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PION DEUTERON SCATTERING AT INTERMEDIATE ENERGIES

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ABSTRACT. A comparison is made of results of calculations of πd elastic scattering cross section using multiple scattering and three-body equations, in relation to their ability to reproduce the experimental data at intermediate energies. It is shown that the two methods of theoretical calculation give quite similar curves for the elastic differential cross sections, and that both fail in reproducing backward scattering data above 200 MeV. The new accurate experimental data on πd total cross section as a function of the energy are confronted with the theoretical values obtained from the multiple scattering calculation through the optical theorem. Comparison is made between the values of the real part of the forward amplitude evaluated using dispersion relations and using the multiple scattering method.

RESUMO. Comparamos os resultados dos cálculos de seção de choque para espalhamento elástico πd, tendo em vista sua capacidade para reproduzir os dados experimentais a energias intermediárias. Mostramos que os dois métodos de cálculo teórico produzem curvas muito semelhantes para a seção de choque diferencial elástico, e que ambos falham na descrição de espalhamento a grandes ângulos em energias acima de 200 MeV. Os dados experimentais recentes em seção de choque total πd como função de energia são confrontados com os valores teóricos obtidos no cálculo de espalhamento múltiplo através do teorema ótico. Fazemos também a comparação entre os valores da parte real da amplitude para a frente calculados usando relações de dispersão e usando o método de espalhamento múltiplo.

1. INTRODUCTION

Scattering and absorption of mesons by deuterons are the simplest non-elementary processes available for studies in strong interaction physics. Here we are obviously restricting ourselves to low and intermediate energies, where hadrons can be considered (at least in the phenomenological level) as structureless objects. These processes are important to particle physics, because the deuteron is the simplest, and hopefully a treatable, system containing neutrons. They are also of interest to nuclear physics, because they set the natural test-ground for ideas and methods which are to be applied in the study of the interaction of particles with nuclear matter. A third important reason for the interest in hadron-deuteron processes is the study of the fundamental three-body problem in quantum mechanics. These varied lines of motivation have given a long standing importance to the physics of hadron-deuteron interactions, which is over twenty years old.[1-5]

From the theoretical point of view, pion-deuteron scattering was the first, and has been the most, extensively studied among the processes in which deuterium is used as a target. The reason for this concentration of effort rests in the fact that, given the charge independence of the strong interaction, the basic ingredients of all two-body forces needed to describe pion-deuteron as a three-body system (in a non-relativistic limit) can be obtained from other two-body experiments. However, measurements of pion-deuteron scattering have not been made, during these twenty years, with the desired comprehensiveness and accuracy. The rate at which experimental data are becoming available has now been increased, as a consequence of the operation of accelerators which produce medium energy pion beams of high intensity (these are the so-called meson factories - LAMPF in California, USA; TRIUMF in Vancouver, Canada; and S.I.N. in Villigen, Switzerland). We refer to these data later.

On the other hand, several accurate kaon-deuteron experiments in the intermediate energy range[6] have been made during the last twenty years. This experimental effort is justified, as one is forced to make use of the K^+d interaction to obtain information on the isotopic spin $I = 0$ state of the kaon-nucleon system.[7,8] However almost all applications of the recent theoretical development in the theory of three-body systems have been directed to the πd system. The reason for this diversification is in that the development of theoretical methods is still in a preliminary stage, and the applications are made as a test on its validity, which requires that the basic knowledge of the two-body interactions be given as an input, and not obtained as an output, of the calculations. Actually, as we shall see later, no theoretical calculation has up to now given a satisfactory description of πd scattering in the intermediate energy range.

In what follows we shall be concerned mainly will pion deuteron elastic scattering, as this process has received most attention in the theoretical work made up to now. In sections 2 and 3 we discuss briefly the two main theoretical methods which have been used at intermediate energies, namely the multiple scattering expansion and the solution of three-body equations of Faddeev type. We review the results of different calculations, showing how they compare among themselves and with the experimental data. In Sec. 4 we discuss the problem of the evaluation of the elastic differential across section at large angles, which has up to now resisted all attempts of solution. In Sec. 5 we describe the existing data, and mention experiments now in progress which will provide more data in a near future. Finally in Sec. 6 we report on the results of an analysis of the angular momentum states contributing to πd scattering at the energy of the πN resonance.

2. THE MULTIPLE SCATTERING METHOD

The multiple scattering method, based in the impulse approximation introduced twenty five years ago by Chew and Goldberger[9], is the classical framework for the description of particle-nucleus interaction. The first applications of the method to pion-deuteron scattering were made in the fifties[4,5], and since then a number of calculations of the same sort followed, varying in details and in sophistication. The availability of large computers has made it possible to include in the calculations double scattering contributions and fermi-motion effects in the evaluation of meson-nucleon amplitudes, and to test the influence of different kinds of deuteron wave-functions, of kinematical ambiguities and of the off-energy-shell extrapolation of the meson-nucleon amplitudes.

Starting from the Faddeev equations, the three-body transition matrix $T(z)$ for the scattering of particle 1 by the bound pair of particles 2 and 3 can be expanded in the form of a multiple scattering series

$$T(z) = t_2(z) + t_3(z) + t_2(z)g_0(z)t_3(z) + t_3(z)g_0(z)t_2(z) + \dots \quad (1)$$

where $t_2(z)$ represents the scattering of particle 1 by particle 3 (in the presence of the spectator particle 2), $t_3(z)$ represents the interaction of particle 1 with particle 2 (in the presence of particle 3 as an spectator), and $g_0(z) = (z - H_0)^{-1}$ is the resolvent for three free particles, all operators being defined in the three-particle Hilbert space. The terms of the above expansion are obviously interpreted as representing single and double scattering of the incident particle. The evaluation of the terms of the series as matrix elements between three-particle states is then made through their reduction to matrix elements of two-body operators. There appears a shift in the argument of the operators, corresponding to the energy of the particle which is an spectator in each collision.[10] Thus, let E be the value of the total pion-deuteron kinetic energy in the centre-of-mass system, let $\vec{p}_1$, $\vec{p}_2$ and $\vec{p}_3$ be the initial momenta of the three particles and $\vec{p}_1'$, $\vec{p}_2'$ and $\vec{p}_3'$ their final values. The term of the series with particle 2 as an spectator then gives

$$\langle \vec{p}_1{}', \vec{p}_2{}', \vec{p}_3{}' | t_2(E) | \vec{p}_1, \vec{p}_2, \vec{p}_3 \rangle = \delta(\vec{q}_2{}' - \vec{q}_2)\,\delta(\vec{K}' - \vec{K})\,\langle \vec{k}_2{}' | t_2(E - \frac{\vec{q}_2^{\,2}}{2m_2}) | \vec{k}_2 \rangle \qquad (2)$$

where

$$\vec{K} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 \qquad (3)$$

is the total momentum of the system in the initial state (with $\vec{K}'$ for the final state),

$$\vec{q}_2 = [(m_1 + m_3)\vec{p}_2 - m_2(\vec{p}_1 + \vec{p}_3)]/(m_1 + m_2 + m_3) \qquad (4)$$

(and $\vec{q}_2{}'$) is the initial (final) momentum of the spectator with respect to the centre-of-mass of the whole system, and $\vec{k}_2$ (or $\vec{k}_2{}'$) given by

$$\vec{k}_2 = (m_3\vec{p}_1 - m_1\vec{p}_3)/(m_1 + m_3) \qquad (5)$$

is the initial (or final) momentum of the meson (particle 1) relative to the centre-of-mass of the interacting meson-nucleon (particles 1 and 3) system. In Eq. (2), t_2 is the usual two-body operator for the interaction of particles 1 and 3. The shift $(E - \vec{q}_2^{\,2}/2m_2)$ in the value of the energy at which this operator is evaluated is very important, and has been shown[10] to have a healthy effect in the results of multiple scattering calculations.

Recent calculations[10] applying the multiple scattering method to all existing data, from the lowest up to about 300 MeV pion incident energy, show the relative success and the limitations of the method. With account for single and double scattering contributions, effects due to nucleon motion, and presence of d-state in the deuteron wave-function, the calculations give a fairly good description of the elastic differential cross section data at 85[2], 142[11] and 182[12] MeV. Below 85 MeV the multiple scattering formalism does not provide an adequate method to describe πd scattering, due to the relatively large importance of the effects of the binding of the two nucleons. At these low energies a three-body calculation based on the solution of Faddeev integral equations provide the recommended framework. At 256 MeV there are discrepancies at large scattering angles (larger than 70^{o}) between the theoretical curves and the experimental points.[13] As we shall see later, all kinds of calculation of πd elastic scattering show these backward angle discrepancies above 200 MeV. Figs. 1, 2, 3 show the theoretical curves[10] at 142, 182 and 256 MeV, together with the experimental data.

The impulse approximation has also been applied successfully[14] to πd break-up reactions ($\pi d \rightarrow \pi pn$) at energies around and below the P33 resonance. Unfortunately the available experimental data at 85[2], 142[11], 182[12] and 224 MeV[15] are rather old, and not as accurate and complete as required for a test of the method of calculation. The values obtained for the cross sections are sensitive to the final state interaction of the two nucleons, particularly at the lowest energies. The results depend also on the treatment given to the kinematical ambiguities present in the problem, which shows that these break-up processes are a valuable tool for the understanding of the behaviour of three-body systems. More accurate experiments measuring πd break-up cross sections at intermediate energies are needed, including the observation of the energy spectrum of the emitted particles. The observation of the momentum distribution of the spectator particle, with proper account taken for the final state interaction effects, is an important instrument for the study of the deuteron wave-function.

3. THREE-BODY CALCULATIONS

The Faddeev equations[16] are the proper theoretical framework for the description of a non-relativistic three-body problem. Pion-deuteron scattering is a natural process where to apply these equations, and attempts have actually been made in this direction.[17] In these applications the Faddeev equations are separated in partial waves, and the two body (πN and NN) t matrices are parametrized in a separable form, so that one arrives at a set of coupled one-dimensional integral equations. These are then solved numerically. The data on πd differential cross section at 47.5 MeV[18] obtained at TRIUMF have been well described within this scheme, in a calculation[17] where the pion kinematics is given a relativistic form.

Very recently[19] the non-relativistic Faddeev equations were applied to describe the πd data at 142 MeV. The pion is treated with relativistic kinematics, and the calculation is very complete in the sense that it accounts for the d-wave part of the deuteron wave function, all s and p πN partial waves are included, the nucleon-nucleon interaction is taken into account, and the couplings between the ℓ = J+1 and ℓ' = J-1 channels of the πd system are not neglected. The fitting of the data is good, as shown in Fig. 1. The authors shown that the couplings of the ℓ = J+1 and ℓ' = J-1 channels give important contributions to backward angle scattering, and that the approximation of restricting the πN interaction to the P_{33} wave only has a strong and harmful effect in the quality of the fitting to the data.

New experiments, presumably of higher accuracy, with pions in the low energy region (from 20 to 65 MeV) are at present being performed at Saclay.[20] This experimental information will be important to test details in the application of non-relativistic Faddeev equations to the πd system.

The relativistic generalization of Faddeev equations[21] is obtained summing up ladder-type diagrams in which only three particles propagate, so as to obtain a set of equations which are similar in form to the Faddeev equations. A three-particle propagator is introduced which guarantees two- and three-particle unitarity, and the four dimensional integral equations are reduced to three-dimensional ones. The equations obtained are then amenable to numerical solutions based in the same techniques already developed for the non-relativistic Faddeev equations. Introducing a partial wave decomposition and assuming separable forms for all two-body amplitudes, the equations are reduced to a system of coupled one-dimensional integral equations. The ingredients of an specific calculation of the scattering cross section include the structure of deuteron wave-function, and assumptions on the off-energy-shell behaviour of the two-body amplitudes.

There are results of two independent calculations[22,23] of πd scattering in the intermediate energy region, with confrontation with the data at 142, 182 and 256 MeV, using these relativistic extensions of Faddeev equations. With the purpose of reducing the amount of involved numerical work, these calculations restrict the πN interaction to the P33 wave. This is a severe limitation at 142 and 256 MeV, which are not quite close to the energy of the P33 resonance. Actually the calculation at 142 MeV by Giraud et al[19] using non-relativistic Faddeev equations has shown that the πN waves other than the P33 wave affect remarkably the results. We can expect these contributions to be important also at 256 MeV. The effect of the nucleon-nucleon interaction in the three-body calculation is shown to be important at 142 MeV, but not so much at 182 and 256 MeV. The authors do not agree, however, on the strength of the effect, which reduces the backward cross section at 142 MeV by a factor 3 in one of the calculations, and by only 30% in the other one. This difference does not seem to be due to the fact that in one of the calculations only the $^3S_1$ and $^1S_0$ waves are taken into account, while in the other case only $^3S_1$ and $^3D_1$ are assumed to contribute, since in either case the dominating effect comes from the same $^3S_1$

wave. The presence of d wave component in the deuteron has its maximum effect in backward scattering, reducing the differential cross section by up to 30% at 182 MeV. Both groups of authors report that pion-nucleon rescatterings give very small contribution.

There are also results of relativistic calculations at these energies around the resonance region, performed by Brayshaw[24] using the boundary condition formalism. In this work only P_{11} and P_{33} πN waves are taken into account, and the scattering parameters are obtained only for $J^P = 0^+$ and 2^+ πd states.

In another relativistic calculation at 182 MeV[25], based in the quasi-potential method, with the πN interaction limited to the P33 wave, and NN binding effect only the ${}^3S_1$ channel, the results do not reproduce the data very well.

From Fig. 1-3 we may conclude that, as a whole, there is not much difference between the results of the multiple (single plus double) scattering calculation reported before[10] and of the relativistic three-body calculations, as far as a fitting to the presently available data is concerned. It seems that the inclusion of all πN partial waves, which is done inexpensively in the multiple scattering calculation is more important to the final results then the sophistications of the relativistic three-body formalism.

An attempt which seems to us worth exploring further is that[26] of combining the good features of both multiple scattering and three-body equations methods in the same calculation. In this mixed framework, the parameters of the low J waves in πd scattering are evaluated with the three-body formalism, while the sum of all higher waves is obtained from the multiple scattering amplitudes (by extracting from these amplitudes the contributions due to those low J waves). It has been shown[26] that the combined set of amplitudes give a better fitting to the data than what has been obtained using either method separately.

It is remarkable that, although different calculations produce differential cross section curves which seem reasonable when compared to the data, and not very different from each other, the individual πd partial waves often differ wildly from one calculation to another. An example of this is the result obtained by Giraud et al.[19], that the couplings of $\ell = J \pm 1$ waves are important, while other authors[22,23] neglect them. This shows that the comparison of theoretical results must be made also in the amplitude level, and not only by checking which calculation fits better the (up to now, poor) differential cross section data. This also shows that it urges a phase shift analysis of πd scattering, which of course requires more accurate data than those available at the present, including polarization measurements.

We believe it would be worthwhile to apply the three-body equations to obtain πd break-up cross sections, so as to have a test of self-consistence of the method. We know of no calculation of this sort, and find they would also be welcomed for providing another line of comparison with multiple scattering calculations.

4. THE BACKWARD SCATTERING PROBLEM

For incident pion energies above 200 MeV, and for scattering angles larger than 70 degrees, there are very large discrepancies between any of the calculations and the experimental data, with the theoretical values lying too high by a factor which varies from 3 to 5. These discrepancies are seen markedly at 256 MeV, and are confirmed[10] by the recent LAMPF data[27] at 234 and 325 MeV. The experimental differential cross sections show a marked dip at 90-100 degrees lab scattering angles, which is not present in the theoretical curves. The effect seems to be increasing with the energy, and at 325 MeV is quite dramatic.

There is a clear theoretical problem to be faced here. The multiple scattering method and the three-body equations seem to be unable to explain the data with our present knowledge of deuteron structure and of other characteristics of the system. It seems that some ingredient is missing, or that some new idea is needed.

A possible cause of the problem may be in our insufficient knowledge of the deuteron wave function. In backward scattering above 200 MeV we may have entered a region of momentum transfer in which the deuteron structure has not been previously investigated with the necessary detail.

The contributions due to meson resonances in the intermediate state (the pion propagating between the first and the second scatterings is replaced by η, ρ, ω and so on) have been evaluated[28] as a possible source of the characteristic momentum dependence observed[29] in the differential cross sections at very large angles, which shows a maximum at around 700 MeV/c. However, in the 250 MeV region, where the backward scattering problem is dramatic, these diagrams contribute very little. For example, the ρ contribution has been evaluated[30] in the P33 resonance region, giving an estimated contribution of less than 10 percent of the differential cross section at large angles.

There are other diagrams, which are not contained in the usual multiple scattering or three-body calculations, and which may play an important role in backward scattering at the energies considered. Thus we may have pion production in a πN collision, with its subsequent absorption by the other nucleon, as in the diagram of Fig. 4a. Or we may have important contributions from u-channel di-baryon ($N\Delta$,NN) resonances, as in the diagram of Fig. 4b. Special cases of this last kind of graphs have been studied by Klemm and Weber [39].

5. COMMENTS ON EXISTING AND EXPECTED DATA

The experimental data on the pion deuteron interaction at low and intermediate energies have been produced at slow pace during twenty years. This has been an infortunate situation, in view of the fundamental importance of this system. Until two years ago only the differential cross section data at 61[3], 85[2], 142[11], 182[12], 224[15], 256[13], and 330 MeV[32,33], and the backward scattering measurements above 375 MeV[29] were available. The occuracy of the measurements has reached the 10 percent level in only one of these experiments[13], and only in a few points of the angular range. It is hoped that this situation has now started to change, with the active operation of three meson factories (LAMPF, TRIUMF and SIN) and also renewed interest at CERN and Saclay for this kind of experiments. There are now new data[18] from TRIUMF at 47.5 MeV, and from LAMPF[27] at energies from 234 to 515 MeV.

A new standard in what concerns the quality of the data may now have been established, as indicated by the excellent measurements taken at SIN[34] of π^+d and π^-d total cross sections with better than 1 percent accuracy in the wide energy range from 70 to 370 MeV. The real part of the nuclear forward scattering amplitude is also obtained, via a forward dispersion relation calculation. Fig. 5 shows the average total cross section $\sigma_{\pi d} = \frac{1}{2} [\sigma(\pi^+d \rightarrow \pi^+d) + \sigma(\pi^-d \rightarrow \pi^-d)]$ as obtained in this experiment. The curve presents a large peak at an energy close to the P33 resonance, but we must notice that this behaviour does not come from an enhancement in the contribution from one particular angular momentum state, as the angular distribution at this energy is highly asymmetrical, as can be seen in Fig. 2. It is interesting to remark that the πd cross section at the peak is $\sigma_{\pi d} \simeq 230$ mb, while for the sum of the cross sections of the two separate nucleons we have $\sigma_{\pi p} + \sigma_{\pi n} = 280$ mb. The 20 percent difference is due to the complications arising from three-body effects such as interference, binding, fermi motion, and so on.

The multiple scattering calculation of the imaginary part of the forward amplitude with proper treatment given to the kinematical variables[10] produce, through the optical theorem, the results shown in Fig. 5. We must remark that in this calculation almost the whole contribution comes from the single scattering term, as double scattering has negligible effect in forward scattering. We see that the multiple scattering method give very good results for the total cross section at energies above 180 MeV, but fails at lower energies, possibly due to an inadequate account of binding corrections. It would be nice to know how well the imaginary forward amplitude obtained with the Faddeev-equation method fit these accurate total cross section data.

The real part of the forward amplitude, derived from the same experimental results[34], is show in Fig. 6 as a function of the energy. In the same figure we draw the curve for the Ref(0) obtained from the multiple scattering calculation. In both cases Ref(0) changes sign, with a shift in the value at which Ref(0) = 0 occur. Again here, it would be interesting to know what the three-body calculations produce for this quantity.

Experiments measuring πd differential cross sections at low and intermediate energies are now in progress in several laboratories. In Saclay measurements are being made in the low energy range (from 30 to 65 MeV)[20], an in LAMPF at energies up to 550 MeV[27]. At CERN and SIN, complementary experiments cover the range from 100 to 300 MeV, with large angle measurements ($\theta > 130^o$) made at CERN, and low angle data ($\theta < 130^o$) obtained at SIN.[35] Another kind of experiment is in progress at SIN[36], in which it will be measured the tensor polarization of the recoiling deuteron after scattering of 300 MeV pions. These polarization experiments are planned to give information on the d-wave component of deuteron wave-function, but it would be nice if they could also become useful for the amplitude analysis of the πd interaction.

6. AMPLITUDE ANALYSIS AT 182 MeV

The full description of pion deuteron scattering requires the knowledge of amplitudes in the individual relevant angular momentum states. For a satisfactory understanding of the problem, experimental results and theoretical calculations should agree in the determination of these quantities, which fix both elastic and break-up cross sections. The complete angular momentum analysis requires results of deuteron polarization measurements, which are not yet available. On the other hand, differential cross section measurements are not accurate enough to allow the determination of the contributions from higher angular momentum waves. In spite of these limitations, useful information can be obtained from the existing data. An example of what can be done in this direction is represented by a recent analysis of the experimental data at 182 MeV in terms of helicity amplitudes [37].

The real part of the forward scattering amplitude goes through zero at 182 MeV [34]. This result was obtained under the assumption that forward scattering is dominated by the single scattering contribution (in which case $f_{11} = f_{00}$, where $f_{\lambda\lambda'}$ represents the amplitude for scattering from helicity state λ to helicity state λ'). Another relation of simple form for the amplitudes is written based on the fact that the elastic backward scattering cross section is very small at this energy. Non-homogeneous equations for the helicity amplitudes are obtained using the experimental values of the integrated elastic cross section [12] and of the total cross section [34]. This last value gives, through the optical theorem, information on the imaginary part of amplitudes.

The scattering amplitude from a state of helicity λ to a state of helicity λ' can be written in the form

$$f_{\lambda\lambda'}(\theta,\varphi) = \frac{1}{k} \sum_J \frac{1}{J+1/2} a^J_{\lambda\lambda'}(W)\, d^J_{\lambda\lambda'}(\theta)\, e^{i(\lambda-\lambda')\varphi} \tag{6}$$

and the experimental values for the total and elastic cross sections at 182 MeV are respectively

$$\sigma_T = (229.1 \pm 0.8)\mathrm{mb} = 3.0 \times \frac{4\pi}{k^2} \tag{7}$$

and

$$\sigma_{elastic} = (65 \pm 3)\mathrm{mb} = \frac{1}{3.5}\sigma_T \tag{8}$$

Assuming that only J = 0,1,2,3 angular momentum states contributes to the scattering at this energy, the following set of equations for the $a^J_{\lambda\lambda'}$ amplitudes are obtained

$$a^0_{00} + a^2_{00} = 0. + i \times 1.5 \qquad (9)$$

$$a^1_{00} + a^3_{00} = 0. + i \times 1.5 \qquad (10)$$

$$a^1_{11} + a^2_{11} + a^3_{11} = 0. + i \times 3.0 \qquad (11)$$

$$a^2_{1,-1} = a^1_{1,-1} + a^3_{1,-1} \qquad (12)$$

and

$$\sum_J \sum_{\lambda\lambda'} \frac{1}{(J+\frac{1}{2})} |a^J_{\lambda\lambda'}|^2 = 5.1 \qquad (13)$$

These equations lead to a set of constraint unequalities for the imaginary parts of the amplitudes

$$2(\mathrm{Im}\ a^0_{00})^2 + \frac{2}{5}(\mathrm{Im}\ a^2_{00})^2 \leq 2.33 \qquad (14)$$

$$\frac{2}{3}(\mathrm{Im}\ a^1_{00})^2 + \frac{2}{7}(\mathrm{Im}\ a^3_{00})^2 \leq 2.04 \qquad (15)$$

$$\frac{1}{3}(\mathrm{Im}\ a^1_{11})^2 + \frac{1}{5}(\mathrm{Im}\ a^2_{11})^2 + \frac{1}{7}(\mathrm{Im}\ a^3_{11})^2 \leq 0.98 \qquad (16)$$

which can be used respectively with the imaginary parts of the constraint equations (9), (10), (11) to determine the possible ranges of values for these quantities. One interesting conclusion of this analysis is that the $J = 3$ wave cannot be neglected in the scattering at 182 MeV.

REFERENCES

1. E. Arase, G. Goldhaber and S. Goldhaber, Phys. Rev. 90 (1953) 160.
2. K. C. Rogers, and L. M. Lederman, Phys. Rev. 105 (1957) 247.
3. A. M. Sachs, H. Winick and B. A. Wooter., Phys. Rev. 109 (1958) 1733.
4. S. Fernback, T. A. Green and K. M. Watson, Phys. Rev. 84 (1951) 1084.
5. R. M. Rockmore, Phys. Rev. 105 (1957) 256.
6. R. G. Glasser et al., Phys. Rev. D15 (1977) 1200; C. J. S. Damerell et al., Nucl. Phys. B94 (1975) 374; W. Cameron et al., Nucl. Phys. B78 (1974) 73; G. Giacomelli et al., Nucl. Phys. B42 (1972) 437; M. Sakitt et al., Phys. Rev. D15 (1977) 1846. Other references to K^+d experiments can be found in these papers.
7. E. M. Ferreira, Phys. Rev. 115 (1959) 1727.
8. G. Giacomelli et al., Nucl. Phys. B71 (1974), 138; B. R. Martin, Nucl. Phys. B94 (1975) 413.
9. G. F. Chew, and M. L. Goldberger, Phys. Rev. 87 (1952) 778.
10. E. M. Ferreira, L. P. Rosa and Z. D. Thomé, Phys. Rev. C16 (1977), 2353. For other multiple scattering calculations for the pion deuteron system see references therein.
11. E. G. Pewitt et al., Phys. Rev. 131 (1963) 1826.
12. J. H. Norem, Nucl. Phys. B33 (1971) 512.

13. K. Gabathuler et al., Nucl. Phys. B55 (1973) 397.

14. E. M. Ferreira, L. P. Rosa and Z. D. Thomé, Nuovo Cimento 21A (1974) 187 and 33A (1976) 216.

15. J. L. Acioli, Ph.D. Thesis, University of Chicago, 1968 (unpublished).

16. L. D. Faddeev, Sov. Phys. JETP 12 (1961) 1014.

17. A. W. Thomas, Nucl. Phys. A258 (1976) 417.

18. D. Axen et al., Nucl. Phys. A256 (1976) 387.

19. N. Giraud, G. H. Lamot and C. Fayard, Phys. Rev. Lett. 40 (1978) 438; N. Giraud, Y. Avishai, C. Fayard, G. H. Lamot, preprint Institut de Physique Nucleâire, Université Lyon, LYCEN/7833.

20. B. Saghai, 4ème Session d'Etudes Biennale de Physique Nucléaire (La Toussuire, 1977) S.15.

21. R. Blankenbecler and R. Sugar, Phys. Rev. 142 (1966) 1051; D. Freedman, C. Lovelace and J. Namylowski, Nuovo Cimento 43 (1966) 248; R. Aaron, R. D. Amado, J.E. Young, Phys. Rev. 174 (1968) 2022.

22. V. B. Mandelzweig, H. Garcilazo and J. M. Eisenberg, Nucl. Phys. A256 (1976) 461 and Nucl. Phys. A264 (1976) 531; J. M. Rivera and H. Garcilazo, Nucl. Phys. A285 (1977) 505.

23. A. S. Rinat and A. W. Thomas, Nucl. Phys. A282 (1977) 365.

24. D. D. Brayshaw, Phys. Rev. C11 (1975) 1196.

25. T. I. Kopaleishvili and A. I. Machavariani, Theor. and Math. Phys. 30 (1977) 130; Nucl. Phys. A302 (1978) 423.

26. D. D. Brayshaw and E. Ferreira, Phys. Lett. 68B (1977) 139.

27. R. H. Cole, J.S. McCarthy, R.C. Minehart and F.A. Wadlinger, Phys. Rev. C17 (1978) 681.

28. L. A. Kondratyuk and F. M. Lev, Sov. J. Nucl. Phys. 23 (1976) 556.

29. L. S. Schroeder et al., Phys. Rev. Lett 27 (1971) 1813.

30. E. Levin and J. M. Eisenberg, Nucl. Phys. A292 (1977) 459.

31. G. J. Klemm and H. J. Weber, Zeit. Phys. 268 (1974) 299.

32. L. S. Dul'kova, I. B. Sokolova and M. G. Shafranova, Zh.Eksp. Teor. Fiz. 35 (1958), 313 [Sov. Phys. JETP 35 (1959) 217].

33. G. B. Brunhart, G. S. Faughn and V. P. Kenney, Nuovo Cimento 24 (1963) 1162.

34. E. Pedroni et al., Nucl. Phys. A300 (1978) 321.

35. J. J. Domingo, Private communication.

36. W. Grüebler et al., SIN Physics Report No. 2, December 1977, p. 27.

37. E. M. Ferreira and G. Perez Munguia, PUC preprint NC-18/78, Rio de Janeiro.

FIGURE CAPTIONS

Fig. 1 - Differential cross section for $\pi d \rightarrow \pi d$ scattering at 142 MeV. The experimental data are from reference 11. The solid curve shows the results of the theoretical calculation in reference 10, using the multiple scattering method. The dashed curve shows the values obtained in the three-body calculations by Giraud et al. (reference 19 and private communication).

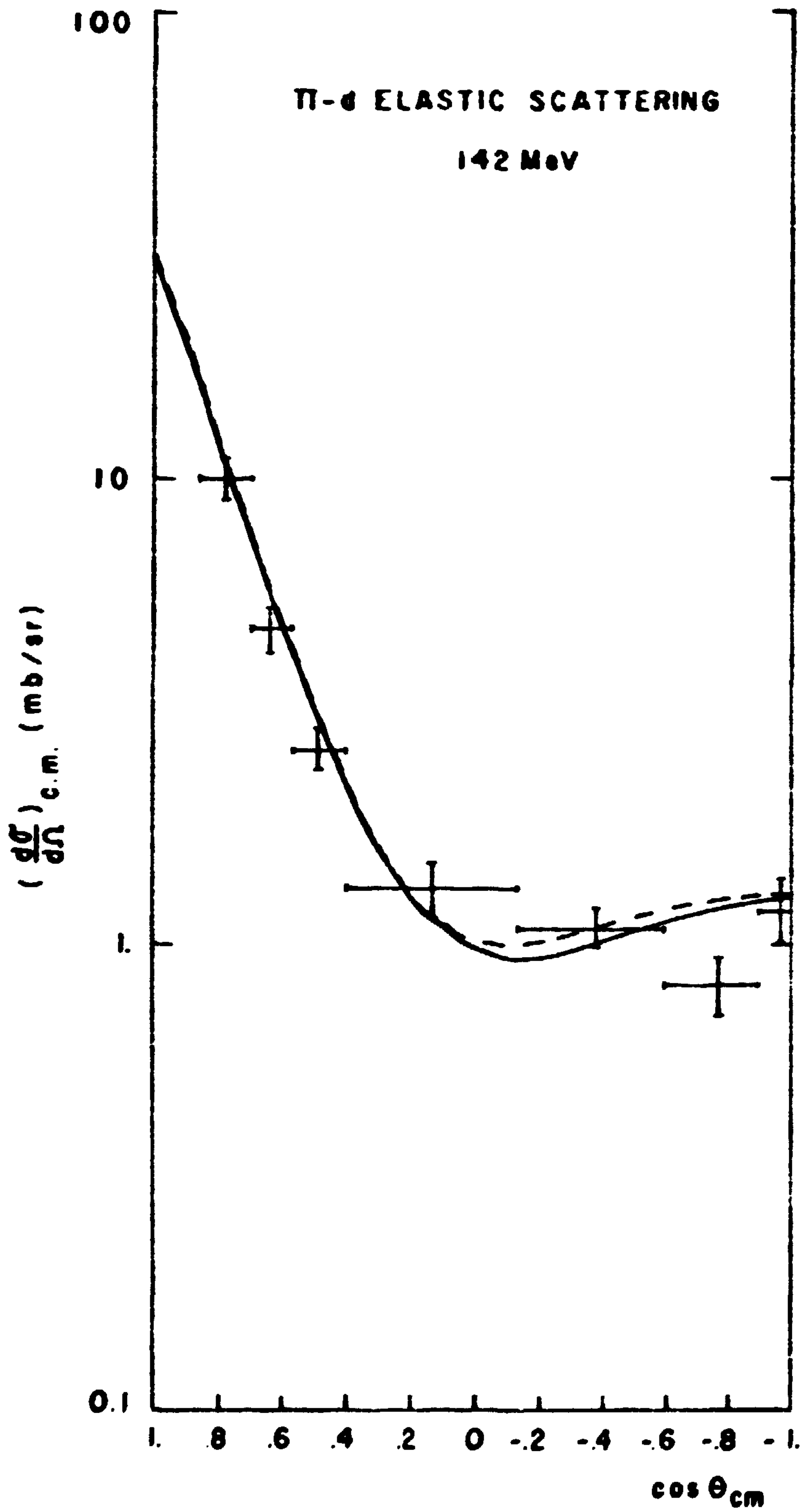
Fig. 2 - Differential cross section for πd elastic scattering at 182 MeV. The experimental data are from reference 12. The solid curve shows the results obtained in the multiple scattering calculation of reference 10. The dashed and dotted curves show respectively the results of the three body calculations of Giraud et al (reference 19 and private communication) and of Rinat and Thomas (reference 23).

Fig. 3 - Differential cross section for πd elastic scattering at 256 MeV. The experimental data are from reference 13. The solid curve shows the results obtained in the multiple scattering calculation of reference 10. The dotted curve represent the results of the relativistic three-body calculation of Rinat and Thomas (reference 23).

Fig. 4 - Diagrams representing contributions not taken into account in the usual multiple scattering and three-body calculations of πd scattering. In Fig. 4a a pion produced in the πN collision is subsequently re-absorbed. Fig. 4b exemplifies contributions of u-channel di-baryon resonances.

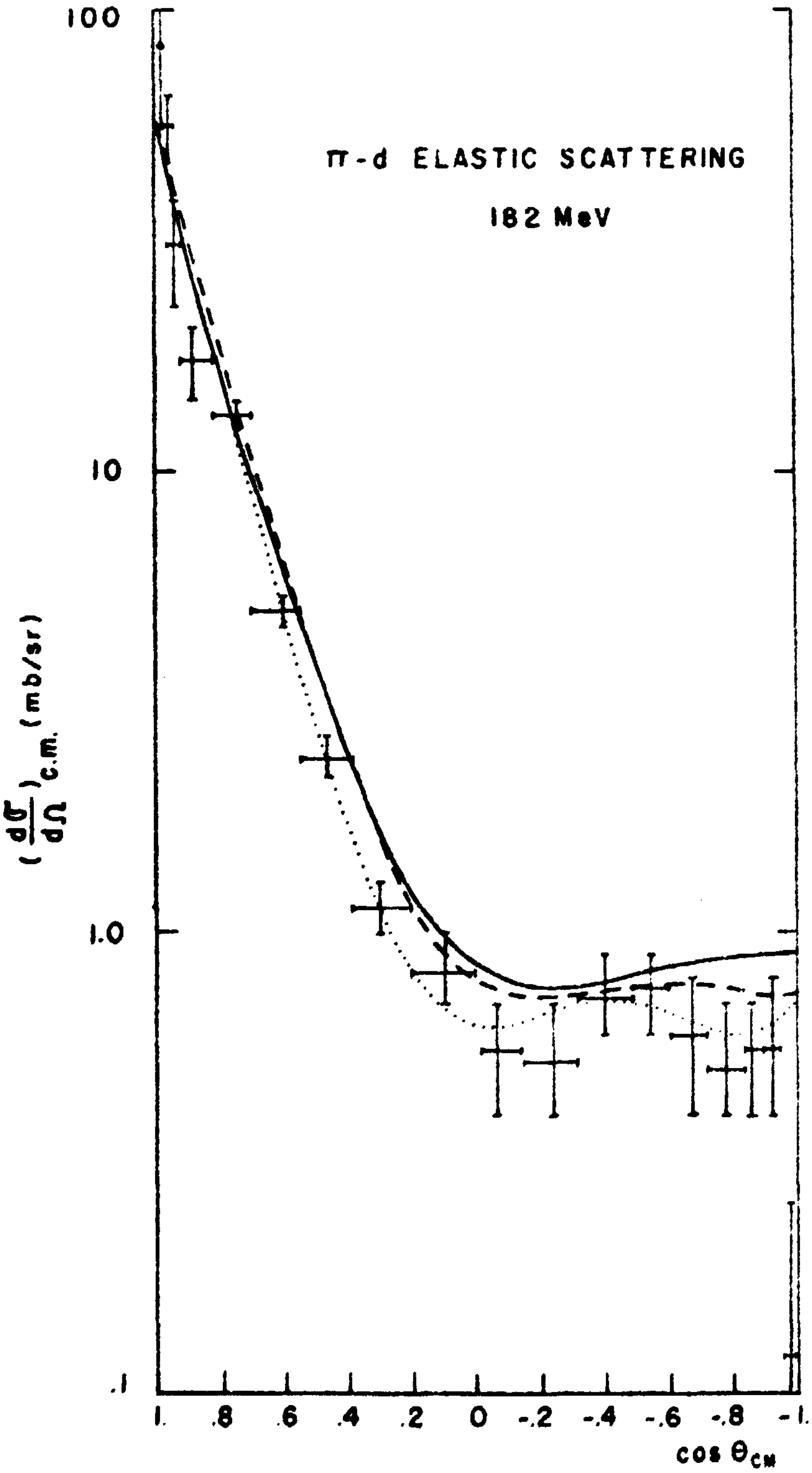
Fig. 5 - Total cross section for πd scattering. The solid curve represents the experimental data of reference 34. The dotted line shows the values obtained through the optical theorem from the multiple scattering calculation of reference 10. It is interesting to notice that the two curves are similar in shape, with one shifted by 10 MeV along the horizontal axis with respect to the other.

Fig. 6 - Real part of the forward amplitude for πd scattering as a function of the energy. The solid curve represents the values obtained (reference 34) through dispersion relations from the measurements of the total cross section (see Fig. 5). The dotted curve shows the values for Ref. 10 obtained in the multiple scattering calculation of reference 10.



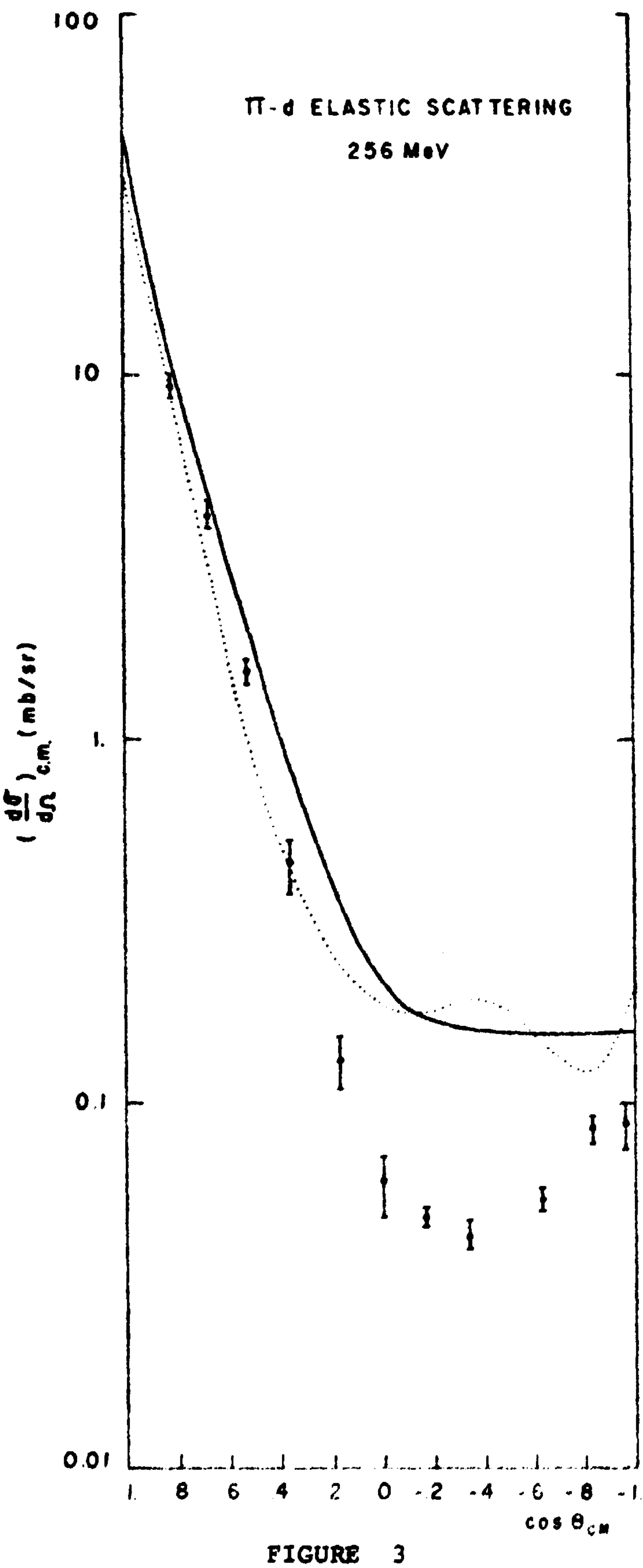
π-d ELASTIC SCATTERING
142 MeV
100
10
1.
0.1
(dσ/dΩ)c.m. (mb/sr)
1. .8 .6 .4 .2 0 -.2 -.4 -.6 -.8 -1.
cos θcm

FIGURE 1



π-d ELASTIC SCATTERING
182 MeV
100
10
1.0
.1
$(\frac{d\sigma}{d\Omega})_{c.m.}$ (mb/sr)
1.
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
-1.
cos θ_{CM}

FIGURE 2



π-d ELASTIC SCATTERING
256 MeV
100
10
1.
0.1
0.01
$(\frac{d\sigma}{d\Omega})_{c.m.}$ (mb/sr)
1. 8 6 4 2 0 -.2 -.4 -.6 -.8 -1.
cos θ$_{CM}$

FIGURE 3

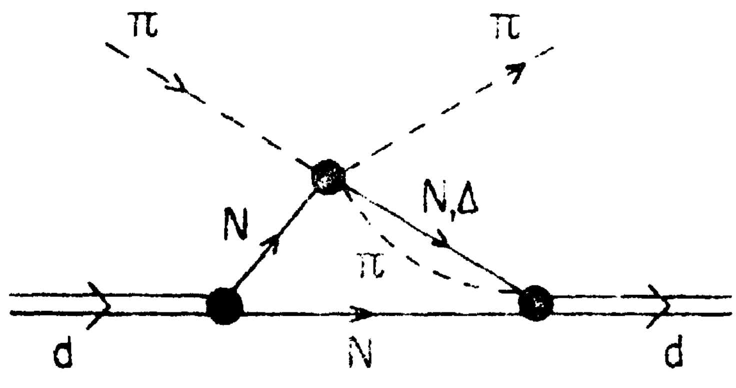


FIGURE 4a

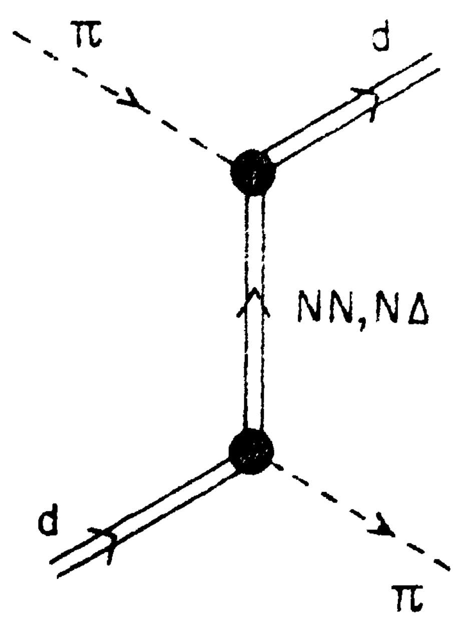


FIGURE 4b

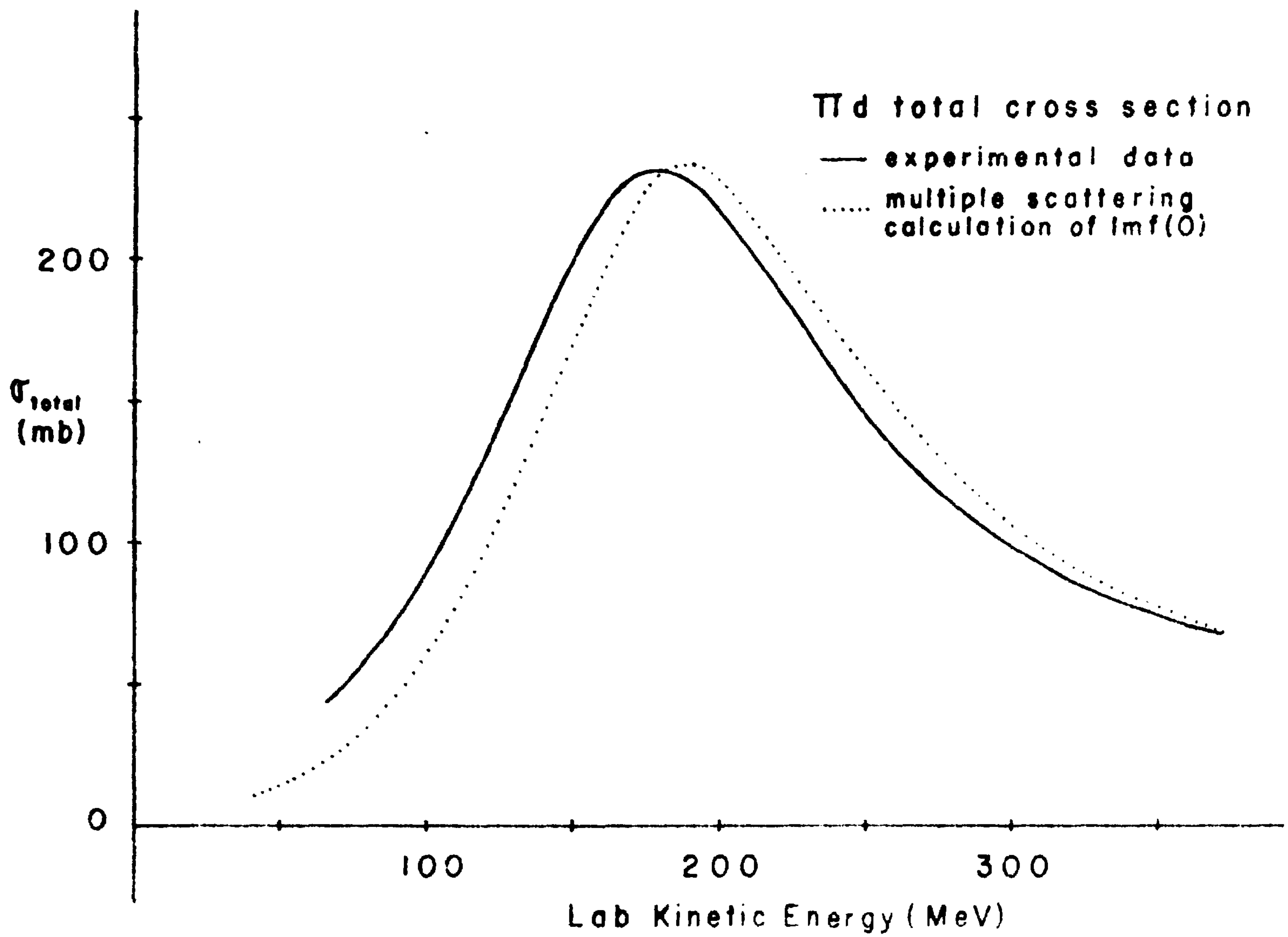
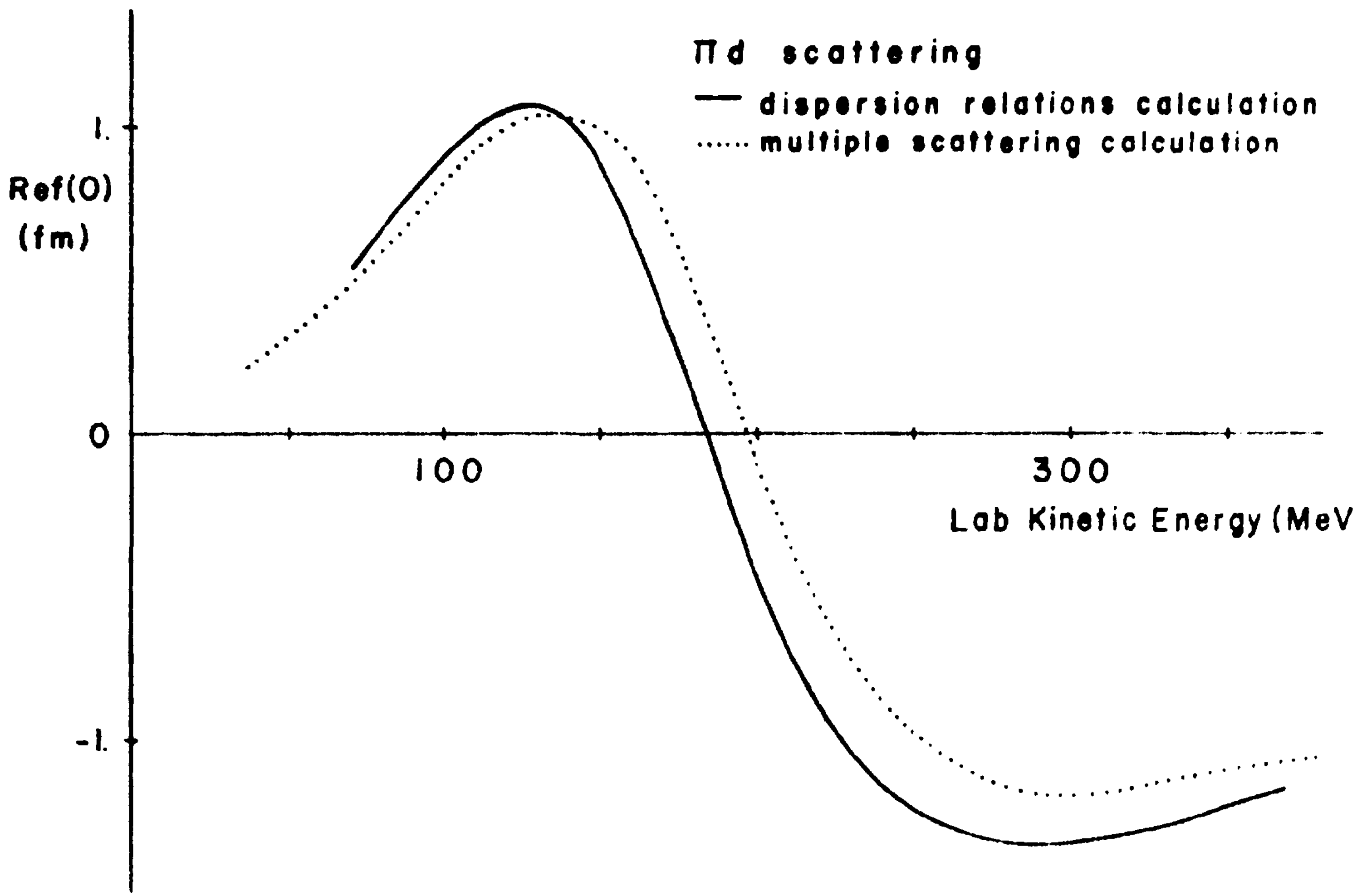

πd total cross section
experimental data
multiple scattering calculation of Imf(0)
σ_{total} (mb)
200
100
0
100
200
300
Lab Kinetic Energy (MeV)

FIGURE 5

FIGURE 6



πd scattering
dispersion relations calculation
multiple scattering calculation
1.
Ref(0)
(fm)
0
-1.
100
300
Lab Kinetic Energy (MeV)