CONF-790802--29

BNL-NUREG-25902

NONLINEAR RESPONSE TO THE MULTIPLE SINE WAVE EXCITATION OF A SOFTENING-HARDENING SYSTEM*

B. Koplik, M. Subudhi, J. Curreri
Structural Analysis Group
Department of Nuclear Energy
Brookhaven National Laboratory
Upton, NY 11973 U.S.A.

Abstract

In studying the earthquake response of the HTGR core, it was observed that the system can display softening-hardening characteristics. This is of great consequence in evaluating the structural safety aspects of the core. In order to obtain a better understanding of the governing parameters, an investigation was undertaken with a single-degree-of-freedom system having a softening-hardening spring characteristic and excited by multiple sine waves. By introducing a cubic nonlinearity in stiffness, the governing differential equation takes on the form:

$$m\ddot{x} + c\dot{x} + \alpha x + \beta x^3 = f(t)$$

This type of system has as many as five roots at a given excitation amplitude and frequency. A parametric study varying the input amplitudes and the spring characteristic were performed. Transients were introduced into the system, and the jump phenomena between the lower softening characteristics to the higher hardening curve was studied.

Numerical results were obtained for the case of two sine wave inputs of different amplitudes applied to a softening spring. In order to generate the resulting curves, it was necessary to sweep down in frequency. By choosing four different combinations of sinusoidal inputs, results were plotted for amplitude vs. frequency. These curves clearly demonstrate the existence of a sizable subharmonic contribution. Examination of several combinations of sine excitations makes it possible to establish large discontinuities in the response. In particular, a large discontinuity was noted in the vicinity of 25 cycles/sec. For this case the jump in response increased by a factor of 3.5. This kind of discontinuity was shown to exist in the case of the hardening spring as well as the softening spring which is of significance because the restoring force curve for the HTGR is actually softening-hardening.

Further investigation into the dynamics of a single-degree-of-freedom system with a cubic nonlinearity was carried out by introducing an exponential decaying term in each of the components of the sinusoidal forcing function such that:

$$f(t) = F_1 e^{-\alpha_1 t} \cos wt - F_2 e^{-\alpha_2 t} \cos 3wt$$

Two sets of runs were made—the first with $\alpha_1 = 0$ and the second with $\alpha_2 = 0$. For this first case as α_2 becomes smaller, the duration of the second term becomes larger. Finally, when $\alpha_2 = 0$ the maximum value of the upper root of the nonlinear response curve is attained. With $\alpha_2 = 0$ and choosing constant values for α_1, the response becomes subharmonic with the 3 wt term putting energy into the third harmonic component of motion. The first term is necessary only to provide enough starting motion to have a third harmonic component that is not masked by the damping energy. For values of α_1 from 0.1 to 0.058, the response is at the lowermost end of the nonlinear curve. Since there is insufficient time for the amplitude to build up, a large subharmonic response cannot develop. However, for $\alpha_1 \lesssim 0.056$ the maximum value of the upper root is attained. The critical deviation of the first term corresponds to the damping coefficient α_1 taken equal to 0.056. The decaying exponential was introduced to shorten the duration of the term to which it is associated. This study was undertaken because of the significant effect that earthquake duration is known to have on the dynamic response of the structure.

MASTER

* Work performed under the auspices of the U. S. Nuclear Regulatory Commission.

1. Introduction

The response of a single-degree-of-freedom system having a softening-hardening spring characteristic and excited by multiple sine waves was investigated. In order to accommodate a nonlinear restoring force the spring stiffness was represented by:

$$F(x) = \alpha x + \beta x^3 \tag{1}$$

By properly choosing the values of the coefficients α and β the overall spring characteristic was used to study hardening, softening, as well as softening-hardening systems. This type of cubic nonlinearity was introduced to simulate the response of a fuel block in a HTGR undergoing an inline collision with other elements separated by clearance gaps in a damped system.

With the cubic nonlinearity in stiffness, F(x) included the governing differential equation of motion becomes:

$$m\ddot{x} + c\dot{x} + \alpha x \pm \beta x^3 = f(t) \tag{2}$$

The various options for choosing the restoring force vs. spring deflection is given in Figure 1. In Case A with α=1000 and β=0, the linear force is shown as a straight line. For Case B with β increased to 100,000, the hardening characteristic is revealed. By changing the sign of β from positive to negative, the spring characteristic changes from hardening to softening as expressed in Case C. The final curve shown as Case D allows for the investigation of a more complex softening-hardening system. As shown in Figure 1, the curve is initially softening for values of x such that $|x| \leq .08$ and then becomes hardening for $|x| \geq .08$. A constant term is introduced in the restoring force to maintain continuity as the spring shifts from softening to hardening.

2. Softening-Hardening Response

At a given excitation amplitude and frequency, the softening-hardening system can yield as many as five roots. A parametric study was carried out in which the input amplitudes and the spring characteristics were varied. Figure 2 shows the dynamic response associated with each of six sinusoidal forcing functions and demonstrates the jump phenomena observed in earlier investigations [1-4]. The forcing function denoted by A represents the first harmonic while B represents the third harmonic. All of the remaining forcing functions from C to F are combinations of the first and third harmonic with varying amplitudes. By sweeping up or down in frequency, the responses for curves C, D, and E remain on the lower or softening curves. However, curve F shows the upper curve of the hardening system. In this case when a sweep of frequency is undertaken and a sudden disturbance is introduced as designated by point P, a jump from the lower (softening) curve to the higher (hardening) curve is observed. As shown in Figure 2, the jump phenomena occurs at approximately 46 cycles per second.

3. Nonlinear Softening Response

In order to study the softening characteristics above, the following governing equation was introduced:

$$\ddot{y} + 1.256\,\dot{y} - 20{,}000\,y^3 = F(t) \tag{3}$$

with F(t) chosen to represent four different combinations of sinusoidal inputs as shown on Figure 3. The curves were generated by sweeping down in frequency and plotting amplitude vs. frequency for all four forcing functions. For the investigation of these responses the coefficient β for the cubic nonlinearity was reduced from 100,000 to 20,000 as given in eq. (3). The curves generated in Figure 3 clearly demonstrate the existance of a sizable subharmonic contribution. By examining several combinations of sinusoidal excitations, it is possible to establish large discontinuities in the dynamic response. In particular, a large discontinuity was noted in the vicinity of 25 cycles per second. For this case the jump in response increased by a factor of 3.5. This kind of discontinuity was shown to exist in the case of the hardening spring as well as the softening spring. Since the restoring force curve for the HTGR is actually softening-hardening, this result is of particular significance.

4. Forcing Function with Decaying Exponential

Further investigation into the dynamics of a single-degree-of-freedom system with a cubic nonlinearity was carried out by introducing an exponential decaying term in each of the components of the sinusoidal forcing function. For this study the governing equation was taken as:

$$\ddot{y} + 1.256\,\dot{y} + 1000\,y + 100{,}000\,y^3 = F(t) \tag{4}$$

with the forcing function given by:

$$F(t) = F_1 e^{-\alpha_1 t}\cos wt - F_2 e^{-\alpha_2 t}\cos 3\,wt \tag{5}$$

The results plotting the amplitude vs frequency are described in Figure 4. Two sets of runs were made, the first with α_1=0 and the second with α_2=0. For the first case as α_2 becomes smaller, the duration of the second term becomes larger. Finally, when α_2=0, the maximum value of the upper root of the nonlinear response curve is attained. With α_2=0 and choosing constant values for α_1, the response becomes subharmonic with the 3 wt term putting energy into the third harmonic component of motion. The first term is necessary only to provide enough starting motion to have a third harmonic component that is not masked by the damping energy. For values of α_1 from 0.1 to 0.058, the response is at the lowermost end of the nonlinear curve. Since there is insufficient time for the amplitude to build up, a large subharmonic response cannot develop. However, for $\alpha_1 \leq 0.056$ the maximum value of the upper root is attained. The critical deviation of the first term corresponds to the camping coefficient α_1 taken equal to 0.056. The decaying exponential was introduced to shorten the duration of the term to which it is associated. This study was undertaken because of the significant effect that earthquake duration is known to have on the dynamic response of the structure.

5. References

[1] REICH, M., BEZLER, P., KOPLIK, B., CURRERI, J. "Nonlinear Dynamic Response of a Multi-Element HTGR Core Structure", Trans. of ANS Meeting, Toronto, Canada, June 1976.

[2] CURRERI, J., BEZLER, P., "Multiple Sine Wave Excitation of a Hard Spring Oscillator", Shock and Vibration Symposium, Albuquerque, NM, October 1976.

[3] KOPLIK, B., REICH, M. "Nonlinear Dynamic Response of a Multi-Mass System with Gaps", Shock and Vibration Symposium, Albuquerque, NM, October, 1976.

[4] CURRERI, J., BEZLER, P., KOPLIK, B., "Higher Order Effects and Their Influence on the HTGR Core", Proceedings of the Japan-U.S. Seminar on HTGR Safety Technology, BNL Report 23196, September, 1977.

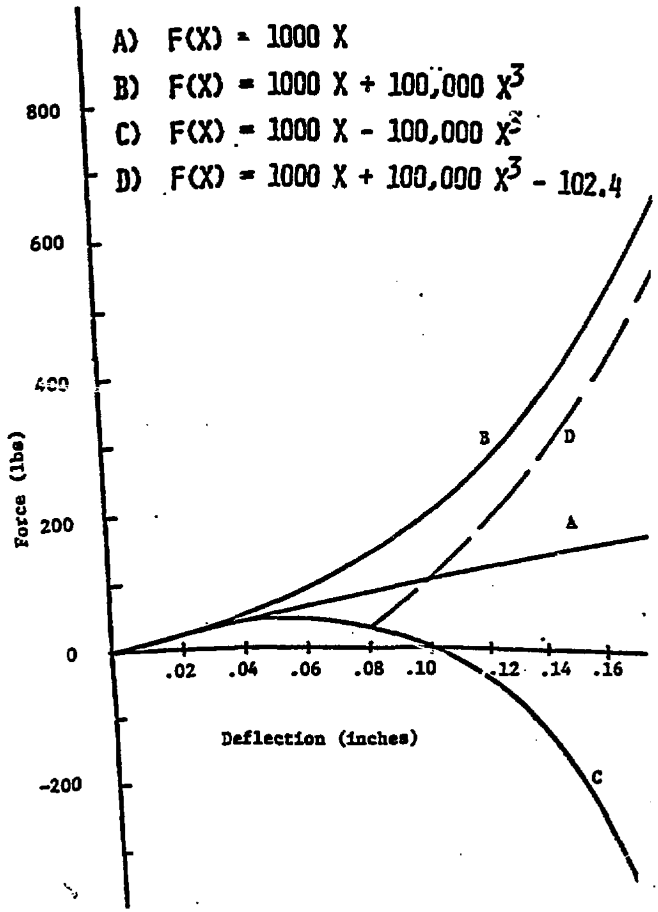

FIGURE 1 FORCE-DEFLECTION CURVE OPTIONS FOR ONE-DEGREE-OF-FREEDOM SYSTEM

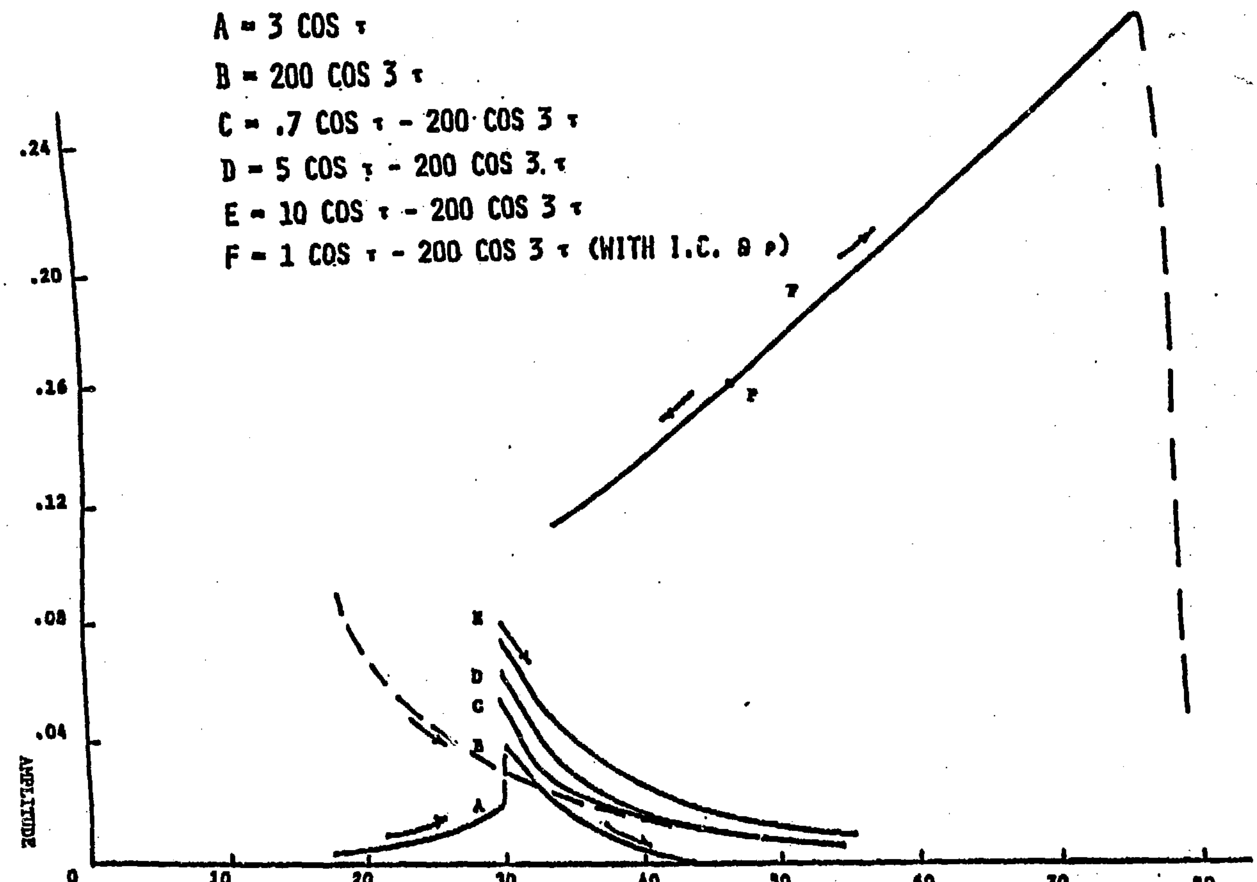

FIGURE 2 SOFTENING-HARDENING RESPONSES

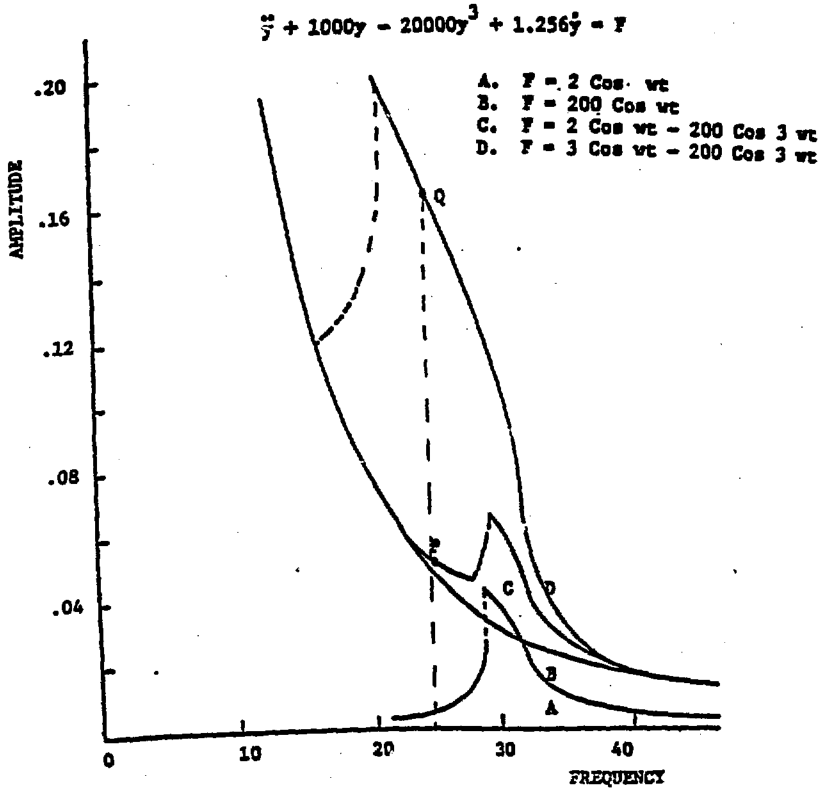

FIGURE 3 NONLINEAR RESPONSE OF A SYSTEM WITH CUBIC SOFTENING CHARACTERISTICS

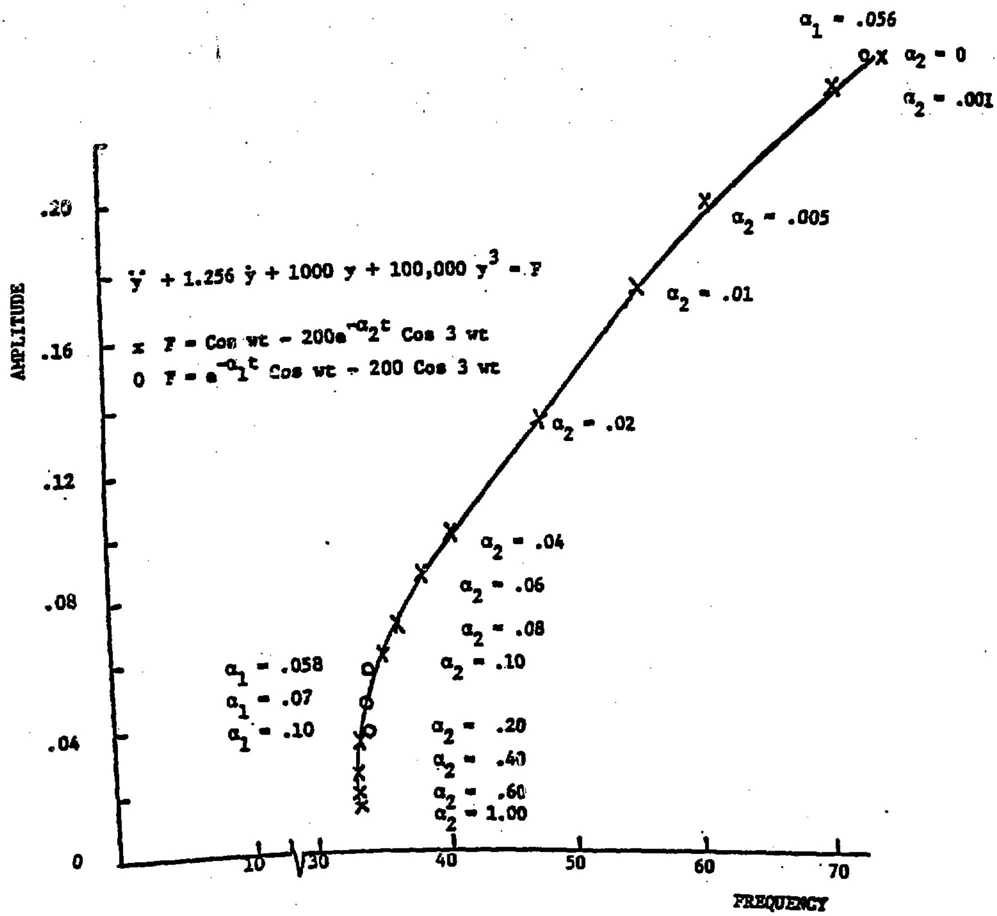

FIGURE 4 EFFECT OF DECAYING EXPONENTIAL ON MAXIMUM RESPONSE