ a & r-\lambda & r'-\lambda' \end{Bmatrix}\begin{Bmatrix} 1 & 1 & 1 \\ a' & a & l \end{Bmatrix} C^{l0}_{a010}\, C^{l0}_{a'010}\, P_l(\cos\widehat{\vec{n}\vec{q}}) \tag{63}$$

where

$$N_j = (2j+1)^{\frac{1}{2}}\ ,\quad N_{r+\lambda} = [2(r+\lambda)+1]^{\frac{1}{2}},\ \ldots$$

The functions $f(j, L, \lambda)$ in Eqs.(62), (63) are related with the functions G_i in Eq.(44) as follows:

$$G_1 = f(1,0,0)/4\pi\,, \quad G_2 = f(1,1,0),$$

$$G_{4+\lambda} = f(1,2,\lambda) \;(\lambda = -1, 0, +1)\,, \quad G_6 = f(0,1,0)$$

In order to derive the formulae (62), (63) it is convenient to use the graphic methods of summation of the Clebsch-Gordan coefficients [19].

From Eqs.(62), (63) one can easy see that A and B are real functions.

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