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ON THE SCALING OF MAGNETIC PLASMA CONFINEMENT UNDER CLASSICAL CONDITIONS

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ABSTRACT

Present magnetic confinement schemes based on tokamaks and similar devices are characterized by relatively large losses and low beta values. As a consequence, thermonuclear conditions can only be reached in such devices at large linear dimensions or by means of very strong magnetic fields, in combination with large heating powers. This does not rule out the possibility of realizing the same conditions on a smaller scale, i.e. by finding alternative schemes which provide classical and stable confinement of a pure plasma in a closed magnetic bottle.

1. Introduction

Extensive theoretical and experimental investigations have so far been made on the plasma balance of several magnetic confinement schemes, with special emphasis on tokamaks and similar devices. The scaling laws obtained for the latter indicate that large linear dimensions and considerable heating powers are required to reach the parameter regime of thermonuclear conditions.

This paper presents a simple discussion on the idealized case of classical plasma confinement in a closed magnetic bottle, i.e. at the minimized plasma losses which would prevail if such a scheme could be realized. No suggestion is made here for specific field geometries which could meet these requirements. Reactor technological aspects are excluded from the present discussion.

2. The Plasma Balance

The present analysis is performed as an order-of-magnitude calculation based on the following simplified classical model:

(i) A fully ionized pure hydrogen plasma with A=1 is considered. The spatial mean values of particle, density, and temperature over the plasma body are denoted by n and $T \equiv (T_i + T_e)/2$.

(ii) The plasma is classically and stably confined in a closed bottle by a magnetic field, the strength of which has the average modulus B. The field is strong enough for the gyro frequencies to become much larger than the collision frequencies of each particle species.

(iii) The transverse average "radius" of the plasma body, in the direction being perpendicular to its boundary surface or surfaces, is denoted by a.

(iv) Plasma-neutral gas interaction is neglected within the hot central parts of the plasma body being considered here. Thus, the ion density n is chosen large enough for the plasma to become "impermeable" to neutral gas, i.e. na has to exceed 5×10^{18} m^{-2} by a substantial factor [1]. Possibly the present estimates should also apply to "permeable" plasmas in cases where the neutral gas flow from surrounding walls has a negligible influence on the plasma heat balance.

(v) For temperatures of practical interest in this connection, the beta value should be kept below a certain maximum limit, i.e. $\beta = 4\mu_0 nkT/B^2 \leq \beta_{max}$.

The plasma balance is described by a steady one-fluid model for which

$$\underline{j} \times \underline{B} = \underline{\nabla} p \qquad (1)$$

$$\eta \underline{j} = \underline{E} + \underline{v} \times \underline{B} - (1/en)\underline{j} \times \underline{B} - (1/2en)\underline{\nabla} p + $$

$$+ (3nk\eta/2B^2)\underline{B} \times \underline{\nabla} T \qquad (2)$$

$$\frac{3}{2}\,\mathrm{div}(p\underline{v}) + p\,\mathrm{div}\underline{v} + \mathrm{div}\underline{q} + \Pi_r = \eta \underline{j}^2 + \Pi_o \qquad (3)$$

where $p = 2nkT$, $\underline{q}$ is the heat flow vector, Π_r the radiation loss, Π_o an externally applied heating power, and the rest of the symbols have their conventional meaning. SI-units are used throughout this paper.

2.1. Particle Losses

Solving for $\underline{v}_\perp \equiv \underline{B} \times (\underline{v} \times \underline{B})/B^2$ in eqs. (1) and (2), and assuming ∇n, ∇T, and $\underline{E}$ to be perpendicular to the boundary surfaces of the plasma body, the power loss due to particle diffusion across the field $\underline{B}$ can be represented by

$$\Pi_\eta \equiv \left|\frac{3}{2}\, \mathrm{div}(p\underline{v}_{\perp\eta})\right| = 3 f_\eta c_\eta n^2 k^2 \sqrt{T} k_\eta / 2B^2 a^2 \qquad (4)$$

where $\underline{v}_{\perp\eta}$ is the corresponding diffusion velocity, f_η is a coefficient of order unity being related to the spatial profile of $\mathrm{div}(p\underline{v}_{\perp\eta})$, c_η is a coefficient representing in a semi-empirical way possibly existing anomalous diffusion losses, and $k_\eta = 129(\ln\Lambda)$ is related to the resistivity η and the associated ratio Λ between the Debye distance and the impact parameter [2].

In addition, the term $p\,\mathrm{div}\underline{v}$ in eq. (3) gives rise to a compression or expansion work. For certain steady cases this work is cancelled by the heating power $\eta j_\perp^2$ due to the component $j_\perp$ of the current density which balances the pressure gradient of eq. (1). In the present simple estimate of orders of magnitude we neglect $p\,\mathrm{div}\underline{v} - \eta j_\perp^2$ in eq. (3).

It should also be mentioned that discharges may exist in which an electric field runs parallel to the plasma boundary, in such a way that the corresponding $\underline{E} \times \underline{B}$ drift fully or partly compensates the diffusion velocity $\underline{v}_{\perp\eta}$. Such a situation is excluded here and does, in any case, not rule out the heat conduction losses.

2.2. Heat Conduction Losses

For the heat conduction across $\underline{B}$ we write

$$\Pi_\lambda \equiv |\mathrm{div}\underline{q}| \simeq |\mathrm{div}(\lambda^{*}\underline{\nabla}_{\perp} T)| \simeq$$

$$\simeq f_\lambda c_\lambda k_\lambda n^2 (\ln\Lambda)\sqrt{T}/B^2 a^2 \qquad (5)$$

where λ^{*} stands for the transverse heat conductivity in a strong magnetic field with $k_\lambda = 1.5 \times 10^{-42}$ according to Spitzer [2], and f_λ and c_λ are the corresponding "profile" and "anomalous" coefficients. From eqs. (4) and (5) the total power loss due to plasma transport now becomes

$$\Pi_p = \Pi_\eta + \Pi_\lambda \equiv f_p c_p k_\lambda (\ln\Lambda) n^2 \sqrt{T}/B^2 a^2 \qquad (6)$$

where

$$f_p c_p = f_\lambda c_\lambda [1 + (6m_e \nu_{ei}/5m_i \nu_{ii})] \qquad (7)$$

ν_{ei} and ν_{ii} are the electron-ion and ion-ion collision frequencies, and $6m_e\nu_{ei}/5m_i\nu_{ii} \simeq 5 \times 10^{-2}$.

2.3. Radiation Losses

For the radiation losses we write [3]

$$\Pi_r = \Pi_b + \Pi_c \equiv f_b c_b k_b n^2 \sqrt{T} + f_c c_c k_c n B^2 T \equiv$$

$$\equiv f_b c_b k_b n^2 \sqrt{T}[1 + k_{cb}(B^2\sqrt{T}/n)] \qquad (8)$$

where $k_b = 1.7 \times 10^{-40}$ and $k_c = 8 \times 10^{-24}$ represent bremsstrahlung and cyclotron radiation. The coefficients c_b and c_c stand for contributions from impurities and other possible enhancements of radiation. In addition, it is also possible that $c_c < 1$ in cases where reabsorption of cyclotron radiation becomes important [3,4].

3. Heating Power and Energy Containment Time

Assuming a closed bottle with the equivalent major radius R and plasma volume $V = 2\pi^2 Ra^2$, the total power input of eqs. (3), (6), and (8) becomes

$$P = 2\pi^2 Rn^2\sqrt{T}\{C_p\left[(\ell n\Lambda)/B^2\right] + C_r a^2\left[1 + k_{cb}(B^2\sqrt{T}/n)\right]\} \quad (9)$$

where $C_p = f_p c_p k$ and $C_r = f_b c_b k_b$. This corresponds to the energy containment time τ_E as given by

$$n\tau_E = \frac{3f_V kB^2a^2\sqrt{T}/C_p(\ell n\Lambda)}{1 + (C_r B^2 a^2/C_p \ell n\Lambda)(1 + k_{cb}B^2\sqrt{T}/n)} \quad (10)$$

where f_V is the "profile coefficient" of nT.

4. Numerical Example

As a numerical illustration of classical confinement in a closed bottle of small size at a moderately large magnetic field strength, we now choose $\beta_{max} = 0.5$, $a = 0.05$ m, $R = 0.15$ m, $B = 2.5$ tesla, and $n = 5 \times 10^{20}$ m^{-3}. Further the values $c_p = 1$, $c_b = 1$, $f_p = f_b = f_V = f_c = 0.5$, and $\ln\Lambda = 10$ are adopted, as well as $c_c = 0.1$ in a case of reflecting walls with 90% plasma reabsorption of cyclotron radiation [4]. The parameter $n\tau_E$ then behaves as shown by Fig.1. Here the full line represents the case $c_p = 1$ of purely classical confinement, and the broken lines simulate increased losses of the same bottle as given by $c_p > 1$.

For comparison the corresponding data of some recent large tokamak experiments [5-11] have further been included, as given by the dots in Fig.1 and listed in Table 1. In the largest of these experiments the volume V, the quantity aB, and the magnetic energy $B^2V/2\mu_o$ exceed the corresponding values of the present small idealized experiment by about two, one and three orders of magnitude, respectively. Equivalent values of c_p have been estimated from eq. (10) and are given in Table 1 as well as within brackets in Fig.1. These values can be taken as a crude measure of the enhancement of the energy losses due to plasma transport, as compared to those which would prevail under idealized classical conditions. In particular, the highest so far achieved temperature with the PLT device has been obtained at a relatively low ion density and with energy losses being about five orders of magnitude larger than those of an idealized classical case. On the other hand, the corresponding enhancement of losses in Alcator becomes limited to somewhat more than two orders of magnitude, on account of the relatively high ion density and the increase of confinement time with density.

Finally, at temperatures in the range $10^7 < T < 10^8$K, the power losses given by eq. (9) would become about 10^4W for the present idealized small experiment, whereas the power loss of the tokamak devices listed in Table 1 ranges from 5×10^5 to 5×10^6 W.

5. Conclusions

On account of the comparatively large plasma losses and low beta values of present tokamaks and similar devices, thermonuclear conditions can be reached in these devices only at large linear dimensions or by means of very strong magnetic fields, in combination with heating powers of considerable magnitude. However, this does not rule out the possibility of alternative schemes for which the same conditions could be realized more efficiently and on a smaller scale. In particular, if a closed confinement scheme could be found which is stable at high beta values and where a pure plasma can be made to obey the laws of classical transport, thermonuclear conditions should also be available in relatively small devices and by means of moderately large technical resources. Even a scheme being less efficient than that of an idealized case, but being substantially better than those so far achieved with tokamaks and similar devices, should have a decisive importance.

The great potentialities of fusion energy are obvious both with respect to its energy resources and environmental properties, whereas several now existing programmes on fusion research may have to be reconsidered. Needless to say, serious efforts to find new and more powerful confinement schemes are therefore strongly justified at the present situation.

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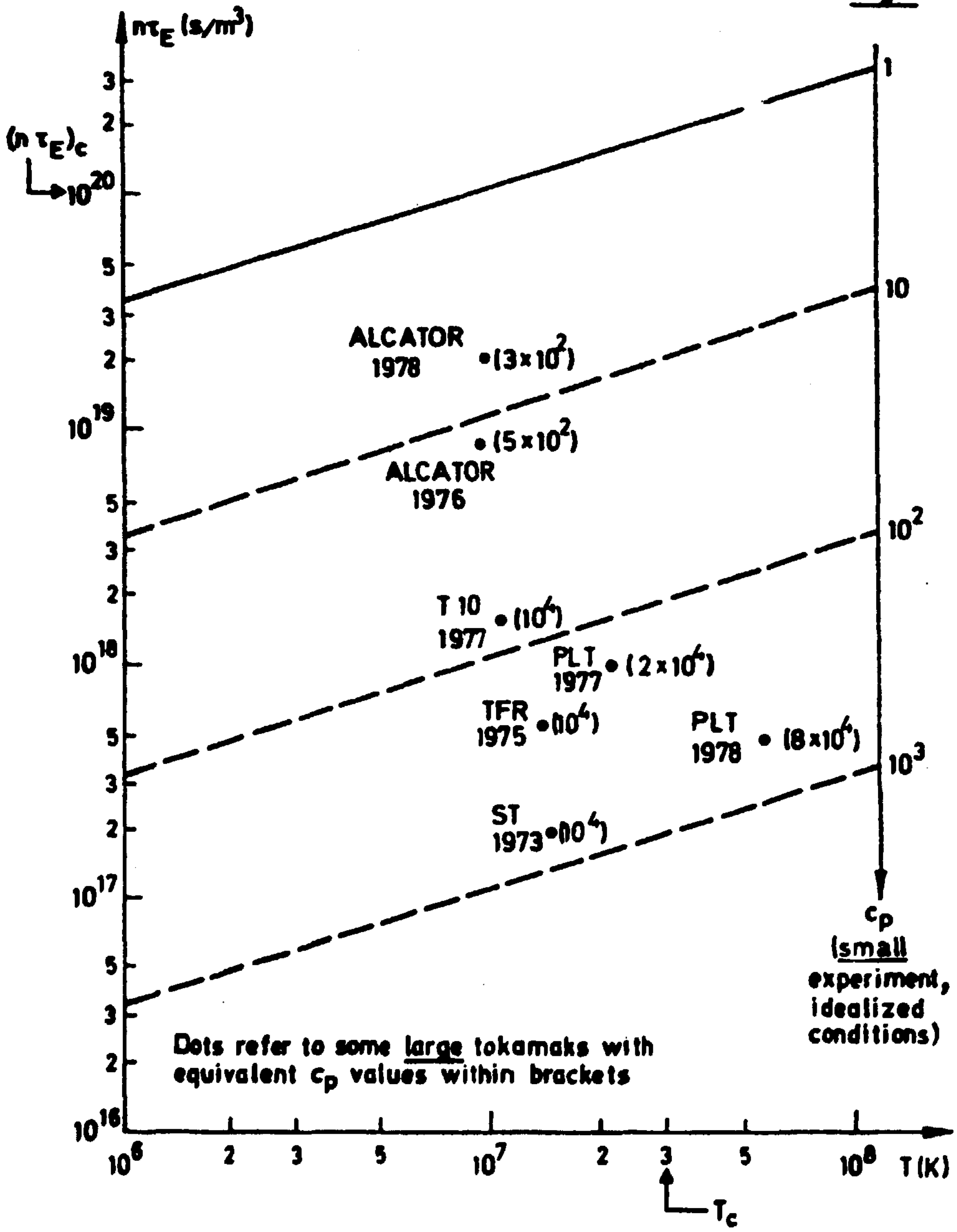
Table 1. Basic data of experiments plotted in Fig. 1

Experiment	R (m)	a (m)	B (tesla)	n (m^{-3})	τ_E (ms)	T (K)	V (m^3)	aB (Vs/m)	c_p (estimated)
ST [5] 1973	1.09	0.13	3.7	2.4×10^{19}	7.5	1.5×10^{7}	0.36	0.481	10^{4}
TFR [6] 1975	0.98	0.20	4.0	2.5×10^{19}	20.5	1.4×10^{7}	0.77	0.800	10^{4}
ALCATOR [7] 1976	0.54	0.095	7.5	4.0×10^{20}	20	0.9×10^{7}	0.10	0.713	500
T 10 [8] 1977	1.50	0.37	3.5	3.7×10^{19}	40	1.2×10^{7}	4.05	1.295	10^{4}
PLT [9] 1977	1.30	0.40	3.5	2.6×10^{19}	40	2.0×10^{7}	4.10	1.400	2×10^{4}
ALCATOR [10] 1978	0.54	0.095	9.0	7.5×10^{20}	25	10^{7}	0.10	0.855	300
PLT [11] 1978	1.30	0.40	3.5	2.0×10^{19}	25	5×10^{7}	4.10	1.400	8×10^{4}
Small idealized experiment	0.15	0.05	2.5	5×10^{20}	see Fig.1	see Fig.1	0.0074	0.125	1

Figure Caption

Fig.1. Values of $n\tau_E$ as a function of T are given by full line for a small experiment with a = 0.05 m, B = 2.5 tesla, and n = 5 x 10^{20} m^{-3}, under the idealized conditions of classical and stable confinement ($c_p \approx 1$). Dots represent some recent large tokamak experiments listed in Table 1 for which estimated equivalent values of c_p are given within brackets. The approximate positions of the "ignition temperature" and the limit of the Lawson criterion for a deuterium-tritium mixture are indicated by T_c and $(n\tau_E)_c$ in the figure.



$n\tau_E$ (s/m³)
$(n\tau_E)_c$
ALCATOR 1978 (3×10²)
ALCATOR 1976 (5×10²)
T 10 1977 (10⁴)
PLT 1977 (2×10⁴)
TFR 1975 (10⁴)
PLT 1978 (8×10⁴)
ST 1973 (10⁴)
1
10
10²
10³
c_p (small experiment, idealized conditions)
Dots refer to some large tokamaks with equivalent c_p values within brackets
T (K)
T_c

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Key words: Magnetic bottles, transport, scaling laws.