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H. FANCHIOTTI, C.A. GARCIA CANAL and H. VUCETICH

CENTRE DE RECHERCHES NUCLEAIRES
UNIVERSITE LOUIS PASTEUR
STRASBOURG

Institut National de Physique Nucléaire et de Physique des Particules

Université Louis Pasteur de Strasbourg

67037 STRASBOURG-CEDEX FRANCE

ANGULAR ASSYMETRIES IN A SHIELDED PAIR LINE

H. FANCHIOTTI †, C.A. GARCIA CANAL ‡ § and H. VUCETICH ‡ §

Abstract

We present the capacitance matrix and surface charge density distributions of an unbalanced pair line with both longitudinal and balanced excitations. We discuss in particular the case in which the axes of the inner wires are not restricted to lie on a line through the axis of the shield

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† Departamento de Física, Universidad Nacional de La Plata, La Plata, Argentina.

‡ Physique Théorique des Hautes Energies, Centre de Recherches Nucléaires, F67037 Strasbourg Cedex, France , on leave of absence from Universidad Nacional de La Plata, La Plata, Argentina.

§ Member of the Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina.

1. INTRODUCTION

There has recently been an increasing interest in the resolution of electromagnetic problems involving multiwire conductors (Nordgard 1976, (Lenahan 1977). In particular the case of the shielded balanced and unbalanced pair has been treated with the proximity effects (Nordgard 1977), (Belevitch 1977), (Paul and Feather 1976).

In previous papers on these topics (Alessandrini, Fanchiotti, Garcia Canal and Vucetich 1974 and 1976), we have presented an alternative and general procedure of solving the multiconductor problem based on the theory of automorphic functions. This procedure allows us to give concise representations of the electric field distributions and capacitance matrices, as well as impedances, in terms of series expansions whose elements depend on the two-dimensional realization of the group of automorphisms. This approach does avoids the inversions of infinite matrices in order to describe the capacitances and charge distributions.

In this paper we present results of our approach concerning the shielded pair paramaters and charge distributions where the geometrical configuration ot the wires is not totally symmetric. In particular, we present the case in which the axes of the inner wires are not contained in the axis of the shield. This is the next natural extension of the calculations in Nordgard (1976 and 1977), which due to the series expansion technique used there cannot be easily extended to the asymmetric case presented here. This computation contributes to the knowledge of the variations of the capacitances due to possible innacuracies in the manufacture of the cable that have not been computed previously. In Lenahan (1977) the case of "off axis" conductors in a shield was considered from a different theoretical point of view, but there is no explicit results of the effect of asymmetries quoted there.

The electrostatic problem posed by a system of parallel cylindrical conductors can be reduced to the solution of the Laplace equation in a two-dimensional multiply-connected domain defined by the intersection of the conductors with the complex plane. This solution is found from a generalization of the well-known method of images. It was discussed at length in our previous paper (Alessandrini, Fanchiotti, Garcia Canal and Vucetich 1974) to which we refer the reader for the details concerning the mathematical techniques. Nevertheless, we briefly summarize the necessary steps for building the desired functions. First one must determine the images of the inner circular boundaries with respect to the shield. Then one introduces N projective transformations (if there are N boundaries) called the generators T_i that map the i-th boundary to its image. The next step is the construction of an infinite set of projective transformations $\{ T_\alpha \}$ by multiplying the basic T_i's, as well as their inverses, in all possible ways. This set is called the group of automorphisms of the domain and its elements generate all possible images of the line charges that lie within the domain. The second step is to construct a set of N analytic functions whose real parts are the solution of the electrostatic problem in which the i-th conductor has a net charge per unit length λ_i and all the remaining conductors have zero net charge. From the knowledge of these functions, one can obtain the capacitance coefficients and charge density of the conductors as well defined series involving the T'_α s .

2. THE SHIELDED PAIR LINE.

We have applied the above method to the calculation of the characteristic parameters and charge distributions of the shielded pair cable of Fig. 1. We begin by obtaining the necessary two generators (of matrices T_i) of the corresponding group of automorphisms. They have the general form

$$T_i = \begin{pmatrix} \dfrac{R}{r_i} & - \dfrac{R J_i}{r_i} e^{i\alpha_i} \\ \dfrac{J_i}{R r_i} e^{-i\alpha_i} & \dfrac{r_i}{R} \left(1 - \dfrac{J_i^2}{r_i^2} \right) \end{pmatrix} \qquad (1)$$

where the parameters are defined in Fig. 1.

When the radii of the inner conductors are small compared with the radius of the shielding cylinder, and when the inner conductors are not too close to each other or to the shield, the series involved in the calculations are very rapidly convergent and only the first terms are dominant. Notice that by keeping only the very first terms in our series in the balanced case one obtains the approximate expressions for the capacitance parameters already in use (King 1955).

We have choosen in our example a realistic cable geometry for the symmetrical case, already treated in the literature (Nordgard 1976) in order to compare our results. There geometrical parameters of the nominal multiwire cable are the following :

$$r_1 = r_2 = 12.587 \text{ mils} \quad ; \quad R = 73.175 \text{ mils}$$

$$J_1 = J_2 = 25.149 \text{ mils} \quad ; \quad \alpha_1 = 0° \quad ; \quad \alpha_2 = 180°$$

and the effective relative permitivity of the dielectric surrounding the cable is taken to be

$$\varepsilon_r = 2.026$$

For manufacturing information it is interesting to have details of the variations of the capacitances and charge densities when the geometry of the cable is changed. We have carried out calculations taking into account separate and simultaneous variations in r_2, J_2 and α_i $(i = 1,2)$. We have considered the following six cases :

Case I : One of the cables of the pair (the one labeled 2) either has 10% smaller or larger radius than the other cable of the pair.

Case II : One of the cables of the pair (the one labeled 2) is either 10% closer to or farther from the axis of the shield than the other cable of the pair.

Case III : One of the cables of the pair (the one labeled 2) has either 10% smaller or larger radius and is either 10% closer to or farther from the axis of the shield than the other cable of the pair.

Case IV : One of the cables of the pair (the one labeled 2) has its angular position varied 10% from the nominal case.

Case V : Both cables of the pair have their angular position varied 10% above the upper half plane (or below the lower half plane).

Case VI : All possible simultaneous variations of 10% in the radius, the position of the center and the angular position.

In Table I, we present the values of the capacitances to ground C_{g1} and C_{g2} together with the mutual capacitance C_m and the other elements of the capacitance matrix, which are defined as usual, for the six mentioned cases. We can ensure four significant digits, as stated in Table I, whenever one considers group elements of the set $\{ T_\alpha \}$ containing products up to four generators.Obviously, if one needs more precision it is sufficient to take more terms in the corresponding series. We remark that our precision is independent of the particular asymmetrical case considered.

3. DISCUSSION

The variations in the different capacitances shown in Table I are due to the proximity effects when the nominal parameters of the cable are changed. Our Table I can be analyzed in two parts. The first one, related to the changes in the radius of conductor 2 and in the separation between the inner cables, presents the same aspects as those reported in Nordgard (1977). In particular, we observe that the direct capacitance measured by C_{12} (or C_{21}) is larger when the radius increases with respect to the nominal case. The same effect is found when the separation of the inner conductor is smaller. These kind of variations are due to the fact that the conductors closely approach each other. Certainly, whenever simultaneous geometrical changes are opposite in the sense of approaching or not approaching the conductors involved, the effects tend to cancel the variations in the capacitances. Similar considerations are valid in discussing the results for the other capacitances. For example we find that the capacitance to ground increases whenever the change in the nominal parameters tend to displace the inner conductor closer to the shield.

The second part of the Table I is devoted to presenting our results concerning the angular variations in the positions of the inner cables with respect to the nominal values. If the only change is a variation of 10% in the angular position of cable 2, the capacitances remain almost unchanged (they vary at most 0.5%) since the separation between the inner cables or between one of them and the shield remains practically unchanged. When both inner conductors suffer an angular variation of 10% in such a way that they approach each other ($\alpha_1 = 18°$; $\alpha_2 = 162°$), the direct capacitance shows an incrase of the same order of magnitude as that due to changes in r_2 or J_2, as was commented on above.

Finally, for the case of simultaneous variations in all of the parameters, the situation is enterely similar to that discussed in relation with the first part of Table I, except for the case of the direct capacitances where, for the reasons already mentioned, the variations are greater. The maximum variation obtained was for the case of simultaneous changes in α_1, α_2 and r_2 by plus 10% and in J_2 by minus 10% where C_{12} (or C_{21}) changes by 30%.

Our results concerning the surface charge density on the different conductors are summarized in Fig. 2. We have presented only the results for the case in which the variations of the geometry of the cable are related with the angular position of the inner wires with respect to the diameter of the shield. We have considered both balanced and longitudinal excitations. Our results show that the maximum and the minimum of the corresponding curves are shifted with respect to the position for the nominal case as can be understood easily from geometrical considerations.

4. CONCLUSIONS

The capacitance matrix and charge distributions of the shielded pair line, for the balanced and longitudinal excitations, were computed using our general method for solving the electrostatic problem of a system of cylindrical parallel conductors. We have considered the various asymmetries of the system, by treating the cases in which there are variations in the radii of the wires and in their relative positions. In particular, the angular position of the inner conductors were varied in order to quantify these asymmetric effects. This computation contributes to the knowledge of the variations in the multiconductor capacitances due to possible innacuracies in the manufacture of the cable that have not been computed previously. It also shows the potentialities of our method of solution of the Laplace equation in a multiply-connected domain.

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	nominal values	Case I		Case II		Case III			
		changes in r_2		changes in J_2		changes in $\frac{r_2}{J_2}$			
		+ 10%	- 10%	+ 10%	- 10%	$\pm$ 10% (+, +)	$\pm$ 10% (+, -)	$\pm$ 10% (-, +)	$\pm$ 10% (-, -)
$C_{g1} = C_{11}$	0.5487	0.5415	0.5560	0.5552	0.5420	0.5482	0.5345	0.5622	0. 5496
$C_{12} = C_{21}$	0.2259	0.2422	0.2103	0.2065	0.2482	0.2209	0.2669	0.1926	0.2306
$C_{g2} = C_{22}$	0.5487	0.5869	0.5118	0.5686	0.5305	0.6087	0.5671	0.5300	0.4951
C_m	0.5002	0.5238	0.4768	0.4874	0.5163	0.5094	0.5420	0.4654	0.4911

Cases IV and V		Case IV							
changes in α(10%)		changes in α(10%) and in $\frac{r_2}{J_2}$							
α_2	α_1, α_2	α_2, (+, +) 10%	α_2, (+, -) 10%	α_2, (-, +) 10%	α_2, (-, -) 10%	α_1, α_2, (+, +)	α_1, α_2, (+, -)	α_1, α_2, (-, +)	α_1, α_2, (-, -)
0.5470	0.5419	0.5465	0.5328	0.5606	0.5481	0.5411	0.5275	0.5557	0.5434
0.2308	0.2467	0.2258	0.2730	0.1967	0.2356	0.2416	0.2830	0.2096	0.2515
0.5470	0.5419	0.6070	0.5655	0.5284	0.4935	0.6018	0.5604	0.5234	0.4885
0.5043	0.5177	0.5134	0.5473	0.4687	0.4952	0.5265	0.5647	0.4792	0.5087

FIGURE CAPTIONS

Fig. 1 Geometry of the shielded pair cable.

Fig. 2a Surface charge density versus azimuthal angle for balanced excitation. Changes in α_2.

Fig. 2b Surface charge density versus azimuthal angle for longitudinal excitation. Changes in α_2.

Fig. 2c Surface charge density versus azimuthal angle for balanced excitation. Changes in α_1 and α_2.

Fig. 2d Surface charge density versus azimuthal angle for longitudinal excitation. Changes in α_1 and α_2.

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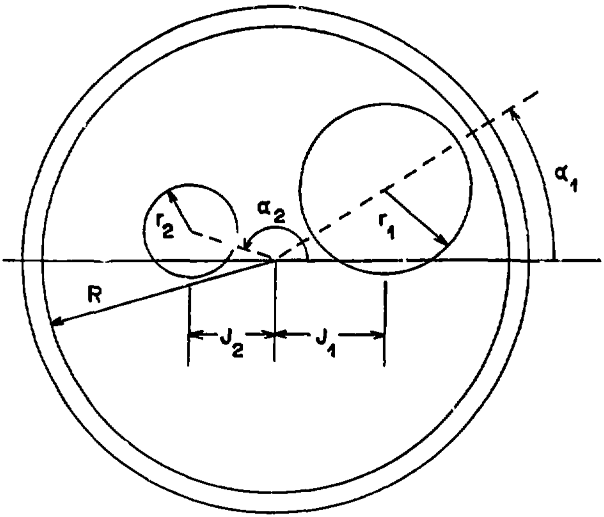
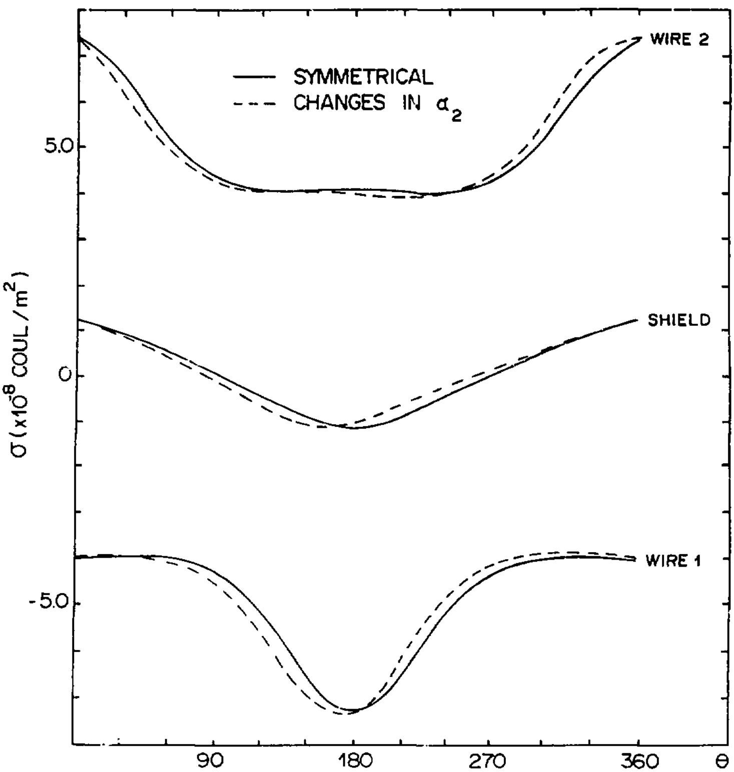
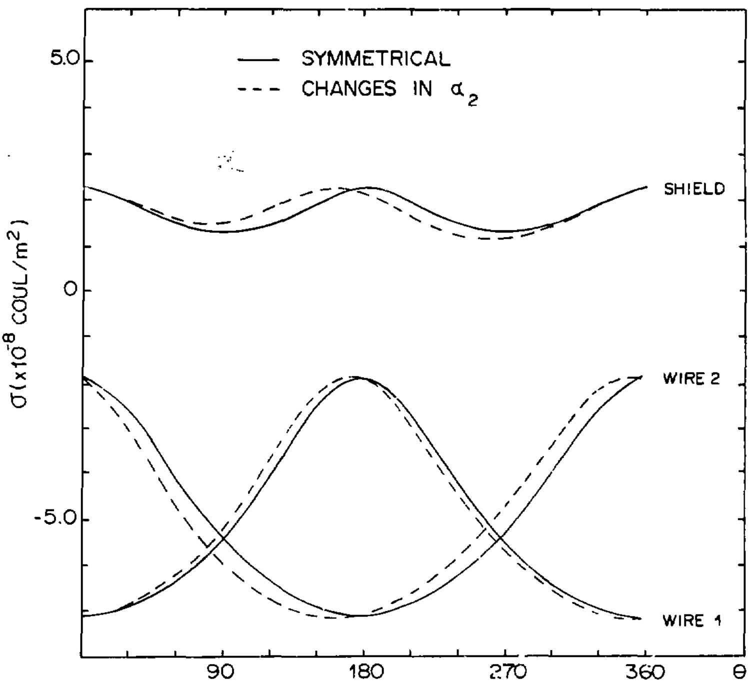


Fig. 1



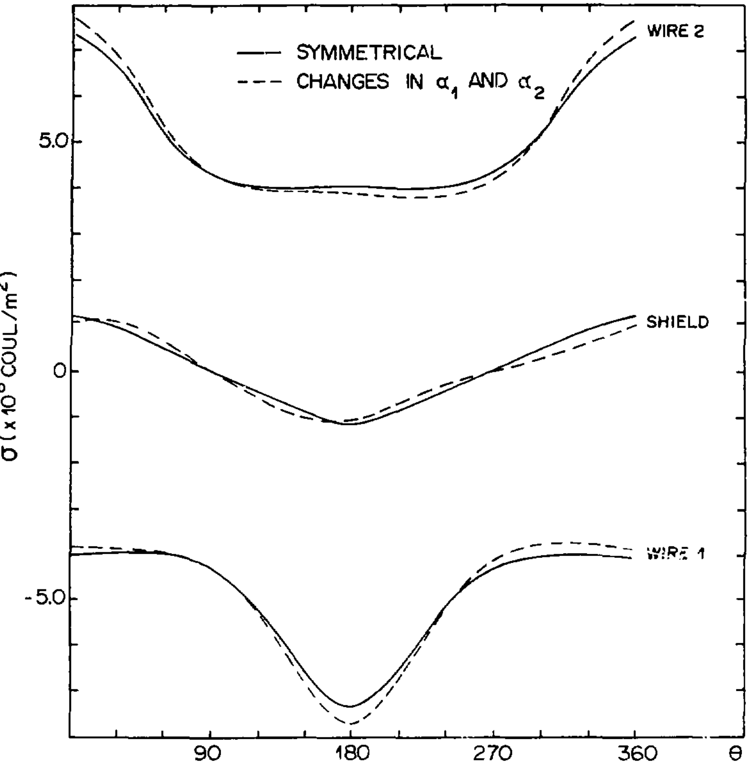
SYMMETRICAL
CHANGES IN α_2
WIRE 2
SHIELD
WIRE 1
5.0
0
-5.0
σ (x10^{-8} COUL/m^2)
90
180
270
360
θ

Fig. 2a



SYMMETRICAL
CHANGES IN α 2
5.0
0
-5.0
σ(x10⁻⁸ COUL/m²)
SHIELD
WIRE 2
WIRE 1
90
180
270
360
θ

Fig. 2b



SYMMETRICAL
CHANGES IN α_1 AND α_2
WIRE 2
SHIELD
WIRE 1
5.0
0
-5.0
σ (x 10^{-8} COUL/m^2)
90
180
270
360
θ

Fig. 2c

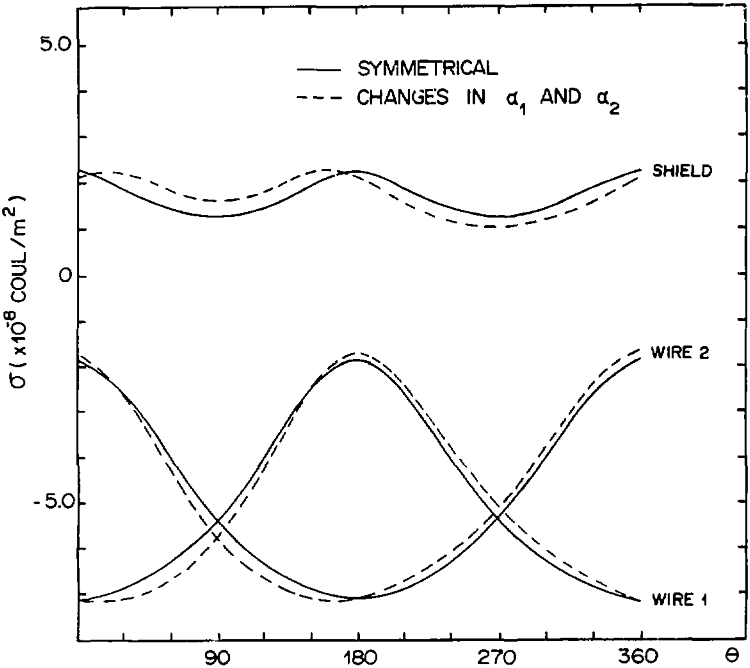

SYMMETRICAL
CHANGES IN α_1 AND α_2
SHIELD
WIRE 2
WIRE 1
σ (x10^{-8} COUL/m^2)
5.0
0
-5.0
90
180
270
360
θ

Fig. 2d

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