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ON THE ASYMPTOTICS OF BARYON FORM FACTORS IN
QUARK-GLUON MODEL

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Specific asymptotic relations for the nucleon form factors are obtained in the quark-gluon model in the lowest order of the perturbation theory. Changing of signs of the nucleon form factors and strong violation of the scaling law are predicted at $Q^2 \to \infty$. The relations between the $\gamma N \to N$ and $\gamma N \to \Delta$ transition form factors are also obtained.

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ОБ АСИМПТОТИКЕ БАРИОННЫХ ФОРМФАКТОРОВ В КВАРК-ГЛЮОННОЙ МОДЕЛИ

В кварк-глюонной модели получены конкретные асимптомотические соотношения для формфакторов нуклонов. При $Q^2 \rightarrow \infty$ предсказывается изменение знаков нуклонных формфакторов и сильное нарушение "масштабного" закона. Получены также соотношения между формфакторами переходов $\chi N \rightarrow \Delta$ и $\chi N \rightarrow N$.

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ON THE ASYMPTOTICS OF BARYON FORM FACTORS IN
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1. Introduction.

Some interesting predictions at high energies and large momentum transfer $Q^2$ have been obtained from the quark structure of hadrons. That is, the quark-counting rules for the exclusive [1] and inclusive [2] processes at large angles as well as asymptotic properties of form factors at large $Q^2$ [3-5] and the properties of hadronic structure functions in the $X \rightarrow 1$ limit in the quark-gluon model.

At the same time it is reasonable to assume in addition, that quarks in nucleon are nonrelativistic. Indeed, the successful application (see, for instance, Ref. 8) of the additive quark model apparently shows that nucleons are weakly bound systems. As is shown in Ref. 9, the correct values of nucleon magnetic moments can be obtained only in the nonrelativistic approximation; for relativistic quarks there are no definite relations between magnetic moments of proton and neutron. The additive quark model gives quite acceptable values of the magnetic and electric quadrupole moments for $\gamma N \rightarrow \Delta$ transition, that is, an evidence in favour of the nonrelativistic description of quarks in the $\Delta$-isobar as well.

In this paper the definite relations between the nucleon form factors, as well as those between the nucleon and the $\gamma N \rightarrow \Delta$ transition form factors,
are obtained in the quark-gluon model based on the interaction of coloured
quarks with the octet of coloured vector gluons. Following Refs. 3-7, we
suppose that in the $Q^2 \to \infty$ limit the $Q^2$-dependence of the form factors
is given by gluon exchanges between quarks in the lowest order of the per-
turbation series, namely, by the graphs shown in Fig. 1, the
incoming and outgoing quarks carrying the finite fraction of the correspond-
ing hadron momenta. As far as the four-momentum squares of virtual quarks
and gluons are large ($\sim Q^2$), the application of the perturbation theory
is justified in accordance with the idea of asymptotic freedom. In addition
we assume that quarks in nucleons (as well as in $\Delta$) can be considered as
nonrelativistic, so one can write $p_i \approx p_2 \approx p_3 \approx P/3$ and $p'_i \approx p'_2 \approx p'_3 \approx P/3$
for the four-momenta of quarks, where $P$ and $P'$ are initial
and final baryon momenta, respectively.

2. Vertices of the $N(\Delta)$ transitions into three quarks and
diagrams of the $\gamma N \to N$ and $\gamma N \to \Delta$ transitions.

Let us write the $3q \to P$ and $3q \to \Delta^+$ vertices in the form

$$i \sqrt{3} f_{N(p_1, p_2, p_3)} \epsilon_{ijk} \vec{U}_P(p_3) U^k(p_3) [\bar{U}^i(p_1) C \gamma_5 U^j(p_2) - \bar{U}^i(p_1) C \gamma_5 U^j(p_2)] +$$

transmutations of $(p_1, p_2, p_3)$,  \(1\)
\[ f_{\Delta}(p_1, p_2, p_3) \epsilon_{ijk} \left[ \bar{U}^\Delta(p) U^k_d(p_3) (\bar{U}^i_u(p_1) C^\gamma_i U^j_u(p_2)) + \bar{U}^\Delta_u(p_3) (\bar{U}^i_u(p_1) C^\gamma_i U^j_u(p_2)) + \bar{U}^i_u(p_1) C^\gamma_i U^j_u(p_2) \right] \]

transmutations of \((p_1, p_2, p_3)_\Delta\),

where \(i, j, k\) are colour indices, the bottom indices \((u, d)\) of spinors denote the type of the quark, \(\bar{U}(p)\) is the wave function of \(\Delta\)-isobar, \(f_{N(\Delta)}(p_1, p_2, p_3)\) is the vertex function, describing the distribution of quarks in baryon, \(C\) is the charge conjugation matrix. In the case of exact \(SU(6)\)-symmetry the functions \(f_N\) and \(f_\Delta\) should be equal and symmetric relative to the transmutations of arguments. In the nonrelativistic limit \((p_1 \approx p_2 \approx p_3 \approx P/3)\) all other forms of \(3q \rightarrow P\) and \(3q \rightarrow \Delta\) vertices are reduced to (1) and (2), respectively. The coefficient \(i\sqrt{3}\) in (1) is due to the relative phase and the normalization of nonrelativistic wave functions of the nucleon and isobar.

The graphs for \(\gamma P \rightarrow P\) and \(\gamma P \rightarrow \Delta^+\) transitions corresponding to the vertices (1) and (2) are shown in Fig. 2. The other graphs for these transitions can be obtained by the replacements of the second and third quark lines. The dashed blocks in Fig. 2 include all possible gluon exchanges in the lowest order of perturbation series according to the graphs in Fig. 1, and are symmetric relative to the above mentioned replacements. So the matrix elements corresponding to all possible diagrams of \(\gamma P \rightarrow P\) and \(\gamma P \rightarrow \Delta^+\) transitions are reduced to those of graphs shown in Fig. 2.

When writing the matrix elements in accordance with the usual Feynman rules, the part of those between \(C\gamma_5(\gamma_B)\) and \(\gamma_5 C\) matrices are
transposed. For instance, if the matrix elements \( M_i \) \((i = a, b, c, d, e)\) corresponding to the graphs of Fig.2 are written in the form

\[
M_i = \frac{3}{(2\pi)^6} \frac{8}{3} \left\{ \prod_{r=1,2,3} \left( p_r^2 - m^2 + i\epsilon \right) \prod_{s=1,2,3} \left( p_s^{\prime 2} - m + i\epsilon \right) \right\} f_N(p_1, p_2, p_3) f_N'(p_1', p_2', p_3') K_i (p_1, p_1') d^4 p_2 d^4 p_3 d^4 p_2' d^4 p_3',
\]

(2)

where \( m \) is the quark mass, then we have for the graph of Fig.2c

\[
K_c^\mu = \frac{8\sqrt{3}}{3} U(P)(\hat{p} + m) \Gamma_1 (\hat{p} + m) \gamma_5 \left[ \gamma_2 (\hat{p} + m) \Gamma_2 (\hat{p} + m) \right]^T \gamma_5 (\hat{p} + m) \Gamma_3 (\hat{p} + m) U(P),
\]

(4)

The coefficient \( 8/3 \) in (3) is common to all diagrams of the lowest order in \( \alpha_s Q^2 \) and arises due to \( \lambda_{\alpha_s}/2 \) matrices in the quark-quark-gluon vertices. Note, that when writing matrix elements of graphs of Figs.2a,2b, there arise traces corresponding to closed loops. For instance, for the graph of Fig.2a we have:

\[
K_a^\mu = \frac{8\sqrt{3}}{3} U(P)(\hat{p} + m) \Gamma_1 (\hat{p} + m) \gamma_5 \left[ \gamma_2 (\hat{p} + m) \Gamma_2 (\hat{p} + m) \right]^T \gamma_5 (\hat{p} + m) \Gamma_3 (\hat{p} + m) U(P),
\]

(5)

In (4) and (5) the factors \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \) denote the parts of the matrix elements (including propagators) corresponding to the first, second and third quark lines in the dashed blocks, respectively.

For the \( \gamma_P \rightarrow \Delta^\mu \) transition one should make in (3)-(5) the replacements \( U_N(P) \rightarrow \tilde{U}_\beta(P) \gamma_5 \gamma_\beta \) (after the square bracket) and \( f_N(p_1, p_2, p_3) \rightarrow f_\Delta (p_1, p_2, p_3) \).

We suppose that the region \( p_1 \approx p_2 \approx p_3 \approx P/3 \) and \( p_1' \approx p_2' \approx p_3' \approx P/3 \) dominates in integrals; therefore, the quantities \( K_i^\mu \) may be taken out of the integral sign at the aforementioned values of quark four-momenta. Denote the remaining integral by \( \hat{A} \). In our assumption of zero binding ener-
(δ ≈ 0) it is convenient to evaluate the \( p_2^0, p_3^0 \) and \( p_2^0', p_3^0' \) integrals in the corresponding rest frames \( (\vec{P} = 0 \) and \( \vec{P}' = 0 \), respectively).

Closing the contour in the lower half-plane and noting that only one pole

\[
p_i^0 = \sqrt{p_i^2 + m^2} = E_i; \quad p_i^0' = \sqrt{p_i'^2 + m^2} = E_i'; \quad i = 1, 2
\]

contribute to each integral when \( \delta \to 0 \), we find

\[
A = \frac{(2\pi i)^4}{8(2\pi)^{16}} \int \frac{d^3p_2 \, d^3p_3 \, f(\vec{p}_2, \vec{p}_3)}{E_2 \, E_3 (m\delta + p_2^2 + p_2^2 + p_3^2)} \frac{d^3p_2' \, d^3p_3' \, f(\vec{p}_2', \vec{p}_3')}{E_2' \, E_3' (m\delta + p_1'^2 + p_2'^2 + p_3'^2)}. 
\]

Thus \( A > 0 \). The expressions for \( M_{a}^{\mu} \) and \( M_{c}^{\mu} \) take the form:

\[
M_{a}^{\mu} = A(2M)^{3/2} \overline{U}_{P} \Gamma_1^{\mu} U_{P} Tr((\hat{P} + M) \Gamma_2 (\hat{P}' + M) \Gamma_3),
\]

\[
M_{c}^{\mu} = A(2M)^{3/2} \overline{U}_{P} \Gamma_1^{\mu} (\hat{P}' + M) \Gamma_2 (\hat{P}' + M) \Gamma_3 U_{P},
\]

where \( M = m \) is the mass of both nucleon and \( \Delta \) -isobar in our approximation, \( I = 1 \) for \( \gamma P \to P \) and \( I = -i\sqrt{3} \gamma_5 \gamma_\beta \) for \( \gamma P \to \Delta^+ \). In the case of \( \gamma P \to \Delta^+ \) transition one should make the replacement \( \overline{U}_{P} \to \overline{U}_{\Delta^+} \) as well.

### 3. Nucleon form factors.

The proton electromagnetic current corresponding to the graphs of Fig.2, has the form:

\[
(P'J^\mu P) = 2q_u (M_{a}^{\mu} + M_{b}^{\mu} + M_{c}^{\mu} + M_{d}^{\mu}) + 2q_d (M_{e}^{\mu} + M_{f}^{\mu}),
\]

where \( q_u \) and \( q_d \) are charges of up and down quarks, respectively.

In the frame with fermion momentum directed along the third axis we have
\[ U_\alpha^\lambda(P) U_{\beta'}^{\lambda'}(P') = \left[ (\not{P} + M)/2 \cdot (1 + i\tau^5 \gamma_5 \not{P} + M/2 - (\gamma + 2i\gamma_5 \gamma \delta_{52} \not{P} + M/2 \gamma_5 \not{P} - M/2) \right] \alpha \beta, \quad (9) \]

where \( a_\mu \) is the polarization pseudovector of fermion with the helicity \( \lambda \). Using this relation we find

\[ 2M_c^\mu = 2M_d^\mu = M_a^\mu \]
\[ M_c^\mu + M_e^\mu = M_b^\mu \]

and the proton electromagnetic current takes the form

\[ (P'^{ij} P) = (4q_u - q_d)M_a^\mu + 2(q_u + 2q_d)M_b^\mu. \quad (11) \]

It is worth noting, that the well known SU(6)-symmetry relations between proton and neutron charges and magnetic moments can be easily obtained using expression (11) in the \( Q^2 \rightarrow 0 \) limit when no gluon exchanges are taken into account in the dashed blocks of graphs in Fig.2.

In the \( Q^2 \rightarrow \infty \) limit we take into account all kinds of gluon exchanges in the lowest order of the perturbation series (Fig.1). Keeping only the leading in \( Q^2 \) terms we find the following asymptotic expressions for the electric and magnetic form factors of proton and neutron:

\[ G_m^P(Q^2) = -3A(2M)^4(4\pi\alpha_s(Q^2))^2/Q^4 \quad (12) \]
\[ G_e^P(Q^2) = -8A(2M)^4(4\pi\alpha_s(Q^2))/Q^4 \quad (13) \]
\[ G_m^n(Q^2) = -G_m^P(Q^2) \quad (14) \]
\[ G_e^n(Q^2) = -3/2 G_e^P(Q^2) \quad (15) \]

Note, that the magnetic Pauli form factor \( F_2 = 4M^2(G_m - G_e)/4M_e^2 + Q^2) \)
is suppressed relative to the electric \( F_1 = \frac{4M^2 G_E + Q^2 G_M}{4M^2 + Q^2} \) when \( Q^2 \to \infty \). This is a consequence of \( V_5 \) -invariance of the vector gluon theory.

Eqs. (12-15) have been obtained in the lowest order of the perturbation series and are valid, generally speaking, only when \( \alpha_s(Q^2) \ln(Q^2) \ll 1 \) because the contribution of higher orders in \( \alpha_s(Q^2) \) results in logarithmic corrections *).

If, as is shown in Ref. 11, \( \alpha_s(Q^2) \) is sufficiently small (\( \sim 0.3 \)) at already \( Q^2 \sim M_P^2 \), then Eqs. (12-15) are valid in the intermediate region of not very large \( Q^2 \).

It is interesting to note that Eqs. (12-15) indicate to the strong deviation from the scaling law for the form factors when \( Q^2 \to \infty \).

*) We are indebted to V.L. Chernyak, V.G. Serbo and A.R. Zhitnitsky, who informed us that the method derived in Ref. 5 was applicable for the nucleon form factors as well, the anomalous dimension (\( \gamma \)) being zero for operators connected with \( F_1 \), and nonzero for operators connected with \( F_2 \). Thus, (12), (14) and (15) remain valid in the \( Q^2 \to \infty \) limit when one takes into account the higher order corrections, while the electric form factors (13) are modified by the logarithmic factor \( (\ln(Q^2))^\gamma \). The asymptotic signs of the form factors obtained in this paper, of course, are not changed by higher order corrections.
The most striking feature of our result (12-15) is, of course, the negative sign of the proton form factors and the positive sign of the neutron ones. The signs obtained are due to the contributions of the graphs of Figs. 1b and 1c, which turn out numerically larger than the contributions of graphs of Fig. 1a. To demonstrate this let us write the relative contributions of the graphs of Figs. 1a, 1b and 1c, respectively (in the units \(4\pi\alpha_s(Q^2)^2A(2M)\delta/Q^4\)): (3, -•, 0) for \(G^P_M\); (1, -6, -3) for \(G^P_E\); (-1, 2, 2) for \(G^n_M\) and (2/3, 16/3, 6) for \(G^n_E\).

The changing of sign of the electric form factor is connected with the following fact. The coefficient at \(q_u\) in (11) becomes negative, and the coefficient at \(q_d\) remains positive when \(Q^2 \to \infty\). In the case of the magnetic form factor \(G^P_M\), both of the coefficients change the signs (remind that at \(Q^2 = 0\), \(G^P_M = 4q_u - q_d\)).

It is interesting that the changing of signs of the nucleon form factors is independent of the type of gluons and takes place for the scalar and pseudoscalar gluons as well. In the case of pions the form factor turns out positive for the vector gluons (in contrast to the result of Ref. 5), while for scalar and pseudoscalar gluons the pion form factor is negative in the \(Q^2 \to \infty\) limit. The experimental confirmation of the changing of signs of the nucleon form factors when \(Q^2 \to \infty\) would be a strong argument in favour of vector gluons if the pion form factor remains positive in this limit.

We should like to note the following fact. The module of the ratio of the coefficient at \(q_u\) to that at \(q_d\) in (11) is essentially less (8 times less for \(G^P_E\) and 4 times less for \(G^P_M\)) when \(Q^2 \to \infty\) as compared with \(Q^2 = 0\). Thus, in the model under consideration the comparative weight of the down quarks in the proton (and of the up quarks in the neut-
ron) increases in the $Q^2 \rightarrow \infty$ limit in comparison with their weights in charge and magnetic moment at $Q^2 = 0$.

Unfortunately, the available experimental data cannot be used for finding the $Q^2$-dependence of the form factors the tendency to the fulfillment of the above obtained asymptotic relations, because the $Q^2 > 3 \text{(Gev)}^2$ data do not allow to separate $G_E^P$ and $G_M^P$ [12]. From these data it is even impossible to make definite conclusions on the $Q^2$-dependence of $G_E^P$ [13].

The changing of signs of the nucleon form factors may lead to the dips in the differential cross section. It may be also revealed in polarization experiments, for instance, in experiments on measurement of asymmetry proportional to $G_E \cdot G_M$ [13], which appears when the longitudinal polarized leptons are scattered on the target proton with polarization orthogonal to the recoil proton momenta in the laboratory frame.

In the lowest order of the perturbation theory we have found also the terms of the order:

$$G_M^P(Q) = -3A(2M)^2/Q^2(4\pi\alpha_s)^2(1 - 22M^2/Q^2) ,$$

$$G_E^P(Q) = -8A(2M)^2/Q^2(4\pi\alpha_s)^2(1 - 11M^2/4Q^2) ,$$

$$G_M^D(Q) = 3A(2M)^2/Q^2(4\pi\alpha_s)^2(1 - 44M^2/3Q^2) ,$$

$$G_E^D(Q) = 12A(2M)^2/Q^2(4\pi\alpha_s)^2(1 + 4M^2/3Q^2) .$$

(16)
The higher order contributions may distort these results; nevertheless, it is interesting that the coefficients at $M^6/Q^6$ terms are large and differ strongly from the dipole formula prediction contrary to the assumption by Bredsky and Chertok [8]. Terms of order $M^6/Q^6$ and the leading terms are opposite in signs (except for the case of $G^n$), and one can also see the changing of signs of the form factors as $Q^2$ increases.

Note, that for large $Q^2$ the vector and axial vector weak form factors are equal in vector-gluon theory due to $\gamma_5$-invariance [10]. Thus, we also predict the changing of signs of $g_A$ and $g_V$ in the asymptopia $(q^2, G_P^M - G_P^n)$.

4. Form factors of the $\gamma P \rightarrow \Delta^+$-transition.

We define the $\gamma P \rightarrow \Delta^+$ transition current as follows [15]:

$$
(\Delta^+ | J_{\mu} | P) = i \bar{U}(P') (G_1(Q^2) H^{(1)}_{\beta\mu} + G_2(Q^2) H^{(2)}_{\beta\mu} + G_3(Q^2) H^{(3)}_{\beta\mu}) U(P)
$$

(17)

where

$$
H^{(1)}_{\beta\mu} = (q_{\beta} \gamma_\mu - \hat{q} \delta_{\beta\mu}) \gamma_5
$$

$$
H^{(2)}_{\beta\mu} = (q_{\beta} P_{\mu} - (q P) \delta_{\beta\mu}) \gamma_5
$$

$$
H^{(3)}_{\beta\mu} = (q_{\beta} q - q^2 \delta_{\beta\mu}) \gamma_5
$$

(18)

$$
P = (P' + P)/2; \quad q = P' - P; \quad Q^2 = -q^2$$
As the second terms in the square brackets in (1) and (2) are opposite in signs, the total contribution of the graphs of Figs. 2a, b turns out zero while the other graphs have opposite signs for the up and down quarks. Thus we have

\[(\Delta^{+}\gamma_{\mu}I P) = 2(q_{u} - q_{d}) (M_{b}^{\mu} + M_{c}^{\mu} + M_{e}^{\mu})\]  

(19)

One can obtain the following relation among matrix elements

\[M_{b}^{\mu} = M_{c}^{\mu} = M_{e}^{\mu}\]  

(20)

and so the \(\gamma P \rightarrow \Delta^{+}\) transition matrix element actually reduces to:

\[(\Delta^{+}\gamma_{\mu}I P) = 6(q_{u} - q_{d}) M_{b}^{\mu}\]  

(21)

In the asymptotic region we obtain for the \(G_{i}(Q^{2})\) form factors:

\[G_{1}(Q^{2}) = 6\sqrt{3}AM(4\pi\alpha_{s}(Q^{2}))^{2}(2M)^{4}/Q^{6}\]

\[G_{2}(Q^{2}) = -16\sqrt{3}AM^{2}(4\pi\alpha_{s}(Q^{2}))^{2}(2M)^{4}/Q^{8}\]  

(22)

\[G_{3}(Q^{2}) = -28\sqrt{3}AM^{2}(4\pi\alpha_{s}(Q^{2}))^{2}(2M)^{4}/Q^{8}\]

The suppression of \(G_{2}\) and \(G_{3}\) relative to \(G_{1}\) at \(Q^{2} \rightarrow \infty\) is due to the \(V_{5}\) -invariance.

In the terms of commonly used magnetic dipole \(G_{M}^{*}\), electric quadrupole \(G_{E}^{*}\) and Coulomb \(G_{C}^{*}\) form factors the obtained results take...
the form:

\[ \frac{G_M}{G_E} = - \frac{G_E}{G_p} = \sqrt{3}/8 \]

\[ \frac{G_C}{G_E} \sim 1/Q^2 \]  \hspace{1cm} (23)

Contrary to the nucleon form factors the magnetic form factor \( G_M^* \) does not change the sign when \( Q^2 \to \infty \) (\( G(0) = 3 \)).

The available experimental data on \( \gamma N \to \Delta \) are carried out at not large enough values of \( Q^2 \) (\( \lesssim 1.6 \text{ GeV}^2 \)) (see, e.g. Ref. 16) and do not allow to test our predictions for \( G_M^*, G_E^* \) and \( G_C^* \).

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Fig. 1
Fig. 2
FIGURE CAPTIONS

Fig. 1. The Feynman graphs corresponding to gluon exchanges in the lowest order of the perturbation series. The other seven graphs may be obtained by the replacing of the second and the third quark lines.

Fig. 2. The graphs for the $\gamma P \rightarrow P$ and $\gamma P \rightarrow \Delta^+$ transition form factors corresponding to the (1) and (2) vertices. The dashed blocks include the gluon exchanges shown in Fig. 1.
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