BNL-25710
CONF-790318--1

Symposium on Nonlinear Dynamics and the Beam-Beam Interaction,
Brookhaven National Laboratory, March 19-21, 1979.

The Beam-Beam Force and Storage Ring Parameters *

J.C. Herrera,
Brookhaven National Laboratory

*Research carried out under the auspices of the United States Department of Energy under Contract No. EY-76-C-02-0016.

I. Introduction

In this brief report, we wish to outline the fundamental aspects of the beam-beam force as it occurs in Intersecting Storage Rings. We shall indicate how the effect of the beam-particle electromagnetic force (weak-strong interaction) is quite different in the case of unbunched proton beams which cross each other at an angle (as in the ISR and in ISABELLE), as compared to the case of electron-positron beams where bunches collide head-on (as in the Spear II Ring and in PETRA and PEP).

II. Unbunched Intersecting Beams

In Fig. (1) we show a particle trajectory crossing a continuous beam at an angle α_o. It is clear from the figure that, during this passage, the particle will sample a series of gradient fields which result in a change in its betatron tunes. These changes can be calculated by perturbation theory and are:

$$\Delta\nu_V = \frac{-\beta_V^* r_p I}{\sqrt{2\pi}\; ec\gamma\beta^2 \sigma_V^* \tan(\alpha_o/2)} \qquad (1)$$

and

$$\Delta\nu_H = 0 \quad , \qquad (2)$$

where

α_o = horizontal total crossing angle (rad),

σ_V^* = Gaussian rms (vertical) size at crossing (m),

β_V^* = vertical amplitude function at crossing (m),

r_p = classical radius of proton (1.535×10^{-18}m),

(γ,β) = relativistic particle factors of colliding protons,

I = average current per beam (A),

$(\Delta\nu)_{V,H}$ = vertical or horizontal tune shift per crossing,

e = charge on proton (1.602×10^{-19}C),

c = velocity of light (2.9979×10^{8} m/s).

A pertinent question to ask is : "What is the deflection of the particle trajectory as it crosses the unbunched beam?". This change in slope occurs in the vertical plane. Thus we have

$$\Delta\left(\frac{dy}{ds}\right) = \frac{2\pi r_p I}{ec\gamma\beta^2 \tan(\alpha_o/2)} \Phi\left(\frac{y}{\sqrt{2}\,\sigma_v}\right) \quad , \tag{3}$$

where the error function is defined by

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_o^x dt\ e^{-t^2} \quad . \tag{4}$$

In Fig. (2), we have sketched the variation of this deflection as a function of the distance of the particle path from the horizontal symmetry plane of the colliding beam. We note that the magnitude of the maximum deflection tends toward

$$\left[\Delta y'\right]_{max} = \frac{2\pi r_p I}{ec\gamma\beta^2 \tan(\alpha_o/2)} \tag{5}$$

for large vertical amplitudes of the incident particle. It is important to mention that since the linear term obtainable from Eq. (3) is proportional to the change in betatron tune expressed by Eq. (1), this value of $\Delta\nu$ has been conventionally designated as the beam-beam strength parameter. Thus, the change in tune calculated from first order perturbation theory serves as a measure of the strength of the nonlinear beam-particle interaction specified by the net angular deflection of an intersecting particle trajectory.

III. Head-on Collisions for Bunched Beams

For an electron traversing a bunch of positrons, such as occurs in e^+e^- storage rings, the situation is that depicted in Fig. (3). A particle passing though the center of a bunch whose density of charge is Gaussian in the traverse plane will experience a tune shift in both planes. These are calculated to be

$$\Delta\nu_V = \frac{\beta_V^* \, r_e \, N_B}{2\pi\gamma\beta^2 \, \sigma_V^* \, (\sigma_H^* + \sigma_V^*)} \quad , \qquad (6)$$

and

$$\Delta\nu_H = \frac{\beta_H^* \, r_e \, N_B}{2\pi\gamma\beta^2 \, \sigma_H^* \, (\sigma_H^* + \sigma_V^*)} \quad . \qquad (7)$$

We have used a notation such that

$\sigma^*_{H,V}$ = rms horizontal or vertical size at the collision point (m),

$\beta^*_{H,V}$ = machine amplitude function over the collision region (m),

N_B = number of particles per bunch, and

r_e = classical radius of an electron (2.818×10^{-15}m).

For head-on collisions the angular deflection of the particle path is not confined to the vertical plane as is the case for the collision of unbunched beams at an angle. The net deflections in the two planes are:

$$\Delta\left(\frac{dx}{ds}\right) = \frac{-2N_B r_e x}{\gamma\beta^2 r^2} \left(1 - e^{-r^2/2\sigma^2}\right) \quad , \qquad (8)$$

and

$$\Delta\left(\frac{dy}{ds}\right) = \frac{-2N_B r_e y}{\gamma\beta^2 r^2} \left(1 - e^{-r^2/2\sigma^2}\right) \quad , \qquad (9)$$

where x and y are the horizontal and vertical displacement of the particle trajectory and r $(=\sqrt{x^2 + y^2})$ is the radial distance from the center line of the bunch. In Fig. (4) we sketch the variation of this net deflection in one of the planes. In contrast to the unbunched case, the deflection after rising to a maximum value of about

$$[\Delta x']_{max} \simeq \frac{0.9\ N_B r_e}{\gamma\beta^2\sigma} \qquad (10)$$

decreases to zero for particles passing through the outer fringes of a bunch. Though the strength parameters, Eqs. (6) and (7), are applicable to unsymmetrical Gaussian beams, the net deflections, Eqs. (8) and (9), only apply to cylindrically symmetrical beams, with a rms half-width of σ.

IV. Tabulation of Machine Parameters

Table 1 is a presentation of the parameters determining the luminosity and the beam-beam strength for the Cern ISR and the Brookhaven pp rings, ISABELLE. An attempt has been made to give a set of consistent values for these parameters which are derived from the listed equations. It may be of interest to mention that, for ISABELLE at 30 GeV, the beam-beam strength parameter and the maximum angular deflection are significantly higher than those of the ISR.

In Table II, we give an analogous list of parameters for the Spear II machine as well as for the PETRA and PEP machines, which will soon be operational. It is evident that e^+e^- rings operate with a beam-beam strength parameter more than an order of magnitude higher than pp unbunches machines. In addition, these strengths are approximately equal in magnitude in the two transverse planes. The tabulation also indicates that the maximum deflection undergone by a particle in traversing the other beam is three orders of magnitude greater for e^+e^- rings when compared to the corresponding deflection in pp rings.

Table I. Beam Characteristics of pp Storage Rings.

		p-p Machines		
		ISR CERN $(p,p)\equiv(1,2)$	ISABELLE 30x30 GeV "Standard Insertion"	ISABELLE 400x400 GeV "Standard Insertion"
Energy	$E_1=E_2=E$	26.73 GeV	29.4 GeV	400 GeV
Particle Gamma	$\gamma=E/M_p$	28.49	31.3	426.3
Current	$I_1=I_2=I$	28.6A	8A	8A
Normalized Emittence (95% Phase Space)	$E_V=\frac{6\pi\sigma_V^2(\gamma\beta)}{\beta_V}$	$19.6\pi 10^{-6}$ rad m	$15\pi 10^{-6}$ rad m	$15\pi 10^{-6}$ rad m
Beta Functions (Intersection)	β_H^*	~22 m	30 m	30 m
	β_V^*	14.25 m	7.5 m	7.5 m
RMS Vertical Size (Intersection)	$\sigma_V^*=\sqrt{\frac{E_V\beta_V^*}{6\pi(\gamma\beta)}}$	1.278 mm	0.774 mm	0.21 mm
Effective Height	$h_e=2\sqrt{\pi}\,\sigma_V^*$	4.53 mm	2.74 mm	0.743 mm
Crossing Angle	α	14.77°	9.8 mrad	9.8 mrad
Luminosity	$L=\frac{I_1I_2}{2\sqrt{\pi}e^2c\sigma_V^*\tan\frac{\alpha}{2}}$	1.3×10^{31} cm^{-2} sec^{-1}	6.2×10^{31} cm^{-2} sec^{-1}	2.3×10^{32} cm^{-2} sec^{-1}
Circumference	$2\pi R$	942.48 m	3766.6 m	3766.6 m
Particle Revolution Frequency	$f_o=c/2\pi R$	3.181×10^5 Hz	79.59×10^3 Hz	79.59×10^3 Hz
Number of Particles	$N=I/ef_o$	5.61×10^{14}	6.27×10^{14}	6.27×10^{14}
Number of Intersections		8	6	6
Classical Radius of Proton	r_p	1.535×10^{-18} m		
Vertical Beam-Beam Strength Parameter Per Intersection	$\Delta\nu_V=\frac{-\beta_V^*r_pI}{\sqrt{2\pi}ec\gamma\beta^2\sigma_V^*\tan\frac{\alpha}{2}}$	-1.1×10^{-3}	-6.4×10^{-3}	-1.74×10^{-3}
Horizontal Beam-Beam Tune Shift	$\Delta\nu_H$	0.	0.	0.
Maximum Vertical Deflection	$(\Delta y')_{max}=\frac{2\pi r_pI}{ec\gamma\beta^2\tan\frac{\alpha}{2}}$	1.56×10^{-6} rad	1.1×10^{-5} rad	7.7×10^{-7} rad
Betatron Tunes	$\nu_V\approx\nu_H$	8.9	22.6	22.6

Table II. Beam Characteristics of e^+e^- Storage Rings.

e^+e^- Machines

		SPEAR II $(1,2)=(e^-,e^+)$	PEP (e^-,e^-)	PETRA (e^-,e^+)
Energy	$E_1=E_2=E$	3.71 GeV	15 GeV	15 GeV
Particle Gamma	$\gamma=E/M_e$	7260.2	29354	29354
Total Energy in c.m.	$\sqrt{s}=2\sqrt{E_1E_2}$	7.42 GeV	30 GeV	30 GeV
Number of Particles	$N_1=N_2=N$	1.75×10^{11}	2.52×10^{12}	3.84×10^{12}
Number of Bunches	n_B	1	3	4
Particles per Bunch	$\frac{q}{e}=\frac{N}{n_B}$	1.75×10^{11}	8.4×10^{11}	9.6×10^{11}
Crossing Angle	α	0.	0	0
Circumference	$2\pi R$	234.13 m	2200 m	2200 m
Magnetic Radius of Curvature	ρ	12.7 m	165.52 m	197.14 m
Beta Functions	β_H^*	1.197 m	2.8 m	3.0 m
at Intersection	β_V^*	0.0998 m	0.11 m	0.15 m
ETA Function at Intersection	η^*	0.	0.49 m	
Beam Betatron Emittance	$\epsilon=\sigma^2/\beta$ — ϵ_H	0.665×10^{-6} rad m	0.138×10^{-6} rad m	0.15×10^{-6} rad m
	ϵ_V	0.902×10^{-8} rad m	0.933×10^{-8} rad m	0.75×10^{-8} rad m
Coupling Coefficient	$K=\sqrt{\epsilon_V/\epsilon_H}$	0.12	0.26	0.22
Betatron RMS Size (Intersection)	$\sigma^2=\beta\epsilon$ — $(\sigma_H^*)_\beta$	0.892 mm	0.662 mm	0.67 mm
	σ_V^*	0.030 mm	0.032 mm	0.034 mm
Energy Spread	$\frac{\sigma_\epsilon}{E}=\left[\frac{3.84\times10^{-13}\gamma^2}{2\rho}\right]^{1/2}$	0.89×10^{-3}	1.0×10^{-3}	0.91×10^{-3}
Energy RMS Size (Intersection)	$\sigma_\epsilon^*=\eta^*\frac{\sigma_\epsilon}{E}$	0.	0.49 mm	
Total Horz. RMS Size	$\sigma_H^2=\sigma_\epsilon^{*2}+(\sigma_H^*)_\beta^2$	0.892 mm	0.79 mm	0.67 mm
Particle Revolution Frequency	$f_o=c/2\pi R$	1.280×10^6 Hz	1.36×10^5 Hz	1.30×10^5 Hz
Average Current	$I=e(\frac{q}{e})n_Bf_o$	35.9 mA	54.9 mA	80.0 mA
Classical Radius of Electron	r_e	2.818×10^{-15} m		

Table II (Continued).

e^+e^- Machines

		SPEAR II $(1,2)\equiv(e^-,e^+)$	PEP (e^-,e^+)	PETRA (e^-,e^+)
Luminosity Per Bunch Collision	$\mathcal{L}_{BC}=\frac{q_1 q_2}{e^2 4\pi\sigma_V^* \sigma_H^*}$	$9.1\times10^{24}\,cm^{-2}$	$2.22\times10^{26}\,cm^{-2}$	$3.22\times10^{26}\,cm^{-2}$
Average Luminosity	$L=n_B f_c \mathcal{L}_{BC}$	$1.16\times10^{31}\,cm^{-2}sec^{-1}$	$9.1\times10^{31}\,cm^{-2}sec^{-1}$	$1.7\times10^{32}\,cm^{-2}sec^{-1}$
Vertical Beam-Beam Strength Parameter	$\Delta\nu_V=\frac{\beta_V^* r_e (N/n_B)}{2\pi\gamma\beta^2 \sigma_V^*(\sigma_H^*+\sigma_V^*)}$	0.039	0.054	0.092
Horizontal Beam-Beam Strength Parameter	$\Delta\nu_H=\frac{\beta_H^* r_e (N/n_B)}{2\pi\gamma\beta^2 \sigma_H^*(\sigma_H^*+\sigma_V^*)}$	0.016	0.055	0.093
Approx. MAX Particle Deflection Per Collision	$\sim\frac{(N/n_B) r_e}{\gamma\sqrt{\sigma_V^* \sigma_H^*}}$	$\sim10^{-3}$ rad	$\sim10^{-3}$ rad	$\sim10^{-3}$
(Energy Loss/Rest Mass) Per Turn Due to Syn. Radiation	$\left(\frac{\Delta W}{mc^2}\right)_{TURN}=\frac{4\pi}{3}\frac{r_e}{\rho}\gamma^4\beta^3$	2.582	52.94	44.45
Energy Loss (SR) Per Turn	ΔW	1.32×10^6 eV/TURN	27.1×10^6 eV/TURN	22.7×10^6 eV/TURN
Total Power Loss Per Beam	$(\Delta W)I$	47.4×10^3 W	1.5×10^6 W	1.8×10^6 W
Average Radiated Power Per Particle	$P_\gamma=(\Delta W)f_o$	1.69×10^{12} eV/sec	3.68×10^{12} eV/sec	2.95×10^{12} eV/sec
Transverse Damping Time	$\tau_T=2E/P_\gamma$	4.4×10^{-3} sec	8.1×10^{-3} sec	$10.\times10^{-8}$ sec
Betatron	ν_H	5.27	21.25	22.2
Tunes	ν_V	5.19	18.75	22.2

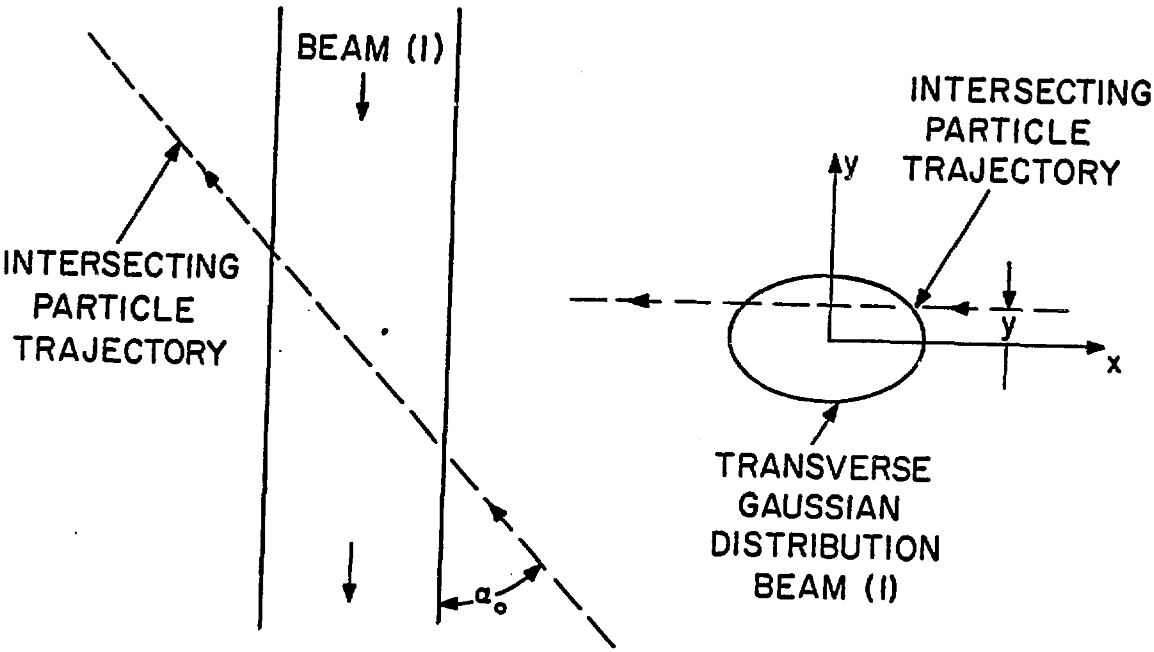
Figure Captions

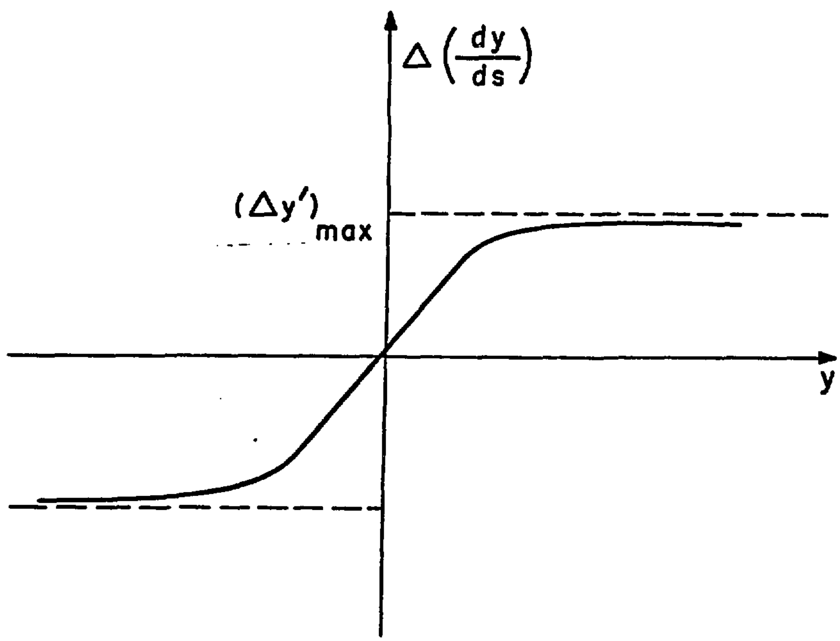
Fig. 1. Particle Crossing a Continuous Beam.

Fig. 2. Angular Deflection for a Particle Crossing an Unbunched Beam at an Angle.

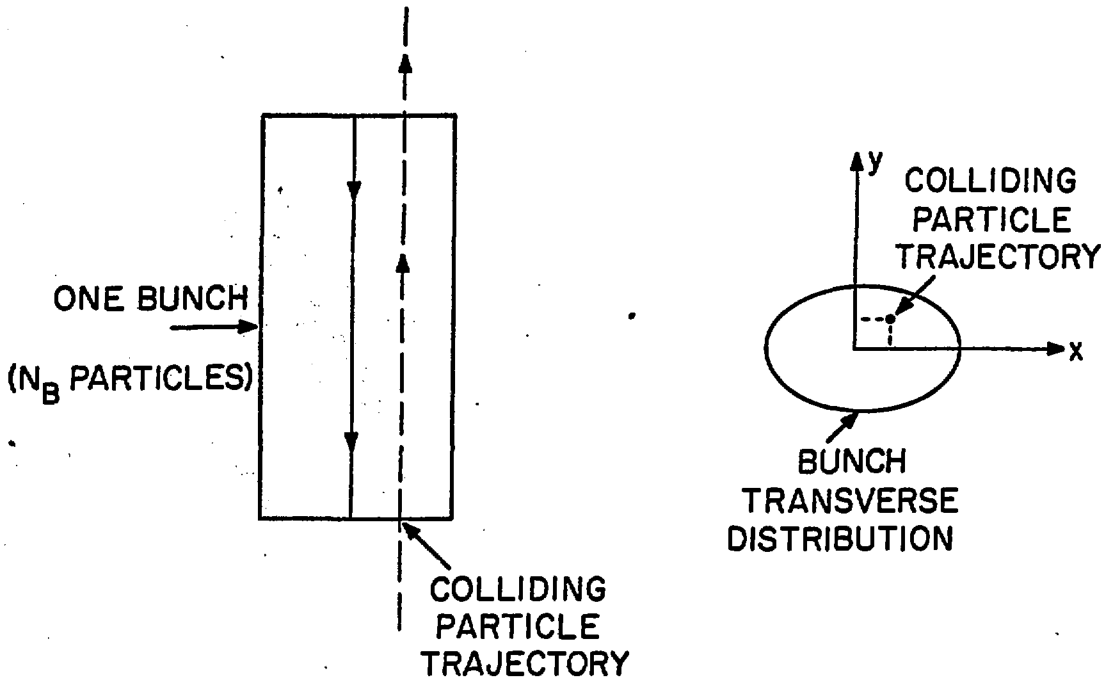
Fig. 3. Particle Colliding Head-on with a Bunch.

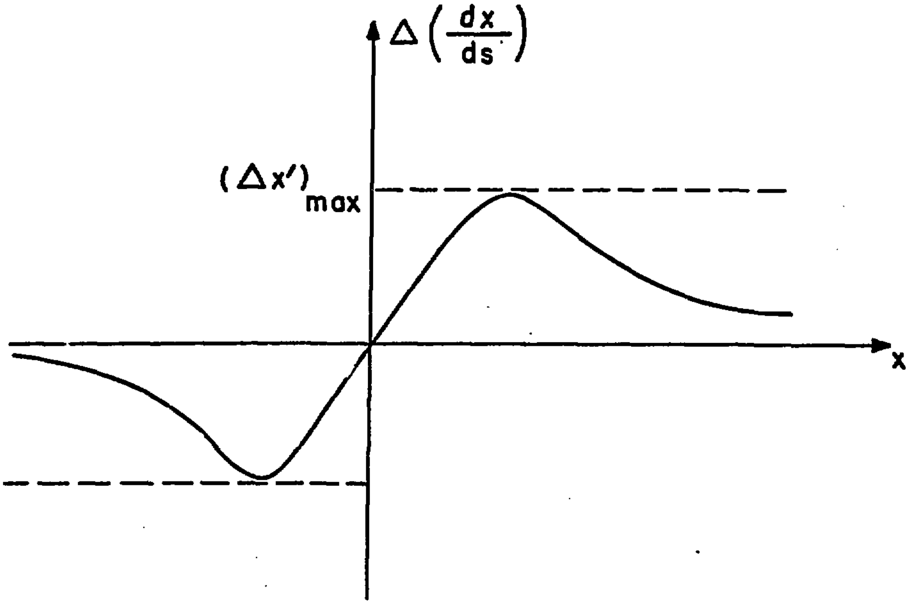
Fig. 4. Angular Deflection for a Particle Colliding Head-on with a Bunched Beam.


BEAM (I)
INTERSECTING
PARTICLE
TRAJECTORY
α_o
INTERSECTING
PARTICLE
TRAJECTORY
y
x
y
TRANSVERSE
GAUSSIAN
DISTRIBUTION
BEAM (I)



$\Delta\left(\frac{dy}{ds}\right)$
$(\Delta y')_{max}$
y


ONE BUNCH
(N_B PARTICLES)
COLLIDING
PARTICLE
TRAJECTORY
y
COLLIDING
PARTICLE
TRAJECTORY
x
BUNCH
TRANSVERSE
DISTRIBUTION



$\Delta\left(\frac{dx}{ds}\right)$
$(\Delta x')_{max}$
x