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Abstract

The collisional-radiative model is applied to a recombining hydrogen plasma in order to investigate the plasma condition in which the population inversion between the energy levels of hydrogen can be generated.

The population inversion is expected in a plasma where the three body recombination has a large contribution to the recombining processes and the effective recombination rate is beyond a certain value for a given electron density and temperature. Calculated results are presented in figures and tables.

1. Introduction

It has been theoretically demonstrated that a strong population-inversion for the laser action is obtained in a recombining plasma which is rapidly cooled after it is initially sufficiently heated(1)(2). Recombination processes in such a plasma are analysed by using rate equations for the populations of the energy levels of atoms or ions in the plasma and also an equation of relaxation of the electron temperature. However, in those reports a rather simple model for the recombining plasma is considered: The initial distribution of the population is assumed to be in local thermodynamic equilibrium and the plasma with high electron temperature to be abruptly cooled down to under 0.5 eV and its temperature to be kept constant afterwards(3)(4)(5). Recently, an obsevation of stationary population inversion between low lying levels of He^+ was made in a recombining plasma(6).

In this reports, general criterion for generating the population inversion in a hydrogen recombining plasma is investigated on the basis of a CR model. As one of convenient quantities describing the recombination process, we introduce the effective recombination rate of the plasma, $-\frac{dn_e}{dt} \equiv -\dot{n}_e$, where n_e is the electron density. The rate, $\dot{n}_e$, will be helpful to show quantitatively that the population inversion in the recombining plasma can be obtained only when the recombination processes occur with a rate beyond a certain degree.

Numerical calculations have been made of the plasma condition, i.e. the electron density, the electron temperature and the recombination rate, for generating the inversion between low-lying levels with the principal quantum number of less than 5. The calculated results are presented in figures and tables, which will be helpful for investigating the possibility of laser action in plasmas. Populating and depopulating processes for the levels are discussed in order to explain the mechanism of the inversion. The dependence of the populating processes upon $-\dot{n}_e$ is also calculated. The recombining hydrogen plasma described in the present report is assumed to be spatially homogeneous and optically thin.

In the second section, the formulation of the problem is presented. Numerical results are given in the third section and the mechanism of the population inversion is discussed in the fourth section. Conclusions are presented in the fifth section.

2. Formulation of the problems

Rate equation, similar to that of Drawin[7], for the population density n_i of a level i is given by

$$\sum_{j=1}^{n^*} a_{ij} n_j + \delta_i = 0, \quad n^* \geq i \geq 2, \tag{1}$$

where the coefficients a_{ij} and δ_i are functions of the electron density and the collisional and the radiative transition rate coefficients[8]. The population density n_1 of the ground state is determined by

$$n_e n_+ \alpha_{CR} - n_e n_1 S_{CR} = \frac{\partial n_1}{\partial t} = - \frac{\partial n_e}{\partial t}, \tag{2}$$

where α_{CR} and S_{CR} are the collisional-radiative recombination and ionization coefficients[7], respectively, and n_+ is the density of the hydrogen ion. When the population inversion takes place between a pair of levels i_1 and i_0, the ratio of the population densities of the two levels should satisfy the following relation:

$$\frac{n_{i_1}}{n_{i_0}} > \left(\frac{i_1}{i_0}\right)^2. \tag{3}$$

Here, the right-hand side of the inequality is the ratio of the statistical weights of the levels. In the present calculation the atom-atom and the atom-ion collisions are neglected.

As is well known, the solution of the Eq. (1) is expressed as

$$n_i = n_i^{(0)} + g_i^{(1)} n_1 \qquad i \geq 2 \tag{4}$$

Here $n_i^{(0)}$ and $g_i^{(1)}$ satisfy the following equations:

$$\sum_{j=2}^{n^*} a_{ij} n_j^{(0)} + \delta_i = 0 \qquad i \geq 2,$$

$$\sum_{j=2}^{n^*} a_{ij} g_j^{(1)} + a_{i_1} = 0 \qquad i \geq 2,$$

where n^* is the maximum principal quantum number of the cut-off level which is chosen to be 20 according to Caciatore and Capetelli[9]. Equation (4) gives the population density of the ground state by

$$n_1 = \frac{\left(\frac{n_{i_1}}{n_{i_0}}\right) \cdot n_{i_0}^{(0)} - n_{i_1}^{(0)}}{g_{i_1}^{(1)} - \left(\frac{n_{i_1}}{n_{i_0}}\right) \cdot g_{i_0}^{(1)}} \tag{5}$$

Since the relation $n_{i_1}^{(0)}/n_{i_0}^{(0)} \geq g_{i_1}^{(1)}/g_{i_0}^{(1)}$ is valid in the optically thin case[10], the ratio n_{i_1}/n_{i_0} is restricted to

$$\frac{g_{i_1}^{(1)}}{g_{i_0}^{(1)}} < \frac{n_{i_1}}{n_{i_0}} \leq \frac{n_{i_1}^{(0)}}{n_{i_0}^{(0)}} \tag{6}$$

Then, equations (2), (3) and (6) yield the conditions for the population inversion between the levels i_1 and i_0:

$$\left(\frac{i_1}{i_0}\right)^2 < \frac{n_{i_1}^{(0)}}{n_{i_0}^{(0)}} \tag{7}$$

$$n_e n_+ \alpha_{CR} - n_e (n_1)_{limit} S_{CR} < -\dot{n}_e < n_e n_+ \alpha_{CR} \tag{8}$$

Eq. (7) gives a condition for the electron temperature and density when the population inversion takes place in the recombining plasma. In Eq. (8), $(n_1)_{limit}$ is the population density of the ground state when the ratio n_{i_1}/n_{i_0} is equal to the ratio of the statistical weights of the levels i_1 and i_0, i.e., $n_{i_1}/n_{i_0} = (i_1/i_0)^2$. When the ratio of the inverted populations takes its maximum value, that is, $n_{i_1}/n_{i_0} = n_{i_1}^{(0)}/n_{i_0}^{(0)}$ holds, the population density of the ground state becomes zero and the recombination rate takes its maximum value, $n_e n_+ \alpha_{CR}$. The population densities of the relevant levels in the plasma are derived from Eqs. (4) and (5). Consequently, one can generate the population inversion in the recombining plasma only when T_e, n_e and recombination rate $-\dot{n}_e$ satisfy Eqs. (7) and (8).

3. Numerical Results

The calculation is carried out for hydrogen plasma with the temperatures (0.5 ∿ 15 eV) and the densities ($10^8 \sim 10^{17}$ cm^{-3}). The coefficients a_{ij}, δ_i, α_{CR} and S_{CR} are obtained from the formula for the collisional and radiative transition rates presented by johnson[8].

Figure 1 shows the upper bound value of T_e as a function of n_e for various pairs of levels (i_1, i_0) when the inversion occurs in the plasma. At a low electron density the inversion can be expected at a low electron temperature, while at a sufficiently high density ($n_e > 5 \times 10^{14}$ cm^{-3}) the inversion can no longer be expected at any temperature. The upper limit density for the inversion decreases with an increase of a lower level i_0 against the same upper level i_1. At a relatively low density ($n_e < 5 \times 10^{12}$ cm^{-3}) the upper bound temperature increases with the level i_1 against the same lower level i_0. On the other hand, this tendency is reversed at a high density.

Figure 2 gives the population density devided by the statistical weight, $n_i/2i^2$, ($i = 1 \sim 5$) as a function of the recombination rate, $-\dot{n}_e$, for hydrogen plasma at T_e = 2 eV and $n_e = 10^{12}$ cm^{-3}. The population inversion between levels i_0 and i_1, is expected at the recombination rate, $-\dot{n}_e$, larger than that corresponding to the intersection (the point indicated by a symbol ↑) of the two graphs for n_{i_0} and n_{i_1}. For example, the population inversion between the levels $i_0 = 4$ and $i_1 = 5$ takes place when the recombination rate is larger than 0.206×10^{12} cm^{-3} sec^{-1}, and at $-\dot{n}_e = 0.25 \times 10^{12}$ cm^{-2} sec^{-1}, the inversion appears between the levels i = 3, 4 and 5, while each of the levels i_1 = 3, 4 and 5 is not overpopulated against the level $i_0 = 2$. The allowance of the recombination rate decreases for the presence of the inversion between lower lying levels with an increase of the energy difference between the levels. The upper and the lower bound of the recombination rate, $\mathrm{Max}|\dot{n}_e|$ and $\mathrm{Min}|\dot{n}_e|$ for several line pairs are given in Table I. From Eq. (8), $\mathrm{Max}|\dot{n}_e| = n_e n_+ \alpha_{CR}$ and $\mathrm{Min}|\dot{n}_e| = n_e n_+ \alpha_{CR} - n_e (n_1)_{limit} S_{CR}$. Here, $\mathrm{Max}|\dot{n}_e|$ which is a function of T_e and n_e is independent of the level pair (i_0, i_1). Blanks in the column of $\mathrm{Min}|\dot{n}_e|$ indicate that the inversion can not be expected at the corresponding plasma parameters. Table 1 shows that the difference between $\mathrm{Min}|\dot{n}_e|$ and $\mathrm{Max}|\dot{n}_e|$ increases as the energy difference between the levels concerned decreases.

Table 2 gives the population density of the ground state when the recombination rate is equal to $\mathrm{Min}|\dot{n}_e|$ in the plasma. The population density of the ground state decreases as the recombination rate is raised, and the population inversion becomes intensified. When the recombination rate takes its maximum value ($\mathrm{Max}|\dot{n}_e|$), n_1 is reduced to zero and the inversion between the two levels of interest reaches its most dominant state.

4. Discussion

In the present calculation, only the radiative and the three body recombination are taken into account. The wall recombination which plays an important role in a small laboratory plasma is neglected here because a uniform and infinite plasma is considered. When the rate of the three body recombination is equal to that of the radiative recombination, the electron density should satisfy the following relation:

$$n_e = \sum_{j=1}^{n^*} \beta_j / \sum_{j=1}^{n^*} K_{cj} \tag{9}$$

where K_{cj} and β_j are the rate coefficients of the three body and the radiative recombination to the level j, respectively. Figure 3 shows the calculated density as a function of the electron temperature. The three body recombination becomes more dominant than the radiative recombination in the upper region of the graph shown in Fig. 3, while the radiative recombination is important in the lower region.

We will discuss populating and de-populating processes of the lower-lying levels in connection with the recombination rates in three different cases: (1) the two processes of the recombination have almost the same contribution to the recombinations (2) the radiative recombination is dominant, and (3) the three body recombination plays a leading process.

The rate equation, Eq. (1), for the level i ($i \geq 2$), expresses a balance between populating and de-populating rates. The coefficients, a_{ij} and δ_i, are given by the rate coefficients of the elementary processes concerned, i.e., the collisional rate coefficient for exciting from the level i to the level j, C_{ij}, the collisional-de-excitation coefficients, F_{ji}, the collisional ionization rate coefficients from the level i to the continuum, K_{ic}, Einstein's coefficient for the spontaneous emission from the level j to the level i, A_{ij}, and the recombination rate coefficients, K_{ci} and β_i. The population density of the level i is given by[6]

$$n_i = \frac{\sum_{j \neq 1} a_{ij} \cdot n_j + \delta_i}{-a_{ii}}$$

$$= \frac{n_e^2 \cdot (n_e \cdot K_{ci} + \beta_i) + \sum_{j > i} n_e \cdot n_j \cdot C_{ji} + \sum_{j > i} (A_{ji} + n_e \cdot F_{ji}) \cdot n_j}{n_e \cdot (K_{ic} + \sum_{j > i} C_{ij} + \sum_{j < i} F_{ij}) + \sum_{j < i} A_{ji}} \quad (10)$$

The numerator of Eq. (10) expresses the total populating rate to the level i, U_{in}, and the denominator expresses the total depopulating frequency ν_{out}. The populating rate depends upon the recombination rate $-\dot{n}_e$, but the depopulating frequency does not. Relative values of each component of the populating and depopulating rates are presented in percentage for the three different cases described above in Table 3(a), (b) and (c) which also have a column for $n_i/2i^2$. Here, we take the levels i = 4 and 5 as examples and discuss the populating processes from the calculated results:

(1) Low electron density and temperature (kT_e = 1 eV, $n_e = 10^9$ cm^{-3}): The radiative recombination rate is comparable with the three body recombination (Table 3(a)), if $-\dot{n}_e$ is small, and the ground state is sufficiently populated as shown in Fig. 2. Therefore, the lower-lying levels are mostly populated by the electron impact from the ground state, which is clearly shown at $-\dot{n}_e = 0.3552 \times 10^6$ cm^{-3} sec^{-1} in Table 3(a). When $-\dot{n}_e$ becomes

larger, the population density of the ground state is reduced. Then, the populating rates into the excited levels from the ground level decrease. In this case, dominant processes contributing to the population of the levels i = 4 and 5 are the direct radiative recombination and the spontaneous transition from the upper levels which are mostly populated by the three body recombination. Hence, the ratio of the populating rate of the levels i = 4 and 5 are reduced as is shown in Table 3(a) ($-\dot{n}_e = 0.3854 \times 10^6$ cm^{-3} sec^{-1}). On the other hand, the depopulating rate from the level i = 5 is smaller than that from the level 4, since only the spontaneous transition is responsible for the outflow of electron from the levels. Therefore, the population inversion takes place when $-\dot{n}_e$ exceeds a certain value at a given electron density and temperature.

(2) Low electron density and high electron temperature (kT_e = 10 eV, $n_e = 10^9$ cm^{-3}): The populating rate is almost due to the radiative recombination of which the rate is shown in Fig. 4. The rate B_j for lower level is larger than that for higher level. Consequently, the inversion can hardly be expected (Table 3(b)).

(3) Relatively high electron density plasma ($n_e = 10^{12}$ cm^{-3}) in which the recombination takes place almost due to the three body collision (Table 3(c)): The recombining electrons are captured firstly into the highly excited levels and transferred to the lower levels by the collisional de-excitations and the spontaneous transitions. When $-\dot{n}_e$ is small, these electrons are accumulated preferentially to the ground state, so that the levels i = 4 and 5, are populated mostly by electron impact excitation from

the ground level. Therefore, the lower level i = 4 is more populated than the higher one i = 5, and the inversion hardly takes place. When $-\dot{n}_e$ is large, the population of the levels are dominated by the collisional de-excitation from the higher levels and the spontaneous transitions. The flow-in rate of the electrons to the level i = 5 is larger than that to the level i = 4. The decay rate of the level i = 4 by the spontaneous transition is almost the same in order of magnitude as that of the level i = 5 by the collisional excitation and de-excitation processes. Therefore, the population inversion between the levels can be generated. These results may be compared with that of the experiment on the *TPD-1 plasma*[6], *if the* usual reduced electron density and temperature are introduced. When the electron density is raised up beyond a certain value (Table 3(d)), the decay rate of the level i = 5 by the collisional excitation is considerably larger than that of the level i = 4. Therefore, the inversion can not take place.

5. Conclusion

In a recombining hydrogen plasma, the calculation based on the CR model in which the three body and the radiative recombinations are taken into account reveals: In order to generate the population inversion, the electron density and temperature should lie in the region shown in Fig. 1, provided that the recombination rate $-\dot{n}_e$ is sufficiently large as indicated in Table 1. Especially, in the high density plasma where the recombination processes are dominated by the three body recombination, the populating rate of the higher level is larger than that of the lower level due to the large value of $-\dot{n}_e$, while the decay rate of both the levels are almost the same.

In a low density and high electron temperature, where the radiative recombination is dominant, the population inversion can not be generated.

In a dense plasma where the decay rate of both the levels are mostly determined by the electron impact excitation, the inversion can no more be expected.

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Table Captions

Table 1 Upper and lower bounds of the recombination rate $\mathrm{Max}|\dot{n}_e|$ and $\mathrm{Min}|\dot{n}_e|$ for the population inversion.

Table 2 Upper bound of the population density of the ground state $(n_1/2)$ for the population inversion.

Table 3 Populating rate and depopulating frequency for various processes.

Figure Captions

Fig. 1 Upperbound electron temperature versus electron density for various line pairs when the population inversion is expected in the recombining plasma. The pair of numbers in parentheses marked on each of the curves indicates the pair of the principal quantum numbers.

Fig. 2 Population density, n_i ($i = 1,2,3,4$ and 5), as a function of the recombination rate, $-\dot{n}_e$, in the recombining hydrogen plasma at $n_e = 10^{12}$ cm^{-3} and $kT_e = 2$ eV.

Fig. 3 Electron density and temperature, at which the three body recombination process occurs equally as the radiative one.

Fig. 4 Radiative recombination rate coefficient as a function of electron temperature.

Table 1(a)

N_e (cm^{-3})	1.0+08				1.0+09			
(i_0, i_1)		(2,3)	(3,4)	(4,5)		(2,3)	(3,4)	(4,5)
kT_e (eV)	Max $\lvert\dot{n}_e\rvert$ x10^3	Min $\lvert\dot{n}_e\rvert$ x10^3	Min $\lvert\dot{n}_e\rvert$ x10^3	Min $\lvert\dot{n}_e\rvert$ x10^3	Max $\lvert\dot{n}_e\rvert$ x10^5	Min $\lvert\dot{n}_e\rvert$ x10^5	Min $\lvert\dot{n}_e\rvert$ x10^5	Min $\lvert\dot{n}_e\rvert$ x10^5
0.5	6.388	6.388	6.384	6.355	6.689	6.689	6.684	6.648
1.0	3.760	3.759	3.744	3.683	3.854	3.853	3.833	3.765
2.0	2.203	2.200	2.173	2.125	2.233	2.230	2.200	2.144
3.0	1.599		1.578	1.559	1.614		1.591	1.565
4.0	1.267		1.260	1.263	1.277		1.268	1.265
5.0								

N_e (cm^{-3})	1.0+10				1.0+11			
(i_0, i_1)		(2,3)	(3,4)	(4,5)		(2,3)	(3,4)	(4,5)
kT_e (eV)	Max $\lvert\dot{n}_e\rvert$ x10^7	Min $\lvert\dot{n}_e\rvert$ x10^7	Min $\lvert\dot{n}_e\rvert$ x10^7	Min $\lvert\dot{n}_e\rvert$ x10^7	Max $\lvert\dot{n}_e\rvert$ x10^9	Min $\lvert\dot{n}_e\rvert$ x10^9	Min $\lvert\dot{n}_e\rvert$ x10^9	Min $\lvert\dot{n}_e\rvert$ x10^9
0.5	7.398	7.398	7.391	7.330	9.029	9.029	9.010	8.779
1.0	4.060	4.059	4.031	3.930	4.495	4.494	4.442	4.139
2.0	2.292	2.290	2.252	2.168	2.409	2.406	2.341	2.115
3.0	1.643	1.643	1.613	1.566	1.697	1.696	1.644	1.498
4.0	1.294		1.280	1.260	1.325		1.293	1.198
5.0	1.073			1.071	1.093		1.079	1.016
6.0					0.9325			0.8898
7.0								

N_e (cm^{-3})	1.0+12				1.0+13			
(i_0, i_1)		(2,3)	(3,4)	(4,5)		(2,3)	(3,4)	(4,5)
kT_e (eV)	Max $\lvert\dot{n}_e\rvert$ x10^{11}	Min $\lvert\dot{n}_e\rvert$ x10^{11}	Min $\lvert\dot{n}_e\rvert$ x10^{11}	Min $\lvert\dot{n}_e\rvert$ x10^{11}	Max $\lvert\dot{n}_e\rvert$ x10^{13}	Min $\lvert\dot{n}_e\rvert$ x10^{13}	Min $\lvert\dot{n}_e\rvert$ x10^{13}	Min $\lvert\dot{n}_e\rvert$ x10^{13}
0.5	13.07	13.07	12.94	11.63	24.90	24.89	23.83	
1.0	5.458	5.454	5.223	4.405	7.765	7.735	6.903	
2.0	2.650	2.642	2.424	2.062	3.167	3.115	2.688	
3.0	1.803	1.796	1.636	1.435	2.020	1.975	1.725	
4.0	1.383	1.381	1.268	1.139	1.500	1.466	1.306	
5.0	1.129		1.052	0.9607	1.200	1.178	1.070	
6.0	0.9573		0.9075	0.8382	1.003	0.9914	0.9152	
7.0	0.8322		0.8027	0.7469	0.8641	0.8599	0.8044	
8.0	0.7368		0.7220	0.6752	0.7598		0.7201	
9.0	0.6615		0.6575	0.6169	0.6786		0.6532	
10.0	0.6003			0.5683	0.6133		0.5984	
12.0	0.5067			0.4912	0.5146		0.5133	
14.0	0.4383			0.4325				
16.0								
18.0								

Read: 1.0+08 = 1.0 x 10^8, etc.

Table 1(b)

$N_e (cm^{-3})$	1.0+14				1.0+15			
(i_0, i_1)		(2,3)	(3,4)	(4,5)		(2,3)	(3,4)	(4,5)
kT_e (eV)	Max $\lvert\dot{n}_e\rvert$ $\times 10^{15}$	Min $\lvert\dot{n}_e\rvert$ $\times 10^{15}$	Min $\lvert\dot{n}_e\rvert$ $\times 10^{15}$	Min $\lvert\dot{n}_e\rvert$ $\times 10^{15}$	Max $\lvert\dot{n}_e\rvert$	Min $\lvert\dot{n}_e\rvert$	Min $\lvert\dot{n}_e\rvert$	Min $\lvert\dot{n}_e\rvert$
0.5	71.19	71.07						
1.0	14.21	13.94						
2.0	4.334	4.106						
3.0	2.476	2.324						
4.0	1.734	1.634						
5.0	1.338	1.273						
6.0	1.091	1.052						
7.0	0.9235	0.9007						
8.0	0.8014	0.7906						
9.0	0.7083	0.7064						
10.0								
12.0								

Read: 1.0+14 = 1.0 x 10^{14}, etc.

Table 2

N_e (cm^{-3})	1.0+08			1.0+09		
(i_0, i_1)	(2,3)	(3,4)	(4,5)	(2,3)	(3,4)	(4,5)
kT_e=0.5eV	2.936+10	5.579+12	4.298+13	3.175+11	6.095+13	4.829+14
1.0	5.498+05	1.925+07	7.731+07	5.869+06	2.065+08	8.526+08
2.0	8.675+02	1.941+04	5.085+04	9.584+03	2.087+05	5.682+05
3.0		1.015+03	1.982+03		1.128+04	2.400+04
4.0		8.412+01	4.748+01		1.114+03	1.455+03
5.0						

N_e (cm^{-3})	1.0+10			1.0+11		
(i_0, i_1)	(2,3)	(3,4)	(4,5)	(2,3)	(3,4)	(4,5)
kT_e=0.5eV	3.793+12	7.549+14	6.668+15	5.537+13	1.293+16	1.760+17
1.0	6.767+07	2.440+09	1.120+10	9.203+08	3.822+10	2.561+11
2.0	1.163+05	2.450+06	7.534+06	1.696+06	3.744+07	1.609+08
3.0	9.696+02	1.393+05	3.604+05	5.963+04	2.286+06	8.631+06
4.0		1.732+04	4.135+04		3.690+05	1.459+06
5.0			8.194+02		6.985+04	3.856+05
6.0						1.201+05
7.0						3.458+04
8.0						2.704+03
9.0						
10.0						

N_e (cm^{-3})	1.0+12			1.0+13		
(i_0, i_1)	(2,3)	(3,4)	(4,5)	(2,3)	(3,4)	(4,5)
kT_e=0.5eV	1.276+15	5.070+17	5.826+18	7.374+16	1.703+19	
1.0	1.897+10	1.209+12	5.424+12	8.593+11	2.450+13	
2.0	3.826+07	1.016+09	2.646+09	1.620+09	1.489+10	
3.0	2.556+06	6.253+07	1.380+08	1.296+08	8.355+08	
4.0	2.594+05	1.178+07	2.489+07	2.691+07	1.551+08	
5.0		3.494+06	7.594+06	7.932+06	4.744+07	
6.0		1.276+06	3.054+06	2.533+06	1.872+07	
7.0		5.008+05	1.446+06	6.055+05	8.502+06	
8.0		1.824+05	7.590+05		4.157+06	
9.0		3.812+04	4.257+05		2.078+06	
10.0			2.485+05		9.976+05	
12.0			8.753+04		6.444+04	
14.0			2.603+04			
16.0						

N_e (cm^{-3})	1.0+14			1.0+15		
(i_0, i_1)	(2,3)	(3,4)	(4,5)	(2,3)	(3,4)	(4,5)
kT_e=0.5eV	3.992+18					
1.0	2.776+13					
2.0	3.556+10					
3.0	2.493+09					
4.0	5.021+08					
5.0	1.548+08					
6.0	5.775+07					
7.0	2.268+07					
8.0	7.910+06					
9.0	1.071+06					
10.0						

Read: 2.936+10 = 2.936×10^{10}, etc.

Table 3a

******* kT_e = 1 eV, N_e = 1.0+09 cm^{-3}, i_0 = 4, i_1 = 5 *******

Ratio of Recombination (Three Body = 42 %, Radiative = 58 %)

Depopulating Frequency

Level	$\sum_{j<i} A_{ji}$	$n_e \sum_{j<i} F_{ij}$	$n_e \sum_{j>i} C_{ij}$	$n_e K_{ic}$	ν_{out}
2	100 %	0.0	0.0	0.0	-0.47+09
3	100 %	0.0	0.0	0.0	-0.10+09
4	100 %	0.0	0.0	0.0	-0.30+08
5	100 %	0.0	0.0	0.0	-0.12+08

Populating Rate

[$\dot{n}_e$ = -0.3552+06 $cm^{-3}sec^{-1}$, $N(i_1)/N(i_0)$ = 1.348, N(1)/2= 2.9+09]

Level	$\sum_{j>i} A_{ij}$	$n_e \sum_{j>i} F_{ji} n_j$	$n_e \sum_{j<i} C_{ji} n_j$	$n_e{}^3 K_{ci}$	$n_e{}^2 \beta_i$	$U_{in}/2i^2$	$N(i)/2i^2$
2	3 %	0.0	96 %	0.0	1 %	0.90+06	1.91-03
3	17 %	0.0	69 %	0.0	14 %	0.16+05	1.60-04
4	26 %	0.0	43 %	0.0	31 %	0.28+04	0.93-04
5	29 %	0.0	30 %	0.0	41 %	0.92+03	0.77-04

[$\dot{n}_e$ = -0.3854+06 $cm^{-3}sec^{-1}$, $N(i_1)/N(i_0)$ = 1.742, N(1)/2= 0.0]

Level	$\sum_{j>i} A_{ij}$	$n_e \sum_{j>i} F_{ji} n_j$	$n_e \sum_{j<i} C_{ji} n_j$	$n_e{}^3 K_{ci}$	$n_e{}^2 \beta_i$	$U_{in}/2i^2$	$N(i)/2i^2$
2	48 %	0.0	0.0	0.0	52 %	0.17+05	0.36-04
3	42 %	0.0	0.0	0.0	58 %	0.40+04	0.40-04
4	38 %	0.0	0.0	0.0	62 %	0.14+04	0.47-04
5	37 %	0.0	0.0	0.0	63 %	0.59+03	0.49-04

Table 3b

******* kT_e = 10 eV, N_e = 1.0+09 cm^{-3}, i_0 = 4, i_1 = 5 *******

Ratio of Recombination (Three Body = 10 %, Radiative = 90 %)

Depopulating Frequency

Level	$\sum_{j<i} A_{ji}$	$n_e \sum_{j<i} F_{ij}$	$n_e \sum_{j>i} C_{ij}$	$n_e K_{ic}$	ν_{out}
2	100 %	0.0	0.0	0.0	-0.47+09
3	100 %	0.0	0.0	0.0	-0.10+09
4	100 %	0.0	0.0	0.0	-0.30+08
5	100 %	0.0	0.0	0.0	-0.12+08

Populating Rate

[$\dot{n}_e$ = -0.5829+05 $cm^{-3}sec^{-1}$, $N(i_1)/N(i_0)$ = 1.472, N(1)/2= 0.0]

Level	$\sum_{j>i} A_{ij}$	$n_e \sum_{j>i} F_{ji} n_j$	$n_e \sum_{j<i} C_{ji} n_j$	$n_e{}^3 K_{ci}$	$n_e{}^2 \beta_i$	$U_{in}/2i^2$	$N(i)/2i^2$
2	31 %	0.0	0.0	0.0	69 %	0.20+04	4.26-06
3	28 %	0.0	0.0	0.0	72 %	0.37+03	3.70-06
4	27 %	0.0	0.0	0.0	73 %	0.10+03	3.33-06
5	27 %	0.0	0.0	0.0	73 %	0.38+02	3.17-06

Read: -0.3552+06 = -0.3552×10^6, etc.

Table 3c

******* kT_e = 1 eV, N_e = 1.0+12 cm^{-3}, i_0= 4, i_1= 5 *******

Ratio of Recombination (Three Body = 99.9 %, Radiative = 0.1 %)

Depopulating Frequency

Level	$\sum_{j<i} A_{ji}$	$n_e \sum_{j<i} F_{ij}$	$n_e \sum_{j>i} C_{ij}$	$n_e K_{ic}$	ν_{out}
2	100 %	0.0	0.0	0.0	-0.47+09
3	98 %	2.0 %	0.0	0.0	-0.10+09
4	77 %	3.5 %	18 %	1.5 %	-0.39+08
5	30 %	13 %	54 %	3.0 %	-0.38+08

Populating Rate

[$\dot{n}_e$= -0.3965+12 $cm^{-3}sec^{-1}$, $N(i_1)/N(i_0)$= 1.414, N(1)/2= 7.7+12]

Level	$\sum_{j>i} A_{ij}$	$n_e \sum_{j>i} F_{ji} n_j$	$n_e \sum_{j<i} C_{ji} n_j$	$n_e^3 K_{ci}$	$n_e^2 \beta_i$	$U_{in}/2i^2$	$N(i)/2i^2$
2	3 %	0.0	97 %	0.0	0.0	0.24+13	5.11+03
3	15 %	2 %	77 %	0.0	6 %	0.39+11	3.90+02
4	19 %	23 %	44 %	3 %	11 %	0.78+10	2.00+02
5	7.5 %	61 %	21 %	5 %	5.5 %	0.69+10	1.82+02

[$\dot{n}_e$= -0.5458+12 $cm^{-3}sec^{-1}$, $N(i_1)/N(i_0)$= 2.261, N(1)/2= 0.0]

Level	$\sum_{j>i} A_{ij}$	$n_e \sum_{j>i} F_{ji} n_j$	$n_e \sum_{j<i} C_{ji} n_j$	$n_e^3 K_{ci}$	$n_e^2 \beta_i$	$U_{in}/2i^2$	$N(i)/2i^2$
2	64 %	0.0	0.0	0.0	36 %	0.25+11	5.32+01
3	58 %	6 %	0.0	1 %	35 %	0.67+10	6.70+01
4	34 %	39 %	1 %	4 %	22 %	0.39+10	1.00+02
5	9 %	71 %	6 %	7 %	7 %	0.55+10	1.45+02

Table 3d

******* kT_e = 1 eV, N_e=1.0+14 cm^{-3}, i_0= 4, i_1= 5 *******

Ratio of Recombination (Three Body = 99.9 %, Radiative = 0.1 %)

Depopulating Frequency

Level	$\sum_{j<i} A_{ji}$	$n_e \sum_{j<i} F_{ij}$	$n_e \sum_{j>i} C_{ij}$	$n_e K_{ic}$	ν_{out}
2	98 %	0.0	2 %	0.0	-0.48+09
3	34 %	9 %	54 %	3 %	-0.29+09
4	3 %	15 %	77 %	5 %	-0.93+09
5	1 %	18 %	76 %	5 %	-0.27+10

Populating Rate

[$\dot{n}_e$= -0.1421+17 $cm^{-3}sec^{-1}$, $N(i_1)/N(i_0)$= 1.236, N(1)/2= 0.0]

Level	$\sum_{j>i} A_{ij}$	$n_e \sum_{j>i} F_{ji} n_j$	$n_e \sum_{j<i} C_{ji} n_j$	$n_e^3 K_{ci}$	$n_e^2 \beta_i$	$U_{in}/2i^2$	$N(i)/2i^2$
2	59 %	29 %	0.0	2 %	10 %	0.89+15	1.85+06
3	7 %	83 %	0.0	7 %	3 %	0.10+16	3.45+06
4	1 %	82 %	10 %	7 %	0.0	0.26+16	2.80+06
5	0.0	77 %	17 %	6 %	0.0	0.59+16	2.19+06

Read: -0.3965+12 = -0.3965×10^{12}, etc.

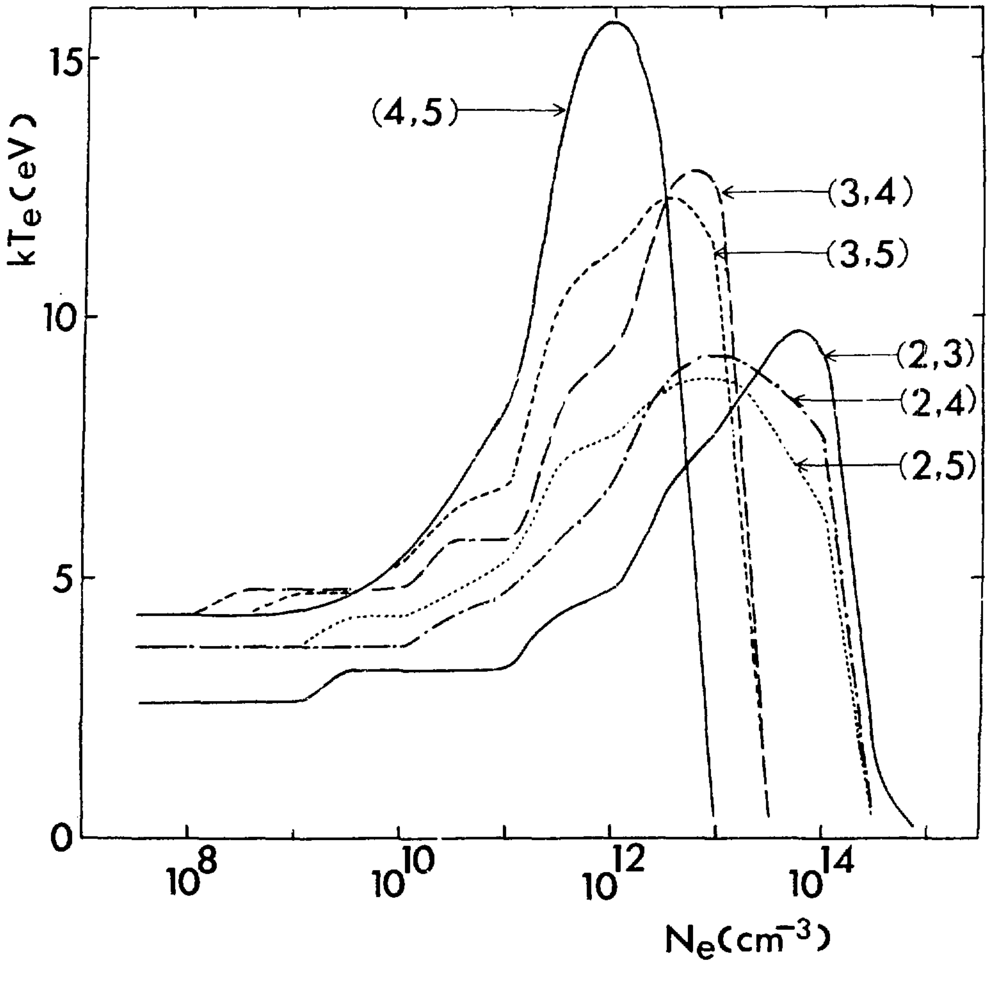
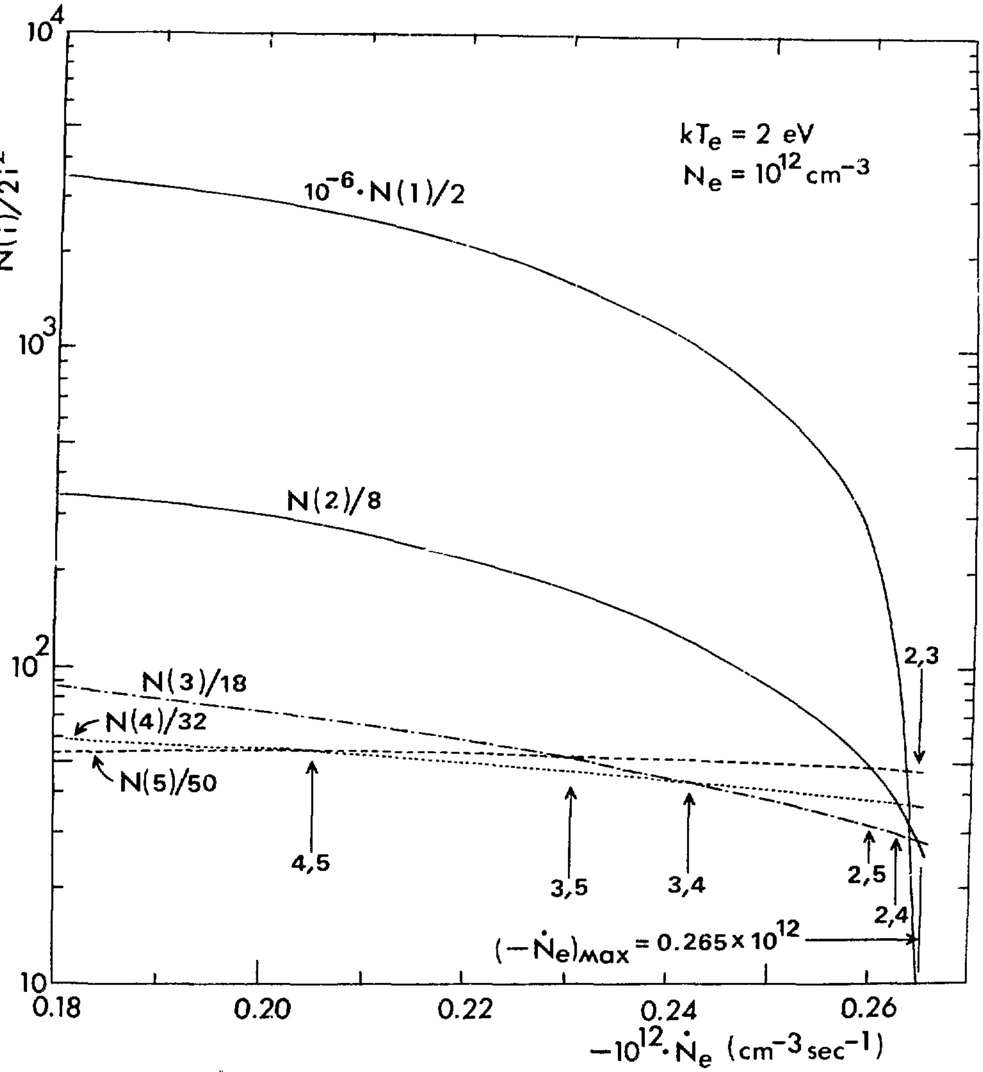
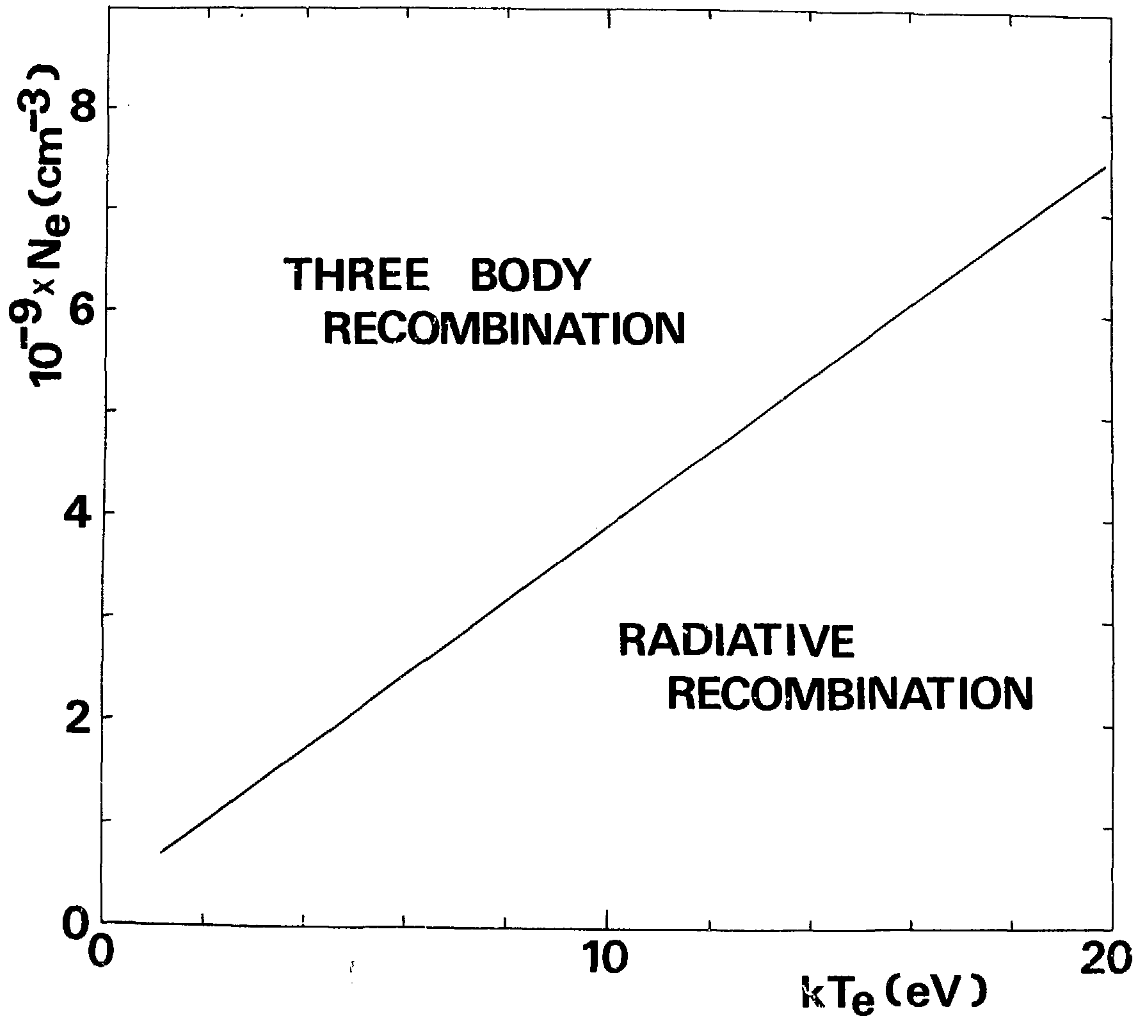


Fig. 1



10^4
10^3
10^2
10
N(i)/2i²
kTₑ = 2 eV
Nₑ = 10¹² cm⁻³
10⁻⁶·N(1)/2
N(2)/8
N(3)/18
N(4)/32
N(5)/50
2,3
4,5
3,5
3,4
2,5
2,4
(−Ṅe)max = 0.265 x 10¹²
0.18
0.20
0.22
0.24
0.26
−10¹²·Ṅₑ (cm⁻³ sec⁻¹)

Fig. 2



$10^{-9} \times N_e\,(cm^{-3})$
8
6
4
2
0
0
10
20
kT_e (eV)
THREE BODY
RECOMBINATION
RADIATIVE
RECOMBINATION

Fig. 3

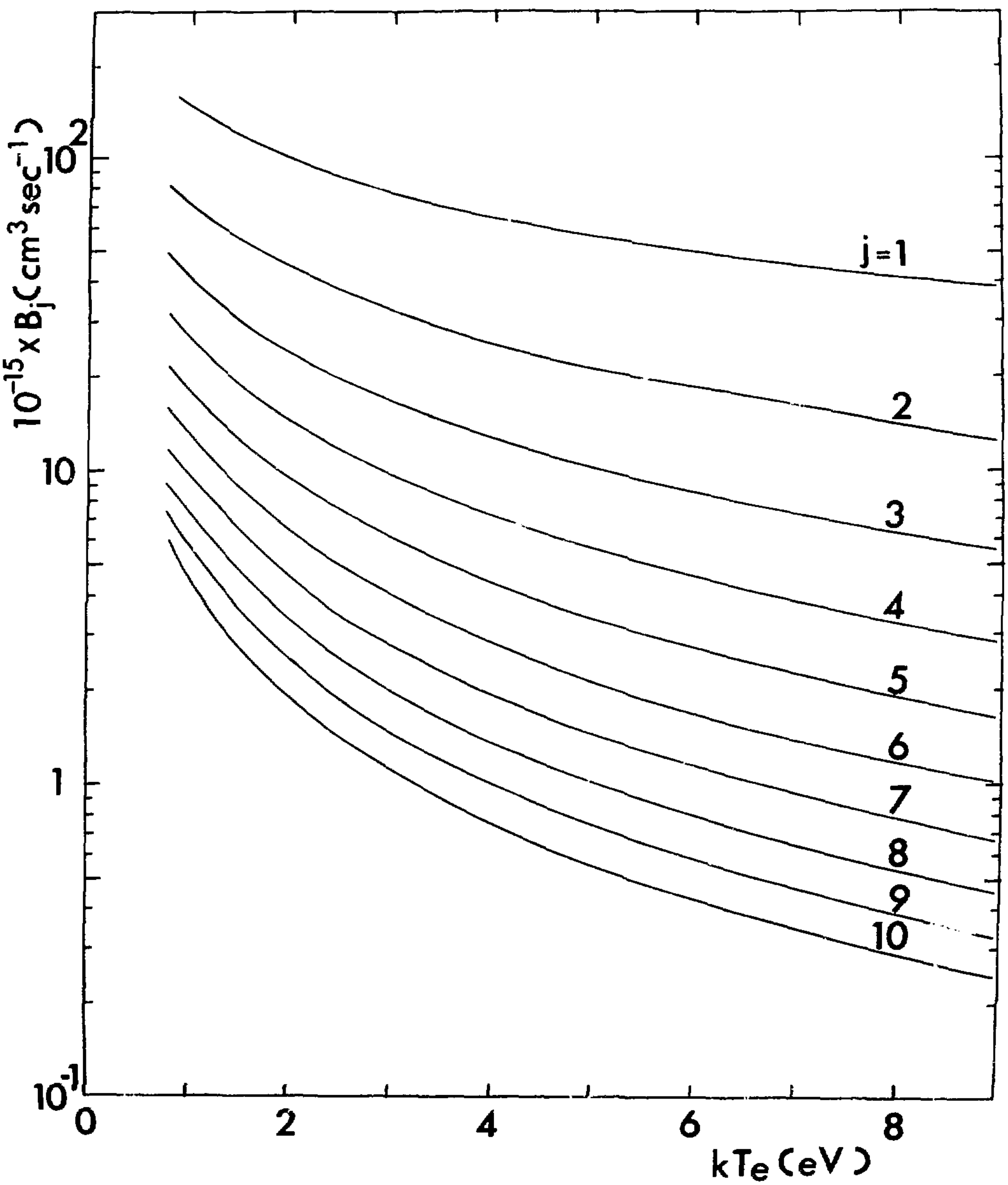


Fig. 4