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104 50 Stockholm

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MAGNETIZED WHIRLS IN PLASMA FOCUS DISCHARGES*

Erik A Witalis

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Abstract

The plasma focus is briefly described with emphasis on its capabilities as a neutron source. The filamentary whirl structures observed in the discharge plasma are described. Starting with a simple, early and particularly well established case of vorticity imparted by a rotational electric field to the plasma in MHD generators, a general derivation is then outlined proving that such magnetically induced rotation is a general feature for the normally Hall-conducting magnetized plasma. Physical interpretations of the effect are given and objections to it are critically reviewed as is also a theory proposing radiation cooling as the cause of plasma filamentation. A more detailed derivation based essentially on the consistent description of the motion and the field generation of the charged plasma particles yields a theoretical model where the specific features of magnetically compressed plasmas are found. In particular, the ion collisionless skin depth is obtained as the key length parameter. This length is identified as roughly the whirl radius. In conjunction with a generalized Bennett relation theoretical whirl properties are predicted and found to agree with observations. Mechanisms that relate the whirls to nuclear fusion reaction conditions are tentatively indicated.

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Sammanfattning

Plasma-fokus-anordningen beskrives kortfattat med tonvikt på dess egenskaper som neutronkälla. En beskrivning gives av de trådformade virvelstrukturer vilka observerats i urladdningsplasmat. Med utgångspunkt från ett enkelt, tidigt och övertygande fall, när ett elektriskt fält av rotationstyp överlagrar en rotationsrörelse på plasmat i MHD-generatorer, presenteras därefter en allmän härledning, vilken bevisar att sådan magnetiskt inducerad rotation är en allmän egenskap för det magnetiserade plasmat med dess normala elektriska Hall-konduktivitet. Fysikaliska förklaringar gives av rotationsrörelsen och invändningar granskas kritiskt liksom även en teori där kylning genom strålning föreslås vara orsaken till plasmats uppdelning i trådstrukturer. En mer detaljerad härledning grundas väsentligen på en samstämmig beskrivning av de laddade plasmapartiklarnas rörelse och fältgenerering och den ger en teoretisk modell där man finner speciella kännetecken hos magnetiskt komprimerade plasmor. Speciellt erhålles jonernas kollisionsfria inträngningsdjup som den avgörande längdparametern. Denna längd identifieras som den ungefärliga radien på virvelstrukturen. Med anknytning till en generaliserad Bennettrelation förutsäges de teoretiskt förväntade virvelegenskaperna och dessa befinnes överensstämma med observationer. Mekanismer vilka relaterar virvlarna till betingelser för nukleära fusionsreaktioner antydes.

Inviterad föreläsning med titeln "Magnetiserade virvlar i plasmafokusurladdningar" vilken gavs vid andra internationella konferensen angående lagring, kompression och omkoppling av energi som ägde rum i Venedig den 5-8 december 1978. Författarens deltagande i konferensen skedde med stöd från institutionen för elektromagnetisk fältteori vid Chalmers tekniska högskola enligt programmet för fusionsforskningen inom EURATOM.

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CONTENTS

1. INTRODUCTION

The plasma focus is a pulsed controlled fusion device in which a discharge between two simply shaped coaxial electrodes sweeps up and then compresses a plasma [1,5]. In contrast to larger and more sophisticated fusion energy arrangements it produces copious amounts of fusion neutrons during a short time, $\approx 10^{-7}$ sec. At present it has yielded emission numbers of $\approx 2 \cdot 10^{12}$ per discharge with an initial deuterium gas filling and $\approx 10^{14}$ with a deuterium-tritium mixture [1]. Empirical but well-established scaling laws indicate that one or more orders of magnitude higher figures may very well be achieved, [1],still plasma foci experiments encompass only a modest part of the total fusion energy research. One reason is that a generally accepted description is still lacking about the violently dynamic processes which create the observed tiny filament regions where extreme electrical fields accelerate ions far beyond the threshold energies for fusion reactions [1]. Another reason is that the plasma focus construction and operation seem incompatible with envisaged fusion reactors like the huge toroidal schemes. However, it should be remembered that "fusion energy" is essentially kinetic energy of fast neutrons. Even if the truly thermonuclear plasma can be created and confined by magnetic or inertial means, the safe and, in particular, economic conversion of such neutron energy into heat and then into other energy forms will present formidable difficulties. In contrast, there is an undisputed knowledge of how to exploit fast neutrons in hybrid schemes where they are used instead for the production of fissile nuclear fuel, i.e. breeding. Crude estimates indicate about an order of magnitude more available energy from hybrid schemes than the direct production of heat from neutrons, but clearly at the expense of the much discussed disadvantages of fission compared to fusion energy.

2. THE WHIRL STRUCTURES

From extensive experimental work [2, 3] carried out during many years by, in particular, the Bostick-Nardi-Prior team there is strong evidence that intensely magnetized, $\approx 10^3$ T, plasma whirl structures, also with extremly high plasma density $\approx 10^{20}$ cm^{-3}, are instrumental in creating the localized regions from which emitted x-rays and nuclear reaction products prove the existence of strong accelerating electric fields. The analysis [4] of the neutron emission from the Jülich 1 device shows the presence of ions with energies above 300 keV up to at least 1 MeV.

In early plasma focus work attention was concentrated on these energetic regions with apparently more controversy than progress in solving the seemingly crucial question whether conditions there could be considered as thermonuclear or, oppositely, of the beam-target character [5]. A more rewarding approach advocated and pursued by Bostick and collaborators [2] was instead to concentrate on the plasma properties and behaviour before the strong radiation phase. By techniques so as to yield the utmost in time and space resolution applied to experimental assemblies of fairly modest size a remarkable discharge fine structure was found, in particular the mentioned magnetized plasma whirls. The diagnostic techniques (image converter, x-ray pinhole camera, x-ray scintillation detector, shadowgraph, spectroscophy, probes, etc.) and the methods for analysing the experimental recordings have been reported [2, 3, 6]. Reference is given to these papaers, here we shall only indicate some whirl structure properties of importance for the reasoning and the theory in the sequel.

i) The whirl structures are formed initially at an early stage of the discharge, probably immediately after the ignition and start of the so-called run-down phase when the discharge moves in the annular space between the coaxial electrodes.

ii) The whirls have then the shape of radially directed filamentary parts of the otherwise fairly smooth discharge current sheath. They are about a tenth of a millimeter in diameter and distributed roughly equally in azimuth.

iii) They retain their identity and azimuthal position also during the subsequent radially implosive sheath-plus-filaments motion towards the focus axis ahead of the center electrode. This filamentary structure exists even up to the instant of nearly total discharge constriction with the coincident rapid decrease of total discharge current always observed.

iv) A remarkable feature of plasma foci is that this maximum constriction of the discharge channel is usually not the same, neither in time nor space, as the region of strong x-ray and neutron emission.

v) The filaments carry an appreciable part of the large discharge current, peak value $\approx$ 1 MA. This filament current, of course electronic, must flow in helical paths as the filaments in addition to the azimuthal, i.e. sheath-driving magnetic field also contain a large magnetic flux. Its field strength is of the same order as the azimuthal, $\approx 10^3$ T. Topologically, the filaments are thus likely to be screw pinches.

vi) A strong mass rotation with the rotation vector directed along the filament axis has been deduced from considerations of the filament geometry and dynamics with respects to the adjacent current sheath.

3. MAGNETICALLY INDUCED MASS ROTATION

In 1966 R. Wienecke informed the author about a not understood pressure and velocity distortion effect which occurred when the plasma flow entered a MHD generating section. Soon after, the explanation was given independently by A.B. Vatazhin [7] and by

E.A. Witalis [8]. The distortion effect was proved to be a strict consequence of the two basic MHD equations for charge and mass transport, respectively, but, of course, with the Hall term retained for the former and the convective part of the acceleration term for the latter. The combined two equations also yield the following physical explanation [8]: Due to the induction law a rotational electric field will be "felt" by the steady and quasi-neutral plasma flow when it enters the strong and transverse MHD generator magnetic field. This inductive electric field will act preferentially on the ions, in spite of the plasma quasi-neutrality, so as to superimpose a mass rotation on the flow, thus distorting its previous symmetric, normally parabolic, velocity profile. Unfortunately, MHD power generation research practically ceased in Western countries about that time when the phenomenon was understood, but extensive experimental investigations carried out in the USSR since then have shown that the effect can be very pronounced, that it is generally harmful to generator performance but also that a technique so as to counteract the distortion has been developed by Y. Emets [9] at the Institute of Electrodynamics in Kiev.

In 1968 I generalized my flow distortion analysis so as to be valid for an arbitrary geometry and to include the explicit time dependence [10]. Evidently, the plasma focus group of the Stevens Institute immediately recognized that the derived mechanism might be central for a plausible explanation of the observed strongly magnetized whirl structures [11].

Omitting intermediate steps the derivation essentially goes as follows. The usual MHD equations for mass and charge transport are taken to be valid. With standard notations, i.e. e is the ion charge and $\underline{V}$ is mass velocity taken to be equal to the average ionic velocity, they are

$$\rho \frac{d\underline{V}}{dt} = \rho_e \underline{E} + \underline{j} \times \underline{B} - \underline{\nabla} \cdot \underline{\underline{P}} \qquad (1)$$

$$\frac{\underline{j}}{\sigma} = \underline{E} + \underline{V} \times \underline{B} + \frac{1}{en_e} \underline{\nabla} \cdot \underline{\underline{P}}_e - \frac{1}{en_e} \underline{j} \times \underline{B} \qquad (2)$$

Consider an arbitrary closed loop, line element $d\underline{s}$, attached to the moving plasma mass frame. The loop defines the boundary of a simple closed surface, element $d\underline{S}$. We apply the identity

$$\frac{d}{dt} \int \underline{a} \cdot d\underline{S} = \int \frac{\partial \underline{a}}{\partial t} \cdot d\underline{S} + \int (\text{div } \underline{a})\, \underline{V} \cdot d\underline{S} - \oint \underline{V} \times \underline{a} \cdot d\underline{s} \qquad (3)$$

to the function

$$\underline{a} = \underline{B} + \frac{\rho}{en_e} \text{ curl } \underline{V} \qquad (4)$$

and assume complete or constant fractional ionization so that the divergence integrand in Eq. (3) vanishes, partly also because of the Maxwell equation

$$\text{div } \underline{B} = 0 \qquad (5)$$

The surface integral of the explicit time derivative in Eq. (3) is then transformed into a line integral by means of Stoke's integral theorem and the Maxwell equation

$$\text{curl } \underline{E} = - \frac{\partial \underline{B}}{\partial t} \qquad (6)$$

The next steps are to expand the convective derivative in Eq. (1) as

$$\frac{d\underline{V}}{dt} = \frac{\partial \underline{V}}{\partial t} + \frac{1}{2} \text{ grad } V^2 - \underline{V} \times \text{curl } \underline{V}, \qquad (7)$$

eliminate the $\underline{j} \times \underline{B}$-terms between Eqs. (1) and (2) and then integrate the resulting equation along the loop. A comparison between this expression and the combined Eqs. (3) and (4) directly yields

$$\frac{d}{dt}\int \underline{B} \cdot d\underline{S} = -\frac{d}{dt}\oint \frac{\rho}{en_e} \underline{V} \cdot d\underline{s} + \oint \left[\frac{\rho_e}{en_e} \underline{E} - \frac{1}{en_e} \underline{\nabla} \cdot (\underline{\underline{P}} - \underline{\underline{P}}_e) - \frac{\underline{j}}{\sigma}\right] \cdot d\underline{s} \qquad (8)$$

This equation contains a number of special cases of which the simplest one is the Kelvin circulation theorem for the aporoximate conservation of vorticity in the purely hydrodynamic case, i.e. when $n_e = \underline{j} = 0$, so that

$$\frac{d}{dt} \oint \underline{V} \cdot d\underline{s} = - \oint \frac{1}{\rho} \underline{\nabla} \cdot \underline{\underline{P}} \cdot d\underline{s} \approx 0 \tag{9}$$

The classical Alfvén theorem of flux conservation, i.e. magnetic field lines "frozen" to mass

$$\frac{d}{dt} \int \underline{B} \cdot d\underline{S} = 0 \tag{10}$$

is seen to require more than $\sigma \to \infty$ because of $n_e \to \infty$ at a finite $\underline{j}$. In particular, for our assumption of complete or constant ionization degree also the particle mass of the heavy species has to taken negligible like, implicitely, the electron mass in Eq. (2).

For a highly ionized and strongly magnetized quasi-neutral plasma, like that in a plasma focus device, effects represented by the last integral in Eq. (8), i.e. field action on excess electrical charge, viscosity and ion-electron friction, respectively, are all small. The remaining time derivatives of integrals imply that magnetic flux variations impart mechanical angular momentum to the plasma. Such a transfer may be considered as a kind of the Einstein-de Haas effect but caused by magnetically, not atomically, bound gyrating electrons. The inverse case whereby mechanical plasma rotation generates magnetic flux is a dynamo mechanism (or a plasma kind of the Barnett effect). In experiments with a coaxial plasma gun this effect was found to be very pronounced [12] and called the flux amplification phenomenon [13]. It is likely that the longlived laser plasma fields which have been observed even to increase after the laser radiation has ceased, are caused by an initial mechanical rotation [14].

The essence of Eq. (8) can also be expressed in another way: Approximate conservation of canonical, i.e. matter-plus-field, angular momentum is retained for the ionic plasma component. One may gain better insight into this result from the very compact and elegant alternative derivation recently presented by V. Nardi [6]. He obtains essentially our Eq. (8) by starting with the Lagrange invariant applied to the microscopic canonical momenta of the plasma particles followed by Hamilton's equations technique.

4. GENERAL VIEW ON PLASMA ROTATION

For completeness some difficulties in plasma rotation theory have to be briefly noted here. It was early observed that when a theta pinch plasma becomes permeated, radially compressed and heated by the fast-rising and externally impressed axial magnetic field the plasma column starts to rotate. The rotation sign, magnitude and time dependence are all as predicted by Eq. (8) with the first two terms dominating. The flux-preserving skin currents assumed in early theory and later often in stability analyses have never been found to exist so that the plasma column rotation is strongly accelerated as the impressed flux rapidly penetrates it. For external peak field strengths larger than ≈ 1T, the rotation becomes very strong, up to the order ion Larmor gyration speed, megacycles per second, so that even the emission of fusion MeV neutrons can be rotationally shifted [5]. Unfortunately, the origin of this strong rotation, always present, has been a most controversial subject of plasma physics and fusion research for almost two decades. The reason for this can be traced back to a short and perfectly correct analysis [15] essentially disproving any electromagnetic torque on a certain fluid taken to represent the quasi-neutral plasma of a theta pinch but, in contrast to any real magnetized plasma, devoid of the total stress tensor assymmetry due to the internal angular momenta from the magnetically gyrating electrons [16]. Explained in another way, there is no equipartition between ions and electrons of the electromagnetic torque on a quasi-neutral plasma.

It is an irony in that identically our basic equations (1), (2), (5), and (6) were presented, but with the normally rather unimportant resistive term in Eq. (1) neglected, as fundamental starting equations in the standard reference work [17] on the theta pinch rotation origin and controversy. The mathematical consequence of these equations, Eq. (8), was not obtained and the essence of it, torque of electromagnetic origin, was refuted. However, the insignificance of plasma excess charge, $\rho_e << en_e$, was correctly assessed as is obvious in Eq. (8). The following integrand term is of some interest in this context. It can hardly be interpreted as anything but essentially a small viscous rotation retardation similar to the action of the following resistive retardation from ion-electron friction. In contrast, Ref. [16] succeeds to present arguments about a viscosity effect as the most probable rotation driving mechanism!

We have reason to believe that magnetically induced plasma rotation is a general feature. For instance, under the name of exhaust swirl it is a well-established property of magneto-plasmadynamic (MPD) accelerators [18]. The investigation on them by Kogelschatz [19] is especially interesting because the ion-slip plasma conditions in his experiment clearly showed, as expected, a stronger rotation of the ion gas than that of the neutral component.

5. THE PLASMA FOCUS AS A Z-PINCH

The plasma focus is usually regarded as basically a Z-pinch-related device because its operation is seen to depend upon the self-constriction of a discharge channel [5]. It may be noted, however, that historically it originates from experiments with the Mather plasma gun, also that its capabilities exceed anything ever produced by straight discharges and further, that its operation so as to produce extreme bursts of neutrons is strongly dependent upon an intricate, necessary and a poorly understood balance of experimental parameters [1], [2], [3], [4]. They include the shape, size and polarity of the electrodes, most properties of the external electrical circuitry, the initial deuterium gas density and purity, and certain curious conditioning processes evidently necessary to obtain the proper discharge ignition. For instance, under non-optimized conditions the deliberate addition of a large atomic percentage of argon may lead to an order-of-magnitude increase of the neutron emission [20].

Z-pinch theory hardly gives support for a belief that Ohmic or compressional heating of a constricting channel create the filamentary x-ray emitting regions, radius ≈ 0.1 mm. The well-known "snow-plow" model for a collapsing Z-pinch does not predict such small final radii, and the inductive voltage across the neck of a m=0, i.e. "sausage", instability is not sufficiently large to explain the observed high energy nuclear reactions. However, a mechanism has recently been proposed for the fast and extreme contraction of a Z-pinch so as to explain the observed filaments [21]. It is based on the Bennett pressure balance and on the energy partition between radiation loss and Ohmic heating. As was found in early Z-pinch theory the combination of these effects leads to a limiting current called the "Pease-Braginskii Current". Only below this limit is the Ohmic heating stronger than the dominating "Bremsstrahlung" radiation losses. For pure deuterium the limit current is 1.5 MA. When impurity ions contribute to this power loss, as they certainly do in plasma foci, the limit is reduced drastically, an order of magnitude and probably even more. One may expect then that the plasma cooling caused by this excessive radiation loss may lead to a decrease in the plasma pressure and hence a plasma channel constriction provided, of course, that the seemingly ever present plasma instabilities have sufficiently slow growth times. Clearly, cooling contradicts any Z-pinch constriction leading to a hot filament and Ref. [21] assumes constant temperature T and constant line density N so, by the Bennett relation also assumed to be valid

$$\mu_o I^2 = 8\pi NkT \qquad (11)$$

the channel power input has to be provided by a constant current source. The power balance of a straight plasma channel with the inductance L is

$$P_{in} = P_\Omega + \frac{1}{2}\frac{d}{dt}\left(LI^2\right) + \frac{1}{2} I^2 \frac{dL}{dt} \qquad (12)$$

with, in order, power input, Ohmic heating, build-up rate of magnetic energy and mechanical compression power made by the magnetic field on the channel. Constant current leads to the wellknown equipartition [22]. ("the magnetic energy acts as a negative potential energy for the mechanical work") between the last two terms. Ref. [21] derives very rapid constrictions, in time ranges 10-100 µsec, to the observed small radii when there is excessive radiation losses. This result is obtained by combining the Bennett relation (11), not with Eq. (12) but with an internal channel power balance

$$P_w = \frac{1}{2} I^2 \frac{dL}{dt} = P_\Omega - P_R \qquad (13)$$

where P_R is the radiation loss power. Eq. (12) shows that such a constriction mechanism imposes remarkable properties of the power input source P_{in} which must operate, because of Eq. (11), as a constant current generator. In addition to cover for the excessive radiation losses it has also to provide power for the compression work on the channel and, further in addition and on the same short time-scale, an equal amount of power has to be delivered to increase the channel magnetic energy.

The Bennett relation, Eq. (11), is often found in plasma focus theory. As it can be shown that it is no more than a rather trivial static limit case of a far more general Bennett relation [23] which includes even the violently dynamic "snow-plow" model, it should be used with caution or be avoided.

6. THEORY OF PLASMA WHIRL STRUCTURES.

The starting point is the equation of motion for singly ionized ions when viscous effects and electon-ion friction, i.e. resistivity, are negligible

$$m_i n_i \frac{d\underline{V}_i}{dt} = en_i(\underline{E} + \underline{V}_i \times \underline{B}) - \text{grad } p_i . \tag{14}$$

With the function $\underline{a}$ as

$$\underline{a} = \underline{B} + \frac{m_i}{e} \text{curl } \underline{V}_i \tag{15}$$

applied to the identity (3) and then the use of Eqs. (5), (6) and (7) it is not too difficult to rewrite Eq. (14) in its integral form pertaining to a closed loop attached to the moving, in particular rotating, ion gas, velocity $\underline{V}_i$

$$m_i \frac{d}{dt} \oint \underline{V}_i \cdot d\underline{s} = -e\frac{d}{dt} \int_{\underline{V}_i} \underline{B} \cdot d\underline{S} - \oint_{\underline{V}_i} \frac{1}{n_i} \text{grad } p_i \cdot d\underline{s} . \tag{16}$$

$d\underline{s}$ is the loop length element and $d\underline{S}$ is the element, directed normally, of a simple closed surface which has its boundary given by the loop.

In the following we shall consider the variations in time and space of a magnetic field of uniform direction, $\underline{B} = B\hat{z}$, and azimuthal symmetry, $B = B(r)$. With an ion density distribution with the same rotational symmetry as the magnetic field Eq. (16) can be simplified as

$$m_i \frac{d}{dt} (V_{i\varphi} r) = - e \frac{d}{dt} \int_0^r B(\xi) \xi d\xi , \quad r = r(t) \tag{17}$$

In the Maxwell equation relating magnetic field and current density

$$\text{curl } \underline{B} = \mu_0 (\underline{j} + \text{curl } \underline{M}) \tag{18}$$

we distinguish between the charge transport by free particles, denoted by $\underline{j}$, and that by the magnetically bound or "static" particle motion, curl $\underline{M}$, where the magnetization $\underline{M}$ is given by the density of the magnetic moments μ_e of gyrating electrons

$$\underline{M} = -p_{e\perp} \underline{B}/B^2, \quad p_{e\perp} = \frac{1}{2} n_e m_e V_{e\perp}^2 = n_e \mu_e B \tag{19}$$

It is important to note the partition of free and bound electron motion, curl $\underline{H}$ and curl $\underline{M}$, respectively, where $\underline{H}$ is the magnetizing field. Such a partition which is fundamental in classical theory of magnetized media is often uncorrectly claimed to be generally unsuitable for plasmas because of the non-linear relation between $\underline{H}$ and $\underline{M}$. A similar partition for the ionic motion, however, is not made here, because the evaluation of the free and definitely non-negligible ion inertia drift current will only bring back a single particle version of Eq. (14).

For the assumed cylindrical geometry the Maxwell equation (18) becomes

$$-\frac{1}{\mu_0} \frac{\partial B}{\partial r} = j_\varphi + \frac{\partial}{\partial r} \left(\frac{p_{e\perp}}{B}\right) \tag{20}$$

where the free particle current j_φ is the sum of the total ion motion and azimuthal drift part of the electronic motion. In the first order orbit theory only the electric field drift and the magnetic field gradient drift enter for these electron drift motions

$$j_{e\varphi} = e n_e \frac{E_r}{B} + \frac{p_{e\perp}}{B^2} \frac{\partial B}{\partial r} \tag{21}$$

and the free particle current becomes

$$j_\varphi = e n_i V_{i\varphi} + j_{e\varphi} \tag{22}$$

Eqs. (18), (21) and (22) give the total current

$$-\frac{1}{\mu_0}\frac{\partial B}{\partial r} = en_i V_{i\varphi} + en_e \frac{E_r}{B} + \frac{1}{B}\frac{\partial p_{e\perp}}{\partial r} \qquad (23)$$

The electronic part of this current can of course also be obtained from the usual equation of motion for electrons described as a massless fluid, i.e.

$$\underline{E} + \underline{V}_e \times \underline{B} + \frac{1}{en_e} \text{ grad } p_e = 0 \qquad (24)$$

Note that the field gradient part $-\frac{p_{e\perp}}{B^2}\frac{\partial B}{\partial r}$ of the "static" magnetization current in Eq. (20) is identically cancelled by the free magnetic gradient drift current in Eq. (21). This paradoxical cancellation was graphically proved by Tonks [24] to be caused by the change in Larmor orbit radius with the magnetic field. A remarkably similar cancellation will occur among the two remaining electron terms in Eq. (23). Consider the bracket below in the expansion of these terms

$$\frac{n_e}{B}\left[eE_r + \frac{\partial}{\partial r}\left(\tfrac{1}{2} m_e V_{e\perp}^2\right)\right] + \mu_e \frac{\partial n_e}{\partial r} \qquad (25)$$

The electron energy gained for a radial displacement, $-eE_r\delta r$, must equal the corresponding change in kinetic energy

$$-eE_r\,\delta r = \frac{\partial}{\partial r}\left(\tfrac{1}{2} m_e V_{e\perp}^2\right)\delta r \qquad (26)$$

and the bracket terms in (25) cancel. A further simplification, not to be made here, would be to neglect electron pressure effects altogether and thus the last term diamagnetic current in (25). Actually, under the name of Vlasov-Fluid Model [25] there is such a model where the ions obey Eq. (14) but the electrons are postulated to be just a massless and charge-neutralizing fluid for which

$$\underline{E} + \underline{V}_e \times \underline{B} = 0 \qquad (27)$$

This Vlasov-Fluid Model has been proved remarkably successful, better than ideal magnetohydrodynamics, to describe stability properties of high-β plasmas.

The azimuthal mass velocity $V_{i\varphi}$ is eliminated between Eqs. (17) and (23). The result is

$$\frac{d}{dt}\left[\lambda^2 r \frac{\partial B}{\partial r} + \mu_o \mu_e \lambda^2 r \frac{\partial n_e}{\partial r} - \int_0^r B(\xi)\xi \, d\xi\right] = 0 \qquad (28)$$

where $\lambda = \lambda(r)$ is an important characteristic length called the collisionless ion skin depth, or ion plasma wavelength, and which depends on the ion density $n_i(r)$ and the ion mass m_i as

$$\lambda^{-2} = \mu_o e^2 n_i / m_i = \omega_{pi}^2 / c^2 \qquad (29)$$

E.g., for a deuterium ion density range 10^{17} to 10^{19} cm^{-3} or singly ionized argon between $2 \cdot 10^{18}$ and $2 \cdot 10^{20}$ cm^{-3} λ will range between 1 and 0.1 mm.

Eq. (28) can be integrated directly

$$\lambda^2 r \frac{\partial B}{\partial r} + \mu_o \mu e \lambda^2 \frac{\partial n_e}{\partial r} - \phi = C_o \qquad (30)$$

where

$$\phi(r) = \int_0^r B(\xi)\xi d\xi \qquad (31)$$

and, as seen from Eq. (17), the integration constant C_o is a constant of motion and is proportional to the canonical, i.e. matter-plus-field, angular momentum of the loop at any reference instant of time t_0 when also $r = r_0$, $n_i(r_0) = n_0$ and $\lambda = \lambda_0$. For compactness, the usual factor 2 π in expressions for flux, Eq. (31), and angular momentum has been dropped.

Eq. (30) can be written as a linear differential equation relating ϕ and r

$$\frac{\partial^2 \phi}{\partial r^2} - \frac{1}{r}\frac{\partial \phi}{\partial r} - \lambda^{-2}(\phi - C_0) = -\mu_0 \mu_e r \frac{\partial n_e}{\partial r} \tag{32}$$

provided that $\lambda(n_i)$ and the right hand side term can be expressed as known functions of r. However, $n_i(r)$ is known only to the extent of total ion number conservation within r when there is balance between ionization and recombination

$$\int_0^r n_i(\xi)\xi \, d\xi = \text{constant in time} \tag{33}$$

A fairly general density distribution is contained by

$$n_i r^\alpha = n_0 r_0^\alpha \quad , 1 < \alpha < \infty \tag{34}$$

$\alpha = 1$ would mean a line distribution of ions, $\alpha = 2$ is the important special case of retained relative ion density distribution everywhere upon plasma whirl compression or expansion, i.e. something like a "breathing" mode. $\alpha>2$ means a "piling up" of ions in the vicinity of r upon compression, and $\alpha=\infty$ means perfect attachment of the ions to a cylindrical piston surface at r. The general solution to Eq. (32) for any value of α within the stated limits has been given recently [26]. It is a rather complicated expression in hyperbolic Bessel functions. This investigation will be limited to the simplest but probably most realistic special case $\alpha = 2$, i.e. uniform compression or expansion. Then, but only then, the solution can also be expressed in simpler power functions.

The last term of Eq. (32) is readily made a known function of r by means of the quasi-neutrality condition $n_e = n_i$ followed by Eq. (34) with $\alpha = 2$ and finally the constancy of motion for electronic magnetic moments. The resulting differential equation becomes

$$\frac{\partial^2\phi}{\partial r^2} - \frac{1}{r}\frac{\partial\phi}{\partial r} - \lambda_o^{-2}\left(\frac{r_o}{r}\right)^2 (\phi - C_o + 2\, m_i\mu_e/e^2) = 0 \qquad (35)$$

Its solution is

$$\phi(r) = C_o - 2\, m_i\mu_e/e^2 + r(A_1 r^q + A_2 r^{-q}) \qquad (36)$$

where

$$q^2 = 1 + r_o^2/\lambda_o^2$$

To determine the space integration constants A_1 and A_2 with use of the time integration constant C_o the latter has to be split up into its flux part ϕ_o and its mechanical angular momentum part L_o

$$C_o = \phi_o + L_o, \quad \phi_o = \int_0^{r_o} B(\xi, t_o)\xi d\xi, \quad L_o = \frac{m_i}{e} V_{i\varphi o}\, r_o \qquad (37)$$

and, further, B(r) has to be calculated from eq. (36). It is found

$$A_1 = \frac{1}{2q}\left[B_o r_o^{-q+1} - (L_o - 2\, m_i\mu_e/e^2)\,(q-1) r_o^{-1-q}\right]$$

$$A_2 = \frac{-1}{2q}\left[B_o r_o^{q+1} + (L_o - 2\, m_i\mu_e/e^2)\,(q+1) r_o^{q-1}\right] \qquad (38)$$

7. PHYSICAL INTERPRETATION OF WHIRL EQUATIONS

It is to be emphasized that the fields and fluxes obtained so far express no more than conservation of canonical angular momentum for that certain circular loop in the moving ion gas frame which is characterized by the set of initial conditions r_0, B_0, ϕ_0 etc. Singularities of $B(r)$ and $\phi(r)$ at $r \to \infty$ and $r = o$, respectively, simply mean that infinite expansion or compression of this loop would require infinite field strengths and fluxes with corresponding singular momenta for the loop mechanical angular momentum L

$$L = 2m_i\mu_e/e^2 - r(A_1\, r^q + A_2\, r^{-q}) \tag{39}$$

The effect of the "static" electronic motion due to the electron density gradient appears as a correction $2m_i\mu_e/e^2$ to the mechanical angular momentum. Under typical whirl conditions, e.g. an electron temperature of 100 eV, B = 10 Mgauss = 10^3T, r = 0.1 mm and deuterium ions this electronic term is about three powers of ten smaller than the flux part $\approx Br^2/2$ of the canonical angular momentum.

The striking feature of Eqs. (36) and (39) is their very strong dependence on the initial ion density n_0 contained in the parameter $r_0/\lambda_0(n_0)$ of the exponent q, see Eqs.(29) and (36). It is illustrating to study the limit $\lambda_0 \gg r_0$. In this case the flux $\phi(r)$ from Eqs. (36) and (39) is obtained as

$$\phi(r) = \frac{r^2}{2}\left[B_0 - \frac{1}{2\lambda_0^2}\,(L_0 - 2\, m_i\mu_e/e^2)\right] \approx r^2\, B_0/2 \tag{40}$$

The physical interpretation of this result is the following:

When $\lambda_0 \gg r_0$ the ions are so few and heavy that the inductive effects associated with their motion from r_0 to r are insufficient to change the initial field B_0.

In the opposite limit, $\lambda_o << r_o$, it can be shown

$$|B_o\, r_o\, \lambda_o| \approx |L_o - m_i\, 2\mu_e/e^2| \tag{41}$$

The factor $\lambda_o << r_o$ in Eq. (41) means that the mechanical part of the total canonical angular momentum is small when n_i is large because of large flux-creating ionic current then carried at small or moderate ionic velocity.

Clearly, any electromagnetic transfer process must obey both momentum and energy laws. As stated, Eq. (36) concerns conservation of the canonical angular momentum of a magnetized plasma and it permits any large value of the magnetic flux provided it is counteracted by a correspondingly large mechanical angular momentum. Transfers between mechanical and magnetic energy densities severly limit this large range. Actually, an inspection of Eqs. (36) and (38) proves that, because of the strong power dependence, the effects followed upon a change in r can be of realistic magnitude only for $r_o^2/\lambda_o^2 \approx 1$. This would mean a border region of strong field-plasma interaction with a depth of about λ_o if one tries to magnetically compress a plasma column. Classical MHD theory predicts a thin and conductivity-dependent skin dept $\delta(T_e)$ as the interaction region. That has never been observed. Experiments [27] show instead a penetration depth equal to $\lambda\ (n_i, m_i)$. As a result, a new species [28] in the plethora of instability theories soon appeared to resolve the conflict. It was so constructed that it modified $\delta(T_e)$ to yield the desired $\delta(n_i, m_i, T_e)$.

8. PLASMA WHIRL PROPERTIES

The theory above strongly supports the view, expressed already in Ref. (11), that the magnetized whirl structures observed in plasma foci are examples of magnetically induced plasma rotation [10]. If we assume that their observed diameter is a few times the derived characteristic length λ (m_i, m_i) for the field and plasma density variations, then there is agreement within the experimentel accuracy between λ as observed and as calculated from measurements of their plasma density [2, 3, 6].

The whirls carry a mass M and have a moment of inertia J of the order

$$M \approx \pi \lambda^2 n_i m_i, \; J \approx M \lambda^2, \quad \lambda^2 = m_i/(\mu_0 e^2 n_i) \qquad (42)$$

With their ion Larmor gyration speed they contain, as expected, a kinetic energy about equal to that of their axial magnetic field B_z

$$W_{kin} = \frac{1}{2} J\omega^2 = \frac{1}{2}\pi\lambda^2 B_z^2/\mu_0 = W_{zmagn}, \; \omega = eB_z/m_i \qquad (43)$$

However, the equality is probably not strict, rather the opposite, i.e. conversions between kinetic and magnetic energy should be possible, consistent with the reasonings presented, which were based on the conservation of only the total kinetic plus-magnetic field angular momentum. It can be noted that such an energy transfer does probably not lead to an expansion or a compression of a magnetized whirl structure. From the following simplified form of the generalized Bennett relation [23] for a whirl structure of radius R

$$\text{Acc. term} = W_{kin} + W_{zmagn} + \frac{2}{3} W_{th} - \frac{\pi}{2} R^2 B^2(R)/\mu_0 \qquad (44)$$

it is seen that the left hand side radial acceleration term, discussed at length in Ref. (23), will not be affected by an energy-conserving transfer between the kinetic energy, W_{kin}, and the axial field energy, W_{zmagn}. The following gas pressure term, actually the right hand side of the original Bennett relation, Eq. (11), can be expected to be small, mainly because of radiation cooling. The pressure at R has been taken negligible in Eq. (44) so that confinement is provided by the magnetic field pressure from the total field $\underline{B}$

$$\underline{B}(R) = B_z(R)\hat{z} + B_\theta(R)\hat{\theta} \qquad (45)$$

In Eqs. (44) and (45) one can recognize the wellknown fact that a necessary, but not sufficient, confinement condition for the plasma in the "magnetically stabilized" Z-pinch is the increase with radius of the total magnetic pressure. Like in the original Bennett relation, Eq. (11), B_θ is generated by the axially flowing electronic current supplied by the electrodes. B_z can also be expected to derive in part from the same external sources in accordance with the general property of strong current discharges to relax towards the energetically favorable force-free helically structured current density and magnetic field strength distributions [29].

In the simplified form given here, Eq. (44), of the generalized Bennett relation [23],effects of radiation fields and excess electrical charge have not been included. It should be noted, however, that these standard simplifications may not be generally good for the fast and complicated processes in plasma focus discharges.

A few additional points can be made about the derived whirl structures. The Stevens group has long argued [2, 3, 6] that these belong to a general discharge fine structure that normally escapes notice in large and powerful experiments and, when observed, little attention is paid to it. Experimentally, the neutron yield from intermediate and large plasma foci has been found to scale with the fourth power, or nearly that, of the total focus plasma current [1]. Regardless of whether the neutron-producing collisions are thermal or not such a power dependence implies a very pronounced deutron density increase by magnetic compression.

However, whereas whirls of the density ≈ 10^{19} cm^{-3} and the corresponding radius ≈ 0.1 mm have been recorded in detail in small devices the theoretically expected and more energetic whirls with radii as small as 10μm and 1 μm for the peak densities 10^{21} cm^{-3} and 10^{23} cm^{-3}, respectively, can hardly be expected to be discerned as individual and separated structures if they, like those actually seen on X-ray pinhole camera photographs in the small experiments, move swiftly to form a radially contracting bundle.

The observed stability of the whirls contrasts with the erratic behaviour of constricted high current gas discharges. It is likely that the mass rotation provides stability. Actually, stabilization by means of a mechanically impressed plasma vortex motion, although poorly understood, has been a technique known and used to stabilize gas discharges since the beginning of this century [30]. A problem closely related to that of the stability is the friction losses, i.e. the Ohmic energy dissipation, in the relative rotational motion between the electrons and ions in a whirl. It can be shown that the strongly anisotropic conductivity reduces this friction, actually, the decay time for vorticity been proved [31] to be increased by up to a factor $1 + \beta^2$ where β is the plasma Hall parameter which safely can be taken $\beta >> 1$ for the considered very strong magnetic fields.

9. NUCLEAR FUSION REACTIONS

There seems to be general agreement that measurements on the emitted neutron distribution in time, space and energy indicate at least two neutron producing mechanisms, probably several. It is doubtful whether the experiments can be interpreted so as to support an assumption that any of these could be considered "thermonuclear". As stressed in a very recent discussion [32] on this matter the observed neutron production is more likely to be the result of several competing and interacting mechanisms, none of which is well known.

An often made observation is that the neutron emission period is preceded by one or more plasma channel constrictions, usually labelled as m = 0 ("sausage") instabilities. Coincident with these constrictions there are violent variations in the total electronic current of the plasma channel. Ref. 6 associates the constrictions with a somewhat complicated build-up of a multiplicity of toroidal structures having closed-loop circulation. Upon their disruption particles are accelerated by either a betatron accelerating process or the appearance of charge separation fields.

Our presented whirl structure description does not provide a solution to the question of the origin of the accelerating processes that lead to fusion reactions. However, it does provide some aspects which may be of relevance.

i) If, as observed in the Stevens experiment [6], the wirl structures are more or less retained during the strong current variation, then the axial magnetic flux from the externally impressed helical electron current flow will act during its rise as the driving field along a theta-pinch coil. Note that the theta pinch remains unsurpassed in transferring kinetic energy to a plasma, and this in accordance with the here described conservation of matter-plus-field angular momentum [26]. (Explained differently, the theta pinch arrangement is a poorly constructed betatron with the so-called 2:1 field rule forgotten so, when it acts on the ions in a plasma, it tends to drive them to the coil axis. For a single ion and perfect coil this focussing to the axis would be perfect, regardless of the ion initial position and velocity! [33])

ii) In one or a few regions of the expanding plasma, long after its maximum compression, a seemingly electrostatic acceleration of the deutrons occurs. This is often stated to be the main neutron production mechanism [1, 4]. The remarkable feature is that the total voltage of this accelerating field seems far higher then the maximum voltage supplied initially to the plasma focus electrodes. A non-repetitive mechanism might be coherent radiation effects, extensively studied as the cause of the electromagnetic pulse (EMP) from high altitude nuclear explosions. Such radiation effects are indeed negligible under more usual plasma conditions but not neccessarily so for the plasma particle motion under those extreme rates of change, nanoseconds, of field strengths, also reported [2, 3, 6] extreme, $\approx 10^3$ T.

The lowest order relativistic correction can indeed also be written as an electric field of gradient character. It is given here for completeness and not too difficult to derive from Ref. 34, Eq. (12.90).

$$\underline{E}^i_{rad} = - \frac{\mu_0}{8\pi} \sum_k e_k \frac{\partial^2}{\partial t^2} \hat{r}_{ik} \qquad (46)$$

$\underline{E}^i$ is the radiation field strength at the field point $\underline{r}_i$ and caused by the acceleration of the source points with charges e_k which are located at $\underline{r}_k$, $\hat{r}_{ik}$ is the unit length vector directed between $\underline{r}_K$ and $\underline{r}_i$.

In conclusion it is remarked that the plasma focus physics is still a rather unexplored field of science. For the experimentalists it requires diagnostic techniques for the space and time resolution exceeding those in better explored areas of fusion and plasma research. For the theoreticians the situation is somewhat similar. Standard simplifications and usual line-of-thought may, or may not, be valid under the observed extreme plasma conditions. To achieve a good understanding is indeed a formidable task but also an exciting challenge!

References

1. A. Bernard, P. Cloth, H. Conrads, A. Coudeville, G. Gourlan, A. Jolas, Ch. Masonnier, J.P. Rager, "The dense plasma focus - a high intensity neutron source". Nucl. Instruments and Methods, 145, 191, 1977.

2. W.H. Bostick, V. Nardi, and W. Prior, "Production and confinement of high-density plasmas". Annals of the New York Academy of Sciences, 251, May 8, 1975.

3. W.H. Bostick, V. Nardi and W. Prior, "X-ray fine structure of dense plasma in a co-axial accelerator". J. Plasma Phys., 8, 7, 1972.

4. P. Cloth and H. Conrads, "Neutronics of a dense-plasma focus - an investigation of a fusion plasma". Nuclear Science and Engineering, 62, 591, 1977.

5. G. Lehner, "Nuclear reactions in laboratory plasmas", Ch. 20 in "Reactions under plasma conditions", Ed. M. Venugopalan, Wiley, New York, 1971.

6. V. Nardi, "Megagauss fields by automodulating currents". Proc. Int. Conf. on Energy Storage, Compression and Switching, Asti-Torino 1974, p. 173. Plenum Press, New York 1976.

7. A.B. Vatazhin, "Deformation of the velocity profile in an inhomogeneous magnetic field". PMM, 31, 72, 1967.

8. E.A. Witalis, "MHD flow distortion caused by strong Hall effect", Plasma Physics, 10, 109, 1968.

9. A.K. Grigoriadi, Yu. P. Emets and A.P. Rashchepkin, "On the circulation currents in a plasma moving in the discontinuity region of a magnetic field". Teplofizika Vysokich Temperatur, 11, 913, 1973.

10. E.A. Witalis, "Rotational motion of magnetized plasmas". Plasma Physics, 10, 747, 1968.

11. W.H. Bostick, L. Grunberger, V. Nardi and W. Prior,"Vorticity in the current sheath of the plasma coaxial accelerator". Proc. 9th Int. Conf. on Ionized Gases, p. 66, Bucharest, 1969.

12. E.A. Witalis, "A magnetic field generating mechanism in the plasma from a coaxial plasma gun". Plasma Physics, 13, 507, 1971.

13. H. Alfvén and C.-G. Fälthammar, "Cosmical Electrodynamics", 2nd ed., p. 125. Oxford University Press, 1963.

14. E.A. Witalis, "Theory of magnetically induced rotation and a dynamo effect in laser plasmas". Proc. Troy Conf. on Laser Interactions and Related Phenomena, p. 237. Eds. Hora, Schwarz, Plenum Press, New York, 1974.

15. J.W. Dungey, "Torque on a plasma in terms of magnetic stress". Nucl. Fusion, 1, 312, 1961.

16. E.A. Witalis, "Magnetically induced plasma rotation". Z. Naturforsch., 29 a, 1138, 1974.

17. M.G. Haines, "On the origin of rotation in magnetically confined plasmas". Advances in Physics (Phil. Mag. Suppl.), 14, 167, 1965.

18. E.A. Witalis, "Theory for oscillations in MPD accelerators". AIAA, J., 7, 1383, 1969.

19. U. Kogelschatz, "Doppler-shift measurements of axial and rotational velocities in an MPD arc", AIAA J., 8, 150, 1970.

20. G. Decker et al. in Forschungsbericht 1977, p. 36. Institut für Plasmaforschung der Universität Stuttgart.

21. J.W. Shearer, "Contraction of Z-pinches actuated by radiation losses". Phys. Fluids, 19, 1426, 1976.

22. R. Becker and F. Sauter, "Theorie der Elektrizität" 19. Aufl., p. 148. Teubner, Stuttgart, 1968.

23. E.A. Witalis, "Z-pinch dynamics from the generalized Bennett relation". Phys. Rev. A, 10, 2373, 1974.

24. L. Tonks, "Particle transport, electric currents, and pressure balance in a magnetically immobilized plasma". Phys. Rev., 97, 1443, 1955.

25. R.L. Morse and J.P. Freidberg, "Vlasov-fluid model for studying gross stability of high-beta plasmas". Phys. Fluids, 15, 1102, 1972.

26. E.A. Witalis, "Theta pinch description from classical electrodynamics". Int. J. of Fusion Energy, 2, No. 2, 1979.

27. A.A. Newton, "Distributions of plasma density and beta in high-beta pinches". Nucl. Fusion, 13, 759, 1973.

28. H.A.B. Bodin, J. McCartan, A.A. Newton and G.H. Wolf, "Diffusion and stability of high-beta plasma in an 8-metre theta pinch". Prof. Int. Conf. on Fusion Research, Novosibirsk 1968. IAEA, Vienna, 2, p. 533, 1969. (See in particular the five assumptions (a)-(f) on p. 542.)

29. J.B. Taylor, "Relaxation of toroidal plasma and generation of reverse magnetic fields". Phys. Rev. Lett., 33, 1139, 1974.

30. O. Schönherr, "Über die Fabrikation des Luftsalpeters nach dem Verfahren der Badischem Anilin- und Sodafabrik".Elektrotechn. Z., 30, 365, 1909.

31. E.A. Witalis, "Vorticity suppression time for a Hall effect plasma". Plasma Physics, 13, 939, 1971.

32. C. Gourlan, H. Kroegler, Ch. Maisonnier, J.P. Rager, B.V. Robouch and A. Gentilini, "Recent progress in the 1 MJ plasma focus dynamics and the scaling for the neutron production". Paper CN-37-U-3-1, IAEA Fusion Conf., Innsbruck, 1978.

33. V.P. Krivets and B.P. Peregood, "Plasma heating and compression when nonadiabatic motion of particles". Phys. Letters, 31A, 177, 1970.

34. J.D. Jackson, "Classical Electrodynamics", p. 409, Wiley, New York, 1962.